CHAPTER 2

A skeletal version of ORANI: Theory, data, computations and results

3. Introduction

In this chapter we try to maximize the accessibility of ORANI by working through a miniature version. We describe the data base, the theory, the computational method and an application of MO (Miniature ORANI). MO leaves out investment, government spending, production taxes and subsidies, agricultural land and technological change. It recognizes only one type of labour and only two industries. It fails to model margins and does not distinguish between purchasers' and producers' prices. It uses a fictitious data base and overly restrictive specifications of various substitution possibilities. Nevertheless, we feel that MO is a useful model of a model. It simplifies ORANI while retaining the main ideas intact. In particular, we hope that MO gives readers a rapid understanding (unencumbered by the detail of ORANI) of each of the following:

(i) the way in which standard microeconomic theory (cost minimizing, utility maximizing, etc.) underlies the ORANI structural equations;

(ii) ORANI's use of multiproduct and nested production functions and nested utility functions;

(iii) the role of input–output data in the estimation of ORANI parameters;

(iv) the computational procedures and the advantages and disadvantages of linearization in the ORANI context;

(v) the way in which model flexibility is enhanced by allowing variables to be shuffled between the endogenous and exogenous categories; and

(vi) some of the principal mechanisms explaining ORANI results.

The chapter is organized as follows. First, in section 4 we describe the input–output data base of MO. Then in section 5 we set out the theoretical structure. As the theory is developed, we refer back to the input–output data to show how the coefficients in each equation are estimated. The complete numerical representation of MO is contained in table 5.1. In section 6 we discuss the selection of endogenous and exogenous variables. We argue that by leaving this selection to model users we increase considerably the range of problems to which ORANI can be applied. Section 7 works through a computed solution of MO. The results are interpreted and used to highlight some of the implications of the model's underlying theory. In section 8 we offer some comments on computational theory, reminding readers of the costs and benefits of ORANI's use of linear approximations. The final section contains brief concluding remarks.

Throughout the chapter we relate each part (the data, the theory, the computations and the results) of MO to the corresponding part of ORANI. Our hope is that the documentation on ORANI, which comprises the bulk of this book, will provide no mysteries for those who have worked through MO. In addition, readers who are familiar with MO should have no difficulty in understanding published results from ORANI. They should, for example, be in a good position to read ORANI application papers, such as those referred to in Chapters 7 and 8.

4. The input–output data base

Figure 4.1 sets out schematically the input–output data required for MO. The data refer to flows in a particular year, the base year for the model. Matrix $\tilde{A}$ shows the flows of the g domestically produced commodities into the h industries.[1] $\tilde{B}$ and $\tilde{C}$ are vectors showing domestic commodity flows to households and exports. $\tilde{D}$ and $\tilde{E}$ refer to flows of imported commodities to industries and households. $-\tilde{Z}$ is the vector showing the negative of duty paid on imports. If we add across $\tilde{D}$, $\tilde{E}$ and $-\tilde{Z}$ we obtain the foreign currency costs (in \$A) of imported commodities. $\tilde{G}$ and $\tilde{H}$ are row vectors showing payments to labour and rentals on capital in each industry. $\tilde{J}$ is a matrix showing the commodity composition of each industry's output. If we add down a column of $\tilde{J}$ we obtain the total value of an industry's output. This could also be computed by adding down the appropriate column of $\tilde{A}$, $\tilde{D}$, $\tilde{G}$ and $\tilde{H}$. If we add across a row of $\tilde{J}$, we obtain the economy's output of a particular commodity. Alternatively, we can obtain commodity outputs by adding across the rows of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$.

The input–output data base for ORANI includes many more categories

[1] The matrices and vectors in fig. 4.1 are marked with tildes. A similar convention is used in describing the input–output data base for ORANI in Chapter 4. The intention is to avoid confusion with the use of the symbols A, B, etc. in the notation appearing elsewhere in the book.

		Industries	Households	Exports	- Duty	Row totals
		$\longleftarrow h \longrightarrow$	$\longleftarrow 1 \longrightarrow$	$\longleftarrow 1 \longrightarrow$	$\longleftarrow 1 \longrightarrow$	
Domestic commodities	$\uparrow g \downarrow$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$		Commodity outputs
Imported commodities	$\uparrow g \downarrow$	$\tilde{D}$	$\tilde{E}$		$-\tilde{Z}$	Foreign currency cost of commodity imports
Labor	$\uparrow 1 \downarrow$	$\tilde{G}$				Payments to labor
Capital	$\uparrow 1 \downarrow$	$\tilde{H}$				Payments to capital
		Total industry outputs	Total household consumption	Total exports	Total duty	
Domestic commodities	$\uparrow g \downarrow$	$\tilde{J}$				Commodity outputs

Figure 4.1. Schematic input–output data base for MO.

of flows than are shown in fig. 4.1. Fig. 4.1 simplifies the ORANI data base by excluding commodity flows to investment and to government. Fig. 4.1 also omits any explicit representation of demands for commodities (or services) to be used as margins, e.g. transport, wholesale and retail trade services. Finally, it shows no disaggregation of labour inputs, no land inputs and no miscellaneous "other" inputs. Nevertheless, fig. 4.1 will be adequate to illustrate the role of input–output data in implementing the ORANI model.

For our illustrative model it will be convenient to assume that $h=2$ and $g=2$.[2] Thus, MO will have only two industries and four commodities (two

[2]In standard runs of ORANI, $h=113$ and $g=115$.

domestically produced commodities and two imported). This will allow us to carry along a numerical example while we explain the MO theory. We will assume that the input–output data base for MO is as shown in fig. 4.2. The numbers in fig. 4.2 are to be interpreted as Australian dollar amounts for the base year, say 1968–69.[3] It is worth pointing out that our data reflect a base-period balance of trade deficit of $A1. The foreign currency cost of imports is 9+12=21, while the foreign currency value of exports is 19+1=20. Alternatively, we can calculate the balance of trade deficit as absorption minus GDP, i.e. household consumption (62) minus factor payments and duty (55+6).

		Industries 1	Industries 2	Households	Exports	-Duty	Row totals
Domestic commodities	1	10	8	17	19		54
	2	15	1	34	1		51
Imported commodities	1	1	8	1		-1	9
	2	5	2	10		-5	12
Labor		20	20				40
Capital		10	5				15
		61	44				
Domestic commodity outputs	1	45	9				54
	2	16	35				51
				62	20		

Figure 4.2. Input–output data base for MO, numerical example

[3]This is the base year for ORANI.

5. The theoretical structure

ORANI is a computable general equilibrium model in the Johansen class. The distinguishing characteristic of a Johansen-type model is that it is written as a system of linear equations in percentage changes of the variables.[4] Rather than writing

$$Y=f(X_1, X_2), \tag{5.1}$$

where Y is output and X_1 and X_2 are inputs, in a Johansen model we use the linear percentage change form

$$y-\varepsilon_1 x_1-\varepsilon_2 x_2=0, \tag{5.2}$$

where ε_i is the elasticity of output with respect to inputs of factor i, and y, x_1 and x_2 are percentage changes in Y, X_1 and X_2.

In matrix notation, a Johansen model can be represented by

$$Az=0, \tag{5.3}$$

where A is a matrix of coefficients and z is the vector of percentage changes in the model's variables. For MO, A is a 39×52 matrix (see table 5.1), i.e. MO has 39 equations and 52 variables. Because the A matrix is assumed fixed, (5.3) provides only a local representation of the equations suggested by economic theory. For example, (5.2) is valid only for "small" changes in X_1 and X_2. This disadvantage must be weighed against the computational advantages and flexibility of linear models. We return to this issue in section 8 (see also Chapter 5).

The equations of a typical Johansen model can be classified into five groups:

(i) equations describing household and other final demands for commodities;

(ii) equations describing industry demands for primary factors and intermediate inputs;

(iii) pricing equations setting pure profits from all activities to zero;

(iv) market clearing equations for primary factors and commodities; and

(v) miscellaneous definitional equations, e.g. equations defining GDP, aggregate employment and the consumer price index.

[4]Examples of Johansen-type models include Johansen (1960), Taylor and Black (1974), Klijn (1974) and Staelin (1976). Examples of computable general equilibrium models outside the Johansen class include Shoven and Whalley (1972, 1973, 1974), Dixon (1975, 1978c), Dervis (1975, 1980), Adelman and Robinson (1978) and Boadway and Treddenick (1978).

In presenting the theory of MO we will use this five-part classification. However, one additional set of equations – those describing the commodity composition of industry outputs – will be required. MO recognizes multi-product production functions for industries. Therefore, in MO, industries have output-composition decisions. This aspect of MO will be treated under heading (ii), i.e. it will be convenient to extend this heading to include the equations explaining the composition of industry outputs as well as those explaining the composition of industry inputs.

A note on notation

In describing MO, we will observe the following notational conventions.

(i) The percentage change in any variable V will be represented by v, i.e. $v=(\mathrm{d}V/V)\,100$.

(ii) $X_{(is)j}$ will denote the demand by user j for input i of type s. The possible values for subscript i are 1, 2 and 3. Where i equals 1 and 2 we refer to commodities 1 and 2 and where i is 3 we refer to primary factors. s can take the values 1 and 2. In the context of commodities, $s=1$ means domestic while $s=2$ means imported. Thus, the subscript (12) indicates imported good 1. The subscript (21) indicates domestically produced good 2, etc. In the context of primary factors ($i=3$), $s=1$ means labour while $s=2$ means capital. The subscript (32), for example, should be read as primary factor type 2, i.e. capital. The subscript j has four possible values: j equals 1 and 2 refer to industries 1 and 2, $j=3$ refers to households, while $j=4$ refers to exports. A few examples should clarify matters:

$X_{(12)1}$ = demand for imported good 1 to be used as an input to industry 1;
$X_{(11)4}$ = export volume for domestically produced good 1;
$X_{(21)3}$ = demand for domestically produced good 2 by households;
$X_{(31)2}$ = demand for primary factor type 1, i.e. labour, by industry 2.

Not all possible combinations of subscript values define valid MO variables. For example, readers will not find $X_{(12)4}$, $X_{(31)3}$ or $X_{(31)4}$ appearing in MO. This is because we assume that imported commodities are not simply re-exported, we assume that households do not use primary factors and we assume that primary factors are not exported.

(iii) Commodity outputs from our two industries will be denoted by $Y_{(i1)j}$, where both i and j can take the values 1 and 2. Thus, $Y_{(11)2}$ is the output of domestically produced good 1 by industry 2. Naturally, our industries can only produce domestic commodities. The symbol $Y_{(12)j}$ has no meaning in MO. Consequently, we could delete the "type" subscript on the Y's. We prefer to retain it, however, so that the subscript (is) immediately indicates good or factor i of type s. If this subscript is appended to an X, we are defining a demand. If it is appended to a Y, we are defining a production level. If it is appended to a P, we are defining a price.

The notation used for MO is suggestive of, although not precisely the same as, that used for ORANI. Since ORANI identifies many more variables and parameters than does MO, the number of symbols required and the number of subscripts and superscripts carried on many of the symbols are much greater in ORANI. The number of available symbols is limited and we have been unable to avoid some overlap in the use of symbols in MO and ORANI. The notation used in this chapter, therefore, must be carefully distinguished by the reader from the notation used in the rest of the book.

5.1. *Household and other final demands*

5.1.1. *Household demands*

We explain household demands via utility maximizing. For our illustrative model, MO, we will assume that the utility function takes the nested form

$$U=\min\left\{\frac{X_{(1\cdot)3}}{A_{(1\cdot)3}},\qquad \frac{X_{(2\cdot)3}}{A_{(2\cdot)3}}\right\}, \tag{5.4}$$

where

$$X_{(i\cdot)3}=X_{(i1)3}^{\alpha_{(i1)3}}X_{(i2)3}^{\alpha_{(i2)3}} \qquad i=1,2, \tag{5.5}$$

and the A's and α's are positive parameters with $\alpha_{(i1)3}+\alpha_{(i2)3}=1$, for i equals 1 and 2. The specification (5.4)–(5.5) implies that consumers derive utility from "effective" units of goods 1 and 2, where an effective unit of good i is an aggregation of commodities $(i1)$ and $(i2)$, i.e. an effective unit of good i is an aggregation of units of domestically produced good i and units of imported good i.[5] The aggregation is defined by eq. (5.5). In the particular case set out here, the household sector is assumed to behave as if effective units of goods 1 and 2 are nonsubstitutes, i.e. (5.4) has the Leontief form. On the other hand, units of imported and domestic commodity i substitute for each other

[5]The idea of using nested utility functions to handle substitution possibilities between domestic and imported commodities is found in Armington (1969, 1970). See also Artus and Rhomberg (1973), Dixon (1976b) and Dervis (1980).

(with unitary elasticity) in the creation of effective units good i, i.e. (5.5) has the Cobb–Douglas form. Of course, (5.4) and (5.5) can take more empirically relevant forms. The equations corresponding to these in ORANI have, respectively, nested additive and CES forms.[6] Eqs. (5.4) and (5.5) are adequate, however, for illustrative purposes.

The next step is to introduce the household budget constraint

$$\sum_{i=1}^{2}\sum_{s=1}^{2} P_{(is)}X_{(is)3}=C, \tag{5.6}$$

where $P_{(is)}$ is the price in the domestic market for commodity (is), and C is the household sector's aggregate expenditure level. Notice that the prices $(P_{(is)},\ i,s=1,2)$ carry no user subscript. In this illustrative model we will abstract from complications caused by the distinction between purchasers' and producers' prices. This distinction (caused by transport, sales taxes and other margins costs) is not of great theoretical interest. On the other hand, it is of practical importance and receives detailed treatment in the ORANI model (see sections 17, 18 and 28).

On maximizing (5.4) subject to (5.5) and (5.6) we can derive the household demand functions. It is rather laborious to obtain their explicit forms and it will be sufficient to denote them by

$$X_{(is)3}=X_{(is)3}(P,\ C), \qquad i,s=1,2, \tag{5.7}$$

where P is the vector of commodity prices.

In linear percentage change form, (5.7) becomes

$$x_{(is)3}=\varepsilon_{(is)}c+\sum_{q=1}^{2}\sum_{r=1}^{2}\eta_{(is)(qr)}p_{(qr)}, \qquad i,s=1,2, \tag{5.8}$$

where $\varepsilon_{(is)}$ is the expenditure[7] elasticity of demand for good i of type s. For example, $\varepsilon_{(12)}$ is the expenditure elasticity of demand for imported good 1. $\eta_{(is)(qr)}$ is the cross price elasticity of demand for good i of type s with respect to changes in the price of good q of type r. For example, $\eta_{(12)(21)}$ is the cross elasticity between imported good 1 and domestic good 2.

With the particular utility specification (5.4)–(5.5), it is clear that

$$\varepsilon_{(is)}=1 \quad \text{for all } i \text{ and } s. \tag{5.9}$$

[6]The theory underlying the ORANI household demand equations is in section 14. The estimation of the parameters for the nested additive utility function is described in subsection 29.5. Subsection 29.1 describes the estimation of the substitution elasticities between imported and domestic commodities.

[7]C is the household expenditure level. Therefore, the ε's are expenditure elasticities, not income elasticities.

The utility function is homothetic (in fact it is homogeneous of degree 1) and therefore, in the absence of price changes, a 1 percent increase total expenditure will be allocated as a 1 percent increase in expenditure on each commodity.

For the price elasticities, we have the well-known Hicks–Slutsky partition – total effect equals income effect plus substitution effect, i.e.

$$\eta_{(is)(qr)} = -\varepsilon_{(is)} S_{(qr)3} + \bar{\eta}_{(is)(qr)}, \qquad i,s,q,r=1,2, \tag{5.10}$$

where $S_{(qr)3}$ is the share of the total household budget devoted to commodity (qr) and $\bar{\eta}_{(is)(qr)}$ is the *compensated* cross elasticity of demand for (is) with respect to changes in price (qr). With utility held constant, (5.4) implies that changes in the prices of domestic or imported good 1 will not affect the demands for domestic or imported good 2, and vice versa. Hence,

$$\bar{\eta}_{(is)(qr)} = 0 \quad \text{if } i \neq q.$$

On the other hand, with utility held constant at $\bar{U}$ say, $X_{(i1)3}$ and $X_{(i2)3}$ will be chosen to minimize

$$\left.\begin{aligned} &\sum_{s=1}^{2} P_{(is)} X_{(is)3} \\ &\text{subject to} \\ &\prod_{s=1}^{2} X_{(is)3}^{\alpha_{(is)3}} = A_{(i\cdot)3}\bar{U}. \end{aligned}\right\} \tag{5.11}$$

The first-order conditions for a solution of this problem are

$$P_{(is)} X_{(is)3} - \Lambda \alpha_{(is)3} (A_{(i\cdot)3}\bar{U}) = 0, \qquad s=1,2, \tag{5.12}$$

and

$$\prod_{s=1}^{2} X_{(is)3}^{\alpha_{(is)3}} = A_{(i\cdot)3}\bar{U}, \tag{5.13}$$

where Λ is the Lagrangian multiplier. Holding utility constant, and linearizing these first-order conditions, we obtain

$$p_{(is)} + x_{(is)3} = \lambda, \qquad s=1,2, \tag{5.14}$$

and

$$\sum_{s=1}^{2} \alpha_{(is)3} x_{(is)3} = 0. \tag{5.15}$$

When we eliminate λ, we find that

$$\bar{\eta}_{(is)(is)} = -1 + \alpha_{(is)3}, \qquad s = 1,2, \tag{5.16}$$

and

$$\bar{\eta}_{(is)(ir)} = \alpha_{(ir)3}, \qquad \text{where } r \neq s.$$

We also note from (5.12) that the α's are expenditure shares. $\alpha_{(is)3}$ is the share of good (is) in the households' total expenditure on good i. Thus, from our input–output data base (fig. 4.2) we can compute the compensated price elasticities:

$$\begin{aligned}
&\bar{\eta}_{(11)(11)} = -1 + 17/18 = -0.06; &\quad& \bar{\eta}_{(21)(21)} = -1 + 34/44 = -0.23,\\
&\bar{\eta}_{(11)(12)} = 1/18 = 0.06; && \bar{\eta}_{(21)(22)} = 10/44 = 0.23,\\
&\bar{\eta}_{(12)(12)} = -1 + 1/18 = -0.94; && \bar{\eta}_{(22)(22)} = -1 + 10/44 = -0.77,\\
&\bar{\eta}_{(12)(11)} = 17/18 = 0.94; && \bar{\eta}_{(22)(21)} = 34/44 = 0.77.
\end{aligned}$$

Now we combine these calculations with formulae (5.9) and (5.10) to obtain the uncompensated elasticities. For example,

$$\begin{aligned}
\eta_{(11)(11)} &= -\varepsilon_{(11)} S_{(11)3} + \bar{\eta}_{(11)(11)}\\
&= -1 \times 17/62 - 0.06\\
&= -0.33.
\end{aligned}$$

Readers who are interested in following the arithmetical example further can read the elasticities from the first four rows of table 5.1. These rows display the household demand equations (5.8) in their computational form, given the utility maximizing model (5.4)–(5.6) and the data base shown in fig. 4.2. For example, row 1 in table 5.1 should be read as

$$1.00x_{(11)3} - 1.00c + 0.33p_{(11)} - 0.04p_{(12)} + 0.55p_{(21)} + 0.16p_{(22)} = 0,$$

i.e.

$$x_{(11)3} = \varepsilon_{(11)} c + \sum_{q=1}^{2} \sum_{r=1}^{2} \eta_{(11)(qr)} p_{(qr)},$$

where

$$\eta_{(11)(11)} = -0.33; \qquad \eta_{(11)(12)} = 0.04; \qquad \eta_{(11)(21)} = -0.55,$$
$$\eta_{(11)(22)} = -0.16 \quad \text{and} \quad \varepsilon_{(11)} = 1.$$

Table 5.1
The A matrix for MO.

Row Nos.	Equations	Column Nos. →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Row Nos.
		Variables	Household demands				Household expenditure level	Commodity prices (local currency)				Commodity prices (foreign currency)				Exports		Shifts in foreign demand		
			$x_{(11)3}$	$x_{(12)3}$	$x_{(21)3}$	$x_{(22)3}$	c	$p_{(11)}$	$p_{(12)}$	$p_{(21)}$	$p_{(22)}$	$p^*_{(11)}$	$p^*_{(21)}$	$p^*_{(12)}$	$p^*_{(22)}$	$x_{(11)4}$	$x_{(21)4}$	$f_{(11)4}$	$f_{(21)4}$	
1	Household demands	(5.8)	1.00				-1.00	0.33	-0.04	0.55	0.16									1
2				1.00			-1.00	-0.67	0.96	0.55	0.16									2
3					1.00		-1.00	0.27	0.02	0.78	-0.07									3
4						1.00	-1.00	0.27	0.02	-0.22	0.93									4
5	Export demands	(5.19)										1.00				0.50		-1.00		5
6													1.00				0.05		-1.00	6
7	Commodity outputs by industry	(5.29)						-0.26		0.26										7
8								0.74		-0.74										8
9								-0.80		0.80										9
10								0.20		-0.20										10
11	Industry demands for primary factors and intermediate inputs	(5.33)						0.09	-0.09											11
12								-0.91	0.91											12
13										0.25	-0.25									13
14										-0.75	0.75									14
15																				15
16																				16
17								0.50	-0.50											17
18								-0.50	0.50											18
19										0.67	-0.67									19
20										-0.33	0.33									20
21																				21
22																				22
23	Zero pure profits in production	(5.41)						0.57	-0.02	0.02	-0.08									23
24								0.02	-0.18	0.77	-0.05									24
25	Zero pure profits in exporting	(5.42)						-1.00				1.00								25
26										-1.00			1.00							26
27	Zero pure profits in importing	(5.43)							-1.00					1.00						27
28											-1.00				1.00					28
29	Market clearing for domestic commodities	(5.45)	-0.31													-0.35				29
30					-0.67												-0.02			30
31	Market clearing for primary factors	(5.48)																		31
32																				32
33																				33
34	Imports	(5.55)		-0.04		-0.33								-0.43	-0.57					34
35	Exports	(5.56)										-0.95	-0.05			-0.95	-0.05			35
36	Balance of trade	(5.57)																		36
37	CPI	(5.58)						-0.27	-0.02	-0.55	-0.16									37
38	Wage rate	(5.59)																		38
39	Real consumption	(5.60)					-1.00													39

.... continued

Table 5.1 (*continued*)

Col. Nos / Row Nos	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
	Industry activity levels		Commodity outputs by industry				Intermediate and primary factor demands by industry												
	z_1	z_2	$y_{(11)1}$	$y_{(21)1}$	$y_{(11)2}$	$y_{(21)2}$	$x_{(11)1}$	$x_{(12)1}$	$x_{(21)1}$	$x_{(22)1}$	$x_{(31)1}$	$x_{(32)1}$	$x_{(11)2}$	$x_{(12)2}$	$x_{(21)2}$	$x_{(22)2}$	$x_{(31)2}$	$x_{(32)2}$	Row Nos
1																			1
2																			2
3																			3
4																			4
5																			5
6																			6
7	-1.00		1.00																7
8	-1.00			1.00															8
9		-1.00			1.00														9
10		-1.00				1.00													10
11	-1.00						1.00												11
12	-1.00							1.00											12
13	-1.00								1.00										13
14	-1.00									1.00									14
15	-1.00										1.00								15
16	-1.00											1.00							16
17		-1.00											1.00						17
18		-1.00												1.00					18
19		-1.00													1.00				19
20		-1.00														1.00			20
21		-1.00															1.00		21
22		-1.00																1.00	22
23																			23
24																			24
25																			25
26																			26
27																			27
28																			28
29			0.83		0.17		-0.19						-0.15						29
30				0.31		0.69			-0.29						-0.02				30
31											0.50						0.50		31
32												1.00							32
33																		1.00	33
34								-0.04		-0.17				-0.35		-0.07			34
35																			35
36																			36
37																			37
38																			38
39																			39

.... continued

Table 5.1 (*continued*)

Col. Nos / Row Nos	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	
	Factor prices			Exchange rate	One plus rate of export subsidy		One plus rate of import duty		Factor employment levels			Aggregate imports, exports and the balance of trade			Aggregate prices	Wage shift	Real consumption	Row Nos
	$p_{(31)}$	$p_{(32)1}$	$p_{(32)2}$	ϕ	v_1	v_2	t_1	t_2	ℓ	k_1	k_2	m	e	ΔB	cpi	$f_{(31)}$	c_R	
1																		1
2																		2
3																		3
4																		4
5																		5
6																		6
7																		7
8																		8
9																		9
10																		10
11																		11
12																		12
13																		13
14																		14
15	0.33	-0.33																15
16	-0.67	0.67																16
17																		17
18																		18
19																		19
20																		20
21	0.20		-0.20															21
22	-0.80		0.80															22
23	-0.33	-0.16																23
24	-0.45		-0.11															24
25				1.00	1.00													25
26				1.00		1.00												26
27				1.00			1.00											27
28				1.00				1.00										28
29																		29
30																		30
31									-1.00									31
32										-1.00								32
33											-1.00							33
34												1.00						34
35													1.00					35
36												0.21	-0.20	1.00				36
37															1.00			37
38	1.00														-1.00	-1.00		38
39															1.00		1.00	39

5.1.2. Exports

We write the export demand functions as

$$P^*_{(i1)} = X_{(i1)4}^{-\gamma_i} F_{(i1)4}, \qquad i=1,2, \tag{5.18}$$

where $P^*_{(i1)}$ is the foreign-currency price of domestic good i, $X_{(i1)4}$ is the export volume, γ_i is a positive parameter (the reciprocal of the foreign elasticity of demand) and $F_{(i1)4}$ is a "shift" variable. For example, if there is an increase in foreign demand, i.e. an upward movement in the demand curve, then $F_{(i1)4}$ increases.

In linear percentage change form (5.18) becomes

$$p^*_{(i1)} = -\gamma_i x_{(i1)4} + f_{(i1)4}, \qquad i=1,2, \tag{5.19}$$

In table 5.1, (5.19) is shown in its computational form in rows 5 and 6. The values chosen for γ_1 and γ_2 are 0.50 and 0.05, respectively, i.e. the foreign elasticity of demand for good 1 is 2.0 while that for good 2 is 20.0. Values such as these are typical of the ORANI data base (see subsection 29.6).

5.2. Industry inputs and outputs

We imagine that industry production functions can be expressed as

$$G_j(Y_{(11)j}, Y_{(21)j}) = Z_j, \qquad j=1,2, \tag{5.20}$$

and

$$H_j(X_{(11)j}, X_{(12)j}, X_{(21)j}, X_{(22)j}, X_{(31)j}, X_{(32)j}) = Z_j, \qquad j=1,2, \tag{5.21}$$

or more compactly as

$$G_j(Y^j) = Z_j,$$

and

$$H_j(X^j) = Z_j,$$

where Y^j is the vector of outputs of industry j and X^j is the vector of inputs. Z_j is a variable reflecting industry j's overall capacity to produce.

Industry j is viewed as buying a production possibilities frontier. More inputs yield a higher Z_j and a higher Z_j corresponds to an expanded production possibilities set (see fig. 5.1).

Notice that under (5.20)–(5.21) inputs are regarded as nonspecific to products. Inputs merely generate a general capacity to produce which can

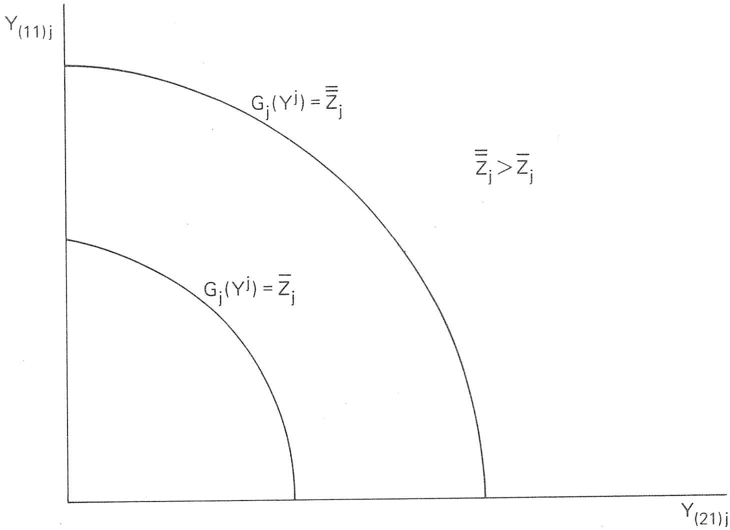

Figure 5.1. Production possibilities frontiers

be used to produce a variety of products. For example, one can think of labour, tractors and fertilizer as being general farm inputs which allow the production of various combinations of wheat, wool, cattle, etc. In ORANI the use of multiproduct production functions is, in fact, confined to four agricultural industries (subsection 28.2.1 and table 28.1). The remaining 109 industries have single-output production functions, i.e. for these industries

$$G_j(Y^j) = Y_{(r1)j},$$

where r is *the* commodity produced by industry j.

For our illustrative model, we will assume that the product transformation frontiers (5.20) have the form

$$(Y^2_{(11)j}\beta_{(11)j} + Y^2_{(21)j}\beta_{(21)j})^{1/2} = Z_j, \qquad j = 1,2, \tag{5.22}$$

where $\beta_{(11)j}$ and $\beta_{(21)j}$ are positive parameters. Under (5.22), the product transformation frontier is a quarter elipse. In our applied work on Australian agriculture we used the more general *CRETH* function to specify product transformation possibilities (see subsection 11.2). For our present purposes an advantage of (5.22) is that it implies comparatively straightforward supply relations.[8] We generate the supply relations for industry j by considering the problem of choosing $Y_{(11)j}$ and $Y_{(21)j}$ to maximize

$$P_{(11)}Y_{(11)j}+P_{(21)}Y_{(21)j},$$

subject to (5.22).

The first-order conditions for this problem are

$$P_{(i1)}-\Lambda Z_j^{-1}\beta_{(i1)j}Y_{(i1)j}=0, \qquad i=1,2, \tag{5.24}$$

and

$$(Y_{(11)j}^2\beta_{(11)j}+Y_{(21)j}^2\beta_{(21)j})^{1/2}=Z_j, \tag{5.25}$$

where Λ is the Lagrangian multiplier. In linear percentage change form we have

$$p_{(i1)}=\lambda-z_j+y_{(i1)j}, \qquad i=1,2, \tag{5.26}$$

and

$$y_{(11)j}R_{(11)j}+y_{(21)j}R_{(21)j}=z_j, \tag{5.27}$$

where

$$R_{(i1)j}=Y_{(i1)j}^2\beta_{(i1)j}/(Y_{(11)j}^2\beta_{(11)j}+Y_{(21)j}^2\beta_{(21)j}).$$

From (5.24) we see that the $R_{(i1)j}$'s are revenue shares, i.e.

$$R_{(i1)j}=P_{(i1)}Y_{(i1)j}/(P_{(11)}Y_{(11)j}+P_{(21)}Y_{(21)j}), \qquad i=1,2. \tag{5.28}$$

Next we eliminate λ from (5.26)–(5.27) to obtain the supply relations

$$y_{(i1)j}=z_j+\left(p_{(i1)}-\sum_{q=1}^{2}R_{(q1)j}p_{(q1)}\right), \qquad i=1,2. \tag{5.29}$$

Equation (5.29) says that, in the absence of price changes, the output of good $(i1)$ by industry j will expand with the overall level of activity, Z_j, in industry j. However, if the price of good $(i1)$ increases relative to the appropriately weighted average of prices $P_{(11)}$ and $P_{(21)}$, then industry j's output of good $(i1)$ will increase more quickly than Z_j, i.e. industry j will transform its product mix in favour of good $(i1)$. With the particular equation (5.29), the elasticity of transformation between goods (11) and (21) in industry j is unity. Under the more general *CRETH* specification for the transforma-

[8] That is, equations of the form

$$Y_{(i1)j}=Y_{(i1)j}(P_{(11)},P_{(21)},Z_j), \qquad i,j=1,2. \tag{5.23}$$

tion frontier, the elasticities of transformation between pairs of products are left as free parameters to be empirically determined (see subsection 29.4).

The computational form of (5.29) is shown in table 5.1, rows 7–10. The $R_{(q1)j}$'s are computed as column shares in the $\tilde{J}$ matrix (fig. 4.1). For example, with the data in fig. 4.2, we have $R_{(11)1}=45/(45+16)=0.74$.

On the input side we assume that H_j has the form

$$\min\left(\frac{X_{(1\cdot)j}}{A_{(1\cdot)j}}, \frac{X_{(2\cdot)j}}{A_{(2\cdot)j}}, \frac{X_{(3\cdot)j}}{A_{(3\cdot)j}}\right)=Z_j, \tag{5.30}$$

where $X_{(1\cdot)j}$ and $X_{(2\cdot)j}$ are Cobb–Douglas combinations of inputs of goods 1 and 2 from domestic and foreign sources and $X_{(3\cdot)j}$ is a Cobb–Douglas combination of primary factor inputs, labour and capital. More specifically,

$$X_{(i\cdot)j}=X_{(i1)j}^{\alpha_{(i1)j}}\, X_{(i2)j}^{\alpha_{(i2)j}}, \qquad i=1,2,3, \tag{5.31}$$

where the α's are positive parameters summing to unity in each equation.[9] Under (5.30) no substitution is allowed between primary factors and intermediate inputs or between intermediate inputs of goods 1 and 2. On the other hand, domestically produced and imported inputs of each good i substitute with unitary elasticity [see (5.31)]. Similarly, labour and capital can be substituted. Although the theory of ORANI allows for considerably more general specifications of H_j (see subsection 11.1), in practice (5.30)–(5.31) is quite close to what is actually implemented. For example, in the case of labour–capital substitution, our empirical work does not support the generalization of the Cobb–Douglas form (5.31) to anything beyond *CES* (see subsection 29.2).

By assuming cost-minimizing behaviour, i.e. by assuming that industry j chooses $X_{(is)j}$, $i=1,2,3$ and $s=1,2$, to minimize

$$\sum_{i=1}^{2}\sum_{s=1}^{2} P_{(is)}X_{(is)j}+P_{(31)}X_{(31)j}+P_{(32)j}X_{(32)j},$$

subject to (5.30)–(5.31), we can obtain input demand functions of the form

$$X_{(is)j}=X_{(is)j}(P, P_{(31)}, P_{(32)j}, Z_j), \qquad i=1,2,3, \quad s=1,2, \tag{5.32}$$

where P is [as in (5.7)] the vector of commodity prices, $P_{(31)}$ is the price of labour and $P_{(32)j}$ is the rental on the use of a unit of capital in industry j. Notice that we have no j subscript on the wage rate, $P_{(31)}$, while we do include a j subscript on the rental on capital. This is consistent with the assumption commonly made in ORANI computations that labour is mobile between

[9]For definitions of the $X_{(qr)j}$'s, see the notes on notation at the beginning of this section.

industries, but that capital is immobile. In other words, capital is industry-specific and labour is not. Thus, labour will have an economy-wide price, while the rental value of any unit of capital will reflect conditions in the specific using industry.

Under the particular specification (5.30)–(5.31), the linear percentage change forms for the input demand functions for industry j are

$$\left.\begin{aligned} x_{(is)j} &= z_j - \left(p_{(is)} - \sum_{r=1}^{2} \alpha_{(ir)j} p_{(ir)}\right), \qquad i,s=1,2, \\ x_{(31)j} &= z_j - [p_{(31)} - (\alpha_{(31)j} p_{(31)} + \alpha_{(32)j} p_{(32)j})], \\ &\text{and} \\ x_{(32)j} &= z_j - [p_{(32)j} - (\alpha_{(31)j} p_{(31)} + \alpha_{(32)j} p_{(32)j})]. \end{aligned}\right\} \tag{5.33}$$

These equations are derived by considering problems similar to (5.11), where the 3's are replaced by j's and the right-hand sides of the constraints become $A_{(i\cdot)j}Z_j$ rather than $A_{(i\cdot)3}\bar{U}$. The α's are again interpretable as shares. For example, $\alpha_{(is)j}$ is the share of commodity (is) in industry j's total expenditure on good i. Thus, from our input–output data (fig. 4.2) we can compute

$$\alpha_{(11)1} = 10/11; \qquad \alpha_{(21)1} = 15/20, \quad \text{etc.}$$

Rows 11–22 of table 5.1 display the computational versions of the input demand functions for MO. Rows 11–16 cover the demands by industry 1 while rows 17–22 refer to demands by industry 2. For each industry the demand equations are listed in the order (11), (12), (21), (22), (31), (32).

5.3. *Zero pure profits for all activities*

The activities recognized in our illustrative model are production, exporting and importing. The zero-pure-profits condition for production implies that

$$P_{(11)}Y_{(11)j} + P_{(21)}Y_{(21)j} = P_{(31)}X_{(31)j} + P_{(32)j}X_{(32)j} + \sum_{i=1}^{2}\sum_{s=1}^{2} P_{(is)}X_{(is)j}, \qquad j=1,2, \tag{5.34}$$

i.e. revenue in industry j equals costs in the industry. It should be emphasized that such an equation does not rule out profits. It does rule out pure profits, i.e. profits not accruing to a factor of production. In models incorporating equations such as (5.34), variations in profits are simulated by variations in the $P_{(32)j}$'s. Adverse events in industry j will reduce the profitability of using capital in industry j, i.e. there will be reductions in the rental value, $P_{(32)j}$, of capital in the industry.

The second set of zero-pure-profit conditions in our illustrative model equates the revenue from exporting to the relevant costs, i.e.

$$P^*_{(i1)}V_i\Phi = P_{(i1)}, \qquad i=1,2, \tag{5.35}$$

where $P^*_{(i1)}$ is, as in eq. (5.18), the foreign currency price of domestic good i, Φ is the exchange rate ($A/$foreign) and V_i is one plus the *ad valorem* rate of export subsidy. Thus, on the left of (5.35) we have the $A value, to the exporter, of exporting a unit of commodity i. On the right we have the cost of doing so, i.e. the domestic price of a unit of commodity i.

The final set of zero-pure-profit conditions equates the selling prices of imported commodities to the costs of importing, i.e.

$$P_{(i2)} = P^*_{(i2)}T_i\Phi, \qquad i=1,2, \tag{5.36}$$

where T_i is one plus the *ad valorem* rate of tariff on imports of good i and $P^*_{(i2)}$ is the foreign currency price.

In linear percentage change form, (5.34) becomes

$$\sum_{i=1}^{2} (p_{(i1)} + y_{(i1)j})R_{(i1)j} = (p_{(31)} + x_{(31)j})S_{(31)j} + (p_{(32)j} + x_{(32)j})S_{(32)j} + \sum_{i=1}^{2}\sum_{s=1}^{2} (p_{(is)} + x_{(is)j})S_{(is)j}, \qquad j=1,2, \tag{5.37}$$

where the $R_{(i1)j}$'s are revenue shares [defined in (5.28)] and the $S_{(is)j}$'s are cost shares. For example, $S_{(12)1}$ is the share in industry 1's total costs accounted for by inputs of imported commodity 1. Given the data in fig. 4.2, this share would have the value 1/61.

Equation (5.37) can be simplified by recalling from (5.27) that

$$\sum_{i=1}^{2} y_{(i1)j}R_{(i1)j} = z_j, \qquad j=1,2, \tag{5.38}$$

and observing from (5.33) that

$$\sum_{s=1}^{2} x_{(is)j}\alpha_{(is)j} = z_j, \qquad i=1,2,3, \quad j=1,2. \tag{5.39}$$

On using (5.38) and (5.39) in (5.37) and on noting that

$$S_{(is)j} = \alpha_{(is)j}S_{(i\cdot)j}, \qquad i=1,2,3, \quad s=1,2, \quad j=1,2, \tag{5.40}$$

where $S_{(i\cdot)j}$, $i=1,2$, is the share of the j's total costs represented by inputs of good i from both domestic and foreign sources and $S_{(3\cdot)j}$ is the primary factor share, we see that (5.37) reduces to

$$\sum_{i=1}^{2} p_{(i1)}R_{(i1)j}=p_{(31)}S_{(31)j}+p_{(32)j}S_{(32)j} + \sum_{i=1}^{2}\sum_{s=1}^{2} p_{(is)}S_{(is)j}, \qquad j=1,2. \tag{5.41}$$

Eq. (5.41) says that for each industry an appropriately weighted average of the percentage changes in output prices equals an appropriately weighted average of the percentage changes in input prices. The fact that it has been possible to eliminate output and input quantities from (5.37) can be traced back to the assumption of constant returns to scale implied by the production specification (5.22) and (5.30)–(5.31). Under constant returns to scale unit costs are independent of the scale of output.

The linear percentage change forms for (5.35) and (5.36) are

$$p^*_{(i1)}+v_i+\phi=p_{(i1)}, \qquad i=1,2, \tag{5.42}$$

and

$$p^*_{(i2)}+t_i+\phi=p_{(i2)}, \qquad i=1,2. \tag{5.43}$$

In table 5.1 the computational forms for the zero-pure-profit equations (5.41)–(5.43) are shown in rows 23–28. The coefficients in eq. (5.41) are computed from the input–output data in fig. 4.2. For example, the coefficient on $p_{(11)}$ in row 23 is

$$R_{(11)1}-S_{(11)1}=\frac{45}{61}-\frac{10}{61}=0.57.$$

The coefficient on $P_{(12)}$ in row 24 is

$$-S_{(12)2}=-\frac{8}{44}=-0.18.$$

5.4. *Market clearing for commodities and factors*

For our two domestically produced commodities we have

$$Y_{(i1)1}+Y_{(i1)2}=\sum_{j=1}^{4} X_{(i1)j}, \qquad i=1,2, \tag{5.44}$$

i.e. the supply of domestic good i equals intermediate demand plus household demand plus export demand. In linear percentage change form (5.44) is written as

$$\sum_{j=1}^{2} y_{(i1)j}Q_{(i1)j}=\sum_{j=1}^{4} x_{(i1)j}W_{(i1)j}, \qquad i=1,2, \tag{5.45}$$

where the Q's are industry market shares for each commodity and the W's are shares of intermediate, household and export demand in aggregate commodity demands. For example, using the data in fig. 4.2, we have

$$Q_{(11)1}=45/54; \qquad Q_{(11)2}=9/54,$$

$$W_{(11)1}=10/54; \qquad W_{(11)3}=17/54, \quad \text{etc.}$$

The computational form for (5.45) is shown in rows 29 and 30 of table 5.1.

The market-clearing equations for primary factors are

$$X_{(31)1}+X_{(31)2}=L, \tag{5.46}$$

and

$$X_{(32)j}=K_j, \qquad j=1,2, \tag{5.47}$$

where L is the aggregate level of employment and $X_{(31)1}$ and $X_{(31)2}$ are labour demands in industries 1 and 2. Thus, (5.46) amounts to saying that employment demands are satisfied, i.e. aggregate employment, L, is the sum of labour demands in each industry. Eq. (5.46) does not, of course, impose the full employment assumption on our model. Although we could set L exogenously at the full employment level, an obvious alternative would be to set the wage rate, $P_{(31)}$, exogenously and to let the model determine L. Under this latter specification our assumption would be that the labour market is slack, i.e. labour supply constraints play no role in determining actual employment.

In eq. (5.47) K_j is the employment of capital in industry j. For short-run applications one would normally set K_j exogenously to reflect the current availability of capital in industry j.[10] Thus, one would impose the assumption that capital stocks are fully employed. This does not exclude the phenomenon of excess capacity. Excess capacity can be interpreted as a situation in which capital stocks are being operated with less labour than was planned when those capital stocks were created. Events unfavourable from the point of view of industry j will decrease $X_{(31)j}/K_j$ but will not invalidate (5.47).

An important difference between eqs. (5.46) and (5.47) is that labour

[10]Since MO omits investment, it is perhaps difficult to imagine an application where the K_j's are endogenous. We consider this point in section 6.

demands are added across industries, whereas for capital there is a separate market clearing equation for each industry. This reflects the assumption that, even in the short run, labour is mobile across industries whereas capital, once installed in an industry, is immobile. These assumptions are maintained in short-run ORANI simulations. In ORANI, however, there are nine types of labour rather than one. Consequently, there are nine equations of the form (5.46).

In linear percentage change form the market-clearing equations for primary factors are

$$\left.\begin{aligned} &x_{(31)1}W_{(31)1}+x_{(31)2}W_{(31)2}=\ell \\ &\text{and} \\ &x_{(32)j}=k_j, \qquad j=1,2, \end{aligned}\right\} \tag{5.48}$$

where $W_{(31)1}$ and $W_{(31)2}$ are the shares of total employment accounted for by industries 1 and 2. Because we assume that the wage rate is uniform across industries, it follows that employment is proportional to wage payments. Therefore, $W_{(31)1}$ and $W_{(31)2}$ can be computed from fig. 4.2 as

$$W_{(31)1}=20/40 \quad \text{and} \quad W_{(31)2}=20/40.$$

The computational forms for (5.48) appear in rows 31–33 of table 5.1.

5.5. *Other useful equations*

ORANI contains many miscellaneous equations which are included simply to facilitate applications. Some of these define summary variables, e.g. the consumer price index, GDP and the balance of trade. Other equations allow for institutional factors, e.g. wage indexation. For our illustrative model, we will append six examples:

$$M=\sum_{i=1}^{2} P^{*}_{(i2)}(X_{(i2)1}+X_{(i2)2}+X_{(i2)3}), \tag{5.49}$$

$$E=\sum_{i=1}^{2} P^{*}_{(i1)}X_{(i1)4}, \tag{5.50}$$

$$B=E-M, \tag{5.51}$$

$$CPI=\prod_{i=1}^{2}\prod_{s=1}^{2} P_{(is)}^{S_{(is)3}}, \tag{5.52}$$

$$P_{(31)} = (CPI)^h F_{(31)}, \tag{5.53}$$

and

$$C_R = C/CPI. \tag{5.54}$$

Equations (5.49)–(5.51) define the foreign currency values of imports (M), exports (E) and the balance of trade (B). Eq. (5.52) defines the consumer price index. The $S_{(is)3}$'s are the weights. They are defined as in (5.10), i.e. $S_{(is)3}$ is the share of the total household budget devoted to commodity (is). Eq. (5.53) allows for wage indexation. For example, if the parameter h is set at unity and the wage-shift variable $F_{(31)}$ is held constant, then wages will move with the CPI, i.e. we will be simulating a situation of 100 percent wage indexation. Exogenous shifts in real wages can be introduced via changes in $F_{(31)}$ and partial wage indexation can be handled by setting h at less than one. The final equation, (5.54), defines real household expenditure, C_R.

In linear percentage change form (5.49) is written as

$$m = \sum_{i=1}^{2} N_{(i2)}(p^*_{(i2)} + x_{(i2)1}W_{(i2)1} + x_{(i2)2}W_{(i2)2} + x_{(i2)3}W_{(i2)3}), \tag{5.55}$$

where $N_{(i2)}$ is the share of commodity ($i2$) in total imports. The data in fig. 4.2 imply that

$$N_{(12)} = 9/(9+12) \quad \text{and} \quad N_{(22)} = 12/(9+12).$$

The $W_{(i2)j}$'s are the shares of imports of good ($i2$) going to industries and households. From the data in fig. 4.2 we have

$$W_{(12)1} = 1/(1+8+1); \qquad W_{(12)2} = 8/10, \quad \text{etc.}$$

The linear percentage change form for (5.50) is

$$e = \sum_{i=1}^{2} N_{(i1)}(p^*_{(i1)} + x_{(i1)4}), \tag{5.56}$$

where $N_{(i1)}$ is the share of commodity ($i1$) in total exports. From fig. 4.2 we have[11]

$$N_{(11)} = 19/20 \quad \text{and} \quad N_{(21)} = 1/20.$$

In the case of the balance of trade equation, (5.51), a strict linear percentage change form is inappropriate. The problem is that B may move through zero and so the percentage change in B may become undefined. In our

[11] We assume that there are no export subsidies in the base period. Therefore, the export column in fig. 4.2 reflects foreign currency values.

computations we use the variable ΔB, the *change* (not the percentage change) in the balance of trade. Thus, we rewrite (5.51) as

$$\Delta B = (Ee - Mm)\frac{1}{100}, \tag{5.57}$$

where E and M are the base-period values for exports and imports. One minor disadvantage of (5.57) is that it requires us to keep track of the units of ΔB. In table 5.1, row 36, we have used base-period local currency values for E and M, i.e. $E=20$ and $M=21$ (see fig. 4.2). Thus, ΔB is the change in the balance of trade in terms of \$A of the base period.

The linear percentage change forms for the final three equations are:

$$cpi = \sum_{i=1}^{2} \sum_{s=1}^{2} S_{(is)3} p_{(is)}, \tag{5.58}$$

$$p_{(31)} = h(cpi) + f_{(31)}, \tag{5.59}$$

and

$$c_R = c - cpi. \tag{5.60}$$

The computational versions of these three equations, together with those for the aggregate trade equations (5.55)–(5.57), are in rows 34–39 of table 5.1. It will seem from row 38 that we have set the wage-indexing parameter h at 1. Thus, $f_{(31)}$ becomes the percentage change in real wages.

6. The choice of endogenous and exogenous variables

We recall from the beginning of section 5 that a Johansen model can be represented by

$$Az = 0, \tag{6.1}$$

where A is an $m \times n$ matrix of coefficients and z is an $n \times 1$ vector of variables. In section 5 we derived the A matrix for the illustrative model MO and set out the result in table 5.1. It can be seen from table 5.1 that the A matrix for MO has 39 rows and 52 columns, i.e. $m=39$ and $n=52$. Thus, to solve this model

$$n - m = 13$$

variables must be declared exogenous.

Once the choice of exogenous variables has been made, (6.1) is rewritten as

$$A_1 y + A_2 x = 0, \tag{6.2}$$

where A_1 is the 39×39 matrix formed by the 39 columns of A corresponding to the endogenous variables and A_2 is the 39×13 matrix formed by the 13 columns of A corresponding to the exogenous variables. y and x are sub-vectors of z. They are, respectively, the 39×1 and 13×1 vectors of endogenous and exogenous variables.

Provided that A_1 is invertible,[12] we can proceed from (6.2) to the solution

$$y = -A_1^{-1} A_2 x, \tag{6.3}$$

Eq. (6.3) expresses the percentage change in each endogenous variable as a linear function of the percentage changes in the 13 exogenous variables. We note that $[-A_1^{-1}A_2]_{ij}$ is the elasticity of the ith endogenous variable with respect to the jth exogenous variable.[13] For example, $[-A_1^{-1}A_2]_{ij}$ could be the percentage change in employment in industry 2 arising from a 1 percent increase in the foreign currency price of imported commodity 1. If this elasticity had the value 1.2, say, this would be interpreted as meaning that a 1 percent increase in the foreign currency price of imported good 1 would cause employment in industry 2 to be 1.2 percent higher than it otherwise would have been.

The 13 exogenous variables can be chosen in many different ways. In table 6.1 we have given one possibility. Under this choice, the A_2 matrix is formed from table 5.1 by selecting columns 12, 13, 15–17, 39, 40, 42, 43, 45, 46, 51 and 52, while the A_1 matrix is made up of columns 1–11, 14, 18–38, 41, 44 and 47–50. In table 7.1 we have presented selected rows and columns of the solution matrix $-A_1^{-1}A_2$. The printed rows relate to the more important endogenous variables and the columns to exogenous variables of interest. However, before we discuss the solution matrix, we will work through table 6.1. It will be useful to consider some alternative selections of exogenous variables. Much of the flexibility of the ORANI model in policy applications arises from the user's ability to swap variables between the exogenous and endogenous categories.

The first group of exogenous variables given in table 6.1 are the foreign currency prices of imports. MO (in common with ORANI) contains no equations describing foreign supply conditions and therefore it is difficult to imagine a plausible experiment in which the $p^*_{(i2)}$ would be endogenous.

[12]We offer no formal theory on the conditions under which A_1 will be invertible. Experience suggests, however, that A_1 will be invertible for all sensible classifications of variables between the exogenous and endogenous categories. We return to this issue at the end of this section.

[13]The variable ΔB leads to an exception. As explained in the previous section, this appears in change (rather than percentage change) form in the model. The elements of the ΔB row or column in the matrix $-A_1^{-1}A_2$ are, therefore, not true elasticities.

Table 6.1
A possible list of exogenous variables for MO.

Variable	Subscript Range	Number	Description	Column no. in Table 5.1
$p^*_{(i2)}$	i=1,2	2	Foreign currency import prices	12, 13
t_i	i=1,2	2	One plus the ad valorem tariffs	42, 43
k_j	j=1,2	2	Current availability of capital stocks	45, 46
$f_{(31)}$		1	Wage shift variable	51
v_1 } $x_{(21)4}$		2	One plus the ad valorem export subsidy for the major export commodity and the export volume for the minor export commodity	40, 15
c_R		1	Real aggregate household expenditure	52
$f_{(i1)4}$	i=1,2	2	Shifts in export demands	16, 17
ϕ		1	The exchange rate, \$A per \$Foreign	39
	Total =	13		

By placing the $p^*_{(i2)}$, $i=1,2$, in the exogenous category, we are adopting the small country assumption for imports, i.e. world prices are independent of Australian demands. We are also allowing for the computation of answers to questions of the form: What were (or will be) the effects of past (or projected) changes in foreign import supply prices?

The second group of exogenous variables are the tariffs or tariff equivalents of quantitative restrictions. The tariffs are among the exogenous variables for any computation directed at the traditional effective protection question: Which industries benefit and which lose from protection? Other questions might concern the effects of protection on employment and on the rate of inflation. Each of these questions could be analysed under exogenously given changes in the t_i. On the other hand, it would be possible to conduct MO, and ORANI, experiments in which some, or all, of the t_i are endogenous. For example, we might wish to compute the level of protection which would be required to maintain current employment levels in the footwear industry, say, in the face of exogenously given movements in foreign prices, domestic wages and the exchange rate. For such a computation, employment in the footwear industry would replace the footwear tariff in the exogenous list.

The third set of variables in table 6.1 are the supplies or employment levels of industry capital stocks. With the k_j's exogenous, our MO solutions are short-run. That is, we are determining the effects of tariff changes, say, over a period sufficiently short such that induced changes in capital availability may be ignored. In the absence of a set of equations describing the costs of capital creation, there is no obvious alternative to exogenizing the k_j's.[14] In ORANI, where capital creation is modelled, we are able to determine rates of return, i.e. rental values of units of capital divided by the costs of creating new units. Then rates of return become a natural replacement for the k_j's on the exogenous list. When the rates of return, rather than the k_j's, are exogenous, our solutions are long-run. For example, we might be investigating the long-run effects of a change in tariffs. Our assumption would be that in the long run, rates of return are independent of tariff changes. Thus, we would set rates of return exogenously. On the other hand, tariff changes will affect industry growth prospects. Thus, we would allow the k_j's to be endogenous. In this way our model would capture the idea that initial disturbances in rates of return induced by the tariff change would be eliminated by changes across industries in their rates of capital accumulation.

The fourth variable on our illustrative exogenous list is $f_{(31)}$. If the parameter h in eq. (5.59) is set at one (which it is in our present computations, see table 5.1, row 38, column 50), then $f_{(31)}$ is the percentage change in real wages. If $f_{(31)}$ is set at zero, then MO will determine the change in aggregate employment, ℓ, arising from changes in tariffs, etc. under conditions of constant real wages and abundant supplies of labour. Alternatively, for a full employment simulation, ℓ would replace $f_{(31)}$ on the exogenous list, and MO would generate the change in real wages which would be required for the achievement of full employment under the influence of proposed policy changes. In ORANI, where there are nine occupational groups recognized, we can allow wages to adjust to cause full employment in some occupations, while allowing wages to be determined exogenously in others.

Our fifth group of typical exogenous variables is a selection of export subsidies and export levels. The export subsidy, v_1, for the major export commodity (see fig. 4.2) is set exogenously. On the other hand, for good 2, of which very little is exported, the export volume is exogenous while the export subsidy is endogenous. In ORANI, the user specifies a set G containing the labels of those commodities for which the model is to be allowed to explain exports. For all other commodities, i.e. $i \notin G$, exports are exogenous and the model produces the export subsidy (or tax) required to achieve the given export level (see section 23 and subsection 29.6). In most ORANI

[14]Nevertheless, in one short-run ORANI application the k_j's were swapped with the rentals, the $p_{(32)j}$'s in the notation of MO. The aim was to set an environment of fixed markup pricing. See Dixon, Parmenter and Powell (1978, appendix) or Dixon, Powell and Parmenter (1979, section 3.5). A brief summary of this material is in subsection 50.2.

computations we have included in G those commodities for which exports are more than 20 percent of total output (see table 29.5). For these commodities, it is reasonable to assume that shifts in world prices, $P^*_{(i1)}$, strongly influence domestic prices $P_{(i1)}$. Notice that, in eq. (5.42), if v_i is exogenous, then $p_{(i1)}$ will move with $p^*_{(i1)}$. By contrast, if v_i is endogenous, then $p_{(i1)}$ will move independently of $p^*_{(i1)}$. Movements in $p^*_{(i1)}$ will be absorbed by offsetting movements in v_i. Note finally that the endogenous v_i can simply be deleted from MO by deleting eq. (5.42) for the relevant i.

The next variable in our exogenous list is c_R, the real aggregate level of household expenditure. By placing c_R on the exogenous list, we are setting an economic environment in which real aggregate demand is controllable independently of other variables appearing in table 6.1. The underlying assumption is that policy makers have available macro instruments, not explained in MO, by which they can influence c_R. Alternatively, model users might set ΔB exogenously in place of c_R. In this case, MO would indicate the change in real domestic absorption which would need to accompany a tariff cut, say, if we are to maintain a target level for the balance of trade.

The seventh group of variables in table 6.1 is the shift in foreign demand curves for local products, $f_{(i1)4}$, $i=1,2$. As was the case with the $p^*_{(i2)}$'s it is difficult to imagine a sensible experiment in which the $f_{(i1)4}$'s are endogenous. MO and ORANI have no equations relating the position of foreign demand curves to variables in the local economy. The role of the $f_{(i1)4}$'s is to allow model users to simulate the effects on the local economy of exogenously specified movements in export demand.

The last variable in table 6.1 is the exchange rate, ϕ. It acts as the numeraire, i.e. it determines the absolute price level. With wages fully indexed and the exogenous variables as in table 6.1, a 1 percent increase in the exchange rate ($\phi=1$) produces zero effect on all real endogenous variables and a 1 percent increase in all domestic price and other nominal variables (see the column marked 39 in table 7.1). Natural alternatives to ϕ as the numeraire include $p_{(31)}$, the wage rate,[15] and cpi, the consumer price index.

We conclude with one final comment on the partitioning of variables into the exogenous and endogenous categories. While our discussion of table 6.1 indicates a wide variety of legitimate possibilities, it is not true that MO (and ORANI) can be closed by the exogenous setting of *any* $n-m$ variables. For example, at least one monetary variable should be included in the exogenous list. If all domestic currency prices, the exchange rate, all wages and all monetary aggregates are treated as endogenous, then our computa-

[15]This was the choice of Johansen (1960) and Taylor and Black (1974).

tions will fail since there is nothing to determine the absolute price level. Similarly, some care is necessary to avoid inconsistencies. For example, if an attempt were made to set all three variables, c_R, c and cpi exogenously, then eq. (5.60) would be violated. Although we can offer no formal theory to guide model users in their choice of exogenous variables, as a working rule, if a price appears on the exogenous list, then a corresponding quantity should be on the endogenous list and vice versa. If wages are exogenous, then employment will be endogenous; if subsidies are endogenous, then exports will be exogenous, etc.

7. Some results from the MO model

In table 7.1 we have printed selected components from the 39 × 13 matrix, $-A_1^{-1}A_2$. The table shows the elasticities[16] of 13 out of the 39 endogenous

Table 7.1
Selected rows and columns from an MO solution matrix.

Variable number		39	43	51	52	Macro package
	Exogenous variables / Endogenous variables	ϕ exchange rate	t_2 tariff on good 2	$f_{(31)}$ real wage rate	c_R real aggregate absorption	2.76% increase in aggregate absorption plus 2.38% cut in real wages
18	z_1 output of industry 1	.0	-.44	-1.35	.15	3.62
19	z_2 output of industry 2	.0	.16	- .57	.85	3.71
20	$y_{(11)1}$ } industry 1's commodity outputs	.0	-.47	-1.40	.13	3.69
21	$y_{(21)1}$	.0	-.39	-1.19	.19	3.35
22	$y_{(11)2}$ } industry 2's commodity outputs	.0	.10	- .73	.80	3.94
23	$y_{(21)2}$	.0	.18	- .52	.86	3.61
28	$x_{(31)1}$ } employment by industry	.0	-.66	-2.01	.22	5.39
34	$x_{(31)2}$	.0	.21	- .71	1.06	4.62
44	ℓ aggregate employment	.0	-.23	-1.36	.64	5.01
47	m aggregate imports	.0	-.15	.33	.96	1.86
48	e aggregate exports	.0	-.36	-1.11	-.26	1.92
49	ΔB balance of trade[(a)]	.0	-.04	- .29	-.25	0.00
50	cpi consumer price index	1.00	.52	1.07	.25	-1.86

(a) Entries in this row have the units $A of the base period. All other entries in the table are pure elasticities.

[16]Again we note the exception involving the balance of trade. See footnote 13 above.

variables with respect to 4 of the 13 exogenous variables. With MO it would be possible to print the entire elasticities matrix. This would not be possible with ORANI where both the numbers of equations and exogenous variables are many thousands. The ORANI programmes are written so that users not only choose which components of $-A_1^{-1}A_2$ to print, but must also choose which components to compute (see section 32). The selection of rows and columns for computation and examination will, of course, depend on the application.

For our illustrative application with MO we will look at the implications of three broad approaches to macroeconomic policy: (i) increased protection, (ii) reductions in real wages and (iii) real demand expansion. Consequently, in table 7.1 we have displayed the elasticities of selected endogenous variables with respect to (a) the rate of protection on the major import commodity, (b) the real wage rate and (c) the real level of household expenditure. Table 7.1 also gives some elasticities with respect to the exchange rate. These were included merely to confirm the role of ϕ as the numeraire under conditions of fixed real wages and the exogenous-variable list as set out in table 6.1.

7.1. *Increased protection*

There are several ORANI studies of the effects of protection, including one presented in this book (see Chapter 7). An overview of these studies is given in subsection 50.1. Much of what ORANI implies about protection is illustrated by the results in the t_2-column of table 7.1.

The t_2-column refers to the effects of a 1 percent increase in one plus the *ad valorem* tariff on commodity 2. From fig. 4.2 we see that the value of imports of good 2 on the domestic market is \$A17, whereas their foreign currency cost is \$A12, i.e.

$$P_{(22)}M_2 = 5 + 2 + 10 = 17,$$

and

$$P^*_{(22)}\Phi M_2 = 12,$$

where M_2 is the volume of imports of good 2 and the remaining notation is as in section 5. Hence, it follows from eq. (5.36) that

$$T_2 = 17/12 = 1.42,$$

i.e. the *ad valorem* rate of protection on good 2 is 42 percent. Thus, a 1 percent increase in T_2 is a $1.42/0.42 = 3.4$ percent increase in the *ad valorem* rate of protection. The entries in the t_2-column are, therefore, to be read as follows: if the *ad valorem* tariff on commodity 2 were increased by 3.4 percent in an environment where tariff changes were not allowed to affect real aggregate demand, the real wage rate or any of the other variables in table 6.1, then in the short run we could expect output in industry 1 to be 0.44 percent less than it would otherwise have been, output in industry 2 to be 0.16 percent more than it otherwise would have been, etc. By the short run we mean a period which is sufficiently short such that we can ignore changes in capital availabilities that may be induced by the tariff change. On the other hand, enough time must be allowed for businessmen and consumers to adjust their input and output decisions to the new relative prices. In most ORANI applications papers we have assumed that such a time is about one to two years. On applying this rule to our MO results, we would say that a sustained 3.4 percent tariff increase on commodity 2 would, eighteen months later, say, cause the rate of output in industry 1 to be 0.44 percent less than it otherwise would have been.

The main implications of the t_2-column accord well with those of numerous ORANI calculations. We see that MO implies that tariffs are an ineffective instrument for stimulating aggregate employment. Our 3.4 percent increase in the tariff on commodity 2 produces a 0.23 percent reduction in total labour demand. The increase in employment in industry 2, whose production is heavily concentrated on commodity 2, is more than offset by the reduction in employment in industry 1. Protection of the import-competing industry imposes cost increases on the rest of the economy. Notice that the tariff increase adds 0.52 percent to the consumer price index. Under full wage indexation this adds 0.52 percent to the wage bill per unit employment in all industries. Industry 1, which specializes in the production of the export commodity and thus faces a highly elastic demand curve, is poorly placed to pass on cost increases. The cost squeeze effect on export production is reflected by the 0.36 percent reduction in the foreign currency value of exports. It is interesting that this reduction in exports is sufficiently large that the simulated net effect of the tariff increase on the balance of trade is a movement towards deficit.

Two results in the tariff column which may need further explanation are those for $y_{(21)1}$ and $y_{(11)2}$. Despite the increased protection for commodity 2, industry 1 cuts its production of commodity 2. Equally curious, at first sight, is industry 2's increase in production of commodity 1. The reason for these results can be understood if we think of industry production decisions in two stages. At the first stage, imagine that industry j produces commodities 1 and 2 in fixed proportions. Then j's reaction to the increased tariff on commodity 2 will depend on what happens to the industry's wage and material costs compared with what happens to the price of its output. The movement in the price of its output is a weighted average of the movements in the prices of commodities 1 and 2, where the weights reflect the shares of these commodities in the industry's total revenue. Under a tariff increase on commodity 2, there is a favourable movement in the product price for industry 2 (which specializes in the production of good 2) relative to industry 2's costs. On the other hand, there is an unfavourable movement in the product price to cost ratio in industry 1 (which specializes in the production of good 1). Thus, on the assumption that the composition of each industry's output is fixed, it is now clear that the increase in T_2 would cause industry 1 to contract its output level (and its output of both commodities) while in industry 2 the output level would expand. This explains why z_1 is negative and z_2 is positive in the t_2-column of table 7.1.

The second stage of the production decision concerns the product mix. Because the price of good 2 increases relative to that of good 1, both industries transform the composition of their output in favour of product 2. However, in the present computations the transformation effects are small relative to the level-of-activity effects. The effect, on industry 1's production of good 2, of the industry's reduction in its overall level of activity ($z_1 = -0.44$) easily outweighs the transformation effect in favour of good 2. Similarly, the effect of industry 2's expansion in overall output on its production of good 1 outweighs the transformation effect against good 1.

7.2. *Reductions in real wages*

The $f_{(31)}$-column of table 7.1 shows the effects on the selected endogenous variables of a 1 percent increase in the real wage rate. As with the tariff results, the MO results for a real wage increase are an accurate guide to the corresponding results from ORANI. In several papers ORANI results have been reported which identify increases in the costs of employing labour as the major factor in causing employment and balance of trade problems. (See, for example, Dixon, Parmenter and Sutton, 1978a.) Looking at the $f_{(31)}$-column of table 7.1, we see that according to MO a 1 percent increase in real wages reduces aggregate labour demand by 1.36 percent. The reduction is especially severe in industry 1 where employment falls by 2.01 percent. This is explained by industry 1's specialization in the production of the export good, good 1. Because the price of good 1 is largely independent of domestic cost conditions, cost increases have a particularly adverse effect on the output of good 1.

The adverse effect on the production of good 1 is reflected in the movement of the balance of trade. The 1 percent real wage increase produces a deterioration in the balance of trade which is equivalent to a loss of 1.45 percent of export revenue (0.29/20 = 0.0145). Most of this is explained by the reduction in exports. Nevertheless, there is also a significant increase in imports. Domestic cost increases reduce the competitiveness of the locally produced good 2, causing substitution towards the imported product. The increase in imports is limited, however, by the reduction in industry activity levels.

The reverse side of this picture is the effect of a *reduction* in the costs of employing labour. By multiplying the $f_{(31)}$-column by minus one, we obtain the effects of a 1 percent wage cut. Thus, according to MO, reductions in real wages cause increases in demands for labour (especially in export oriented industries), an improvement in the balance of trade and a reduction in inflationary pressure. The corresponding results from ORANI are similar and, therefore, lend support to the argument that reductions in real wages are the key to macroeconomic recovery (see subsection 50.2).

7.3. *Real demand expansion*

The c_R-column of table 7.1 shows the effects of a 1 percent expansion in real household expenditure. In MO, household demand is the only form of domestic absorption. Thus, the results here should be compared with ORANI results for a general increase in real aggregate demand rather than with those for an expansion in household expenditure alone.

In a recent publication,[17] ORANI results were used to illustrate some of the difficulties of attempting to implement a macro policy based primarily on demand stimulation. The ORANI computations implied that although demand stimulation would generate increased employment opportunities, it would also involve increased inflationary pressure, problems on the balance of trade and an uneven response across industries. In the c_R-column of table 7.1, we see that MO implies that a 1 percent increase in real aggregate demand will buy a 0.64 percent increase in employment at the cost of an increase of 0.25 percent in consumer prices and a deterioration in the balance of trade which is equivalent to the loss of 1.25 percent of export revenue (0.25/20 = 0.0125). The corresponding tradeoff in the ORANI computa-

[17] Dixon, Powell and Parmenter (1979, ch. 3). See also subsection 50.2.

tions was a 0.58 percent increase in employment for a 1.7 percent increase in consumer prices and deterioration on the balance of trade worth 3.9 percent of total exports. Thus, MO gives a much more favourable picture of the tradeoff than does ORANI. Because MO omits nontraded commodities, it exaggerates the extent to which the domestic price level is held in check by world prices. MO also exaggerates the ratio of trade to GDP and therefore underestimates the percentage impact on the trade accounts of the diversions in exports and the increases in imports required to service expansions in aggregate demand. Nevertheless, the MO results illustrate the proposition that "general demand stimulation cannot, by itself, provide a feasible approach for a return to full employment from a situation of (say) 5 percent unemployment" [Dixon, Powell and Parmenter, 1979, section 3.2(b)].

What aspects of MO (and ORANI) are responsible for these rather pessimistic results? Our theory implies that producers will respond to demand increases with an increase in output and employment only if the demand increase allows an improvement in their price/cost situation. With full wage indexation, prices and costs tend to move together. There is, however, some limited opportunity for improvements in price/cost ratios. Recall that domestic products are modelled as imperfect substitutes for foreign ones. Therefore, increases in demand allow the prices of domestic goods to rise relative to those of foreign substitutes. Thus, because of the import component in both the consumer price index and in material input costs, the appropriate index of wages and materials costs shows a smaller increase than the index of prices of domestically produced commodities. This is the principal explanation of why MO (and ORANI) produce a Keynesian employment response to an increase in aggregate demand under conditions of fixed real wages. It is also an important part of the explanation of the trade and industry results.

Price increases for domestic goods shift both foreign (export) and domestic demand away from local producers. Consequently, a major part of the increase in domestic absorption is provided by a deterioration in the balance of trade. Notice that the c_R-column shows both an increase in imports and a reduction in exports. The resulting movement towards deficit on the balance of trade accounts for about 40 percent of the increase in absorption ($0.25/0.62=0.40$). (The corresponding figure in ORANI computations is about 58 percent.)

On examining the industry results, we again see the effects of a price/cost squeeze in the export sector. Industry 1 benefits from the demand increase to a much smaller extent than does industry 2. In ORANI computations many of the exporting industries are, in fact, shown with negative output and employment responses to general demand stimulation. (See, for example, Dixon, Powell and Parmenter, 1979, table 3.2.) Thus, ORANI computations imply that general demand stimulation has uneven effects across the economy, benefiting those industries where cost increases are easily passed into higher prices while harming some export industries and industries facing intense import competition.

7.4. *A macro package*

Readers will have noticed from the last two subsections that wage cuts and demand stimulation give opposite industry effects. Wage cuts are particularly beneficial for export industries while demand stimulation is particularly beneficial for industries where international trade plays only a minor role. This suggests that a balanced stimulation of the economy might be obtained by a suitable combination of wage reduction and demand stimulation.

One way to investigate such a possibility would be to change the selection of exogenous variables from that shown in table 6.1. We could, for example, ask what would be the implications across industries of a reduction in real wages and an increase in aggregate demand which together were sufficient to cause a 5 percent increase in aggregate employment without a deterioration on the balance of trade. Our two new exogenous variables would be ℓ set at $+5$ and ΔB set at zero. Our new endogenous variables would be $f_{(31)}$ and c_R, i.e. we would be determining the values of $f_{(31)}$ and c_R to be consistent with our exogenously given employment and balance of trade targets.

Rather than repartitioning the A matrix and recomputing $-A_1^{-1}A_2$, we can adopt some short cuts.[18] We note from table 7.1 that if $f_{(31)}$ is α percent and c_R is β percent, then ℓ and ΔB will be given by

$$\ell = -1.36\alpha + 0.64\beta,$$

and

$$\Delta B = -0.29\alpha - 0.25\beta.$$

Hence, if $\ell = 5$ and $\Delta B = 0$, then α and β must be -2.38 and 2.76, respectively. That is to say, according to MO, a 5 percent increase in aggregate employment demand without balance of trade difficulties is achievable by a 2.38

[18]Similar short cuts are available in ORANI computations. See section 36, especially subsection 36.1.

percent reduction in real wages combined with a 2.76 percent increase in real aggregate demand. The corresponding results for ORANI are a 6.15 percent reduction in real wages combined with a 3.21 percent increase in real aggregate demand (see subsection 50.2).

In the final column of table 7.1 we have shown industry output and other results for our MO recovery package. The figures are derived by multiplying the $f_{(31)}$-column by -2.38 and the c_R-column by 2.76 and adding. The most interesting implication of the package (and one which is consistent with the ORANI computations) is that it generates a balanced stimulation of the economy. Similar output and employment expansions are achieved in all industries.

8. The large change problem and the computation of ORANI solutions

8.1. Nonlinear methods

Since the publication in 1960 of Johansen's *Multi-Sectoral Study of Economic Growth* there has been intensive research on procedures for solving general equilibrium models. This has led to the development of several algorithms which do not resort to the linearizations adopted by Johansen. If we were to apply these algorithms to our MO model, we could solve the 39 structural equations (5.7), (5.18), (5.23), (5.32), (5.34)–(5.36), (5.44), (5.46), (5.47) and (5.49)–(5.54) for the levels of the 39 endogenous variables. If we wanted to know the effects of a change in the exogenous variables we would compare the results from two solutions for our 39-equation nonlinear system, the two solutions computed with alternative values for the exogenous variables. Thus, we would avoid the disadvantage of the Johansen procedure, i.e. its inability to cope with large changes in the exogenous variables. Because in the Johansen computations the coefficients in the A matrix [see eq. (6.1)] are treated as parameters, the results are valid only for changes in the exogenous variables which are not sufficiently large to induce significant changes in the sales patterns of commodity outputs, the commodity compositions of industry outputs, the industrial compositions of factor employments, the input compositions of industry costs, etc. Thus, the question arises as to why we retain the Johansen method in our computations for ORANI. Before we answer, however, it will be useful to give a brief overview of the modern alternatives.

Two approaches to solving general equilibrium models can be distinguished in the recent literature. The first exploits the fact that for many economic models the solution can be deduced from the solution of a suitably chosen constrained maximization problem and its dual. In the second approach, various equation-solving methods are applied directly to the structural equations.[19]

We illustrate both approaches by considering the two-household, v-commodity, pure exchange model defined as follows.[20] The $(1 \times 3v)$-vector,

$$Z'_1 = (C'_1, C'_2, P'),$$

is a solution for the endogenous variables if and only if (i) C_i maximizes $U_i(C_i)$ subject to $P'C_i = P'X_i$, for $i = 1,2$, and (ii) $C_1 + C_2 = X_1 + X_2$. P is the $v \times 1$ vector of commodity prices, C_1 and C_2 are the $v \times 1$ consumption vectors for the two households, the U_i are their utility functions[21] and the X_i are the $v \times 1$ vectors giving their initial commodity endowments. These latter variables are set exogenously, i.e. the vector of exogenous variables can be written as

$$Z'_2 = (X'_1, X'_2).$$

Condition (i) requires that each household maximizes its utility subject to its budget constraint while condition (ii) requires that markets clear for commodities.

As an example of the first approach to computing equilibria, we could solve this model by considering constrained maximization problems of the form

$$\left.\begin{array}{l} \text{choose } C_1, C_2 \text{ to maximize} \\ w_1 U_1(C_1) + w_2 U_2(C_2), \\ \text{subject to} \\ C_1 + C_2 = X_1 + X_2, \end{array}\right\} \quad (8.1)$$

[19]The first approach has been applied within the IMPACT Project to solve the SNAPSHOT model. (See Dixon, Harrower and Powell, 1976; Dixon, 1976a; and Dixon, Harrower and Vincent, 1978). Earlier applications appear in Takayama and Judge (1964, 1971), Goreaux and Manne (1973), Dixon (1975, 1978c), Dixon and Butlin (1977) and Ginsburgh and Waelbroeck (1976). For a recent theoretical survey with illustrative applications, see Manne, Chao and Wilson (1978). The second approach has been adopted by, among others, Scarf (1973), Shoven and Whalley (1972, 1973, 1974), Whalley (1978), Adelman and Robinson (1978) and Dervis (1975, 1980).

[20]Note that the notation employed in this illustration is entirely separate from that employed elsewhere in the book.

[21]We assume that the utility functions are strictly concave. This is a convenient assumption and is no more restrictive from an empirical point of view than the utility maximizing model itself (see Dixon, 1975, pp. 96–105). Strict concavity is required to ensure the validity of the w-iteration method to be discussed in the next paragraph (see Dixon, 1975, p. 6).

where w_1 and w_2 are positive parameters normalized so that $w_1+w_2=1$. The first-order conditions for a solution of this problem can be written as

$$\nabla U_i(C_i)=\frac{1}{w_i}\Lambda, \qquad i=1,2, \tag{8.2}$$

and

$$C_1+C_2=X_1+X_2, \tag{8.3}$$

where Λ is the $v\times 1$ vector of Lagrangian multipliers associated with the market-clearing constraint. Now recall that necessary and sufficient conditions for satisfying part (i) of the requirements for a model solution are that there exist β_i such that the C_i, P and β_i jointly satisfy

$$\nabla U_i(C_i)=\beta_i P, \qquad i=1,2,$$

and

$$P'C_i=P'X_i, \qquad i=1,2.$$

Thus, it is apparent that if we are fortunate enough that

$$\Lambda'C_i=\Lambda'X_i \quad \text{for} \quad i=1,2, \tag{8.4}$$

then the solution for the programming problem (8.1) has revealed a solution for our economic model with the price vector P being given by the vector of Lagrangian multipliers Λ. This suggests that we can compute equilibria for our economic model by solving a series of programming problems of the type (8.1), varying the w_i's until the fortunate set of circumstances (8.4) occurs. Intuitively, if in an initial calculation we have[22]

$$\Lambda'C_1>\Lambda'X_1 \tag{8.5}$$

and

$$\Lambda'C_2<\Lambda'X_2, \tag{8.6}$$

then we should decrease w_1 and increase w_2. The expected effect is to reduce the consumption of household 1 and to increase the consumption of household 2, thus moving us closer to satisfying condition (8.4).

As an example of the second approach to the computation of economic equilibria, we could solve our pure exchange model via the excess demand functions. First we would derive the demand functions

$$C_i=C_i(P,\ P'X_i), \qquad i=1,2, \tag{8.7}$$

[22]The market-clearing constraint ensures that if $\Lambda'C_1>\Lambda'X_1$, then $\Lambda'C_2<\Lambda'X_2$.

implied by part (i) of the definition of an equilibrium. Then we would substitute (8.7) into the market clearing equations to obtain the v-equation system

$$\sum_{i=1}^{2} C_i(P, P'X_i) - \sum_{i=1}^{2} X_i = 0, \tag{8.8}$$

where the LHS of (8.8) is the vector of excess demands. We know that these excess demand functions are homogeneous of degree zero in prices and thus one of the prices (say the last) can be set at 1.[23] In addition we can apply Walras' Law to eliminate one of the equations (say the last). Thus, our problem reduces to solving the (v-1) equations

$$E(P^*) = 0, \tag{8.9}$$

where E and P^* are, respectively, the vectors of excess demands and prices for the first (v-1) commodities. At this stage a wide variety of solution techniques can be applied. Among these are the fixed-point procedures pioneered by Scarf (1973). Of greater practical relevance, however, are the simple tâtonnement procedures, e.g. the Gauss–Seidel method. The Newton method and various other approaches which use information on the derivatives of the excess demand functions have also been found effective.

8.2. *The advantages of Johansen's linearization*

In our work on the ORANI model we have found that the main advantage of the Johansen approach is its flexibility. By using the rectangular linear system (6.1) we gain flexibility in terms of (a) model size, (b) model modification and (c) model application.

8.2.1. *Model size*

The term "model size" should be interpreted broadly. A model can be big either because it has a large number of equations or because its equations are highly nonlinear. If we work with the system of linear equations (6.1), our model can remain small in terms of its computing requirements even though the number of equations may be several millions. Of course, in applying (6.3) to solve the model, some condensation may be required. But this is

[23]Alternatively, we can use a normalization rule of the form $\sum_i P_i = 1$.

easily achieved by substituting out equations and variables. For example, if initially we have a three-equation, four-variable system of the form

$$Az = 0,$$

then by using rules from high school algebra we can obtain a two-equation, three variable system of the form

$$A^* z^* = 0,$$

where A^* is a 2×3 matrix and z^* is a 3×1 subvector of z.[24]

When we move to systems of nonlinear equations, size can become a problem. This is despite the rapid advances of the last decade in nonlinear methods for solving general equilibrium models. Under the first approach discussed in subsection 8.1, care must be taken to limit the size of the constrained maximization problem to be solved at each step. Otherwise, even when convergence is very rapid (i.e. only a few solutions of the constrained maximization problem are required) computing costs can become prohibitive. Limiting the size of the constrained maximization problem without reducing the model's economic detail becomes very difficult, especially when it is recognized that nonlinearities in the initial specification of the model must often be handled by piecewise linear approximations involving large numbers of additional variables and constraints. Our own experience at the IMPACT project with the SNAPSHOT model (see footnote 19 above) has been that computing difficulties have constrained our specification of the model. For example, although estimates of the elasticities of substitution between imported and domestic goods of the same input–output classification are available and are used in ORANI (see subsection 29.1), in SNAPSHOT we have been forced to reduce computing costs by treating the shares of imports in domestic markets as exogenous

Recent results using the second approach to computing economic equilibria have looked more promising. Adelman and Robinson (1978, p. 11) comment that "we have not been constrained in our specification of the model by considerations of solution technique". They applied Gauss–Seidel methods to the excess demand functions for commodities and gradient methods to the excess demand functions for factors. Similarly, Whalley (1978) has been able to solve a large model of US, EEC and Japanese trade by applying both modified Scarf and Newton procedures to the excess demand functions.

It is important to emphasize that the successes of Adelman and Robinson,

[24]The condensation process for ORANI is described in section 32.

and Whalley were not achieved via the blind application of standard equation-solving techniques. In both cases they relied on their intimate knowledge of the specific features of their models to improve computational efficiency. That is, their algorithms were tailor-made for their particular models. This reflects these researchers' disenchantment with the performance of general purpose methods (such as Scarf's approach) when applied to models of the size and complexity required to support policy analysis. In the case of the ORANI model, there can be little doubt that the general purpose algorithms which are currently available would be inadequate if applied to the nonlinear structural form. Whether or not a tailor-made algorithm could be devised is an open question. Our opinion is, however, that this would require either an impractically large input of time by a highly skilled team of programmers or a considerable simplification of the model's specification.

Because we have adopted the Johansen linearization, computing considerations have introduced no inflexibilities with regard to ORANI's size and specification. The degree of detail in the industry and commodity classifications and the degree of complexity in the myriad of substitution relationships is limited by data considerations long before computing becomes a constraint.

8.2.2. Model modification

Since its first applications ORANI has undergone continuous modification. While most of these changes have been of a minor nature involving revisions in the input–output data base and in the estimates of various substitution elasticities, there have been some changes (e.g. the inclusion of multiproduct industries in the agricultural sector) which have required a complete re-specification of large blocks of the structural equations.[25]

From a computing point of view, the implementation of revisions in the ORANI model involves no special problems. Most revisions are handled by making the appropriate changes in the input–output and elasticities files and simply rerunning the programmes to form the A matrix. Where new variables or equations are required, the A matrix is expanded by the addition of new columns and rows. None of these procedures involves the rewriting

[25]In the first version of ORANI, all industries were specified according to the usual input–output convention as single-product industries. See Dixon, Parmenter, Ryland and Sutton (1977).

of solution algorithms. The most that is called for is a change in the dimension statement of an inversion routine.

By contrast, in models relying on nonlinear solution routines, computing considerations can play a major role in inhibiting revisions. As we saw in the previous subsection, the success of nonlinear approaches to solving large-scale general equilibrium models depends on the skilful adaptation of general purpose algorithms so that they take advantage of model-specific features. Where a model is undergoing change, even in seemingly minor ways, the rethinking and rewriting of algorithms becomes an energy-sapping chore.

In the context of the ORANI model, the Industries Assistance Commission's aggregation/disaggregation facility provides an interesting example of the advantage of flexibility in the area of model modification (see Hagan, Wright and Smith, 1979). The work of the Commission often involves inquiries into industries at a much finer classification than is given in the ORANI data base.[26] What the Commission's aggregation/disaggregation programmes allow model users to do is to either combine or split[27] the rows and columns of the ORANI input–output accounts. Simultaneously, the programmes make the required adjustments in various dimension statements and reform the A matrix. Thus, when the Commission wishes to use the ORANI model with a revised industrial classification, the necessary model modifications are quite routine.

8.2.3. *Model application*

In section 6 we discussed the advantages for policy applications of being able to switch variables between the exogenous and endogenous categories. This flexibility is greatly reduced in models where nonlinear solution algorithms are adopted. In such models the replacement of one endogenous variable with another will, in general, constitute a major model revision and will require extensive rewriting of solution algorithms.

With the Johansen approach we can simply reallocate the columns of the A matrix between the A_1 and A_2 matrices [see eq. (6.2)] and recompute the matrix $-A_1^{-1}A_2$ [see eq. (6.3)]. However, even this much computing may be

[26]The ORANI data base is described in Chapter 4.

[27]In the case of aggregation, the model user need supply no additional information. For disaggregation, the model user can supply information at varying levels of detail on how the relevant rows and columns should be split. The disaggregation programmes provide convenient default options where users have incomplete information on the input or sales structure of the sub input–output industries.

unnecessary. For example, in subsection 7.4 we used a few hand calculations to move from a solution for MO where the percentage changes in real wages ($f_{(31)}$) and aggregate real absorption (c_R) were exogenous to one in which these variables were replaced on the exogenous list by the change in the balance of trade (ΔB) and the percentage change in the level of employment (ℓ). It can be shown that the principal step required in the swapping of r variables between the endogenous and exogenous lists reduces to the inversion of an $r \times r$ matrix. [See subsection 36.1 and particularly eq. (36.9).] Various other short cuts are available for changing ORANI solutions where there are only a limited number of changes in the A matrix. Thus, in practice, ORANI users store a few $-A_1^{-1}A_2$ matrices from standard runs. Then when new solutions are required, these can often be computed at trivial cost by modifying an earlier solution.

8.3. *The elimination of the Johansen linearization errors*[28]

Given the advantages of working with the linear system (6.1), it is understandable that we have retained the Johansen approach in our work with the ORANI model. In fact, as was mentioned in subsection 8.2.1, we doubt the practicality of applying nonlinear methods to ORANI's structural form. An alternative approach is to derive true ORANI solutions (i.e. solutions which are free from significant linearization errors) by applying a series of n Johansen-style computations with n updates of the A matrix. The procedure we have in mind can be described as follows.

We start by rewriting eq. (6.3) as

$$dY = -\hat{Y}A_1^{-1}(X, Y)A_2(X, Y)\hat{X}^{-1}\,dX \tag{8.10}$$

or, in simpler notation,

$$dY = B(X, Y)\,dX, \tag{8.11}$$

where

$$B(X, Y) = -\hat{Y}A_1^{-1}(X, Y)A_2(X, Y)\hat{X}^{-1}. \tag{8.12}$$

The A and B matrices are written as functions of X and Y to emphasize that their components can be expressed as functions of prices and quantities, i.e. as functions of X and Y. This follows from the fact that the A matrix is constructed from the input–output flows and each input–output flow is a

[28] This topic is dealt with in much greater detail in Chapter 5 and section 47.

product of a price and a quantity. It should be noted that (8.10) involves no approximations. It is an exact implication of the structural equations, i.e. if we denote the exact solution to the structural equations by[29]

$$Y = G(X), \tag{8.13}$$

we then have

$$G_X(X) = B(X, G(X)),$$

where $G_X(X)$ is the Jacobian of G.

Now recall that

$$\lim_{n\to\infty} \left\{ G_X(X_I) + G_X\left(X_I + \frac{1}{n}\Delta X\right) + G_X\left(X_I + \frac{2}{n}\Delta X\right) + \cdots + G_X\left(X_I + \frac{(n-1)}{n}\Delta X\right)\right\}\frac{1}{n}\Delta X = G(X_I + \Delta X) - G(X_I),$$

provided only that the second derivatives of G remain bounded as we move from X_I to $X_I + \Delta X$. Thus, if we have a means of computing the G_X matrix for all values of X, it is apparent that we can evaluate the change in Y caused by the movement of X from X_I to $X_I + \Delta X$ by computing the sum

$$\Delta Y_n = \sum_{t=0}^{n-1} G_X\left(X_I + \frac{t}{n}\Delta X\right)\frac{1}{n}\Delta X, \tag{8.14}$$

where n is chosen to be sufficiently large to ensure the desired degree of accuracy.

The application of these ideas to the problem of computing exact solutions for the ORANI model should be clear. Although we cannot solve the structural equation in the form (8.13), we do know how to evaluate G_X, at least for the initial situation, i.e. we know $B(X_I, Y_I)$. The obvious analogue to (8.14) is to calculate the change in Y caused by the movement in X from X_I to $X_I + \Delta X$ by computing

$$\Delta Y_n = \sum_{t=0}^{n-1} B\left(X_I + \frac{t}{n}\Delta X,\ Y_n^t\right)\frac{1}{n}\Delta X, \tag{8.15}$$

where

$$Y_n^0 = Y_I \tag{8.16}$$

[29] Given X, we assume that the structural equations imply a unique solution for Y.

and

$$Y_n^t = Y_n^{t-1} + B\left(X_I + \frac{(t-1)}{n}\Delta X,\ Y_n^{t-1}\right)\frac{1}{n}\Delta X, \qquad t=1,\ldots,n-1. \quad (8.17)$$

Equations (8.15)–(8.17) describe an n-step procedure. If we wish to compute the effects of changing the exogenous variables from X_I to $X_I + \Delta X$, then we divide the change into n parts. First, the effect of moving the exogenous variables from X_I to $X_I + (1/n)\Delta X$ is computed as

$$\Delta Y_n^0 = B(X_I,\ Y_I)\frac{1}{n}\Delta X.$$

Then the B matrix is re-evaluated at the point $(X_I + (1/n)\Delta X,\ Y_I + \Delta Y_n^0)$. In practice this re-evaluation involves

(i) updating the input–output flows to take account of the changes in prices and quantities implied by the change in the exogenous variables from X_I to $X_I + (1/n)\Delta X$;

(ii) recomputing the A matrix using the updated flows; and

(iii) recomputing the matrix $-A_1^{-1}A_2$.

Having re-evaluated $-A_1^{-1}A_2$, we compute the effect of moving the exogenous variables from $X_I + (1/n)\Delta X$ to $X_I + (2/n)\Delta X$ by

$$\Delta Y_n^1 = B\left(X_I + \frac{1}{n}\Delta X,\ Y_I + \Delta Y_n^0\right)\frac{1}{n}\Delta X.$$

The B matrix is again re-evaluated, this time at the point $(X_I + (2/n)\Delta X,\ Y_I + (\Delta Y)_n^0 + (\Delta Y)_n^1)$. Then this latest value for B is used to compute the effects of changing the exogenous variables from $X_I + (2/n)\Delta X$ to $X_I + (3/n)\Delta X$, etc.

The first question regarding this n-step procedure is one of pure mathematics. Can we be sure that

$$\lim_{n\to\infty} \Delta Y_n = G(X_I + \Delta X) - G(X_I),$$

where ΔY_n is defined by (8.15)–(8.17)? The answer is yes, provided only that the first derivatives of B with respect to Y and the second derivatives of G with respect to X are bounded over the relevant domains in the $(X,\ Y)$ *space.*[30] Because we fail to generate exact values for Y as we move from X_I to $X_I + \Delta X$, we fail to generate exact values for $G_X(X)$. However, we can still be sure that (8.15) will provide an accurate evaluation of the change in Y if n is sufficiently large.

[30] The relevant proposition is proved in section 35.

The second question is one of practical computing. Can the n-step procedure be applied to a model as large as ORANI? The ORANI input–output files identify about half a million flows. The updating of these flows and the recomputing of the A and B matrices generates considerable computer costs.[31] It is clear that unless n can be kept small, the procedure could be too expensive for routine use. Fortunately, our experience suggests that for most purposes n can be very small.[32] We expect that $n=2$ will normally be more than adequate. In fact, computations with MO and with ORANI provide a strong justification for the Johansen method (i.e. $n=1$).

8.4. The n-step update procedure applied to MO

As a preliminary step before attempting to implement our n-step update procedure in ORANI, we applied it in MO. Because the MO results are illustrative of those eventually obtained in ORANI, we have presented some of them in tables 8.1 and 8.2.

Table 8.1 contains the effects of a 25 percent increase in the tariff on good 2 under conditions of fixed real wages, fixed real aggregate demand and a fixed exchange rate – the exogenous variables are those listed in table 6.1. In the 1-iteration column, the computations were carried out by the usual Johansen method, i.e. we computed

$$y=-A_1^{-1}(X_I, Y_I)A_2(X_I, Y_I)x.$$

The initial A matrix is given in table 5.1 and the components of x were set at zero with the exception of t_2, which was set at 7.35. (Recall from subsection 7.1 that T_2 is one plus the *ad valorem* rate of protection and that the initial *ad valorem* rate is 42 percent. To increase the *ad valorem* rate by 25 percent, we increase T_2 by 7.35 percent, i.e. we increase T_2 from 1.42 to 1.52.) Thus, apart from rounding errors, the 1-iteration column of table 8.1 can be obtained by multiplying the t_2-column of table 7.1 by 7.35.

The n-iteration column of table 8.1 was computed as follows. First we noted that

$$\Delta T_2=1.52-1.42=0.10.$$

Then we broke the increase in T_2 into n equal steps where the rth step was

[31] Details are given in section 34.

[32] Computational experience with MO is reported in the next subsection. Experience with ORANI is reported in section 47.

Table 8.1

The implications for selected variables in MO of a 25 percent increase in the tariff on good 2.

Variable number	Variable name \ Number of iterations		1	2	4	8	16	32	64	Infinity[a]	2-iteration extrapolation[b]	Johansen error (per cent)[c]
18	z_1	output for industry 1	-3.2658	-3.1826	-3.1431	-3.1238	-3.1143	-3.1096	-3.1073	-3.1050	-3.0994	5.2
19	z_2	output for industry 2	1.2093	1.1817	1.1684	1.1618	1.1586	1.1570	1.1562	1.1554	1.1541	4.7
20	$y_{(11)1}$	industry 1's commodity outputs	-3.4110	-3.3232	-3.2815	-3.2612	-3.2512	-3.2462	-3.2437	-3.2412	-3.2354	5.2
21	$y_{(21)1}$		-2.8575	-2.7876	-2.7545	-2.7384	-2.7304	-2.7265	-2.7245	-2.7225	-2.7177	4.9
22	$y_{(11)2}$	industry 2's commodity outputs	0.7690	0.7466	0.7360	0.7307	0.7282	0.7269	0.7262	0.7255	0.7242	6.0
23	$y_{(21)2}$		1.3226	1.2934	1.2793	1.2724	1.2690	1.2673	1.2664	1.2655	1.2642	4.5
28	$x_{(31)1}$	employment by industry	-4.8987	-4.7549	-4.6868	-4.6536	-4.6373	-4.6291	-4.6251	-4.6211	-4.6111	6.0
34	$x_{(31)2}$		1.5117	1.4782	1.4620	1.4541	1.4502	1.4482	1.4473	1.4464	1.4447	4.5
44	ℓ	aggregate employment	-1.6935	-1.6384	-1.6124	-1.5997	-1.5935	-1.5904	-1.5889	-1.5874	-1.5833	6.7
47	m	aggregate imports	-1.1110	-1.0574	-1.0316	-1.0190	-1.0127	-1.0096	-1.0081	-1.0066	-1.0038	10.4
48	e	aggregate exports	-2.6773	-2.6231	-2.5972	-2.5845	-2.5782	-2.5751	-2.5735	-2.5719	-2.5689	4.1
49	ΔB	balance of trade	-0.3021	-0.3025	-0.3027	-0.3029	-0.3029	-0.3030	-0.3030	-0.3030	-0.3028	0.3
50	cpi	consumer price index	3.8077	3.7868	3.7767	3.7718	3.7694	3.7682	3.7676	3.7670	3.7659	1.1

(a) Computed by adding the change in the result as we go from 32 to 64 iterations to the result after 64 iterations.

(b) Computed by adding the change in the result as we go from 1 to 2 iterations to the result after 2 iterations.

(c) Computed by comparing the 1-iteration result with the result in the infinity column.

concerned with the effects of increasing T_2 from $1.42+0.10(r-1)/n$ to $1.42+0.10r/n$. The effects of the first increase in T_2, i.e. the increase from 1.42 to $1.42+0.10/n$, were computed using the initial $-A_1^{-1}A_2$ matrix, with the percentage change in T_2 being given by

$$t_2=\frac{0.10/n}{1.42}\,100$$

and percentage changes in all other exogenous variables being set at zero. Then the input–output flows were updated according to the formula

$$(\text{flow})^1=(\text{flow})^0\left(1+\frac{p(0)}{100}\right)\left(1+\frac{q(0)}{100}\right),$$

where $(\text{flow})^0$ is the initial value in the input–output tables and $(\text{flow})^1$ is the value appearing after the first update. $p(0)$ and $q(0)$ are the percentage changes in the relevant price and quantity variables generated in the first step, i.e. as a result of increasing T_2 from 1.42 to $1.42+0.10/n$. Both $p(0)$ and $q(0)$ may, of course, be either endogenous or exogenous. On completing the first update of the input–output flows, we recomputed the A and $-A_1^{-1}A_2$ matrices. To compute the effects of moving T_2 from $1.42+0.10/n$ to $1.42+0.20/n$ we used this new $-A_1^{-1}A_2$ matrix, with t_2 given by

$$t_2=\frac{0.10/n}{1.42+0.10/n}\,100.$$

The percentage changes in all other exogenous variables remained at zero. The input–output flows were again updated. The matrix $-A_1^{-1}A_2$ was recomputed and used in calculating the effects of moving T_2 from $1.42+0.20/n$ to $1.42+0.30/n$. The final results in the n-iteration column of table 8.1 are of the form

$$\frac{\text{result}}{100}=\left(1+\frac{v(0)}{100}\right)\left(1+\frac{v(1)}{100}\right)\cdots\left(1+\frac{v(n-1)}{100}\right)-1,$$

where $v(r-1)$ is the percentage change in the variable arising at the rth step.

A glance at table 8.1 reveals that the results in every row conform very closely to the rule

$$R(2m)-R(m)=2(R(4m)-R(2m)), \tag{8.18}$$

where $R(s)$ is the result from the s-iteration computation and m is a non-negative integer power of 2. For example, when we look at the results for variable 18, we see that

$$R(16)-R(8)=-3.1143+3.1238=0.0095$$

and

$$R(8)-R(4)=-3.1238+3.1431=0.0193,$$

i.e.

$$R(8)-R(4)\simeq 2[R(16)-R(8)].$$

This suggests an easy way to compute

$$R(\infty)=\lim_{q\to\infty}\{R(2^q)|q=\text{positive integer}\},$$

where $R(\infty)$ is the MO result without linearization error. We simply note that (8.18) implies that[33]

$$R(\infty)=R(2m)+(R(2m)-R(m)). \qquad (8.19)$$

In the "infinity" column we have applied (8.19) with $m=32$. For example, in row 18 we have

$$\begin{aligned} R(\infty) &= R(64)+(R(64)-R(32)) \\ &= -3.1073+0.0023 \\ &= -3.1050. \end{aligned}$$

There can be no doubt that these calculations provide highly accurate estimates of the true MO results. We will in fact accept them as being free from linearization error. This leaves us in a position to answer two important questions. How close were we to the true results after a 1-iteration calculation? How close could we get by applying (8.19) with $m=1$?

In the final column of table 8.1 we have expressed the absolute differences between the $R(\infty)$'s and the $R(1)$'s as percentages of the absolute values of the $R(\infty)$'s. The results are certainly encouraging for users of the Johansen method. The linearization errors generated in the particular experiment under consideration average about 5 percent, with the largest being 10 percent. Even the 10 percent error could hardly be of any practical concern. It would be a brave model user who would express a strong preference between -1.0066 percent and -1.1110 percent as alternative projections of the effect on aggregate imports of a 25 percent increase in a particular tariff. Neverthe-

[33]From (8.18) we have

$$\begin{aligned} R(2m)-R(m) &= 2(R(4m)-R(2m)) \\ R(4m)-R(2m) &= 2(R(8m)-R(4m)) \\ \vdots\qquad & \qquad\quad \vdots \\ \hline R(\infty)-R(m) &= 2(R(\infty)-R(2m)) \end{aligned}$$

On rearranging we obtain (8.19).

less, if there were a need to eliminate linearization errors, it appears that this could be achieved with only a single update of the input–output flows and a single recomputation of the A and $-A_1^{-1}A_2$ matrices. The column marked "2-iteration extrapolation" was generated by applying (8.19) with $m=1$. The results are almost indistinguishable from those in the infinity-column.

Cautious readers will be wondering whether there is any basis for expecting these very encouraging results to be applicable outside MO or for experiments apart from a 25 percent increase in the tariff on good 2. We have run many different experiments with MO including a complete removal of the tariff on good 2. In all the cases examined so far we have found that rule (8.18) is an excellent approximation and that linearization errors are almost completely eliminated by extrapolation from the results for the 1- and 2-iteration computations. For example, in table 8.2 we have redone the macro package (last column of table 7.1) using various numbers of iterations.[33] For these calculations the list of exogenous variables given in table 6.1 was modified by the addition of ℓ and ΔB and the deletion of $f_{(31)}$ and c_R. (The short-cut method described in subsection 7.4 was no longer convenient.) Labour demand was increased by 5 percent using one, two, four, etc. steps. The table implies that there were only small linearization errors associated with our initial computation of the macro package and that these become barely detectable when we apply (8.19) with $m=1$.

Our guess is that rule (8.18) [which is the basis for the extrapolation technique (8.19)] means that the solution equations (8.13) for MO can be closely approximated over the policy-relevant domain of X by the quadratic equations

$$G_i(X)=U_i+V_iX+\tfrac{1}{2}X'W_iX, \qquad i=1,\ldots,39,^{34} \tag{8.20}$$

where $G_i(X)$ is the value for the ith endogenous variable and U_i, V_i and W_i are, respectively, 1×1, 1×13 and 13×13 matrices of coefficients. This conjecture is based on the fact that if we applied (8.14) in the context of (8.20), then we would observe the relationship (8.18) as we varied the number of steps (n) in our computations, i.e. if (8.20) were precisely valid and we were making precise evaluations of the Jacobian $G_X(X)$ as we moved X from X_I to $X_I+\Delta X$, then (8.18) would hold exactly (see subsection 31.5). If, on the

[33] The small differences between the results in the macro package column of table 7.1 and those in the 1-iteration column of table 8.2 are caused by rounding. The computations in table 7.1 were made with the A matrix as in table 5.1, i.e. with each coefficient correct to two decimal places. By the time table 8.2 was generated, the process of computing A from the input–output flows had been computerized and a higher level of accuracy was achieved.

[34] Recall that MO has 52 variables and 39 equations.

Table 8.2

The implications for selected variables in MO of a 5 percent increase in labour demand without a deterioration in the balance of trade.

Variable number	Variable name \ Number of iterations		1	2	4	8	16	32	64	Infinity[a]	2-iteration extrapolation[b]	Johansen error (per cent)[c]
18	z_1	output for industry 1	3.5870	3.5765	3.5714	3.5689	3.5676	3.5670	3.5667	3.5664	3.5660	0.6
19	z_2	output for industry 2	3.6956	3.6805	3.6731	3.6694	3.6675	3.6666	3.6662	3.6658	3.6654	0.8
20	$y_{(11)1}$	industry 1's commodity outputs	3.6787	3.6679	3.6626	3.6600	3.6587	3.6581	3.6578	3.6575	3.6571	0.6
21	$y_{(21)1}$		3.3292	3.3192	3.3145	3.3122	3.3110	3.3105	3.3102	3.3099	3.3092	0.6
22	$y_{(11)2}$	industry 2's commodity outputs	3.9736	3.9581	3.9505	3.9466	3.9447	3.9438	3.9433	3.9428	3.9426	0.8
23	$y_{(21)2}$		3.6241	3.6090	3.6016	3.5980	3.5962	3.5952	3.5948	3.5944	3.5939	0.8
28	$x_{(31)1}$	employment by industry	5.3805	5.3888	5.3929	5.3950	5.3960	5.3965	5.3968	5.3971	5.3971	0.3
34	$x_{(31)2}$		4.6195	4.6112	4.6071	4.6050	4.6040	4.6035	4.6032	4.6029	4.6029	0.4
47	m	aggregate imports	1.8364	1.8229	1.8163	1.8131	1.8115	1.8107	1.8103	1.8100	1.8094	1.5
48	e	aggregate exports	1.9282	1.9142	1.9073	1.9038	1.9021	1.9012	1.9008	1.9004	1.9002	1.4
50	cpi	consumer price index	-1.8612	-1.8286	-1.8127	-1.8049	-1.8010	-1.7990	-1.7980	-1.7970	-1.7960	3.6
51	$f_{(31)}$	real wage rate	-2.3795	-2.3397	-2.3200	-2.3102	-2.3053	-2.3028	-2.3016	-2.3004	-2.2999	3.4
52	c_R	real aggregate absorption	2.7564	2.7385	2.7299	2.7256	2.7235	2.7224	2.7219	2.7214	2.7206	1.3

(a) Computed by adding the change in the result as we go from 32 to 64 iterations to the result after 64 iterations.
(b) Computed by adding the change in the result as we go from 1 to 2 iterations to the result after 2 iterations.
(c) Computed by comparing the 1-iteration result with the result in the infinity column.

other hand, (8.20) were merely a good approximation and/or we were only approximating the Jacobian as we moved X, then we would expect (8.18) to be only approximately valid. This has been the case with our MO computations. Thus, our computations are consistent with (although not definitive evidence for) the hypothesis that MO solution equations are very closely approximated by the quadratic form (8.20) in the relevant domain of X.

The equations which make up the structural form of ORANI involve non-linearities of the same general nature as those encountered in the structural equations of MO. Since there is strong evidence to suggest that the solution equations for MO are approximately quadratic, it is reasonable to suppose that the solution equations for ORANI are approximately quadratic. The application of the n-step procedure and the extrapolation rule (8.19) to ORANI support this supposition. The results reported in section 47 indicate that the elimination of linearization errors from ORANI computations can be achieved with very small numbers of recomputations of the A and $-A_1^{-1}A_2$ matrices.

9. Concluding remarks

In our work at the IMPACT Project, we have found it rewarding to build models of models.[35] These miniature models have had several purposes. First, they have provided our professional colleagues with easy access to our main models. For example, by presenting MO in this book, our objective has been to introduce readers in a comparatively painless way to the key ideas and techniques [see (i)–(vi) in section 3] underlying ORANI. Secondly, we have found miniature models to be effective teaching aids. MO has been successfully used in advanced undergraduate courses. Students find it reassuring to be able to look at a complete set of input–output flows on a single page (see fig. 4.2). They enjoy using pocket calculators to check the evaluation of the A matrix (see table 5.1) and they quickly come to understand the complementary roles of theory and data in model building.

Finally, our miniature models have played an important part in our research. They have enhanced our understanding of various results from our main models. For example, it was our examination of results from MO that led us to recognize why ORANI, even when set up in neoclassical mode, will, nevertheless, produce a Keynesian employment response to an increase in aggregate demand (see subsection 7.3). MO was also important in our

[35]Apart from MO, other miniature models built at IMPACT include Dixon (1978a, b).

development of the n-step procedure (described in section 8) for eliminating Johansen linearization errors. The very encouraging performance of the n-step procedure when applied to MO gave us the confidence to commit the required resources to programme it for the main model. As can be seen from section 47, our computational experience with MO was a valid guide to what we would find with ORANI.