The USAGE labor-market extension for the study of illegal immigration

by

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1. Introduction

The six key ingredients in the labor-market specification in USAGE are:

(1) the division of the workforce into categories at start of each year reflecting workforce functions in the previous year;

(2) the identification of workforce activities, that is what people do during the year;

(3) the determination of labor supply from each category to each activity;

(4) the determination of demand for labor in employment activities;

(5) the specification of wage adjustment processes reflecting demand and supply; and

(6) the determination of everyone's activity: who gets the jobs and what happens to those who don't?

A broad picture of the specification can be obtained from Figure 1. We divide the workforce at the start of year t into categories. These categories reflect the activities that people undertook in year t-1, with the main activities being employment in occupations. The number of people in a given category that undertake a particular activity in year t is determined mainly by the category's supply to that activity, relative to supply from people in other categories, and by demand for the services of that activity.

Table 1 lists the equations explained in this note that form the labor-market module of the version of USAGE used in our studies of illegal immigration.

2. Workforce, categories, functions and activities

We adopt two concepts of workforce: the U.S. workforce and the extended workforce. The U.S. workforce is everyone of working age in the U.S excluding people in full-time education and those who are ruled out of work by disabilities. Under this definition, the U.S. workforce includes discouraged workers and other people who are not actively seeking employment. The extended workforce is the U.S. workforce plus potential foreign illegal migrants working outside the U.S. For convenience we refer to these potential foreign illegal migrants as working in Mexico.

Figure 1. Labor-market dynamics in USAGE

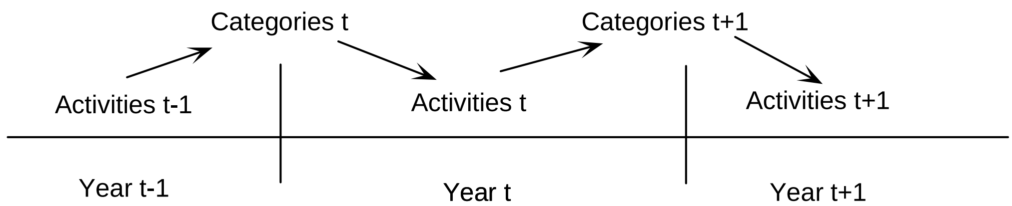


Table 1. USAGE representation of the labor market

Numbers in each category at the beginning of year t

$$\mathrm{CAT}_t\left((b,s,\ell)\right) = \sum_{ss\in \mathrm{Legal\,status}} \mathrm{ACT}_{t-1}\left((b,ss,\ell)\right) * T(b,ss,\ell,s) \tag{1}$$

for all b, s and $\ell \neq$ New

$$\mathrm{CAT}_t(b,s,\mathrm{New}) = \text{exogenous} \qquad \text{for all b and s .} \tag{2}$$

Planned labour supply

$$L_t(c\,;a) = \mathrm{CAT}_t(c) * \left[\frac{\left(B_t(c;a) * \mathrm{ATW}_t(a)\right)^{\eta}}{\sum_q \left(B_t(c;q) * \mathrm{ATW}_t(q)\right)^{\eta}} \right] \quad \text{for all categories c and activities a} \tag{8}$$

$$L_t(a) = \sum_c L_t(c;a) \quad \text{for all activities a} \quad . \tag{9}$$

Demand for labor and employment in the U.S.

$$D_t^1(j) = f_j^1\left(\mathrm{BTW}_t^1(j)\,; K_t(j); A_t(j)\right) \text{ for all U.S. industries j} \tag{11}$$

$$\mathrm{BTW}_t^1(j) = g_j^1\left(\mathrm{BTW}_t(b,s,o)\ \forall\ b,s \text{ and U.S. occupations o}\right) \text{ for all U.S. industries j} \tag{12}$$

$$D_t(b,s,o,j) = D_t^1(j) * h_{b,s,o,j}\left(\mathrm{BTW}_t(bb,ss,oo)\ \ \forall\ bb,ss \text{ and U.S. occupations oo}\right)$$

$$\text{for all b,s and U.S. occupations and industries o and j} \tag{17}$$

$$D_t(b,s,o) = \sum_j D_t(b,s,o,j) \text{ for all b,s and U.S. occupations o} \quad . \tag{18}$$

$$E_t(b,s,o) = D_t(b,s,o) \text{ for all b,s and U.S. occupations o} \quad . \tag{19}$$

Relationship between after-tax and before-tax wage rates

$$\mathrm{ATW}_t(b,s,o) = \mathrm{BTW}_t(b,s,o) * \left(1 - T_t(b,s)\right) \quad \text{for all b,s and U.S. occupations o} \tag{26}$$

$$\mathrm{ATW}_t(b,s,u) = \mathrm{BTW}_t^{ave}(b,s) * F_t(b,s) \quad \text{for all b,s and unemployment functions u} \tag{27}$$

Wage adjustment

$$\frac{\mathrm{ATW}_t(b,s,o)}{\mathrm{ATW}_t^{base}(b,s,o)} - \frac{\mathrm{ATW}_{t-1}(b,s,o)}{\mathrm{ATW}_{t-1}^{base}(b,s,o)} = \alpha \left(\frac{D_t(b,s,o)}{D_t^{base}(b,s,o)} - \frac{L_t(b,s,o)}{L_t^{base}(b,s,o)} \right) ,$$

$$\text{for all (b,s) and all U.S. occupations o} \tag{28}$$

Vacancies, and movements into employment activities

$$V_t(a) = E_t(a) - H_t\left[a\,;a\right] \quad \text{for all U.S. employment activities a} \tag{29}$$

$$H_t(c\,;a) = V_t(a) * \left[\frac{L_t(c;a)}{\sum_{s\neq a} L_t(s;a)} \right],$$

$$\text{for all categories } c \neq a \text{ and all U.S. employment activities a.} \tag{30}$$

$$H_t(c;c) = \mathrm{CAT}_t(c) - \sum_{a\neq c} H_t(c;a) \quad ,$$

$$\text{for all employment categories c (including Mexico)} \tag{31}$$

Table 1 continues …

Table 1 continued

Movements into unemployment and Mexican activities

$$H_t(c;u)=\begin{cases}L_c(c;u)+\mu(c)*CAT_t(c) & \text{for short-run unemployment activities u}\\ 0 & \text{for long-run unemployment activities u}\end{cases}$$

$$\text{for all U.S. employment categories c}\quad, \tag{32}$$

$$H_t(c;u)=\begin{cases}0 & \text{for short-run unemployment activities u}\\ CAT_t(c)-\sum\limits_{a\in \text{employment activ}} H_t(c;a) & \text{for long-run unemployment activities u}\end{cases}$$

$$\text{for all U.S. unemployment categories c and all unemployment activities u} \tag{33}$$

$$H_t(c;u)=\begin{cases}CAT_t(c)-\sum\limits_{a\in \text{U.S. employ activ}} H_t(c;a) & \text{for c not foreign, illegal and for short-run unemploy activ u}\\ CAT_t(c)-\sum\limits_{a\in \text{U.S. employ activ}} H_t(c;a) & \text{for c foreign, illegal and for Mexican activities u}\\ 0 & \text{otherwise}\end{cases}$$

$$\text{for all New categories c and all unemployment or Mexican activities u}\quad. \tag{34}$$

$$H_t(c;u)=\begin{cases}L_t(c;u) & \text{for c non-Mexican}\\ CAT_t(c)-\sum\limits_{a\in \text{U.S. employ activ}} H_t(c;a) & \text{for c Mexican}\end{cases}$$

$$\text{for all non-New categories c and for Mexican activities u} \tag{35}$$

$$\sum_c H_t(c,a)=E_t(a)\quad \text{, for all U.S. unemployment activities and Mexican activities a} \tag{36}$$

Notation

$CAT_t((b,s,\ell))$ is the number of people at the start of year t who are from birthplace b, have legal status s and who performed workforce function ℓ in year t-1.

$CAT_t((b,s,New))$ is the number of people at the start of year t who are from birthplace b, have legal status s and were not in the extended workforce in year t-1.

$ACT_{t-1}((bb,ss,\ell))$ is the number of people in activity (bb,ss,ℓ) in year t-1.

$T(b,ss,\ell,s)$ is the proportion of people in activity (b,ss,ℓ) in year t-1 who are allocated to category (b,s,ℓ) at the start of year t.

$L_t(c;a)$ is the labor supply that people in category c make to activity a. Both c and a are (b,s,ℓ) triples.

$L_t(a)$ is total labor supply to activity a.

$L_t^{base}(a)$ is the base or forecast value of $L_t(a)$.

α is a positive parameter.

$ATW_t(a)$ is the real after-tax wage rate of labor in activity a (for non-employment activities it is a social security payment or other support).

$ATW_t^{base}(a)$ is the base or forecast value of $ATW_t(a)$.

η is a parameter reflecting the ease with which people feel that they can shift between activities.

$B_t(c;a)$ is a variable reflecting the preference of people in category c for earning money in activity a in year t.

$K_t(j)$ is industry j's capital stock.

$BTW_t^1(j)$ is the overall real before-tax wage rate to the industry.

$A_t(j)$ is a vector of variables that influence industry j's demand for labor.

$D_t^1(j)$ is labor input to industry j.

Table 1 continues …

Table 1 continued

$BTW_t(b,s,o)$ is the real before-tax wage rate of workers of birthplace b, legal status s and U.S. occupation o.

$D_t(b,s,o,j)$ is j's input of labor of birthplace b, legal status s and U.S. occupation o.

$D_t(b,s,o)$ is aggregate demand for (b,s,o) labor.

$D_t^{base}(b,s,o)$ is the base or forecast value of $D_t(b,s,o)$.

$E_t(b,s,o)$ is employment of (b,s,o) labor.

$T_t(b,s)$ is the payroll and income-tax rate applying to all (b,s) workers in the U.S.

$BTW_t^{ave}(b,s)$ is the average real before-tax wage rate of (b,s) workers in the U.S.

$F_t(b,s)$ is the fraction of $BTW_t^{ave}(b,s)$ that (b,s) people receive in unemployment activities from social security payments or other support.

$V_t(a)$ is vacancies in activity a.

$H_t[c\,;\,a]$ is the flow of people from category c to activity a.

$\mu(c)$ is the fraction of people of category c people who become involuntarily unemployed.

At the beginning of each year, we allocate people in the extended workforce to categories according to their birthplace, legal status and recent labor market function. We allow for two birthplaces, domestic[1] and foreign, and two legal statuses, legal and illegal. All people with birthplace "domestic" have the status "legal". Some foreign residents of the U.S. are legal while others are illegal. We classify all workers in Mexico as illegal. This of course does not mean that Mexicans are working illegally in Mexico. It means that from the point of view of the U.S., Mexican workers in Mexico are potential foreign illegal migrants.

A person's recent workforce function refers to what he or she did in the labor market in the previous year, year t-1. The functions we identify are:

employed in occupation m, where m is one of the 50 U.S. occupations identified in USAGE;
short-run unemployed in the U.S., that is unemployed for a substantial amount of year t-1 but not unemployed in year t-2;
long-run unemployed in the U.S., that is unemployed for a substantial amount of year t-1 and also of year t-2;
living in the U.S. but not in the workforce[2];
employed in the single Mexican occupation recognized in USAGE;
living in Mexico but not in the workforce[3].

A final concept that we need to explain before setting out the algebra of the labor-market specification is activity. Activities are defined by birthplace, legal status and workforce function in the current year. Examples of activities in year t are: working in the U.S. as a domestic-legal construction laborer; working in the U.S. as a foreign-illegal cook; and experiencing short-run unemployment in the U.S. as a foreign legal resident. Another activity is working as a foreign illegal in Mexico. As already mentioned, we do not wish to imply that Mexicans are working illegally in Mexico. As we will see, Mexican workers in Mexico will be modeled as potential workers in foreign illegal activities in the U.S.

[1] By domestic we mean people born in the U.S. or people who entered the U.S. as dependents of legal residents of the U.S.

[2] The people we are concerned with are in the workforce at the start of year t. Consequently the people with this workforce function in t-1 are new entrants in year t.

[3] Again, these people are new entrants.

The link (the upward-sloping arrows in Figure 1) between the number of people in different activities in year t-1 and the number of people in each category at the start of year t is specified by the equations:

$$\mathrm{CAT}_t\left((\mathrm{b},\mathrm{s},\ell)\right)=\sum_{\mathrm{ss}\in\mathrm{Legal\,status}}\mathrm{ACT}_{t-1}\left((\mathrm{b},\mathrm{ss},\ell)\right)*\mathrm{T}(\mathrm{b},\mathrm{ss},\ell,\mathrm{s}) \quad (1)$$

for all b, s and non-new functions, i.e. $\ell \neq \mathrm{New}$

$$\mathrm{CAT}_t\left((\mathrm{b},\mathrm{s},\mathrm{New})\right)=\text{exogenous} \qquad \text{for all b and s .} \quad (2)$$

In these equations,

$\mathrm{CAT}_t\left((\mathrm{b},\mathrm{s},\ell)\right)$ is the number of people at the start of year t who are from birthplace b, have legal status s and who performed workforce function ℓ in year t-1.

$\mathrm{CAT}_t\left((\mathrm{b},\mathrm{s},\mathrm{New})\right)$ is the number of people at the start of year t who are from birthplace b, have legal status s and were not in the extended workforce in year t-1, that is the number of new (b,s) entrants to the extended workforce. If b is domestic and s is legal, then we have in mind high school and college graduates entering the job market in the U.S. If b is foreign and s is legal, then we have in mind newly admitted legal migrants of working age. If b is foreign and s is illegal, then we have in mind high school and college graduates entering the workforce in Mexico. There is no-one in the category domestic-illegal-new. As indicated in (2), the numbers of people in "New" categories are set exogenously, reflecting demographic factors.

$\mathrm{ACT}_{t-1}\left((\mathrm{bb},\mathrm{ss},\ell)\right)$ is the number of people in activity $(\mathrm{bb},\mathrm{ss},\ell)$ in year t-1, that is the number of people who, in year t-1, belonged to birthplace bb, had legal status ss and labor-force function ℓ.

$\mathrm{T}(\mathrm{b},\mathrm{ss},\ell,\mathrm{s})$ is the proportion of people in activity $(\mathrm{b},\mathrm{ss},\ell)$ in year t-1 who are allocated to category $(\mathrm{b},\mathrm{s},\ell)$ at the start of year t: we assume that people never change their birthplace.[4]

In the simulations reported in sections 3 to 6 of Dixon *et al.* (2008), we set

$$\mathrm{T}(\mathrm{b},\mathrm{ss},\ell,\mathrm{s})=\begin{cases}0.99 & \text{for all } \ell \text{ and b, and for all s, ss such that s = ss}\\ 0.00 & \text{otherwise}\end{cases} \quad . \quad (3)$$

Under (3), we assume that no-one changes legal status, and that one per cent of people in every activity in year t-1 drop out of the extended workforce at the beginning of year t, either through retirement or death. More sophisticated transition assumptions are possible. To allow for legalization of some foreign illegals in the U.S. and for differences in retirement/death rates across activities, we could set $\mathrm{T}(\mathrm{b},\mathrm{ss},\ell,\mathrm{s})$ according to:

$$\mathrm{T}(\mathrm{b},\mathrm{ss},\ell,\mathrm{s})=\mathrm{Survive}(\mathrm{b},\mathrm{ss},\ell)*\mathrm{P}(\mathrm{s}\mid\mathrm{ss},\ell) \quad (4)$$

where

$\mathrm{Survive}(\mathrm{b},\mathrm{ss},\ell)$ is the proportion of people in activity $(\mathrm{b},\mathrm{ss},\ell)$ in year t-1 who remain in the extended workforce in year t; and

[4] Consistent with the definition of a category, only people from activities with workforce function ℓ in year t-1 are allocated to categories with workforce function ℓ at the start of year t.

$P(s|ss,\ell)$ is the probability of a surviving person who had legal status ss and workforce function ℓ in year t-1 achieving legal status s at the start of year t.

In (4), we continue to assume that people cannot change their birthplace but we allow for the possibility of changes in legal status. In future research we could investigate the implications of legalization programs by simulating the effects of suitable shocks to $P(s|ss,\ell)$ for ss= illegal, s = legal and $\ell \neq \text{Mexico}$.

3. Labor supply from each category to each activity

USAGE specifies labor supply from people in each category to each activity. Via these specifications, we ensure that people in a category with birthplace b and legal status s make offers only to activities with these characteristics. Thus, people in the category domestic-legal construction laborer can offer only to activities with the domestic and legal characteristics. Most of these people offer to the activity domestic-legal construction laborer, that is they offer to continue their employment of last year. However, some will offer to change occupation in response to changes in relative wages and a few will offer to unemployment. Some people in the category foreign-illegal Mexico will offer to foreign-illegal occupations in the U.S., that is they will seek to enter the U.S. as illegal immigrants, and some people in foreign-illegal categories operating in the U.S. will make offers to the activity foreign-illegal Mexico, that is they will offer to return home. In making these decisions, people in these foreign-illegal categories compare wages in Mexico with wages for foreign-illegal occupations in the U.S.

In developing the labor-supply functions for USAGE, we assume that at the beginning of year t, people in category c [where c is a (birthplace, legal status, function) triple] decide their offers to activity a [where a is also a (b,s,ℓ) triple] for the year by solving a problem of the form: choose $L_t(c;a)$, for all activities a

$$\text{to maximize } U_c\left[ATW_t(a) * L_t(c;a) \ \ \forall \text{ activities a}\right] \tag{5}$$

$$\text{subject to } \sum_a L_t(c;a) = CAT_t(c) \tag{6}$$

where

$L_t(c;a)$ is the labor supply that people in category c make to activity a;

$CAT_t(c)$ is the number of people in category c;

$ATW_t(a)$ is the real after-tax wage rate of labor in activity a (for non-employment activities, that is short-and long-run unemployment, $ATW_t(a)$ can be thought of as a social security payment or other support); and

U_c is a homothetic function with the usual properties of utility functions (positive first derivatives and quasi-concavity).

In (5) and (6), people in category c treat dollars earned in different activities as imperfect substitutes. This is a convenient and flexible specification through which we can allow labor supplies to shift between activities in response to changes in after-tax rewards. By specifying a separate utility function for each c, we can ensure that each category makes supplies to activities that are compatible with the category's birthplace, legal status and occupational characteristics.

In the applications presented in Dixon *et al.* (2008), U_c has the CES form:

$$U_c = \left[\sum_a \left(B_t(c;a) * ATW_t(a) * L_t(c;a) \right)^{\frac{\eta}{1+\eta}} \right]^{\frac{1+\eta}{\eta}} \quad . \tag{7}$$

where

η is a parameter reflecting the ease with which people feel that they can shift between activities; and

$B_t(c;a)$ is a variable reflecting the preference of people in category c for earning money in activity a in year t.

The $B_t(c;a)$'s play two roles in our analysis. The first is via their initial settings, that is the values assigned to them in our base-year data [2004 in the applications in Dixon *et al.* (2008)].

- By setting $B_{2004}(c;a)$ at 0 if the birthplace and legal characteristics of c differ from those of a, we ensure that people in categories with birthplace b and legal status s offer labor only to (b,s) activities.
- By setting $B_{2004}(c;a)$ at relatively high values when c and a agree in their (b,s) characteristics and have a functional characteristic referring to the same occupation, we ensure that most people employed in year t-1 in occupation m (including the Mexican occupation) offer to continue to work in m in year t.
- By setting $B_{2004}(c;a)$ at suitably chosen positive values when c and a agree in their (b,s) characteristics but have functional characteristics referring to different occupations, we ensure that people make offers to work in occupations compatible with their skills.
- By setting $B_{2004}(c;a)$ at zero where the functional characteristic of c is either short-run or long-run unemployment and the functional characteristic of a is short-run unemployment, we ensure that no-one can stay in short-run unemployment in successive years or move from long-run unemployment back to short-run unemployment.
- By setting $B_{2004}(c;a)$ at a moderately large value where c and a agree in their (b,s) characteristics and c has the functional characteristic of short-run unemployment and a has the functional characteristic long-run unemployment, we introduce a mild discouraged-worker effect for people suffering short-run unemployment.
- By setting $B_{2004}(c;a)$ at a larger value where c and a agree in their (b,s) characteristics and where c and a both have the functional characteristic of long-run unemployment, we introduce a stronger discouraged-worker effect for the long-run unemployed.

The second role of the $B_t(c;a)$'s is to carry shocks in policy runs. In section 3 of Dixon *et al.* (2008), we represent the impact of tighter border security by reductions in the $B_t(c;a)$'s where c and a both have the (b,s) characteristics foreign illegal and c has the functional characteristic of either Mexico or New and a has the functional characteristic of a U.S. occupation.

Under (7), problem (5) - (6) generates labor-supply functions of the form:

$$L_t(c\,;a) = CAT_t(c) * \left[\frac{\left(B_t(c;a) * ATW_t(a) \right)^{\eta}}{\sum_q \left(B_t(c;q) * ATW_t(q) \right)^{\eta}} \right] \quad . \tag{8}$$

Total supply of labor to activity a is obtained as

$$L_t(a) = \sum_c L_t(c;a) \quad \text{for all activities a} \quad . \tag{9}$$

In the main simulations in Dixon *et al.* (2008) we set η in (8) at 2. For understanding what this means, it is useful to express (8) in percentage change form as:

$$\ell_t(c\,;a) = cat_t(c) + \eta * \left(atw_t(a) - atw_t^{ave}(c) \right) + \eta * \left(b_t(c;a) - b_t^{ave}(c) \right) \quad . \tag{10}$$

In (10), the lowercase symbols $\ell_t(c\,;a)$, $cat_t(c)$, $atw_t(a)$ and $b_t(c;a)$ are percentage changes in the variables denoted by the corresponding uppercase symbols, and $atw_t^{ave}(c)$ and $b_t^{ave}(c)$ are weighted averages of the $atw_t(q)$s and $b_t(c;q)$s with the weights reflecting the share of activity q in the offers from people in category c. Thus (10) implies that people in category c will switch their offers towards activity a if the wage rate in activity a rises relative to an average of the wage rates across all the activities in which category-c people could participate. With η set at 2, we assume that the number of people who wish to change jobs, in particular the number of people who wish to move from Mexico to U.S. occupations, is quite sensitive to changes in relative wage rates. However, an increase in $ATW_t(a)$ does not have much affect on $L_t(a;a)$. This is because the bulk of offers from people in category a are to activity a, so that $atw_t(a) - atw_t^{ave}(a)$ is always close to zero. The major part of the supply of labor to any work activity a is from incumbents [that is, $L_t(a;a)$ is a very large fraction of $L_t(a)$]. Thus, even with η as high as 2, the elasticity of supply of labor to activity a with respect to the wage rate in a is relatively low. Dixon *et al.* (2008, subsection 5.4) provides analysis of the sensitivity of the principal results to variations in η.

4. Demand for labor in the U.S.

The labor input, $D_t^1(j)$, to U.S. industry j in year t is specified in USAGE along conventional CGE lines as a function of: the industry's capital stock, $K_t(j)$; the overall real before-tax wage rate to the industry, $BTW_t^1(j)$; and other variables, $A_t(j)$, that influence industry j's demand for labor, including technology and commodity prices:

$$D_t^1(j) = f_j^1\left(BTW_t^1(j)\,; K_t(j); A_t(j) \right) \quad . \tag{11}$$

The overall real wage rate to industry j is determined as a suitable average of the real wage rates applying to the types of labor that the industry employs:

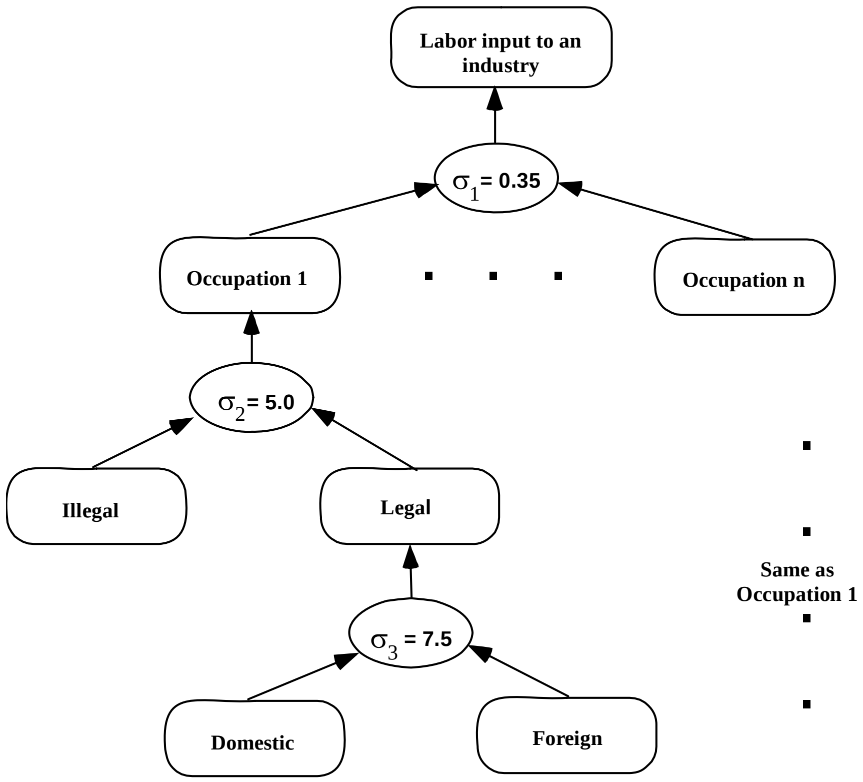
$$BTW_t^1(j) = g_j^1\left(BTW_t(b,s,o) \text{ for all b,s and U.S. occupations o} \right) \quad , \tag{12}$$

where

$BTW_t(b,s,o)$ is the real before-tax wage rate of workers of birthplace b, legal status s and U.S. occupation o.

Within industry j's labor input, the demand for labor by birthplace, legal status and occupation is determined by a nested CES cost minimization problem. The nesting and the substitution elasticities are indicated in Figure 2. As discussed at the end of this section, we assume that there are low substitution possibilities between occupations such as Cooks, Grounds maintenance workers, etc (substitution elasticity of 0.35) but high substitution possibilities between legal and illegal workers of the same occupation (substitution elasticity of 5) and between domestic and foreign legal workers of the same occupation (substitution elasticity of 7.5).

Figure 2. Nesting assumptions in the creation of the labor input to each industry



In algebraic terms, we assume that industry j satisfies its labor requirements by choosing:

$D_t(b,s,o,j)$, j's input of labor of birthplace b, legal status s and U.S. occupation o,

$D_t^3(s,o,j)$, j's input of labor of legal status s and U.S. occupation o, defined as a CES aggregate over b of (b,s,o,j) inputs, and

$D_t^2(o,j)$, j's input of labor of U.S. occupation o, defined as a CES aggregate over s of (s,o,j) inputs,

$$\text{to minimize } \sum_{b,s,o} BTW_t(b,s,o) * D_t(b,s,o,j) \tag{13}$$

$$\text{subject to} \quad D_t^1(j) = CES_o^j\left[D_t^2(o,j)\right] \tag{14}$$

$$D_t^2(o,j) = CES_s^j\left[D_t^3(s,o,j)\right] \text{ for all U.S. occupations o,} \tag{15}$$

$$\text{and} \quad D_t^3(s,o,j) = CES_b^j\left[D_t(b,s,o,j)\right] \tag{16}$$

for all U.S. occupations o and legal status s[5].

[5] As shown in Figure 2, there is no level 3 CES nest when s = illegal (only foreigners can provide illegal labor). However, it is unnecessarily clumsy to include this detail in the algebraic overview of our theory. We can simply assume that industries use tiny amounts of domestic illegal labor.

The CES functions in (14) to (16) incorporate the elasticities shown in Figure 2 and are calibrated to reflect the data on the occupational, birthplace and legal status of workers in U.S. industries.[6]

From problem (13) – (16) we obtain demand functions of the form

$$D_t(b,s,o,j) = D_t^1(j) * h_{b,s,o,j}\left(BTW_t\ (bb,ss,oo)\ \ \forall\ bb, ss \text{ and U.S. occupations } oo\right)$$
$$\text{for all } b,s \text{ and U.S. occupations and industries } o \text{ and } j. \qquad (17)$$

These can be aggregated across industries to determine aggregate demand for (b,s) workers in U.S. occupation o as

$$D_t(b,s,o) = \sum_j D_t(b,s,o,j) \quad \text{for all } b,s \text{ and U.S. occupations } o\ . \qquad (18)$$

We assume that employment of (b,s) workers in U.S. occupation o, $E_t(b,s,o)$, is determined by demand:

$$E_t(b,s,o) = D_t(b,s,o) \text{ for all } b,s \text{ and U.S. occupations } o\ . \qquad (19)$$

Discussion of demand-side substitution elasticities

The value, 0.35, adopted for the occupation-occupation substitution elasticity is a rather old Australian estimate.[7] Fortunately, substitution between occupations is not important for the results in Dixon *et al.* (2008). Our choice of 7.5 for the domestic/foreign substitution elasticity is suggested by the econometric work of Ottaviano and Peri (2006). We set the substitution elasticity between legal and illegal workers in the same occupation at a somewhat lower value, namely 5, because, from the point of view of employers, legality is likely to be an important characteristic. For many employers who do not currently use illegals, it may require a considerable reduction in the wage of illegals relative to that of legals to tempt them to switch to a given number of illegals.

Our use of the Ottaviano and Peri (2006) estimate for the elasticity of substitution between immigrant and native workers could be questioned on two grounds. The first is that we use the estimate as applying to substitution within occupations when the original econometric work looked at immigrant/native substitution within education/experience groups. Is this really an important problem? Let's assume that employers are concerned with occupations. We can include experience and education in an occupation. For example, we can define an occupation as economist with a PhD and five to ten years experience. Another occupation is economist with a PhD and more than ten years experience, etc. Now let's assume that employers substitute between native and immigrant workers within an occupation with a substitution elasticity σ. Then,

$$\ell_{M1} - \ell_{M2} = \sigma * \left(w_{M2} - w_{M1}\right) \qquad (20)$$

where ℓ_{M1} & ℓ_{M2} and w_{M1} & w_{M2} are percentage changes in employment and wage rates of native and immigrant workers in occupation M. Percentage changes in employment and wage rates of natives and immigrants classified by

[6] The Bureau of Labor Statistics (2006) gives detailed data on employment and wage rates by occupation and industry. These were processed into USAGE categories by Dixon and Rimmer (2006). The birthplace and legal status dimensions were added by using Van Hook *et al.* (2005) and by assuming that wage rates for legal and illegal migrants in any occupation are 0.9 and 0.8 times those of native workers. Support for the 0.9 is provided by Rector and Kim (2007, Table 2, page 11). The 0.8 is an assumption.

[7] See Higgs *et al.* (1981).

experience/education groups can be obtained as suitably weighted averages of percentage changes at the occupational level:

$$\ell_{(eq)} = \sum_{M} S_{(eq)M} * \ell_{Mq} \quad \text{, for all e and q = 1, 2} \tag{21}$$

$$w_{(eq)} = \sum_{M} S_{(eq)M} * w_{Mq} \quad \text{, for all e and q = 1, 2} \tag{22}$$

where $\ell_{(eq)}$ and $w_{(eq)}$ are the percentage changes in employment and the wage rate of native (q=1) and immigrant (q=2) workers in experience/education group e, and $S_{(eq)M}$ is the share of eq's employment accounted for by occupation M. After a fair amount of tedious but elementary algebra we can derive the equation

$$w_{(e1)} - w_{(e2)} = \left(\frac{1}{\sigma}\right) * \left(\ell_{(e2)} - \ell_{(e1)}\right) + \varepsilon_e \tag{23}$$

where

$$\varepsilon_e = \sum_{M} \left(S_{(e1)M} - S_{(e2)M}\right) * \left(\frac{\ell_M}{\sigma} + w_M\right) \tag{24}$$

and ℓ_M and w_M are percentage changes in overall employment and wage rate in occupation M defined by

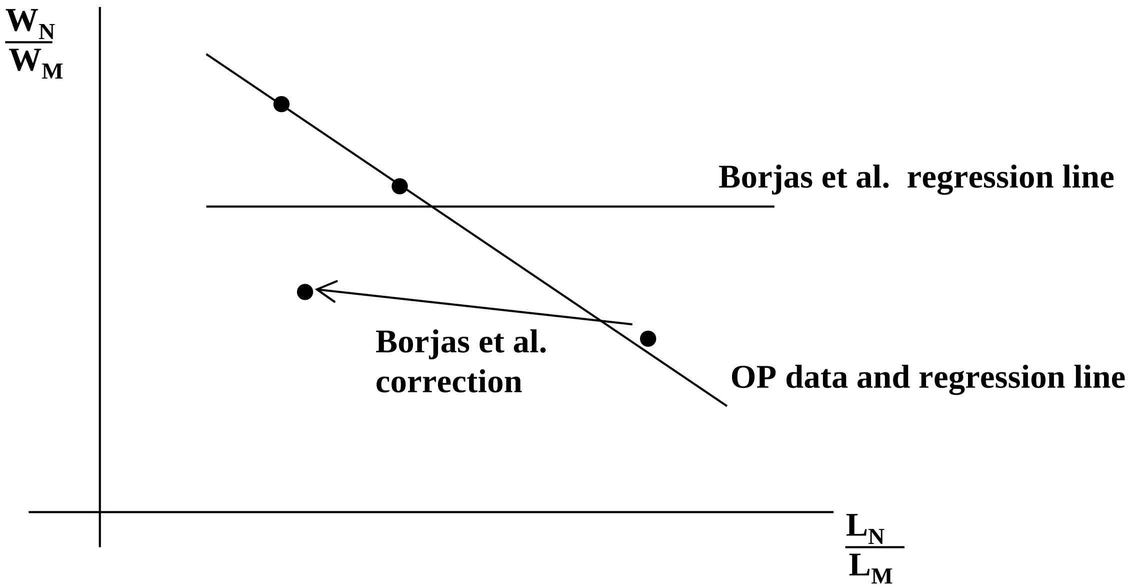
$$\ell_M = \frac{\ell_{M1} + \ell_{M2}}{2} \text{ and } w_M = \frac{w_{M1} + w_{M2}}{2} \quad . \tag{25}$$

Ottaviano and Peri (2006) and other contributors to this literature estimate equations that are versions of equation (23) with some other complications that are not essential to the present discussion. Notice that if natives and immigrants have the same occupational attachment within experience/education groups [$S_{(e1)M} = S_{(e2)M}$ so that $\varepsilon_e = 0$][8] or if differences in occupational attachment are not correlated with growth in employment or wages at the occupational level, then estimation of σ in (23) will reveal the elasticity of substitution between natives and immigrants within an occupation. In this case, our use of the Ottaviano and Peri estimate applied to occupations is not invalidated by the fact that the original estimation was done with data on experience/education groups.

The second ground for questioning to our use of the Ottaviano and Peri (2006) estimate of the elasticity of substitution between immigrant and native workers is that it has recently been criticized by Borjas *et al.* (2008). Borjas *et al.* redo the Ottaviano and Peri paper but make what they consider to be various corrections. The most important of these concerns the number of high-school dropouts. Ottaviano and Peri treated people who are earning money while in high school as though they are high-school dropouts. This inflated the ratio of natives to immigrants in the high-school-dropout/low-experience cell. At the same time, the native/immigrant wage rate in this cell was deflated. This is because high-school students in the cell received very low wages. Hence, when Borjas *et al.* took out the high-school students from the analysis, the native/immigrant numbers ratio looked lower in the critical group and the native/immigrant wage ratio looked higher. However, the correction to the numbers ratio is much larger than that to the wages ratio. What happened is illustrated by Figure 3.

[8] Borjas (2003, pages 1344-47) looks at the occupational mix of natives and immigrants in 1990. He finds that the mix for immigrants in any given education/experience cell is quite similar to that for natives in the same cell.

Figure 3. Schematic representation of Borjas *et al.*'s correction of Ottaviano and Peri



Our conclusion from all of this is that econometric estimates of native/immigrant substitution elasticities are flaky. As illustrated in Figure 3, the Borjas *et al.* correction suggests that Ottaviano and Peri's estimate of $1/\sigma$ in (23) is too large, or equivalently that their estimate of the native/immigrant substitution elasticity is too low. However, on looking at equations (23) and (24), we wonder whether native/immigrant substitution estimates based on (23) might be too high. For example, consider the situation in which as immigrants come into their new country, they tend to go into occupations (compatible with their experience/education group) in which employment and wages are growing rapidly. In this case, ε_e would be negative for most e. In a period such as 1960 to 2005 (Ottaviano and Peri's data period) in which the immigrant population grew rapidly relative to the native population [$\ell_{(e2)} > \ell_{(e1)}$] we suspect that this would lead to an underestimate of $1/\sigma$ in (23): by leaving out the negative term defined by (24) we force the first term on the right hand side of (23) to be too small. If we underestimate $1/\sigma$ then we overestimate σ.

We are not in a position to resolve these econometric issues. Consequently, in Dixon *et al.* (2008, subsection 5.3) we discuss the sensitivity of the principal simulation results to variation in demand-side substitution elasticities.

5. Relationship between after-tax and before-tax wage rates in the U.S.

As can be seen from the previous sections, after-tax wage rates are important in motivating labor supply while before-tax wage rates motivate demand. To relate after-tax wage rates to before-tax wage rates we include in USAGE:

$$ATW_t(b,s,o) = BTW_t(b,s,o) * \left(1 - T_t(b,s)\right) \quad \text{for all b,s and U.S. occupations o} \qquad (26)$$

$$ATW_t(b,s,u) = BTW_t^{ave}(b,s) * F_t(b,s) \quad \text{for all b,s and unemployment functions u.} \qquad (27)$$

In these equations,

$T_t(b,s)$ is the payroll and income-tax rate applying to all (b,s) workers in the U.S.;

$BTW_t^{ave}(b,s)$ is the average real before-tax wage rate of (b,s) workers in the U.S.; and

$F_t(b,s)$ is the fraction of $BTW_t^{ave}(b,s)$ that (b,s) people receive in unemployment activities from social security payments or other support. In the simulations described in Dixon *et al.* (2008), we assume that the $F_t(b,s)$'s are unaffected by changes in immigration policies, that is we assume that percentage movements in unemployment benefits match those in average before-tax wage rates.

In the DR1 simulation reported in section 4 of Dixon *et al.* (2008), the policy shock is an increase in $T_t(b,s)$ for b = foreign and s = illegal.

6. Wage adjustment

In policy runs, we assume that wage rates adjust according to the equation:

$$\frac{ATW_t(b,s,o)}{ATW_t^{base}(b,s,o)} - \frac{ATW_{t-1}(b,s,o)}{ATW_{t-1}^{base}(b,s,o)} = \alpha \left(\frac{D_t(b,s,o)}{D_t^{base}(b,s,o)} - \frac{L_t(b,s,o)}{L_t^{base}(b,s,o)} \right) ,$$

for all (b,s) and all U.S. occupations o (28)

where the superscript "base" refers to values in the basecase forecast and α is a positive parameter.

This equation implies that if a policy causes the market for (b,s,o) employment in year t to be tighter than it was in the basecase forecast (i.e., if the policy causes a larger percentage deviation in demand than supply), then there will be an increase between years t-1 and t in the deviation in (b,s,o)'s real after-tax wage rate. In other words, in periods in which a policy has elevated demand relative to supply, real wages will grow relative to their basecase values. Figure 4 illustrates the operation of equation (28) for a model with a single employment activity.

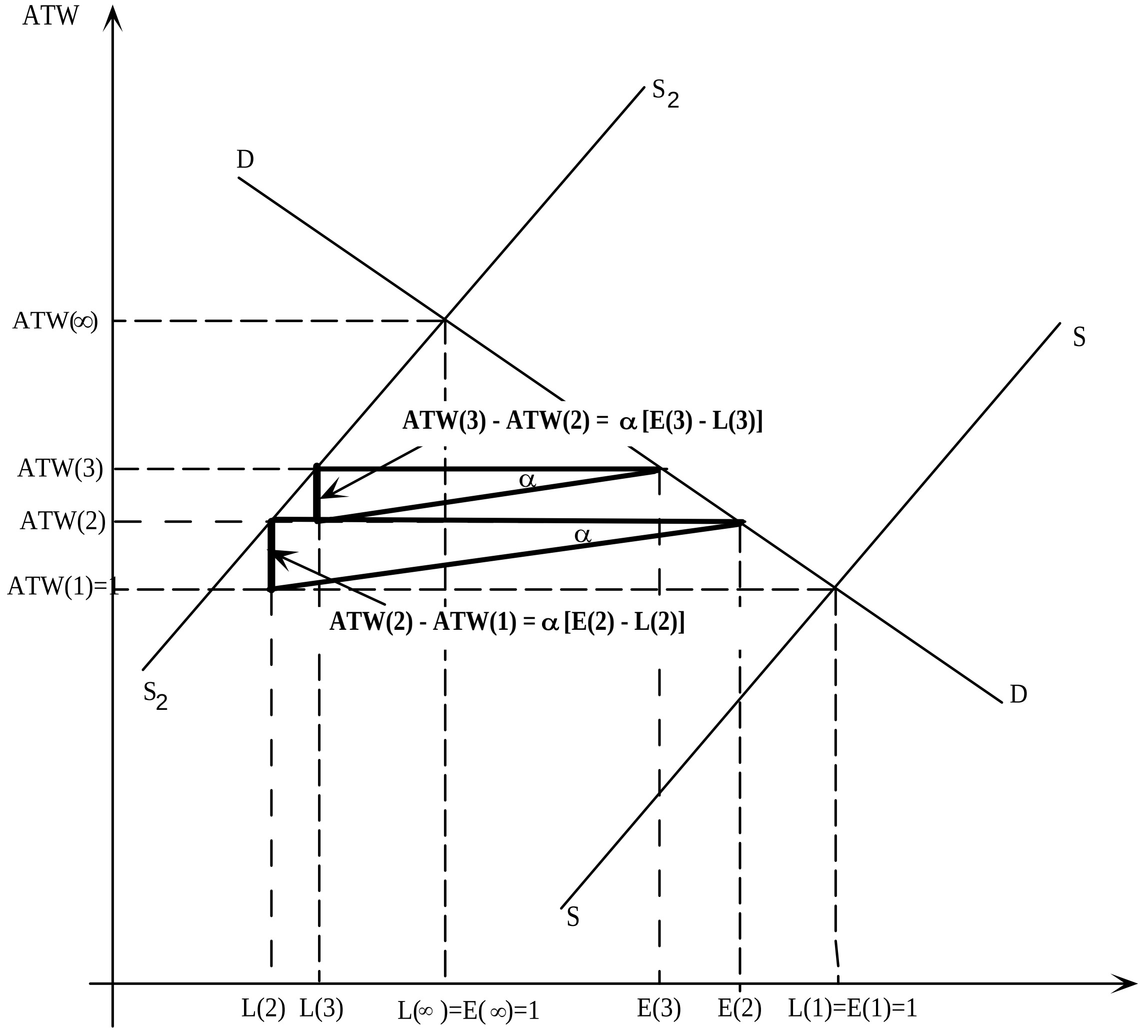
Our assumed wage-adjustment process is compatible with a search model [see for example, Bohringer *et al.* (2005)] in which reductions in labor supply, and resulting reductions in the unemployment rate, generate decreases in the value of having a job relative to the value of not having a job, thereby emboldening workers to demand higher wage rates. It is also compatible with efficiency-wage theory, see for example, Layard *et al.* (1994, pp. 33-45). Under this theory, employers offer wage rates that optimise worker effort per dollar of wage cost. The theory suggests that the effort-optimising wage rate rises when there is a decrease in labor supply and a consequent temporary decrease in unemployment.

In the context of USAGE, we can think of equation (28) as having the role of determining after-tax wage rates for occupations in the U.S. Then at given tax rates, equations (26) and (27) determine before-tax wage rates for these occupations and for unemployment. The only other wage rate in our model is the after-tax wage rate in Mexico. We set this exogenously.

7. The determination of everyone's activity: who gets the jobs and what happens to those who don't?

Under (28), markets for U.S. occupations do not clear. Consequently, we need to specify which offers to employment are accepted and what activities are undertaken by those whose offers to employment are not accepted. In terms of Figure 1, we need to specify the downward sloping arrows.

Figure 4. Wage adjustment in a steady state with one type of employment: a supply shock



In this illustration, but not in USAGE, we assume that there is only one type of labor and that the basecase was generated under steady-state assumptions in which technology, consumer tastes, foreign prices, capital availability, taxes, the size of the labor force and other variables affecting the demand for and supply of labor are unchanged from year to year. In this steady state the demand curve for labor (drawn for a given tax rate) is DD and the supply curve is SS. For convenience we assume that the after-tax wage rate, employment and the supply of labor are one in the steady state, allowing us to eliminate the basecase forecasts from equation (28). Now consider a policy simulation (e.g. a decrease in immigrant inflow) involving a shift in the supply curve in year 2 to S_2S_2, where it remains for all future years. Assuming that there is no change in tax rates (so that changes in after-tax wage rates on the vertical axis are also changes in pre-tax wage rates), then employment decreases from E(1) to E(2) to … E(∞), labor supply decreases from L(1) to L(2) and then rises from L(2) to L(3) to … L(∞), and wages rise from ATW(1) to ATW(2) to … ATW(∞).

In linking categories at the start of year t to activities in year t, we specify an equation for the flow from each category c to each activity a, $H_t(c;a)$.

Flows from all categories to employment in U.S. occupations (area 1 in Figure 5)

We start by defining vacancies in U.S. employment activity a in year t as employment, $E_t(a)$, less the number of jobs filled in the activity by people in category a, that is vacancies in a are jobs less those filled by incumbents:

$$V_t(a) = E_t(a) - H_t\left[a\,;a\right] \quad \text{for all U.S. employment activities a} \tag{29}$$

Figure 5. Specifying the flows from categories to activities

Categories \ Activities	Employment in U.S. occupation	Unemployed in U.S.	Employed in Mexico
Employment in U.S. occupation	1	2	5
Unemployed in U.S.	1	3	5
Employed in Mexico	1	zero	5
New entrant	1	4	4

where

$V_t(a)$ is vacancies and

$H_t[a\,;a]$ is employment of people in category a in activity a.

The flow of people from category c to U.S. employment activity a, a ≠ c, is modeled as being proportional to the vacancies in a and to the share of category c in the supply of labor to activity a from people outside category a. Thus, if people in category c account for 10 per cent of the people outside category a who want jobs in employment-activity a, then people in category c fill 10 per cent of the vacancies in a. That is,

$$H_t(c\,;a) = V_t(a) * \left[\frac{L_t(c;a)}{\sum\limits_{s \neq a} L_t(s;a)} \right],$$

for all categories c ≠ a and all U.S. employment activities a. (30)

In (30), we assume that there is always competition for jobs, that is we assume that the number of people from outside category a who plan to work in employment-activity a, $\sum_{s\neq a} L_t(s;a)$, is greater or equal to the number of vacancies $[V_t(a)]$ in a.[9] This ensures that $H_t(c;a)$ is less than or equal to $L_t(c;a)$ for all categories c ≠ a and all U.S. employment activities a.

A familiar idea in labor economics is that unemployed people, especially long-term unemployed people, have a lower probability of filling vacancies than employed people wanting to move. This idea could be handled in (30) by attaching weights to the L's appearing on the RHS. We achieve a similar effect by assuming that the

[9] While this condition is not guaranteed by the equations in Table 1, it was in fact satisfied in the applications reported in Dixon *et al.* (2008).

unemployed, especially the long-term unemployed, make comparatively weak offers to employment. [Recall the last two dot points in our discussion of the $B_t(c;a)$'s.] That is, $\sum_{a\in \text{U.S. emloy activ}} L_t(c;a)/CAT_t(c)$ is low for people in unemployment categories c.

The number of incumbents in employment-category c who remain in activity c $[H_t(c;c)]$ is defined as the number of people in category c less the number who move out of activity c:

$$H_t(c;c) = CAT_t(c) - \sum_{a\neq c} H_t(c;a) \quad \text{, for all employment categories c (including Mexico)} \tag{31}$$

With $[H_t(c;a)]$ being less than or equal to $[L_t(c;a)]$ for a ≠ c, $H_t(c;c)$ is greater than or equal to $L_t(c;c)$. People in employment-category c who planned to work in activity a ≠ c but who are unable to move to a due to insufficient vacancies simply remain in c.

Flows from all U.S. employment categories to U.S. unemployment activities (area 2 in Figure 5)

People in a U.S. employment category at the start of year t cannot move to a long-run unemployment activity. If they move into unemployment it must be to short-run unemployment. The number of people who make the move to short-run unemployment is the sum of two parts: voluntary moves, $L_t(c;u)$, and involuntary moves. We model involuntary moves from U.S. employment category c as a fraction, $\mu(c)$, of the number of people in the category:

$$H_t(c;u) = \begin{cases} L_c(c;u) + \mu(c) * CAT_t(c) & \text{for short} - \text{run unemployment activities u} \\ 0 & \text{for long} - \text{run unemployment activities u} \end{cases}$$
$$\text{for all U.S. employment categories c} \quad , \tag{32}$$

Normally, $\mu(c)$ is exogenous. However, it is possible that (32) in conjunction with (30) will give values for $H_t(c;c)$ in (31) that exceed $E_t(c)$. In this case, $V_t(c)$ would be negative. We avoid this situation by treating $\mu(c)$ as an endogenous variable. If $V_t(c)$ is greater than zero, then $\mu(c)$ equals an exogenously given minimum value determined by the rate at which individuals are dismissed because of their performance or other factors unrelated to overall demand for people in activity c. Alternatively, $\mu(c)$ moves sufficiently above its minimal value to ensure that $V_t(c)$ equals zero. When $\mu(c)$ is above its minimum value, then there are involuntary flows from employment category c to unemployment caused by overall shortage of jobs.

Flows from U.S. unemployment categories to U.S. unemployment activities (area 3 in Figure 5)

Next we deal with flows between unemployment categories and unemployment activities. We ensure that short-term unemployed people who fail to obtain a job flow to long-term unemployment; and that long-term unemployed people who fail to obtain a job remain in long-term unemployment:

$$H_t(c;u) = \begin{cases} 0 & \text{for short} - \text{run unemployment activities u} \\ CAT_t(c) - \sum_{a\in \text{employment activ}} H_t(c;a) & \text{for long} - \text{run unemployment activities u} \end{cases}$$

$$\text{for all U.S. unemployment categories c and all unemployment activities u} \quad . \tag{33}$$

Flows from New categories to U.S. unemployment activities or to Mexico (area 4 in Figure 5)

New legal entrants (either domestic or foreign) who fail to get a U.S. job are allocated to a short-run unemployment activity. New illegal entrants who fail to get a U.S. job are allocated to employment in Mexico. These allocations are specified by:

$$H_t(c;u) = \begin{cases} CAT_t(c) - \sum\limits_{a \in \text{U.S. employ activ}} H_t(c;a) & \text{for c not foreign, illegal and for short} - \text{run unemploy activ u} \\ CAT_t(c) - \sum\limits_{a \in \text{U.S. employ activ}} H_t(c;a) & \text{for c foreign, illegal and for Mexican activities u} \\ 0 & \text{otherwise} \end{cases}$$

for all New categories c and all unemployment or Mexican activities u . (34)

Flows from non-New categories to Mexico (area 5 in Figure 5)

We assume that flows to Mexico from non-Mexican, non-New categories are voluntary. This means that foreign illegal workers in the U.S. can go home if they want to. Finally, the flow from the category of working in Mexico to the activity of working in Mexico is determined as the number of people in the category less the number that obtain jobs in the U.S. Thus we have:

$$H_t(c;u) = \begin{cases} L_t(c;u) & \text{for c non} - \text{Mexican} \\ CAT_t(c) - \sum\limits_{a \in \text{U.S. employ activ}} H_t(c;a) & \text{for c Mexican} \end{cases}$$

for all non-New categories c and for Mexican activities u (35)

Completing the link from categories to activities

To complete the link from categories at the start of year t to activities in year t we include the equation:

$$\sum_c H_t(c,a) = E_t(a) \quad \text{, for all U.S. unemployment activities and Mexican activities a} \tag{36}$$

A similar equation is not required for U.S. employment activities. Such an equation is implied by (29) and (30).

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