

Eleventh Floor, Menzies Building
Monash University, Wellington Road
CLAYTON Vic 3800 AUSTRALIA

Telephone:
(03) 9905 2398, (03) 9905 5112

Fax:
(03) 9905 2426
e-mail:

Internet home page:

from overseas:
61 3 9905 2398 or
61 3 9905 5112

61 3 9905 2426
impact@buseco.monash.edu.au
<http://www.monash.edu.au/policy/>

Solution Software for CGE Modeling

by

Mark HORRIDGE and Ken PEARSON

Centre of Policy Studies, Monash University

General Paper No. G-214 March 2011

ISSN 1 031 9034

ISBN 978 1 921654 21 3

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Abstract

We describe the progress of computable general equilibrium (CGE) modeling software since the 1980s and contrast the main systems used today: GAMS, MPSGE, and GEMPACK. The development of these general-purpose modeling systems has underpinned rapid growth in the use of CGE models and allowed models to be shared and their results replicated. We show how a very simple model may be implemented and solved in all 3 systems. We note that they produce the same numerical results but have different strengths. We conclude by considering some challenges for the future.

JEL classification: C63, C68, D58.

Keywords: CGE, software, GAMS, GEMPACK, MPSGE

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1. Introduction

Since 1980 computable general equilibrium (CGE) modeling has evolved from a frontier technique practiced by a few specialists into a tool widely used by researchers in universities and other organizations. On the demand side, the growth has been driven by policy-makers' appetite for detailed analyses of issues which affect various sectors or interest groups in different ways. Increased supply of CGE analyses has been enabled by more powerful computers and better software. The latter, software, contribution is examined in this paper.

CGE pioneers were required to develop their own model theory; construct their own database; devise a solution strategy and hire programmers to implement that strategy in a model-specific program. Thus new entrants to the field faced large upfront costs.

General equilibrium models require the numerical solution of large systems of equations, which in turn requires non-trivial computing power. Since the 1960s, a variety of mainframe computers offered this capacity. However, operating systems and computer language (eg, Fortran) implementations differed between manufacturers. It was not easy to "port" a program from one computer to another. This hindered the spread of CGE modeling between sites, and made it hard for other modelers to replicate simulation results. Since CGE results rest upon a host of assumptions (not always documented completely or accurately), replicability—and the potential to recompute results with slightly different assumptions—underpins model credibility. Without these, the model remains an unconvincing "black box".

Standardization has been key to the spread of CGE modeling¹. This includes:

1. Consensus about many (but not all) of the model's mechanisms; eg, the widespread use of nested CES to model substitution.
2. Standard data sources, such as the GTAP world dataset which provides data for several different CGE models; and accepted file formats to distribute such data.
3. Widespread use of x86 PCs, usually running Microsoft Windows. Such machines are now powerful enough to run nearly all CGE models, and offer a standard computer environment: the program developed in Palo Alto will run and give the same results in Bonn.
4. The use of a few software packages (GEMPACK, GAMS and GAMS/MPSGE²) to code and run most CGE models.
5. Training in CGE methods and software at special courses, in graduate study programs, or on-the-job in some large organizations, leading to a pool of CGE operatives with transferable skills.
6. A modest job market for CGE modelers—so that project staff can be replaced. Earlier modeling projects often withered when staff moved on.

These advances have allowed models to be widely shared, and have lowered development costs for new models. Standard CGE modeling software has contributed directly, and indirectly has supported advances 5 and 6 above. We concentrate here on the most widely used software—GAMS and GEMPACK—used by 80% or more of CGE modelers.

2. Early days

By 1980 CGE was already established as a field, with models described by Shoven & Whalley (1972), Taylor & Black (1974), Hudson & Jorgenson (1974), Dixon et al (1977), and Adelman & Robinson (1978). These models were solved with special-purpose programs, usually written in Fortran. Modelers wrote the theory using algebraic notation and then had to hire a computer programmer who converted the theory of the model to Fortran programs. There were often errors and other problems since the modeler could not under-

¹ However, enough heterodoxy and dispute remain to seed further advances.

² MPSGE may be considered as a high-level language used to specify CGE models, which is in principle independent of solution software. It is however nearly always used as a subsystem within GAMS—an experimental version which instead runs as a subsystem within GEMPACK has never gained traction.

stand the Fortran and the programmer could not understand the model. Model changes were difficult and costly. And it was almost impossible for models to be transferred from the originator to other research groups for checking, validation or use. Indeed, often the originator was unable to reproduce results obtained as little as 6 months previously.

These were serious problems.

The solution recognized gradually in both USA and Australia was to develop **general-purpose software** which eliminated the need to write model-specific programs in Fortran. Such software could:

- handle a wide range of models (that is what we mean by "general-purpose"),
- accept input from modelers directly in a form which is intelligible to modelers,
- allow modelers to make changes quickly,
- allow modelers to concentrate on the economics (not the computing),
- allow modelers to reliably reproduce results obtained earlier,
- be used on a range of different computers.

The first general-purpose software used for CGE modeling was GAMS (General Algebraic Modeling System). GAMS was developed starting in the mid-1970s by Alex Meeraus and Jan Bisschop working at the World Bank. Its focus was the solution of large-scale, non-linear optimization problems.³ GAMS was adapted to CGE modeling at the World Bank in the mid-1980s. The first GAMS-based CGE model was by Condon et al (1987). Although GAMS was designed initially to handle non-CGE problems, notably optimization problems, we will only discuss the CGE capabilities of GAMS in this paper.

Coming out of Rutherford's PhD thesis (1987) was a second implementation of general-purpose software, MPSGE (Mathematical Programming System for General Equilibrium analysis). This operates as a subsystem within GAMS.

Meanwhile, in Australia, the ORANI model was being used widely for policy analysis. In the early 1980s ORANI was solved on the CSIRO mainframe computer. Modelers had to submit their jobs on cards and wait overnight for their simulation to run. As will be seen below (section 4.2), solving ORANI required the repeated solution of large systems of linear equations. Pearson suggested that sparse solver techniques would speed up the solution of the model and delivered software to implement these techniques. Then, realising that model implementation and changes were very difficult (because the model was coded in model-specific Fortran), he undertook to develop general-purpose software GEMPACK (General Equilibrium Model PACKage) aimed especially at solving ORANI and related models.

The first version of GEMPACK was used for teaching in 1984, and shortly after that was adopted by Australian CGE modelers. The first GEMPACK manuals were published in 1986 (see Codsi and Pearson, 1986). Early journal descriptions of GEMPACK are Pearson (1988) and Codsi and Pearson (1988). Early versions of GEMPACK only produced Johansen (1-step) solutions which are not accurate, especially for large changes. The first version of GEMPACK which was able to produce accurate solutions of the underlying nonlinear model was Release 4.2.02 (April 1991). Hertel et al (1992) compared the North American levels methods with the Australian GEMPACK-based methods for implementing and solving GE models and showed that the differences were those of style rather than substance since the alternative model representations (linear or levels) and solution methods produce the same results. Early standard GEMPACK-based models producing accurate solutions were documented in Hertel et al (1997) and Horridge, Parmenter and Pearson (1993).

By the mid-1990s, nearly all policy-important CGE models were implemented and solved using one of GAMS, MPSGE or GEMPACK. For this reason we only discuss these 3 software suites below.

Of course it is still possible to implement and solve CGE models using other software, including spreadsheets, Mathematica, Matlab and special-purpose code. But if you want your model to be understood by other policy modelers, it pays to use software familiar to them.

GAMS, MPSGE and GEMPACK make it easy to transfer models around the world and to reproduce model results. Most models implemented with one of these could, at least in principle, be implemented with either of the other two and yield the same results.

³ The early history of GAMS is described in Kallrath (2004, pp. 20-21). The first published technical descriptions of GAMS appeared in Bisschop and Meeraus (1982) and Meeraus (1983).

3. General-purpose software

As mentioned above, a key motivation in designing and adopting general-purpose CGE software was to allow economists to construct and run models without being (or hiring) computer programmers or algorithm experts. So the software should make the normal tasks of specifying models, setting up and solving simulations, looking at results and writing reports as straightforward as possible. If modelers can do these things without too much effort, they are able to concentrate on the economics of their model and simulations. Thus they are able to do qualitatively better policy analysis.

We describe below some of the main features common to today's CGE modeling systems.

3.1 Allow input intelligible to modelers

Each of GAMS, MPSGE and GEMPACK requires that model equations be specified in a text file, using a special syntax.

For GAMS and GEMPACK the syntax resembles standard algebraic notation used in economic text books. For example, the Leontief equation for demands for commodities by industries:

$$X_{ij} = A_{ij}Z_j \quad i \in \text{COM}, j \in \text{IND}$$

might be rendered for GAMS as:

```
E_X   X(i,j) = A(i,j) * Z(j);
```

where subscripts i and j have been previously assigned to the sets COM and IND.

GEMPACK was originally developed for modelers used to working with the ORANI model, in which equations were written down in linearized form (relating percentage changes rather than the levels values of the variables involved). Following this tradition, for GEMPACK we would normally⁴ write down the percent-change (log-change) form:

```
E_x (all,i,COM)(all,j,IND) x(i,j) = a(i,j) + z(j);
```

where the lowercase symbols x , a , and z are understood to be percent changes from the initial values of X , A and Z .

Often it is clearer to explain the main features of CGE models in words, together with diagrams that show the structure of data and substitution (nesting). MPSGE builds on this way⁵ of explaining models, using a concise text summary of accounting constraints and nesting structure. It assumes that most substitution behavior can be described using nested CES functions. The MPSGE representation of a model is typically rather compact.

All three systems make it fairly easy to specify (or change details of) a CGE model.

The use of the three different modeling languages is further illustrated in Section 6, which shows GAMS, MPSGE and GEMPACK representations of the same basic model.

3.2 Handle a wide range of models

While most CGE models bear a strong family resemblance, there are many differences between them. General-purpose software should be able to handle the range of CGE models used for policy analysis. Both GAMS and GEMPACK satisfy this requirement, since they allow nearly any equation to be represented. Indeed neither system contains any economics, or enforces any preconception of how a CGE model works. Either system could be used for non-CGE purposes: to compute, say, the stresses in a bridge design. The modeler is obliged to encode all economic theory used by his model into the text specification—making every assumption explicit. He is of course free to include equations which are wrong, perhaps because of typos.

By contrast MPSGE does enforce a framework or template for models—which is however flexible enough to represent nearly all of the equations used in most CGE models. Within this framework, a beginner has more chance of specifying even a fairly complex model correctly. To model behavioral specifications which

⁴ GEMPACK also accepts equations written in levels form.

⁵ GAMS/HERCULES [Drud *et al.*, 1986, 1989] took this approach even more strongly since the structure of the SAM drove model representation. However GAMS/HERCULES is no longer available.

lie outside the basic framework, MPSGE allows exceptions to be made—at some cost to the brevity and clarity of the model code.

All three systems can handle models which are large—either because there are many equation blocks and/or because the number of sectors or regions is large. In either case such a model may be computationally demanding.

All three systems can handle models with varying levels of aggregation. For example, GTAP supplies a world model dataset with 57 sectors and >100 regions—too big to solve quickly. Modelers must first reduce the size of the database by combining sectors and regions into groups, defined according to the needs of a particular simulation. The same model code can be used with any aggregation of the database—sets (ie, lists) of regions and sectors can be read from the database at runtime.

3.3 *Insulate modelers from the solution method*

A CGE model is a large system of simultaneous non-linear equations—to solve it presents a considerable computational challenge. Hence early CGE modelers were, perforce, amateur practitioners in the field of numerical algorithms. During the 1970s several approaches contended, including:

- Programming approaches, motivated by the wide success of the Simplex algorithm, and by the idea that a CGE model can be recast as a single-optimand optimization problem. While attractive for very simple models with no distortions, this approach becomes less practical where there are multiple optimizing agents facing a range of tax and other distortions.⁶
- Scarf (1967) and related algorithms which constructed a set of equilibrium prices. These were briefly favored but discarded as slow.
- Iterative methods such as Jacobi, which required (a) a model-specific assignment of variables to equations, and (for Gauss-Seidel) a suitable ordering of equations.
- Other methods, such as Newton-Raphson or conjugate gradient.

Today practice has converged to a common paradigm:

- The system of equations is square: the number of equations equals the number of endogenous variables.
- The solver allows a flexible assignment of variables between exogenous and endogenous groups.
- The modeler is not required to understand the internals of the solving engine: it is a "black box" to him.
- The solving engine does not understand the economic intent of the modeler; eg, it does not know which variables are prices and which are quantities.
- Some or all of the model equations may be expressed as complementarities.
- An initial solution is provided which satisfies all (or nearly all) of the equations.

GEMPACK provides a single solution algorithm to satisfy these requirements. GAMS is designed to interface with a range of solvers, but most of these are optimizing⁷ packages. However the PATH solver (and its lighter-duty cousin, MILES) are direct equation solvers. Alternatively, optimizers such as MINOS or CONOPT could be used with a dummy optimand.

3.4 *Windows programs to control simulations and examine data and results*

Prior to the introduction of MsWindows, command-line programs were used to solve CGE models. The modeler needed to remember the right operating system commands, and the names and location of files used. To examine input data and simulation results, voluminous printouts were needed. The computing experience was laborious and annoying.

Both GAMS and GEMPACK now provide Windows programs which greatly facilitate these tasks. For example, the GAMS IDE allows the user to run and manage simulations without ever seeing the command line. It remembers the right file names, and offers a data viewer to directly view 2- or higher-dimensional matrices contained in binary files. GEMPACK offers a similar general-purpose IDE (WinGEM), which is supplemented by half a dozen more specialized programs—for viewing and editing code, data or results. There is a special IDE for running recursive dynamic programs.

⁶ Dixon (1975) proposed a method of constructing an artificial optimand, which corrected for distortions and multiple agents. Rutherford (1999) explores a similar idea.

⁷ GAMS optimizers continue to play an important role in preparing data for CGE models, for example using entropy methods.

These programs, particularly the data viewers, have greatly increased modelers' productivity. Models with many sectors, regions, or periods produce a volume of results which it would be impossible to inspect in print form.

Some of the GEMPACK IDE's have another useful feature: they can identify and zip up all of the original or source files needed to reproduce a simulation. Hence, modelers can archive all ingredients of their results, can retrieve and reproduce them fairly quickly and can send them to others (perhaps in different countries) who can do the same.

3.5 Run on different computers

While today most CGE modelers work on a Windows PC, in the 1980s GAMS and GEMPACK needed to be portable across a range of computers and operating systems. Even up to the mid 1990s, the limited capacity of Windows PCs meant that larger models were sometimes solved on mainframes or UNIX computers.

For example, the Australian Industry Assistance Commission (IAC) was the custodian of the ORANI model in the 1980s. Policy modelers were able to run ORANI on one mainframe computer, using model-specific and operating-system-specific Fortran code⁸. Around 1989, the cost of using this computer became prohibitive. But the ORANI solution code would only run on it and rewriting it would have taken several months. The IAC approached Pearson to see if the fledgling GEMPACK could solve ORANI on the new (and relatively inexpensive) IBM-like mainframe NAS computer they had purchased. Because GEMPACK was written using ANSI standard Fortran instructions, it was possible to port GEMPACK to the NAS and solve ORANI there within two weeks.

Portability remains in 2011, at least for the core, command-line, GAMS and GEMPACK solution programs, which can be run under, eg, Linux or Mac environments. Unfortunately the almost-indispensable Windows IDE/viewer programs are not so portable. At the same time, the introduction of 64-bit Windows means that Windows users need suffer no performance penalty. Consequently non-Windows use is mostly confined to a small band of Linux or Mac enthusiasts—and most of these will still use Windows viewer programs (either on another computer, or within a Windows emulator running under their preferred OS).

However, Windows' dominance is not quite complete. Where a CGE model is part of a larger software system—such as the 'Integrated Assessment' systems which link economic and physical models—it may be necessary to run the combined system under Linux⁹. Computer clusters, which usually run Linux, might occasionally¹⁰ be useful for, say, running Monte Carlo simulations. In each case, economists would require additional IT help to use the system.

Finally, portability remains an insurance policy against future changes in computing environments.

4. Levels and change solution methods

A CGE model may be represented as a system of N simultaneous non-linear equations:

$$\mathbf{F}(\mathbf{Z}) = \mathbf{0}$$

The variables \mathbf{Z} may be divided into N endogenous variables \mathbf{Y} , which are determined by the system, and the remaining variables \mathbf{X} (the exogenous ones). So we may write:

$$\mathbf{F}(\mathbf{Y}, \mathbf{X}) = \mathbf{0}$$

As explained below, in CGE practice we assume that we know an initial solution $[\mathbf{Y}^0, \mathbf{X}^0]$:

$$\mathbf{F}(\mathbf{Y}^0, \mathbf{X}^0) = \mathbf{0}$$

We wish to find another solution with different settings of some exogenous variables:

$$\mathbf{F}(\mathbf{Y}^1, \mathbf{X}^1) = \mathbf{0}$$

⁸ The story here is in no sense a criticism of the original ORANI code written by John Sutton. On the contrary, to be able to solve ORANI with the tiny (by today's, or even late 1980s, standards) memory available on computers in the late 1970s was a veritable *tour de force*. Computers improved massively during the 1980s.

⁹ Essentially because some scientists and their programs will only run under Linux.

¹⁰ X86 Windows machines can contain up to 12 CPU cores, while GEMPACK is able to parallelize simulations using up to 9 cores. A typical cluster offers the modeler a *share* of 100-1000 CPUs (each is which is individually much *slower* than those on a fast Windows PC), wrapped up in an unfamiliar operating system. So far, such offerings have attracted only a few.

and to report percentage differences between \mathbf{Y}^1 and \mathbf{Y}^0 .

Some or all equations may be defined via *complementarities*. Each complementarity equation F_i has an associated variable Z_i which is bounded between U and L . Then *just one* of these three relations will hold:

$$\begin{aligned} F_i(\mathbf{Z}) = 0 & \quad \text{and} \quad L_i \leq Z_i \leq U_i \\ F_i(\mathbf{Z}) > 0 & \quad \text{and} \quad Z_i = L_i \\ F_i(\mathbf{Z}) < 0 & \quad \text{and} \quad Z_i = U_i \end{aligned}$$

Complementarities are a convenient way to represent the first-order or Kuhn-Tucker conditions which arise from optimization by agents. But they may arise in many other contexts (eg, quotas, or stepped tax schedules).

4.1 Levels strategy and GAMS

The levels strategy is to search through various \mathbf{Y} values to find a solution. The process is iterative; for example we might improve the current values via a Newton-Raphson step:

$$\mathbf{F}_Y(\mathbf{Y}, \mathbf{X}^1) \cdot d\mathbf{y} = -\mathbf{F}(\mathbf{Y}, \mathbf{X}^1)$$

where \mathbf{F}_Y is the Jacobian matrix or derivative of \mathbf{F} w.r.t. \mathbf{Y} , and $d\mathbf{y}$ is the suggested set of revisions to the current \mathbf{Y} values. We can measure how close we are to a solution with a *merit function*, for example $\mathbf{F}^T \mathbf{F}$ (the sum of squared errors of the equations). The process stops (ie, we have a solution) when the merit function becomes sufficiently tiny. Near to a solution, convergence is usually very rapid (eg, halving of error at each step).

By design GAMS does not prescribe a particular search strategy for \mathbf{Y} : this task is left to the variety of solvers which may be used with GAMS. Industrial-strength solvers like PATH use a range of strategies. Amongst these, variants of Newton-Raphson play an important part—it may serve as an example here.

What GAMS does do is:

- provide the modeler with a convenient interface for specifying the functions \mathbf{F} , setting up values for \mathbf{X}^0 , \mathbf{Y}^0 and \mathbf{X}^1 , and invoking a solver.
- provide to the solver, on request, fresh evaluations of the functions \mathbf{F} and the gradients \mathbf{F}_Y , using the latest estimates of \mathbf{Y} .

Considering the Newton-Raphson step:

$$\mathbf{F}_Y(\mathbf{Y}, \mathbf{X}^1) \cdot d\mathbf{y} = -\mathbf{F}(\mathbf{Y}, \mathbf{X}^1)$$

we see it requires evaluation (by GAMS) of \mathbf{F} and \mathbf{F}_Y , and solution (by the solver) of a large linear system. All three operations are costly; GAMS (and the solvers) work to reduce costs in many ways. For example, because most variables appear in only a few equations the Jacobian matrix \mathbf{F}_Y is mostly zero: GAMS' sparse storage scheme processes only the non-zero elements.

Levels solution algorithms are not indifferent to units of measurement. Suppose we declare that equations are solved if $\mathbf{F}^T \mathbf{F} < 10^{-6}$. The criterion will be affected by the units we choose for \mathbf{Z} (whether, say, thousands or millions of dollar). Consequently:

- GAMS modelers should scale their data appropriately
- We can imagine changes in exogenous variables that are too small to notice. That is, if we have an initial solution such that $\mathbf{F}(\mathbf{Y}^0, \mathbf{X}^0)^T \mathbf{F}(\mathbf{Y}^0, \mathbf{X}^0) < 10^{-6}$, we could find small changes $\boldsymbol{\epsilon}$ in exogenous variables such that $\mathbf{F}(\mathbf{Y}^0, \mathbf{X}^0 + \boldsymbol{\epsilon})^T \mathbf{F}(\mathbf{Y}^0, \mathbf{X}^0 + \boldsymbol{\epsilon})$ was also $< 10^{-6}$. Hence we need to ensure that shocks rise above the background level of numerical noise—we cannot directly compute the dollar change in GDP that might occur if we added one dollar to national health spending. Instead we might simulate the effect of a million dollar increase—then divide simulated changes by one million.

GAMS reduces these problems by working with double precision arithmetic, which is accurate to 16 significant figures. However, compared to single precision arithmetic, double precision approximately doubles memory needs and execution time¹¹.

¹¹ The time is affected because, when using double precision, only half as many numbers will fit into the fast CPU cache, and twice as many bytes must be exchanged with main memory.

4.2 Change strategy and GEMPACK

GEMPACK's solve strategy requires that we start from a solution, ie:

$$\mathbf{F}(\mathbf{Y}^0, \mathbf{X}^0) = \mathbf{0}$$

For small changes $d\mathbf{y}$ and $d\mathbf{x}$, in the neighborhood of a solution, we have the linear system

$$\mathbf{F}_Y(\mathbf{Y}, \mathbf{X}).d\mathbf{y} + \mathbf{F}_X(\mathbf{Y}, \mathbf{X}).d\mathbf{x} = \mathbf{0}$$

which we could solve to give:

$$d\mathbf{y} = -[\mathbf{F}_Y(\mathbf{Y}, \mathbf{X})]^{-1}\mathbf{F}_X(\mathbf{Y}, \mathbf{X}).d\mathbf{x}$$

The above is a linear or first order approximation, accurate only for very small changes.¹² However, we can reduce the error as much as we like by dividing the exogenous changes $\Delta\mathbf{X}=\mathbf{X}^1-\mathbf{X}^0$ into N equal parts and repeatedly imposing $1/N$ th of the shock to the equation above. After each step, \mathbf{X} and \mathbf{Y} change, so that we need to re-evaluate \mathbf{F}_Y and \mathbf{F}_X . This is the Euler¹³ method of solving differential equations, with the following characteristics:

1. As the number of steps N increases, we get a more accurate answer.
2. Unlike the Newton-Raphson method, we *never need to evaluate* \mathbf{F} . However, we still need, at every step, to evaluate the derivative matrices \mathbf{F}_Y and \mathbf{F}_X , and to solve a large linear system.
3. Compared to Newton-Raphson, Euler is more stable, but slower to reach accurate answers. We find usually that to halve the error we need to *double* the number of steps N .

The last point (3) seems to make Euler less attractive than Newton-Raphson, which often halves the error at *every* step. Besides, the comfort of (1) is diminished by (2): without evaluating \mathbf{F} , how can we know how accurate our solution is, or whether we need to increase N ?

The second, accuracy, concern is addressed as follows: we can perform 3 Euler simulations with, say, 4, 8 and 16 steps; then compare results. Then we can use these 3 estimates of the true answer to calculate a new estimate by Richardson extrapolation. The extrapolated estimate is typically far more accurate than any of its components; it is equivalent, say, to a 1000-step Euler calculation. As well, the extrapolation generates error bounds for each results, so we can tell if our estimate is accurate enough (if not, more steps are needed).

The first worry, that so many Euler steps may be slow, is addressed by:

- using industrial strength linear solution routines, prepared at Harwell Labs, UK (see Duff et al, 1989)
- recasting the linearized equation system using percent change variables. This turns out to eliminate a great deal of computation.
- using automatic algebraic substitution of high-dimension linearized equations to enormously reduce the size of the linear system.

4.2.1 Simple linearized equations enable substitution

In a typical GEMPACK model, most of the change variables are expressed as percentage changes (rather than ordinary changes). Thus, the levels GAMS equation:

$$x(c, s, u) = A_ARM(c, s, u) * xcomp(c, u) * [pcomp(c, u) / p(c, s)] ** SIGMA(c);$$

which describes import-domestic substitution by user u for commodity c , becomes, in percent change form:

$$x(c, s, u) = xcomp(c, u) - SIGMA(c) * [p(c, s) - pcomp(c, u)];$$

The percent change form is simpler, and uses cheaper arithmetic operations: powers are replaced by products, and products by sums. Moreover, GEMPACK code will contain commands like:

Backsolve x using E_x;

which instructs the system to drop the above equation, and simultaneously replace each occurrence of x in other equations by $\{xcomp(c, u) - SIGMA(c) * [p(c, s) - pcomp(c, u)]\}$. This can greatly reduce the number of equations in the system, and is easy to do for linearized equations.

¹² But accurate indeed for tiny changes: in this case, we *can* directly simulate a one dollar increase in health spending.

¹³ Press et al.(1992) think that the Euler method is useful for pedagogical purposes, but too crude for real-world use. By contrast, GEMPACK's experience with Euler has been positive; and the potential for users to understand how it works is a useful bonus. Many of the tricks used by GEMPACK to improve or build on Euler are described in Chapter 16 (ibid), "Integration of Ordinary Differential Equations".

5. General features of CGE Models

Although exhibiting great variety, CGE models tend to share various characteristics. Below we describe a common core of features from which most models depart only in a few respects. Naturally the solution software has been adapted to cope efficiently with the most usual model features.

Like their progenitor, input-output modeling, CGE models focus on the production of commodities by industries – distinguishing from 10 to 1000 sectors. The commodities are demanded by industries and by final demanders (household, investment, government and export). Income arising from primary factor rents and from taxes and transfers is distributed to (and drives spending by) "institutions" such as households, corporations, government and foreigners.

5.1 Main equation groups of a model

Industry input demands are usually assumed to be proportional to output (constant-returns to scale is assumed) and sensitive to relative prices. These demand functions arise from the assumptions (a) that production (output) is a function of input quantities and (b) that producers choose the cheapest combination of inputs that will produce a given output.

The assumptions impose some (triad) restrictions on the form of input demands—but leave open a wide range of substitution possibilities. To tie these down, it is usual to impose additional, separability, assumptions. For example, if an industry has multiple inputs and multiple outputs, we may stipulate an activity index Z such that:

$$F(\text{inputs}) = Z = G(\text{outputs})$$

That is, a sector will choose the cost-minimizing mix of inputs to produce Z , and, given Z , will choose (subject to G) a revenue-maximizing output mix. The implication is that input mix is independent of output mix, or that large blocks of cross-price elasticities are zero, greatly simplifying computation and elasticity estimation. Very often the F function will also be separable via:

$$F(\text{inputs}) = H(I(\text{primary factors}), J(\text{material inputs}))$$

Similarly an industry's choice of how much, say, flour, to use, will be modeled independently of the choice whether to use domestic or imported flour. Household, government, and investment demands are modeled in a similar way.

Thus complicated demand systems are built up from a series of sub-aggregator functions, called *nests*. Usually the functional forms used are of the CES type, or are generalizations of CES. Diagrams of the nesting structure is often described by a diagram, like Figure 1 in Section 6.

The remaining core equations of the typical CGE model are fairly simple, consisting of:

- market-clearing equations which add up total demands for each good, and, for domestic goods, equate demand to domestic output.
- price equations which relate tax-inclusive user prices to output or CIF prices.
- income equations which add up total revenue accruing to each agent.

Usually there will in addition be equations defining common macro aggregates such as GDP or absorption; and model-specific equations defining, say, wage-indexation assumptions or rules for investment and capital accumulation.

5.1.1 Block equations: industry detail captured in matrices of numbers

Usually one (or a small number) of behavioral specifications is applied to all sectors, households or regions. That is, the same functional form is used for all, say, household demand equations. Differences between goods or household types are captured in good- or household-specific numbers held on or deducible from the model database. The equations of the model are thus vector or matrix equations—expressing in a few lines of code, for example, demands by every household type for any of the commodities. This scheme enables a huge number of equations to be represented compactly.

5.1.2 Data: SAM plus elasticities

The typical CGE model database consists of:

- Matrices of transaction values, showing, for example, the value of coal used by the iron industry. Usually the database is presented as an input-output table or as a social accounting matrix. In either case, it covers the whole economy of a country (or even the whole world), and distinguishes a number of sectors, commodities, primary factors and perhaps regions or types of household.
- Elasticities: dimensionless parameters that capture behavioral response. These include the CES substitution parameters that underlie nested production and demand structures, and export demand or household expenditure elasticities.

The SAM structure, which arranges all flows into a single table according to the recipient (row) and payer (column) is a useful conceptual and pedagogical tool. In practice it can become inconvenient for large or detailed models since the SAM becomes too large or sparse to view easily. Also, the need to distinguish all flows through only 2 subscripts is constricting. For example, if we distinguish goods by sector and source and households by income and region, the natural way to record household demands is:

$V(i,s,d,r)$ spending on good i from source s by income decile d in region r

ie, to use 4 subscripts. In a 2-subscript SAM schema, this appears as

$V(g,h)$ spending on good g by household type h

where subscript g runs over a compound set containing all combinations of good and source, and subscript h similarly combines all deciles and regions. Often therefore CGE flows data which could in principle be combined into an enormous, mostly-zero, SAM, are instead stored as a series of multidimensional matrices representing the non-zero blocks of a hypothetical SAM.

Usually these data are stored in a binary file of proprietary format¹⁴. While both GAMS and GEMPACK are able to read data in text or spreadsheet formats, binary storage is more compact and faster to process, especially for large models.

5.2 The data-base is consistent with initial truth of the model equations

Typically the flows database represents a single year and is assumed to be consistent with all (or nearly all) of the model equations. This point deserves elaboration.

Suppose that intermediate input demand by sector j for good i (X_{ij}) is given by the Leontief function:

$$X_{ij} = A_{ij}Z_j$$

where Z_j is output of sector j and A_{ij} is a technological parameter. None of these variables appear in the model database, which only supplies:

$$V_j = P_jZ_j \quad \text{the value of output of sector } j,$$

and $V_{ij} = P_iX_{ij}$ the value of good i used by sector j

To obtain initial values for the variables we need to make assumptions—this is called *calibration*. For example, we might assume that both P_j and P_i were 1. Given the flow values, we could then deduce the quantities X_{ij} and Z_j , and thence calculate A_{ij} . Clearly such a procedure guarantees (or assumes) that the demand equation is initially true.

The assumption that initial prices are 1 amounts to no more than an arbitrary choice of physical units for quantities like "light industrial" output or demand for "other services" that have no natural unit of measurement. Because of this, it is meaningless to report simulated levels values of prices and quantities, except in relation to their initial values. So, universally, CGE model results are presented in terms of *percentage changes* from the initial levels values.

One might ask, would results for a given simulation be affected if we had assumed that $P_j=2$ and $P_i=3$ (instead of both 1). Clearly both initial and final levels results would be affected, but the percentage change results would be the same—otherwise we would reject the model or calibration process.

¹⁴ eg, GEMPACK's HAR files and GAMS' GDX files. Tools are available to easily convert between these two, or into other common formats (eg, text).

Like physical units, monetary units are irrelevant to percentage change results. We could halve all initial data flow values (converting from dollars to pounds) without altering the percentage change results from any simulation.

The database also stores various elasticities which affect behavior. By definition, these are unitless—so initial levels values are important. They are not 'free' in calibration. Consider the CES equation:

$$K/L = A.(P_L/P_K)^\sigma$$

Assuming prices to be 1, and given initial flow values of capital and labor demand, we can deduce K and L and so deduce A. The value of σ must be supplied¹⁵.

It follows from this discussion that the *whole* useful content of a CGE model could be contained in a matrix function F:

$$[p, q] = F(D, E, t, a)$$

where p and q represent finite¹⁶ percent changes in endogenous model variables; D represents various matrices of shares or ratios derived from the SAM or IO database; E is the set of elasticities; and t and a are percent changes in exogenous variables such as tax rates or technological coefficients¹⁷. Both the GEMPACK and the MPSGE ways to represent a CGE model draw on this insight.

The format of matrix function F above minimizes computation cost in the sense we only need to compute and store D and E (the initial database) and p, q, t and a (the results we need to report). That is, an average of 3 values per cell in the original IO table (p, q, D)¹⁸. A full levels-based approach is more expensive, since we need to store and compute the:

- original flows values V0, prices P0, quantities X0 and technological coefficients A0
- post-solution values of P and X, (these were initially set to P0 and X0)
- percentage changes p and q calculated as $p=100(P/P_0 - 1)$ and $q=100(Q/Q_0 - 1)$

That is, an average of 8 values per cell in the original IO table. The load is more than doubled¹⁹, just to support the fictional construct of levels values.

5.3 General features of model code

For both of the two main CGE modeling systems, GAMS and GEMPACK, plain text files are used to specify the CGE model. In GAMS the same text file could also specify several individual simulations: GEMPACK requires that each simulation is specified by its own file, separate from that specifying the model.

Both GAMS and GEMPACK provide their own computer languages, to describe model and simulations. As in most computer languages, entities such as sets or variables must be described or 'declared' before they are used. This imposes an overall structure on model specification files:

- The first step is to define, either by direct enumeration or by reading from a file, the size and contents of various key sets—of sectors, regions, and so on.
- Next, data arrays are declared and their values are read from file.
- More data arrays are declared and given values which are derived from file data.

¹⁵ The alert reader will notice that, with initial price ratio 1, a shock to σ will have no effect; yet with non-uniform initial prices, a shock to σ (holding A fixed) *would* have an effect—contradicting the claim that assumed initial prices do not affect percent change results. The short retort might be that this is why mature CGE modelers never shock elasticities. A longer answer might explore the way in which different ways of writing the CES functions, which are generally agreed to be equivalent, in fact act differently if σ is considered to be a variable.

¹⁶ The percent changes are not infinitesimal and F is non-linear.

¹⁷ The conclusion depends only on the need for model results to be independent of arbitrary choice of units, and requires no controversial neo-classical assumptions. For example, money illusion or increasing-returns-to-scale could be incorporated, if desired, via additional elasticities in the database.

¹⁸ We assume that only a tiny proportion of the exogenous variables t and a are shocked. The remainder are by definition zero.

¹⁹ ie 8/3. In practice this ratio is often reduced to 5/3 by such devices as: assuming that $V_0=X_0$; and reporting only a tiny subset of percentage change results. The first device has the disadvantage that we must apply and follow some special convention about initial prices, which might be complicated if prices are user-specific. The second device makes it harder to understand in detail what the model is doing, or if it is working as desired.

- Next, in GAMS code, units of measurement are assigned (usually by setting prices), and numerous multiplicative constants are computed. The logarithmic form of percent-change equations obviates the need for this step in GEMPACK
- Next the model equations are given, in levels or (for GEMPACK) in percent-change form.
- The closure, or choice of exogenous variables is specified.
- A GAMS model usually includes a no-shock simulation as a check; it should reproduce the benchmark data.
- An exogenous variable is changed, and a new solution computed.
- In GAMS, additional code is needed to derive percent-change results from old and new levels values.

6. Three representations of a simple model

We begin this section by showing the code for a fairly standard model which we call SIMPLE. We describe the model using data and nest diagrams, then give details of implementations²⁰ in GEMPACK, GAMS and MPSGE.

Table 1 below shows the model database. 5 competitive single-product industries (Agric ... Service) produce corresponding goods which are used by industries and final demanders. All non-export users also use imported goods, and industries also use Capital and Labor. There are no taxes in the model.

Table 1 Database for example model

		Sectors					Final Demands				
		Agric	Mining	Manuf	Housing	Service	Hou	Gov	Inv	Exp	Total
Dom	Agric	8	1	45	0	0	0	1	0	70	125
	Mining	3	2	33	0	1	4	1	0	92	136
	Manuf	9	25	12	4	15	50	27	98	6	246
	Housing	1	0	0	0	2	54	3	0	0	60
	Service	11	12	11	9	5	30	31	32	22	163
Imp	Agric	1	0	5	0	0	0	0	0		6
	Mining	1	1	10	0	0	1	0	23		36
	Manuf	4	4	18	0	15	30	6	56		133
	Housing	0	0	0	0	0	0	0	0		0
	Service	0	3	3	0	2	1	6	9		24
Capital		43	41	50	47	52					233
Labor		44	47	59	0	71					221
Total		125	136	246	60	163	170	75	218	190	
Sigma	Dom/imp	2	1	2	0	1					

Demands are structured by a series of nests. For industries, the production/demand structure is shown in Figure 1 below. Each industry combines domestic and imported equivalents into commodity-composites via a CES aggregator. The elasticity of substitution varies by commodity and is shown in the last row of the table above. Import/domestic shares vary by good and by user. Industries demand commodity-composites in proportion to output. Similarly, each industry combines capital and labor using a CES with value 0.5 to produce a primary-factor-composite which is also demanded in proportion to industry output.

²⁰ All three model specification files, and their common database are downloadable: item TPMH0103 at <http://www.monash.edu.au/policy/archivep.htm>.

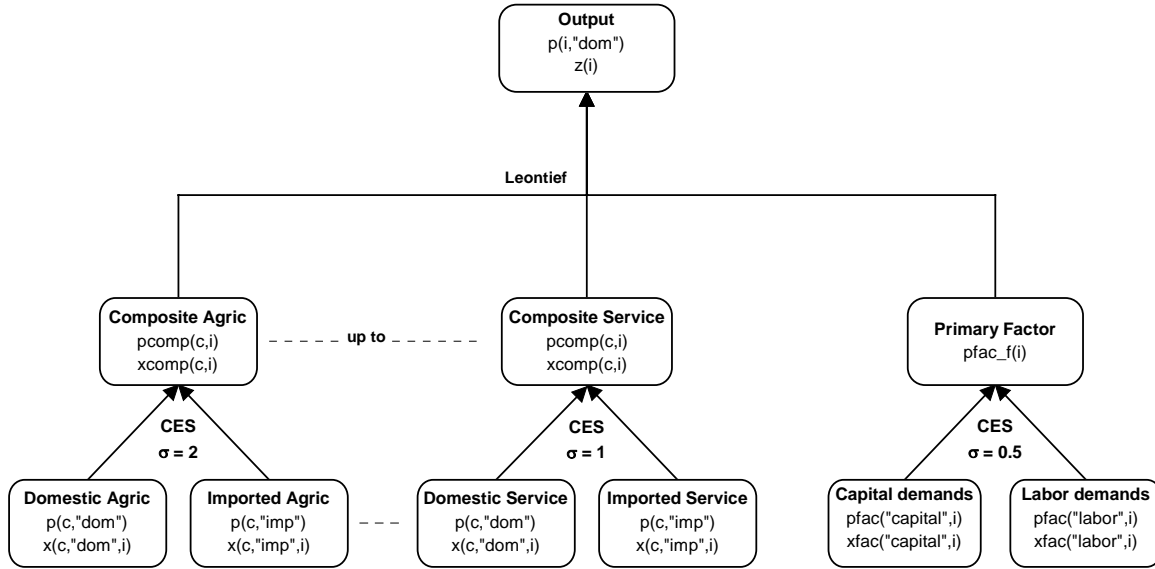


Figure 1 Structure of input demands for industry i

For the non-export final demanders (Hou, Gov and Inv) Figure 2 shows a similar structure except labor and capital are not used and the topmost nest is Cobb-Douglas rather than Leontief. Exports face foreign demand curves with a constant elasticity of -5 .

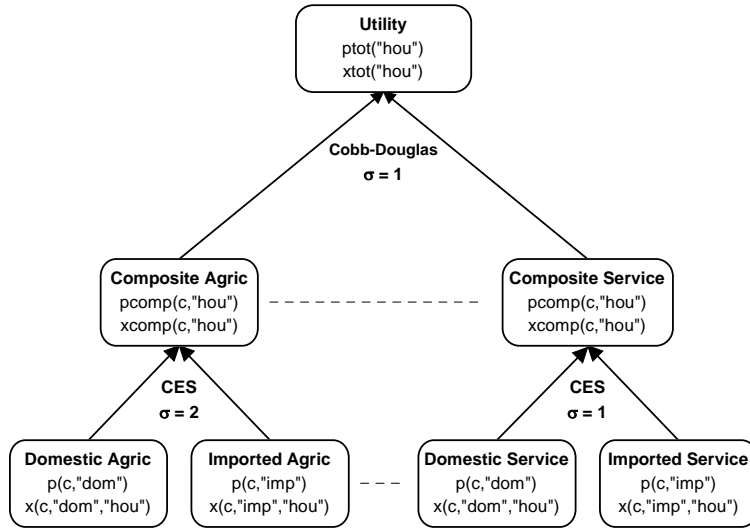


Figure 2 Structure of non-export final demand

To close the model, we will assume that industry capital stocks are fixed, while Labor is mobile between sectors but fixed in aggregate. Aggregate nominal values of the Hou, Gov, and Inv demands are fixed proportions of nominal GDP. Other macro closures would be possible.

To be sure that our database is consistent with model equations (ie, that we can calibrate a solution—see Section 5.2), we need to check only two things:

(a) all the numbers are ≥ 0 .

(b) the first 5 row totals, showing the value of sales of each domestic good, equal the first 5 column totals, showing the total cost of each industry.

6.1 The SIMPLE model in GEMPACK

Table 2 shows how the model is specified using GEMPACK's TABLO language. The file starts by defining various sets. The list of sectors is read from a data file – so the same code could be used for a model with more or different sectors.

Next, the 3 data arrays (USE, FACTOR and SIGMA) are defined and their values read from file. The notation,

```
(all,f,FAC)(all,i,IND) FACTOR(f,i)
```

indicates that FACTOR is a matrix with 2 rows (Capital and Labor, the elements of set FAC) and 5 columns (the 5 sectors which form set IND).

Then, several other coefficients are defined which are all simple addups of USE and FACTOR. The Assertion statement is a safety check – the solver will terminate with an error if DIFF is not tiny.

As for most GEMPACK models, this simple model is specified in percent change form²¹. The model variables are listed next. By default, these are defined as percentage changes from the initial solution, except for the first, delB, which is an ordinary change.

As explained in Section 5.2, the percent change formulation allows us to express the model, and to compute percent changes in prices and quantities, without ever specifying initial values for prices and quantities (we only need initial flows). This enables brevity in model specification, and reduced computation time.

The equations which follow are all linear in percentage change variables, with coefficients that are data-base flows, addups of these flows, or elasticities. The first equation block:

```
E_pcomp # Price: dom/imp composites # (all,c,COM)(all,u,LOCUSR)
ID01[USE_S(c,u)]*pcomp(c,u)=sum{s, SRC, USE(c,s,u)*p(c,s)};
```

is a group of equations that define, for each good and local user, the first-order change in the CES price index of the composite good which combines domestic and imported versions. It states that pcomp(c,u) is a cost-share-weighted average of the percent changes in domestic and imported prices (Roy's Identity). The built-in function ID01 takes care of the case when use of the composite, USE_S(c,u), is zero: ID01(x)=x for all non-zero x, while ID01[0]=1. So in the degenerate case we have pcomp(c,u)=0. The name of the equation block, E_pcomp, is chosen to suggest that these equations can determine values of the variable pcomp.

The next equation block:

```
E_x # Quant: final dom/imp composites# (all,c,COM)(all,s,SRC)(all,l,LOCUSR)
x(c,s,l) = xcomp(c,l) - SIGMA(c)*[p(c,s)-pcomp(c,l)];
```

specifies pairs (for s="dom" or "imp") of demand equations for each good and non-export user. We could subtract the imported from the domestic equation to demonstrate the CES property:

```
(all,c,COM)(all,l,LOCUSR)
x(c,"dom",l) - x(c,"imp",l) = SIGMA(c)*[p(c,"imp")-p(c,"dom")];
```

Many of the remaining equations follow similar patterns, or are simple addups of nominal values. As a check on the model's accounting relations, we have included two variables and equations for nominal GDP: we expect, from Walras' Law, that both variables will end up with the same value.

The equation for real GDP:

```
E_xgdp # Real GDP from income side #
VGDP*xgdp = sum{i,IND, sum{f,FAC, FACTOR(f,i)*[xfac(f,i)-afac(f,i)]}};
```

is a Divisia index, not translated from any levels equation.

²¹ GEMPACK also allows you to specify the model in the levels – this would lead to model code which rather resembled the GAMS version.

Table 2 TABLO Input file for example GEMPACK model

```

File INFILE # base data file #;
Set
  COM # Commodities # read elements from file INFILE header "SEC";
  IND # Industries # read elements from file INFILE header "SEC";
  FIN # Final users # (Hou,Gov,Inv,Exp);
  LOCFIN # Local final users # (Hou,Gov,Inv);
  USR # ALL users # = IND + FIN;
  LOCUSR # Local users # = IND + LOCFIN;
  SRC (dom,imp);
  FAC (Capital,Labor);
  SEC = COM intersect IND;
Subset LOCUSR is subset of USR;
Coefficient ! data on file !
(all,c,COM)(all,s,SRC)(all,u,USR) USE(c,s,u) # Value of inputs #;
(all,f,FAC)(all,i,IND) FACTOR(f,i) # Primary factor costs #;
(parameter)(all,c,COM) SIGMA(c) # CES dom-imp elasticity #;
Read
  USE from file INFILE header "USE";
  FACTOR from file INFILE header "FACT";
  SIGMA from file INFILE header "SIG";
Coefficient ! calculated !
(all,c,COM)(all,u,USR) USE_S(c,u) # Value of import composites #;
(all,i,IND) VALADD(i) # Factor costs #;
(all,u,USR) COSTS(u) # Costs #;
(all,c,COM)(all,s,SRC) SALES(c,s) # Sales #;
(all,q,SEC) DIFF(q) # Costs-Sales #;
VGDP # GDP #;
Formula
(all,c,COM)(all,u,USR) USE_S(c,u) = sum{s,SRC, USE(c,s,u)};
(all,i,IND) VALADD(i) = sum{f,FAC, FACTOR(f,i)};
(all,u,USR) COSTS(u) = sum{c,COM, USE_S(c,u)};
(all,i,IND) COSTS(i) = COSTS(i) + VALADD(i);
(all,c,COM)(all,s,SRC) SALES(c,s) = sum{u,USR, USE(c,s,u)};
(all,q,SEC) DIFF(q) = COSTS(q) - SALES(q,"dom");
VGDP = sum{i,IND,VALADD(i)};
Assertion # Check data balance # (all,q,SEC) ABS[DIFF(q)]<0.001;
Variable
  (change) delB # (Nominal balance of trade)/{nominal GDP} #;
  xgdp # Real GDP #;
  wgdpc # Nominal GDP from income side #;
  wgdpcexp # Nominal GDP from expenditure side #;
  phi # Exchange rate (Local$/foreign$) #;
  (all,c,COM) xexp(c) # Export quantities #;
  (all,c,COM) fqexp(c) # Right shift in export demand curve #;
  (all,i,IND) z(i) # Industry outputs #;
  (all,c,COM) pfimp(c) # Imp goods prices, foreign $ #;
  (all,l,LOCFIN) wtot(l) # Nominal expenditure by local final users #;
  (all,l,LOCFIN) xtot(l) # Real expenditure by local final users #;
  (all,l,LOCFIN) ptot(l) # Price indices for local final users #;
  (all,f,FAC) xfac_i(f) # Total value-weighted factor use #;
  (all,f,FAC) ffac_i(f) # Factor wage shift #;
  (all,c,COM)(all,s,SRC) p(c,s) # Factor prices #;
  (all,c,COM)(all,s,SRC) xdem(c,s) # Total demand for goods #;
  (all,f,FAC)(all,i,IND) pfac(f,i) # Factor prices #;
  (all,i,IND) pfac_f(i) # Factor composite prices #;
  (all,f,FAC)(all,i,IND) xfac(f,i) # Factor use #;
  (all,f,FAC)(all,i,IND) afac(f,i) # Factor-using technical change #;
  (all,f,FAC)(all,i,IND) ffac(f,i) # Factor wage shift #;
  (all,c,COM)(all,l,LOCUSR) pcomp(c,l) # Price dom/imp composites#;
  (all,c,COM)(all,l,LOCUSR) xcomp(c,l) # Quantity dom/imp composites#;
  (all,c,COM)(all,s,SRC)(all,l,LOCUSR) x(c,s,l) # Commodity demands #;

```

Equation

```

! Dom/imp CES BLOCK !
E_pcomp # Price: dom/imp composites # (all,c,COM)(all,u,LOCUSR)
  ID01[USE_S(c,u)]*pcomp(c,u)=sum{s,SRC,USE(c,s,u)*p(c,s)};
E_x # Quant: final dom/imp composites# (all,c,COM)(all,s,SRC)(all,l,LOCUSR)
  x(c,s,l) = xcomp(c,l) - SIGMA(c)*[p(c,s)-pcomp(c,l)];

! Industry demands !
E_pA # Industry cost indices # (all,i,SEC)
  ID01[COSTS(i)]*p(i,"dom")=sum{c,COM,USE_S(c,i)*pcomp(c,i)}+VALADD(i)*pfac_f(i);
E_pfac_f # Factor composite prices # (all,i,IND)
  ID01[VALADD(i)]*pfac_f(i) = sum{f,FAC, FACTOR(f,i)*[pfac(f,i)+afac(f,i)]};
E_xfac # Factor demands # (all,f,FAC)(all,i,IND)
  xfac(f,i) = z(i) + afac(f,i) - 0.5*[pfac(f,i)+afac(f,i)-pfac_f(i)];
E_xcompA # Local final demands (Cobb-Douglas) # (all,c,COM)(all,i,IND)
  xcomp(c,i) = z(i);

! Final demanders !
E_xcompB # Local final demands (Cobb-Douglas) # (all,c,COM)(all,l,LOCFIN)
  xcomp(c,l) + pcomp(c,l) = wtot(l);
E_wtot # Absorption # (all,l,LOCFIN) wtot(l) = wgdppinc;
E_ptot # Price indices for local final users # (all,l,LOCFIN)
  ID01[COSTS(l)]*ptot(l) = sum{c,COM, USE_S(c,l)*pcomp(c,l)};
E_xtot # Real expenditure by local final users # (all,l,LOCFIN)
  xtot(l) + ptot(l) = wtot(l);
E_xexp # Export demands # (all,c,COM)
  xexp(c) = fqexp(c) - 5.0*[p(c,"dom")-phi];

! Total demands and market clearing !
E_ffac_i # Total (value-weighted) factor use # (all,f,FAC)
  sum{i,IND,FACTOR(f,i)*[xfac_i(f) - xfac(f,i)]} = 0;
E_xdem # Total demand for goods # (all,c,COM)(all,s,SRC)
  ID01[SALES(c,s)]*xdem(c,s) =
    sum{l,LOCUSR,USE(c,s,l)*x(c,s,l)} + USE(c,s,"exp")*xexp(c);
E_z # Market clearing # (all,i,SEC) z(i) = xdem(i,"dom");

! Miscellaneous !
E_pB # Import prices # (all,i,SEC) p(i,"imp") = pfimp(i) + phi;
E_pfac # Factor mobility/remuneration # (all,f,FAC)(all,i,IND)
  pfac(f,i) = ptot("hou") + ffac(f,i) + ffac_i(f);
E_wgdppinc # Nominal GDP from income side #
  VGDP*wgdppinc = sum{i,IND, sum{f,FAC, FACTOR(f,i)*[pfac(f,i)+xfac(f,i)]}};
E_xgdp # Real GDP from income side #
  VGDP*xgdp = sum{i,IND, sum{f,FAC, FACTOR(f,i)*[xfac(f,i)-afac(f,i)]}};
E_wgdppexp # Nominal GDP from expenditure side #
  VGDP*wgdppexp= sum{c,COM, sum{l,LOCFIN, USE_S(c,l)*wtot(l)}
    + USE(c,"dom","exp")*[p(c,"dom")+xexp(c)]
    - SALES(c,"imp")*[p(c,"imp")+xdem(c,"imp")]};
E_delB # (balance of trade)/GDP #
  100*VGDP*delB = sum{c,COM,USE(c,"dom","exp")*[p(c,"dom")+xexp(c)-wgdppinc]}
    - sum{c,COM,SALES(c,"imp")*[p(c,"imp")+xdem(c,"imp")-wgdppinc]};

Update
(all,i,IND)(all,f,FAC) FACTOR(f,i)=pfac(f,i)*xfac(f,i);
(all,c,COM)(all,s,SRC)(all,l,LOCUSR) USE(c,s,l) = p(c,s)*x(c,s,l);
(all,c,COM) USE(c,"dom","exp") = p(c,"dom")*xexp(c);
Backsolve x using E_x; pcomp using E_pcomp;

```

The rule for primary factor wages:

```
E_pfac    # Factor mobility/remuneration # (all,i,IND)(all,f,FAC)
pfac(f,i) = ptot("hou") + ffac(f,i) + ffac_i(f);
```

is of a distinctively GEMPACK (or maybe Dixonian) form. At runtime the "f" variables (aka: shifters) will be set exogenous or endogenous to enforce different factor market closures. For example, if for capital we set `ffac_i("Capital")` exogenous and allowed the `ffac(i,"Capital")`, which appear nowhere else, to float endogenously, the equation would be otiose, merely determining changes in `ffac`. We could fix industry capital stocks, `xfac(i,"Capital")`, to determine the capital rents. For a slack Labor market we could hold both `ffac("Labor",i)` and `ffac_i("Labor")` fixed, so indexing wages to the CPI. Or we could fix both total Labor demand, `xfac_i("Labor")`, and the `ffac("Labor",i)` whilst allowing `ffac_i("Labor")` to float endogenously. The effect would be that industry wages would move together, to equate demand and supply for sectorally mobile Labor.

At the end of SIMPLE.TAB are Update instructions which tell GEMPACK how values on the data file are related to model variables. For example, the statement

```
Update (all,i,IND)(all,f,FAC) FACTOR(f,i)=pfac(f,i)*xfac(f,i);
```

indicates that the coefficient `FACTOR` is a product (in the levels) of prices `PFAC` and quantities `XFAC`. GEMPACK uses this information to update data flows after each step in the calculation. `FACTOR` is updated to become:

$$\text{FACTOR}(f,i) * [1 + 0.01 * \text{pfac}(f,i) + 0.01 * \text{xfac}(f,i)]$$

Finally, come backsolve statements such as:

```
Backsolve x using E_x;
```

This tells GEMPACK to perform algebraic substitution on the linear system, everywhere replacing `x(c,s,u)` by `{xcomp(c,u) - SIGMA(c)*[p(c,s)-pcomp(c,u)]}`. This enormously reduces the size of the equation system. The values of `x` are recovered and are available on the solution file. However, no element of `x` can be exogenous.

6.2 An example simulation

In GEMPACK, each simulation is specified using a separate file, suffixed ".CMF". Such a file is shown in Table 3 below. It starts by identifying the model, SIMPLE, described in the file SIMPLE.TAB (listed in Table 2). The name of the input file (simdata.har) is given. A file of identical format will be produced containing post-simulation (updated) values of all the data – this will be named after the CMF file.

On each line, any text following an exclamation mark is treated as a comment.

The next lines cause 3 Gragg simulations to be run, with 4, 6 and 8 steps. The final answers are produced by extrapolating from results of these 3 simulations.

Next comes the list of exogenous variables, or closure. GEMPACK users customarily begin from a well-known or standard closure; then, using "swap" statements, define differences from that known closure. In this case the reference closure is produced automatically by the software – it consists of all those variables which do not have an equation named after them. In this reference closure both labor and capital are mobile between sectors and fixed in aggregate – however we wish to fix industry capital stocks.

The statement:

```
swap xfac("capital",IND) = ffac("capital",IND);
```

causes the capital parts of the `xfac` variable (currently endogenous) to be set exogenous, while the capital parts of the `ffac` variable (currently exogenous) will become exogenous. This mechanism ensures that the number of endogenous variables (which *must* equal the number of equations) does not change. Having fixed the parts, we must endogenize the whole – this is done by a second swap statement.

Last comes a short description of the simulation, followed by the imposed change in shocked variables. In this case we reduce by 10% the amount of labor needed to produce a unit of services.

Table 3 GEMPACK Command file for sample simulation

```

auxiliary files = simple;
log file = yes;
File INFILE = simdata.har; !
Updated File INFILE = <cmf>.UPD; ! will be named after CMF
method = GRAGG ;! Solution method
steps = 4 6 8;

! Automatic closure generated by TABmate Tools...Closure command
!      Variable      Size
Exogenous   afac ; ! FAC*IND   Factor-using technical change
Exogenous   ffac ; ! FAC*IND   Factor wage shift
Exogenous   fqexp ; ! COM      Right shift in export demand curve
Exogenous   pfimp ; ! COM      Imp goods prices, foreign $
Exogenous   phi ; ! 1          Exchange rate (local$/foreign$) [Numeraire]
Exogenous   xfac_i ; ! FAC     Total value-weighted factor use
Rest endogenous;      ! end of TABmate automatic closure
!      new exogenous      old exogenous
swap xfac("capital",IND) = ffac("capital",IND); ! fix industry capital stocks
swap ffac_i("capital")   = xfac_i("capital");   ! unfix total capital stock

Verbal Description = Improvement in Service labor productivity;
shock afac("Labor","Service") = -10;

```

Results from the simulation are shown (as percent changes from base) in Table 4. First note that with aggregate nominal household, government and investment demands following nominal GDP, the residual balance of trade must also be a constant fraction of GDP. Service becomes cheaper, and that industry expands. Labor is released to other sectors, wages fall, and capital increases its share of national income (recall, the labor/capital CES is 0.5). Output increases in most sectors, especially Manuf and Service, while Agric and Mining, which are more export oriented, expand less. Housing output cannot rise because its capital stock is fixed and it uses no labor.

Table 4 Simulation results, 10% Improvement in Service labor productivity

Macro results		Variable	%change				
(balance of trade)/GDP (change)	delB		0				
Exchange rate (local\$/foreign\$)	phi		0				
Nominal GDP from expenditure side	wgdpexp		1.172				
Nominal GDP from income side	wgdpinc		1.172				
Real GDP	xgdp		1.630				
Aggregates for local final users			Hou	Gov	Inv		
Price indices	ptot		0.009	-1.019	-0.281		
Quantity indices	xtot		1.162	2.214	1.457		
Nominal expenditure	wtot		1.172	1.172	1.172		
Sectoral results			Agric	Mining	Manuf	Housing	Service
Factor prices	pfac	Capital	1.037	1.041	2.652	2.240	5.791
		Labor	-0.427	-0.427	-0.427	-0.427	-0.427
Factor use	xfac	Capital	0	0	0	0	0
		Labor	0.732	0.734	1.534	1.330	-2.214
Commodity prices	p	dom	-0.012	-0.028	0.316	1.350	-2.841
		imp	0	0	0	0	0
Commodity demands	xdem	z dom	0.369	0.391	0.824	0	4.817
		imp	0.726	1.024	1.732	0	0.588
Exports	xexp	dom	0.062	0.141	-1.565	-6.484	15.501

The same results will be generated by GAMS and MPSGE with some trivial differences. For example, the GEMPACK results shows changes in the quantity of housing exports – even though Table 1 shows no housing exports. GEMPACK users are accustomed to generating such results, which may be safely ignored, and never affect the percent changes in genuine, non-zero, flows.

6.3 The SIMPLE model in GAMS

Table 5 below shows how the SIMPLE model may be implemented²² in GAMS. In this case a single GMS text file replaces the TAB and CMF file used by GEMPACK. As far as possible the same notation has been used as in the GEMPACK implementation. The chief difference is that although results are again reported as percentage changes, the model is specified in the levels. This means that for all quantities and prices, separate symbols must be defined to represent initial values, final values and percentage changes. In addition, the levels equations require a number of multiplicative constants (which drop out in percentage change form). The need to declare and assign values to all these parameters and variables takes a bit more space than needed in GEMPACK.

Section 1 of the GMS file reads in sets and data from the GDX file, and defines some additional sets. Then, zeroes in the factor payments data are replaced by tiny numbers, while the degenerate case where a CES = 1 is avoided. This simplifies subsequent code, although other approaches are possible. Next some useful add-ups of the input data are defined, and we check that industry costs equal domestic sales.

Section 2 declares initial values of all variables, and assigns values to these variables which are consistent with the base data. By convention, prices are set to 1 – then volumes are equal to initial flow values. Similarly Section 3 deduces technology and distribution parameters in production functions that are consistent with known prices and quantities.

Section 4 declares the model variables and equations. The variables are set equal to their initial values. A dummy variable, OBJ, serves as a maximand in case an optimizing solver is used (even though the system is fully constrained).

Section 5 begins by defining a standard set of exogenous variables (both factors mobile and fixed in aggregate) and performs a trial simulation *without any shock*. The aim is to check that the initial values do indeed constitute a solution. Next, the closure is altered so that industry capital stocks are fixed. A shock is imposed (10% cut in labor needed by services) and the model solved again.

Section 6 uses new and original values to compute a few percent-change results for reporting. The percent change in real GDP is computed as a Fisher (ideal) GDP quantity index—giving the same numerical result as the corresponding Divisia index in the GEMPACK model. Results are stored in a GDX file.

While the GEMPACK modeler automatically receives percent-change results for *all* variables, the GAMS modeler has to prepare formulae for each percent change. The path of least resistance is to compute percent-change results for only a few, summary, variables. In that case, detailed results, which could contain points of interest or even anomalies, may never be viewed.

²² We are grateful to Erwin Corong for preparing the GAMS implementation.

Table 5 GMS Input file for example GAMS model and simulation

```

$title                               Simple Model
$OFFSYMLIST
$OFFSYMREF
$EOLCOM !

* Convention: variables (and initial values of variables) are in lowercase,
* other parameters are in upper case

*===== (1) Set Declaration =====
* Declare and read Sets and Parameters stored in GDX
Set      ! on data file
  USR      Users
  FAC      Factors
  LOCUSR(USR) Local users
  COM(LOCUSR) Commodities
  IND(COM) Industries
  LOCFIN(LOCUSR) Local final users
  SEC(IND) Sectors
  SRC      Sources / dom, imp / ;

Parameter ! on data file
  USE(COM,SRC,USR) Value of inputs
  FACTOR(FAC,IND) Primary factor costs
  SIGMA(COM)      CES dom-imp elasticity ;

$GDXIN input.gdx
$LOADdc USR, FAC, LOCUSR, COM, IND, LOCFIN, SEC, FACTOR, USE, SIGMA

Set      ! Additional Sets
  FIN(USR)      Final users          / Hou, Gov, Inv, Exp /
  EXP(USR)      Exports              / Exp / ;

Alias (IND,i),(COM,c),(SRC,s),(USR,u),(FAC,f),(f,ff),(LOCFIN,lf),(LOCUSR,lu),(FIN,fu);

  FACTOR(f,i)$(FACTOR(f,i)eq 0) = 1E-9; ! alter data to avoid problems
  SIGMA(c) $(SIGMA(c)eq 1) = 1.0001; !NOTE: CES sigma must not equal 1

Parameter ! addups of base data
  VALADD0(i)      Factor costs
  COSTS0(i)       Costs
  SALES0(c,s)     Sales
  DIFF(i)         Costs - sales ;

  VALADD0(i) = SUM[f,FACTOR(f,i)];
  COSTS0(i) = VALADD0(i) + SUM((c,s), USE(c,s,i));
  SALES0(c,s) = SUM(u, USE(c,s,u));
  DIFF(i) = SALES0(i,"dom")- COSTS0(i);

DISPLAY VALADD0, COSTS0, SALES0, DIFF, FACTOR;
ABORT$(ABS{SUM[i, DIFF(i)]} GT 0.01)"!!!DATA BALANCE PROBLEM!!!";

*===== (2) Parameter declarations =====
Parameter ! initial values of variables (and variables) are in lower case
  ffac0(f,i)      Factor wage shift
  pfac0(f,i)      Factor prices
  pfac_f0(i)      Factor composite prices
  xfac0(f,i)      Factor use
  xfac_i0(f)      Total value-weighted factor use
  ffac_i0(f)      Factor wage shift
  xfac_f0(i)      Total factor use by industry
  z0(i)           Industry outputs
  x0(c,s,lu)      Value of inputs
  wtot0(lf)       Nominal expenditure by local final users
  xt00(lf)        Real expenditure by local final users
  xdem0(c,s)      Total demand for goods
  xcomp0(c,lu)    Quant: final dom-imp composites
  ptot0(lf)       Price indices for local final users
  p0(c,s)         Goods prices
  pcomp0(c,lu)    Price: Intermediate dom-imp composites for local users
  wgdpinco       Nominal GDP from income side
  wgdpepxo       Nominal GDP from expenditure side
  delB0          Trade balance
  phi0           Exchange rate
  pfimp0(c)       Imported goods prices-foreign $

```

```

xexp0(c)      Export demand
ftot0(lf)     Absorption parameter
afac0(f,i)    Factor shifter
fqexp0(c)     Initial value of export shifter;

*-- Assign initial values
pcomp0(c,lu) = 1;
p0(c,s)      = 1;
pfac_f0(i)   = 1;
pfac0(f,i)   = 1;
pfimp0(c)    = 1;
ptot0(lf)    = 1;
ffac0(f,i)   = 1;
ffac_i0(f)   = 1;
afac0(f,i)   = 1;
phi0         = 1;

*--- Volumes calculations
x0(c,s,lu)   = USE(c,s,lu)/p0(c,s);
xcomp0(c,lu) = SUM[s,USE(c,s,lu)]/pcomp0(c,lu);
xexp0(c)     = USE(c,"dom","exp")/p0(c,"dom");

*--- Initial Data for use in calibration
xfac0(f,i)   = FACTOR(f,i)/pfac0(f,i);
xfac_f0(i)   = VALADD0(i)/pfac_f0(i);
xfac_i0(f)   = SUM[i, FACTOR(f,i)/pfac0(f,i)];
z0(i)        = COSTS0(i)/p0(i,"dom");
wtot0(lf)    = SUM[(c,s), USE(c,s,lf)];
xtot0(lf)    = wtot0(lf)/ptot0(lf);
xdem0(c,s)   = SUM[lu, x0(c,s,lu)];
xdem0(c,"dom") = xdem0(c,"dom") + xexp0(c);

*--- Other initial values
fqexp0(c)    = xexp0(c)*[p0(c,"dom")/phi0]**(5) ;
wgdpinc0     = SUM{i, SUM[f,FACTOR(f,i)]};
delB0        = SUM[c, USE(c,"dom","exp")] - SUM[(c,u), USE(c,"imp",u)];
wgdpexp0     = SUM[(c,s,lf), USE(c,s,lf)] + delB0;
delB0        = delB0 / wgdpinc0;
ftot0(lf)    = wtot0(lf)/wgdpinc0;

*===== (3) Calibration =====
Parameter    ! Calibrate CES dom-imp
A_ARM(c,s,lu) Armington shares ;
A_ARM(c,s,lu)$xcomp0(c,lu) = x0(c,s,lu)/xcomp0(c,lu)
                        *[pcomp0(c,lu)/p0(c,s)]**SIGMA(c);

Parameter    ! Calibrate CES primary factor
SIGMA_FAC(i) Elasticity in factor use
ALPHA_F(f,i)  Share of factor in CES value added
RHO_F(i)      Rho parameter in CES value added
A_F(i)        Technology parameter in CES value added;
SIGMA_FAC(i)  = 0.5;
RHO_F(i)      = (1-SIGMA_FAC(i))/SIGMA_FAC(i);
ALPHA_F(f,i)  = pfac0(f,i)*xfac0(f,i)**(1+RHO_F(i))/
                SUM{ff,pfac0(ff,i)*xfac0(ff,i)**[1+RHO_F(i)]};
A_F(i)        = xfac_f0(i)/SUM[f,ALPHA_F(f,i)*xfac0(f,i)**(-RHO_F(i))]**(-1/RHO_F(i));

Parameter    ! Calibrate Leontief coefficient
VA_COEF(i)   Value added (Leontief) coefficient for industries
INT_COEF(c,i) Intermediate (Leontief) coefficient for industries;
VA_COEF(i)   = z0(i)/xfac_f0(i);
INT_COEF(c,i) $xcomp0(c,i) = z0(i)/xcomp0(c,i);

Parameter    ! Calibrate Cobb-Douglas for final users
ALPHA_LF(c,lf) CD Shares of local final users
A_LF(lf)       CD Shift parameter for local final users ;
ALPHA_LF(c,lf) = [xcomp0(c,lf)*pcomp0(c,lf)]/wtot0(lf);
A_LF(lf)       = xtot0(lf)/{PROD[c,xcomp0(c,lf)**ALPHA_LF(c,lf)]}

*===== (4) MODEL =====
Variable
pcomp(c,lu)    Price of Composite dom-imp commodity
p(c,s)         Goods' prices
pfac_f(i)      Composite factor prices

```



```

pfac(f,i)      Price of Factors
ptot(lf)       Price indices for local final users
phi            Exchange rate
pfimp(i)       Import prices
xcomp(c,lu)    Composite commodity
x(c,s,lu)      Intermediate demand
xfac_f(i)      Total factor use
xtot(lf)       Total output of local final user
xfac(f,i)      Factor demand
xfac_i(f)      Total factor use
xtot(lf)       Real expenditure of local final users
xexp(c)        Export demand
xdem(c,s)      Total demand for goods
z(i)           Output of industry
wtot(lf)       Nominal expenditure by local final users
wgdpinc        Nominal GDP - income side
wgdpexp        Nominal GDP expenditure side
delB           Trade Balance
afac(f,i)      Factor using technical change
ffac_i(f)      Factor wage shifter
ffac(f,i)      Factor wage shifter
ftot(lf)       Absorption parameter for local final users
fqexp(c)       Export shifter
OBJ            NLP objective function ;

Equation ! Dom-imp Block
E_pcomp        Price of Composite dom-imp commodity
E_xcomp1       Dom-imp Composite for industries (Leontief)
E_xcomp2       Dom-imp Composite for local final users (Cobb-Douglas)
E_x            Intermediate demand ;

E_pcomp(c,lu)$xcomp0(c,lu).. pcomp(c,lu)*xcomp(c,lu) =e= SUM[s, x(c,s,lu)*p(c,s)];
E_xcomp1(c,i)$xcomp0(c,i).. xcomp(c,i) =e= z(i)/INT_COEF(c,i);
E_xcomp2(c,lf).. xcomp(c,lf)*pcomp(c,lf) =e= ALPHA_LF(c,lf)*wtot(lf);
E_x(c,s,lu).. x(c,s,lu)=e= A_ARM(c,s,lu)*xcomp(c,lu)*[pcomp(c,lu)/p(c,s)]**SIGMA(c);

Equation ! Industry demands
E_pA           Industry cost indices
E_pfac_f       Composite factor prices
E_xfac         Demand for factors
E_xfac_f       Composite factor CES ;

E_pA(i,"dom").. p(i,"dom")*z(i) =e= SUM[c,xcomp(c,i)*pcomp(c,i)] + xfac_f(i)*pfac_f(i);
E_pfac_f(i).. pfac_f(i)*xfac_f(i) =e= SUM[f, xfac(f,i)*pfac(f,i)];
E_xfac(f,i).. xfac(f,i)/xfac_f(i) =e= [afac(f,i)*ALPHA_F(f,i)*pfac_f(i)/pfac(f,i)]
**SIGMA_FAC(i)*A_F(i)**(SIGMA_FAC(i)-1);

E_xfac_f(i).. xfac_f(i)*VA_COEF(i) =e= z(i);

Equation ! Final demanders
E_wtot         Nominal expenditure by local final users
E_ptot         Price indices for local final users
E_xtot         Real expenditure by local final users
E_xexp         Export demand ;

E_wtot(lf).. wtot(lf) =e= ftot(lf)*wgdpinc;
E_ptot(lf).. ptot(lf) =e= [1/A_LF(lf)]*
PROD{c$ALPHA_LF(c,lf),[pcomp(c,lf)/ALPHA_LF(c,lf)]
**ALPHA_LF(c,lf)};

E_xtot(lf).. xtot(lf)*ptot(lf) =e= wtot(lf);
E_xexp(c).. xexp(c) =e= fqexp(c)*[p(c,"dom")/phi]**(-5) ;

Equation ! Total demand and market clearing
E_ffac_i       Total factor use
E_xdemA        Total demand for dom goods
E_xdemB        Total demand for imp goods
E_z(i)         Market clearing ;

E_ffac_i(f).. xfac_i(f) =e= SUM[i, xfac(f,i)];
E_xdemA(c).. xdem(c,"dom") =e= SUM[lu, x(c,"dom",lu)] + xexp(c);

```

```

E_xdemB(c)..      xdem(c,"imp") =e= SUM[lu, x(c,"imp",lu)];
E_z(i)..          z(i) =e= xdem(i,"dom");
Equation      ! Miscellaneous equations
E_pB(i)         Import prices
E_pfac(f,i)     Factor remuneration
E_wgdpinc       Nominal GDP income
E_wgdpexp       Nominal GDP expenditure
E_delB         Trade Balance
E_OBJ          NLP objective function      ;
E_pB(i)..       p(i,"imp") =e= pfimp(i)*phi ;
E_pfac(f,i)..   pfac(f,i) =e= ffac_I(f)*ffac(f,i)* ptot("hou");
E_wgdpinc..     wgdpinc =e= SUM{i, SUM[f,xfac(f,i)*pfac(f,i)]};
E_wgdpexp..     wgdpexp =e= SUM[(c,lf)$xcomp0(c,lf), xcomp(c,lf)*pcomp(c,lf)]
                + SUM[c, xexp(c)*p(c,"dom") - xdem(c,"imp")*p(c,"imp")];
E_delB..       delB*wgdpinc =e= SUM[c, xexp(c)*p(c,"dom") - xdem(c,"imp")*p(c,"imp")];
E_OBJ..        OBJ =e= 1;
*--- Initialize levels variables
xfac_f.l(i)     = xfac_f0(i);
xcomp.l(c,lu)   = xcomp0(c,lu);
x.l(c,s,lu)     = x0(c,s,lu);
xfac.l(f,i)     = xfac0(f,i);
xtot.l(lf)      = xt0(lf);
xexp.l(c)       = xexp0(c);
xfac_i.l(f)     = xfac_i0(f);
xdem.l(c,s)     = xdem0(c,s);
xtot.l(lf)      = xt0(lf);
z.l(i)          = z0(i);
pcomp.l(c,lu)   = pcomp0(c,lu);
pfac_f.l(i)     = pfac_f0(i);
pfac.l(f,i)     = pfac0(f,i);
p.l(c,s)        = p0(c,s);
p.l(i,s)        = p0(i,s);
ptot.l(lf)      = ptot0(lf);
wtot.l(lf)      = wt0(lf);
wgdpinc.l       = wgdpinc0;
wgdpexp.l       = wgdpexp0;
delB.l          = delB0;
pfimp.l(i)      = pfimp0(i);
phi.l           = phi0;
fqexp.l(c)      = fqexp0(c);
ftot.l(lf)      = ftot0(lf);
afac.l(f,i)     = afac0(f,i);
ffac_I.l(f)     = ffac_I0(f);
ffac.l(f,i)     = ffac0(f,i);
OBJ.l           = 1;
model simple /all/;
*===== (5) Simulation Closure and Shocks =====
* Exogenous Variables in standard closure
afac.fx(f,i)    = afac0(f,i);
ffac.fx(f,i)    = ffac0(f,i);
fqexp.fx(c)     = xexp0(c);
ftot.fx(lf)     = ftot0(lf);
pfimp.fx(i)     = pfimp0(i);
phi.fx          = phi0;
xfac_i.fx(f)    = xfac_i0(f);
pcomp.fx(c,lu)$ (xcomp0(c,lu)=0) = 1;
xcomp.fx(c,lu)$ (xcomp0(c,lu)=0) = 0;
option sysout=on;
simple.holdfixed=1;      ! assist check that n.equ = n.endogenous var
simple.iterlim=0;        ! dummy simulation to test calibration
simple.tolinfrep=1E-8;
solve simple maximizing obj using NLP; ! OR: solve simple using mcp;

```

```

*--- Closure changes/Swaps
xfac.fx("capital",i) = xfac0("capital",i);    ! fix industry capital stocks
ffac.lo("capital",i) = -INF;
ffac.up("capital",i) = +INF;
ffac.l("capital",i) = ffac0("capital",i);

ffac_I.fx("capital") = ffac_I0("capital");    ! unfix total capital stock
xfac_i.lo("capital") = -INF;
xfac_i.up("capital") = +INF;
xfac_i.l("capital") = xfac_i0("capital");

*--- SHOCKS
afac.fx("labor","Srv") = 0.98; !Improvement in Service factor productivity

*--- Solve model
simple.reslim=19000;
simple.iterlim=200;
simple.holdfixed=1;
solve simple maximizing obj using NLP; ! OR: solve simple using mcp;

*===== (6) Simulation Results =====
Parameter
LGDP          Laspeyeres GDP quantity index
PGDP          Paasche GDP quantity index
CH_XGDPINC    Percentage change in Fisher (ideal) GDP quantity index
CH_Z(i)       Percentage change in output of industries
CH_X(c,s,lu)  Percentage change in intermediate demand
CH_XFAC(f,i)  Percentage change in factor demand
CH_PFAC(f,i)  Percentage change in price of factors
CH_PFAC_F(i)  Percentage change in composite price of factors
CH_P(c,s)     Percentage change in basic prices
CH_WGDPINC    Percentage change in nominal GDP income side
CH_WGDPEXP    Percentage change in nominal GDP expenditure side
CH_DELB       Ordinary change in ratio nominal trade balance to GDP;

LGDP = SUM[i,xfac_f.l(i)*pfac_f0(i)] / SUM[i,xfac_f0(i)*pfac_f0(i)];
PGDP = SUM[i,xfac_f.l(i)*pfac_f.l(i)] / SUM[i,xfac_f0(i)*pfac_f.l(i)];
CH_XGDPINC = [SQRT(LGDP*PGDP)-1]*100;
CH_Z(i)      = (z.l(i)/z0(i)-1)*100;
CH_X(c,s,lu) $x0(c,s,lu) = (x.l(c,s,lu)/x0(c,s,lu)-1)*100;
CH_XFAC(f,i) $xfac0(f,i) = (xfac.l(f,i)/xfac0(f,i)-1)*100;
CH_PFAC(f,i) $pfac0(f,i) = (pfac.l(f,i)/pfac0(f,i)-1)*100;
CH_PFAC_F(i) $pfac_f0(i) = (pfac_f.l(i)/pfac_f0(i)-1)*100;
CH_P(c,s)    = (p.l(c,s)/p0(c,s)-1)*100;
CH_WGDPINC   = (wgdpcinc.l/wgdpcinc0-1)*100;
CH_WGDPEXP   = (wgdpcexp.l/wgdpcexp0-1)*100;
CH_DELB      = (delB.l-delB0);

Display CH_Z,CH_XFAC,CH_PFAC,CH_PFAC_F,CH_P,CH_WGDPINC,CH_XGDPINC,CH_WGDPEXP,CH_DELB;

*---Export results to GDX
EXECUTE_UNLOAD 'RESULTS', CH_Z,CH_XFAC, CH_PFAC, CH_PFAC_F,CH_P, CH_WGDPINC,
CH_XGDPINC, CH_WGDPEXP, CH_DELB;

```

6.4 The SIMPLE model in MPSGE

Table 6 shows how the SIMPLE model might be represented²³ in MPSGE code. In a GAMS/MPSGE model the code sections for calibration and model equations are replaced by a section written in MPSGE's own syntax. Prior to this, the GMS file starts with standard GAMS code for declaring sets and data, reading in the data, and defining some consequential totals, such as GDP and output by industry.

The MPSGE syntax extends from the line "\$ontext" till the line "\$offtext". An MPSGE model revolves around three types of entity:

- constant-returns-to-scale producers, listed under \$sectors.
- goods and primary factors (defined by a unique price), defined in the \$commodities section.
- consumers (either government or private), in the \$consumers section

The \$Prod section compactly expresses the production technology—it is the text equivalent of Figure 1. It says that each industry is the sole producer of a domestic commodity, price $P(j)$. Domestic and imported commodities are combined to form a CES composite (the 'tl' nest), while primary factors are likewise combined in the nest 'va'. At the top level, production is Leontief ('s:0') in composite commodities and composite value-added.

Similarly, the second \$demand block says that each of the 'fd' set (hou, gov, inv) is a Cobb-Douglas consumer (s:1) of composite commodities which are again CES combinations of domestic and imported commodities.

These statements would suffice for the simplest type of MPSGE model. Unfortunately the SIMPLE model defined in previous sections breaks the MPSGE paradigm in three respects:

- Export demand curves are not flat (the MPSGE expectation) but are of a constant elasticity.
- Not all the primary factors are perfectly mobile (the MPSGE expectation); capital may be sector-specific.
- The value of absorption is not equal to factor earnings (the MPSGE expectation) but is rather constrained by a rule that the trade balance must move in proportion to GDP.

Luckily it is always possible (with some ingenuity and a good understanding of MPSGE) to accommodate such unorthodoxies. The \$auxiliary and \$constraint sections do this job. For example, the first \$constraint defines the constant elasticity export demand curves.

Two types of capital are defined: mobile capital has a common price [$W(\text{"capital"})$], while rents vary for sector-specific capital [endowment $ssk(j)$ with price $RK(j)$]. Changing the closure from mobile to fixed capital is accomplished by altering the database: moving all capital earnings to accrue to the fixed type of capital.

The implementation of the balance of trade constraint is more complicated. The first \$demand block states that absorption is, as usual, equal to income; but that income includes not only primary factor income, but also a gift from abroad of value $PFX*bopdef*GDP$, where $bopdef$ is the initial BOT/GDP ²⁴ ratio.

Even with these complications the MPSGE code is only half the length of the corresponding straight GAMS code for calibration and model equations. And indeed, it does *more* than the conventional GAMS code that it replaces:

- the implicit calibration is done in the most efficient possible manner.
- equations and variables for intermediate use are effectively substituted out, reducing system size.
- while both GEMPACK and straight GAMS implementations assume (not unusually) that all outputs and prices remain strictly positive, MPSGE sets up a full system of complementarities which allows for inactive sectors and slack markets. This considerably extends the scope of the system.²⁵

The remainder of the GMS file is ordinary GAMS code. There are three simulations: a test simulation to reproduce the benchmark data; and two simulations of improved Services labor productivity, the first with mobile capital, the second with sector-specific capital.

A small selection of percent-change results from the last two simulations are stored in the matrix "impact". The formulae presuppose a knowledge of MPSGE's default rules for price and quantity normalization. For

²³ We are grateful to Tom Rutherford for preparing the MPSGE implementation.

²⁴ Note the later \$constraint stating that GDP is a 1-based index!

²⁵ Both GEMPACK and straight GAMS allow complementarities to be set up—at the cost of more complex code.

example, sector outputs are initially 1, so $100 \cdot (Y.L(j)-1)$ is the percent change in outputs. Similarly most prices are initially 1.

Similarly some levels results are stored in the matrix "summary". In this case, the role of "summary" is mostly to check that the BOT/GDP ratio does in fact remain constant.

Table 6 GMS Input file for example MPSGE model and simulation

```
$title A Simple General Equilibrium Model
* Next line enables end-of-line comments following #
$eolcom #

set
    i      Goods and sectors      # COM, or IND
    fd      Components final demand /hou,gov,inv/,      # LOCFIN
    f      Primary factors /labor,capital/,      # This capital is mobile      # FAC
    src      Sources /dom,imp/;      # SRC

$gdxin input.gdx
$load i
parameter
    use(i,src,*)      Value of inputs,
    factor(f,i)      Factor demands,
    sigma(i)      CES domestic-import elasticity;
$loaddc use factor sigma
alias (i,j);
parameter
    output(i)      Aggregate output      COSTS(c)
    expend(fd)      Final demand expenditure      COSTS(L)
    export(i)      Benchmark exports      USE(c:dom:exp)
    epsilon(i)      Export demand elasticity      5.0
    bopdef      Base year current account deficit
    sigmaf(i)      Elasticity of substitution among factors      one half
    prd(f,j)      Productivity index      AFAC(f:i)
    ssk(j)      Sector-specific capital
    endow(f)      Primary factor endowment vector      SUM(i:FACTOR(f:i))
    gdp0      Benchmark gross domestic product      VGDP      ;

prd(f,j) = 1;
output(j) = sum((i,src), use(i,src,j)) + sum(f,factor(f,j));
export(i) = use(i,"dom","exp");
expend(fd) = sum((i,src),use(i,src,fd));
epsilon(i) = 5;
sigmaf(i) = 0.5;
ssk(j) = 0;      # start with no sector-specific capital
bopdef = sum((i,j), use(i,"imp",j)) + sum((i,fd),use(i,"imp",fd)) - sum(i,export(i));
endow(f) = (sum(j,factor(f,j)));
gdp0 = sum(f,endow(f));

$ontext
$model:simple
* Comment within MPSGE code
$sectors:
    Y(j)      ! 1-based index of production      ZINDEX
$commodities:
    P(j)      ! Price of domestic output      p(c:dom)
    PFX      ! Exchange rate (Dollar domestic per foreign dollar) PHI
* There are 2 kinds of capital -
* Mobile has price W("capital") and fixed capital in sector j has price RK(j)
* In SIMPLE.TAB factor price is pfac(f:i) which might depend on using industry i.
    W(f)$endow(f)      ! Factor price      PFAC(f:i)
    PGNP      ! Price index for factor income
    RK(j)$ssk(j)      ! Rental rate on sector-specific capital      PFAC(capital:i)
$consumers:
    GNP      ! Aggregate national income
    RA(fd)      ! Expenditure by representative agent      WTOT(L)
$auxiliary:
    X(i)      ! Export quantity      XEXP(c)
```

```

    VX                ! Value of exports (Foreign currency)  COSTS(exp) in domestic dollars
    GDP               ! 1-based index of the value of GDP in foreign dollars

$Prod:Y(j)           s:0 va:sigmaf(j) i.tl:sigma(i)
    o:P(j)           q:output(j)
    i:W(f)           q:(factor(f,j)/prd(f,j)) p:prd(f,j)    va:
    i:RK(j)          q:ssk(j)                                va:
    i:PFX#(i)         q:use(i,"imp",j)                        i.tl:
    i:P(i)           q:use(i,"dom",j)                        i.tl:

$report:
    v:wd(f,j)        i:W(f) prod:Y(j)
    v:VINTIMP(j)      i:PFX#(i) prod:Y(j)

$demand:GNP
* Next two lines add up to primary factor income
    e:W(f)           q:endow(f)
    e:RK(j)          q:ssk(j)
* Next 2 lines seem to cancel each other out
    e:P(i)           q:(-1)                                r:X(i)
    e:PFX            q:1                                    r:VX
* Next says there is a foreign gift of PFX*bopdef*GDP local dollars
    e:PFX            q:bopdef                               r:GDP
    d:PGNP

$demand:RA(fd) s:1 i.tl:sigma(i)
* Next says income(fd)=PGNP*expend(fd) for each of the 3 fd's
    e:PGNP           q:expend(fd)
* Next 2 lines say all fd have armington demands for import v domestic with elasticity sigma
* and that elasticity is 1 between different commodities
    d:P(i)           q:use(i,"dom",fd)                      i.tl:
    d:PFX#(i)        q:use(i,"imp",fd)                      i.tl:

$report:
    v:VFINIMP(fd)    d:PFX#(i) demand:RA(fd)

$constraint:X(i)
    X(i) =e= export(i) * (PFX/P(i))**epsilonX(i);

$constraint:VX
    PFX * VX =e= sum(i, P(i)*X(i));

$constraint:GDP
* GDP is a 1-based index of the value of GDP in foreign dollars
* Actual GDP is LHS of next equation.
    GDP*gdp0*PFX =e= sum(f, W(f)*endow(f)) + sum(j, RK(j)*ssk(j));

$offtext
$sysinclude mpsgeset simple

* NOTE. Foreign gift divided by actual GDP is equal to
* PFX*bopdef*GDP/GDP*gdp0*PFX = bopdef/gdp0 which is constant
GDP.L = 1;
X.L(i) = export(i);
X.FX(i)$(export(i)=0) = 0;
VX.L = sum(i, X.L(i));

* Assign a numeraire to simplify comparison with the GEMPACK solution:
PFX.FX = 1;

simple.iterlim = 0;
simple.workspace = 50;
$include simple.gen
solve simple using mcp;
abort$(simple.objval>1e-2) "Benchmark without SSK inconsistent.";

* Swap the closure and verify
ssk(j) = factor("capital",j);
factor("capital",j) = 0;
endow("capital") = 0;

simple.iterlim = 0;
simple.workspace = 50;
$include simple.gen
solve simple using mcp;
abort$(simple.objval>1e-2) "Benchmark with SSK inconsistent.";

```

```

display VINTIMP.L;

parameter summary Summary of model results ;
summary("ActualGDP", "initial") = sum(f, W.L(f)*endow(f)) + sum(j, RK.L(j)*ssk(j)) ;
summary("VIMP", "initial") = sum(j, VINTIMP.L(j)) + sum(fd, VFINIMP.L(fd)) ;
summary("VEXP", "initial") = sum(i, P.L(i)*X.L(i)) ;
summary("BOP", "initial") = summary("VEXP", "initial") - summary("VIMP", "initial") ;
summary("BOPRatio", "initial") = summary("BOP", "initial")/summary("ActualGDP", "initial") ;

*      Simulate a productivity shock for labor inputs to the service sector:
prd("labor", "srv") = 1/0.98;

*      First simulation is based on mobile capital:
factor("capital", j) = ssk(j);
endow("capital") = sum(j, ssk(j));
ssk(j) = 0;

simple.iterlim = 20000;
$include simple.gen
solve simple using mcp;

parameter      impact Economic impact;
impact("Y%", j, "mobileK") = 100 * (Y.L(j)-1);
impact("X%", j, "mobileK")$export(j) = 100 * (X.L(j)/export(j)-1);
impact("P%", j, "mobileK") = 100 * (P.L(j)/PFX.L-1);
impact("P%", f, "mobileK")$endow(f) = 100 * (W.L(f)/PFX.L-1);
impact("RK%", j, "mobileK")$ssk(j) = 100 * (RK.L(j)/PFX.L-1);

summary("ActualGDP", "mobileK") = sum(f, W.L(f)*endow(f)) + sum(j, RK.L(j)*ssk(j)) ;
summary("VIMP", "mobileK") = sum(j, VINTIMP.L(j)) + sum(fd, VFINIMP.L(fd)) ;
summary("VEXP", "mobileK") = sum(i, P.L(i)*X.L(i)) ;
summary("BOP", "mobileK") = summary("VEXP", "mobileK") - summary("VIMP", "mobileK") ;
summary("BOPRatio", "mobileK") = summary("BOP", "mobileK")/summary("ActualGDP", "mobileK") ;

*      Second simulation is based on sector-specific capital:
ssk(j) = factor("capital", j);
factor("capital", j) = 0;
endow("capital") = 0;

simple.iterlim = 20000;
$include simple.gen
solve simple using mcp;

impact("Y%", j, "SSK") = 100 * (Y.L(j)-1);
impact("X%", j, "SSK")$export(j) = 100 * (X.L(j)/export(j)-1);
impact("P%", j, "SSK") = 100 * (P.L(j)/PFX.L-1);
impact("P%", f, "SSK")$endow(f) = 100 * (W.L(f)/PFX.L-1);
impact("RK%", j, "SSK")$ssk(j) = 100 * (RK.L(j)/PFX.L-1);

summary("ActualGDP", "SSK") = sum(f, W.L(f)*endow(f)) + sum(j, RK.L(j)*ssk(j)) ;
summary("VIMP", "SSK") = sum(j, VINTIMP.L(j)) + sum(fd, VFINIMP.L(fd)) ;
summary("VEXP", "SSK") = sum(i, P.L(i)*X.L(i)) ;
summary("BOP", "SSK") = summary("VEXP", "SSK") - summary("VIMP", "SSK") ;
summary("BOPRatio", "SSK") = summary("BOP", "SSK")/summary("ActualGDP", "SSK") ;

option impact:3:1:1;
display impact;

option summary:5:1:1;
display summary;
EXECUTE_UNLOAD 'RESULTS', Impact;

```

6.5 All three give the same results

As expected, the same results are generated by GEMPACK, GAMS and MPSGE. You can check this by downloading the models from <http://www.monash.edu.au/policy/archivep.htm#TPMH0103>.

6.6 Understanding the results

Understanding the results means working out how the shocks, together with the starting data and the theory of the model, produce changes in endogenous variables. For each important endogenous result, you want to understand both the sign and the approximate magnitude (why is it 3 and not 0.1, for example).

This is a complicated topic which we cannot hope to cover in detail in this paper. To give some of the flavor, consider the following question drawn from our example simulation.

Question. Does improved service-worker productivity mean service-workers get fired? How come there is a 10% productivity increase but only a 2% job loss in the Service industry?

This question is motivated by the result that employment in the Service industry falls by 2.214% in the simulation. This is variable `xfac("Labor","srv")` in the GEMPACK and GAMS implementations.

It turns out that the linear or percentage-change form of the equations is usually more helpful for understanding results than the levels form. GEMPACK provides an viewing program, AnalyseGE, which streamlines this kind of analysis. Below we illustrate this to (partly) answer the question above.

The equation in SIMPLE.TAB which "explains" the variable `xfac` is the CES factor demand equation

```
E_xfac # Factor demands # (a11,f,FAC)(a11,i,IND)
xfac(f,i) = z(i) + afac(f,i) - 0.5*[pfac(f,i)+afac(f,i)-pfac_f(i)];
```

You can ask AnalyseGE to evaluate the variable `xfac` (it tells you -2.214 for the "Labor","srv" part) and to decompose the right-hand side. The decomposition (when `f="Labor"` and `i="srv"`) shows you the table

```
z      4.817
afac   -10.000
e1_pfac  3.359
Total   -1.824
```

The shock value `afac` is directly in this equation. Productivity has increased by 10%. Now overall activity "z" in this industry has increased by 4.817%. The `e1_pfac` value of 3.359 is the value of the last, relative-price, term $-0.5[\dots]$ on the right-hand side of the equation. The $[\dots]$ term is about -6.7 because the effective cost of labor in this industry, $\text{pfac}(f,i)+\text{afac}(f,i)$, falls faster the average cost of factor inputs $\text{pfac}_f(i)$ in this industry, resulting in a boost of 3.359% to industry employment. That boost arises because with capital fixed, an increase in output must raise the labor/capital ratio, and thus output-per-worker falls.

Although the productivity increase would seem to decrease employment by 10%, this is offset by the increased employment needed because activity in the industry increases by 4.8% and because productivity falls with output, resulting in a further 3.36% increase in employment. Overall employment then falls only by about 2%. Notice that the elasticity value of 0.5 enters into these calculations. We may surmise that if the elasticity of substitution between capital and labor was 1 (rather than 0.5) the net effects on employment would be positive.

The "Total" result shown by AnalyseGE, -1.824 (which is obtained by adding the `z`, `afac` and `e1_pfac` components above) is not exactly equal to the result -2.214 for `xfac("Labor","srv")`, since the linearized equations in the TAB file are not satisfied exactly by the accurate results.²⁶

The above analysis could also be carried out by GAMS users. But probably they would need to explicitly add into the post-solution analysis of their GMS file the linearized version of equation `E_xfac` and formulae to calculate the different parts of the decomposition discussed above.

7. The curse of dimensionality

Policy-oriented CGE modelers are constantly tempted to increase the numbers of commodities (C), industries (I), regions (R) or household types (H) distinguished in their models. More sectors allow for greater technological detail, distinguishing, for example, between several technologies for producing electricity. Again, to model a tariff reform, where benefits arise from reducing the dispersion of tax rates, it helps to distinguish commodities finely; ideally at the HS6 level by which tariff rates vary. Dividing households by income decile, urban/rural, and ethnicity, we can easily arrive at $H=100$. It may be politically unacceptable to reduce the number of regions, R, by combining, say, the Dakotas, even if simulation results were not much affected.

But this detail, the special strength of CGE, comes at a high price: it can prohibitively increase the computing resources (memory and especially time) needed to solve the model. The size of equations and variables is often the *product* of various dimensions: household demands may be dimensioned $C \times H \times R$ while intermediate demands might be size $C \times I \times R$ —ie, would increase as the square of the number of sectors.

²⁶ See, for example, <http://www.monash.edu.au/policy/gp-lineq.htm>

Further, theory suggests that the cost of solving a linear system (an important subtask in solution algorithms) follows the *cube* or the *square* of the number of equations. Hence, we may guesstimate that doubling the number of commodities and industries in a model may increase solution time by a factor of 2^5 (between 2^4 and 2^6).

To illustrate this idea we solved the SIMPLE model presented earlier with databases of several sizes, and using several solution methods. The resulting solution times are shown in Table 7 below²⁷. The first 2 rows use GEMPACK in conjunction with the SIMPLE.TAB of Table 2. For the first row, the GEMPACK program TABLO has written a model-specific Fortran program, SIMPLE.FOR, which has been compiled using an optimizing compiler to produce SIMPLE.EXE. This is about twice as fast as the alternative GEMPACK approach (row 2), which uses an interpreter, GEMSIM. The next 2 rows use the "straight GAMS" code of Table 5. Row 3 uses the PATH MCP solver while row 4 uses the CONOPT NLP solver. Finally row 5 shows times²⁸ for the MPSGE specification of Table 6.

Table 7 Solution time (seconds) versus no. of sectors

Sectors:	100	150	200	250	300	350	400	500	Elasticity of time w.r.t. sectors
GEMPACK exe	1	2.2	3.7	5.8	8.6	11.7	16.4	26.0	2.9
GEMSIM	1.88	4.0	7.1	11.2	16.2	22.4	29.7	49.3	3.3
GAMS MCP	2.4	8.1	31.5	111.5	324.6	979.6	1763.0	7550.4	16.4
GAMS NLP	1.6	3.9	8.6	14.0	115.5	235.2	400.7	1107.0	8.8
MPSGE	2.1	7.1	18.2	40.1	78.9	138.5	225.9	524.3	6.6

While during the 1980's the solution time for a 200-sector model might well allow the modeler to enjoy a night's sleep, today's tools do not allow enough time to make a cup of coffee! Only as the number of sectors approaches 400 do solution times become a real concern. And which sane CGE modeler could want more than 200 sectors?

In fact, multi-regional and multi-temporal CGE models, which comprise a collection of interlinked models rather like SIMPLE for each region and timeperiod, can easily reach dimensions much larger than the 500-sector SIMPLE. Such models solve slowly. Modelers are forced to adopt compromises to keep compute times within limits.

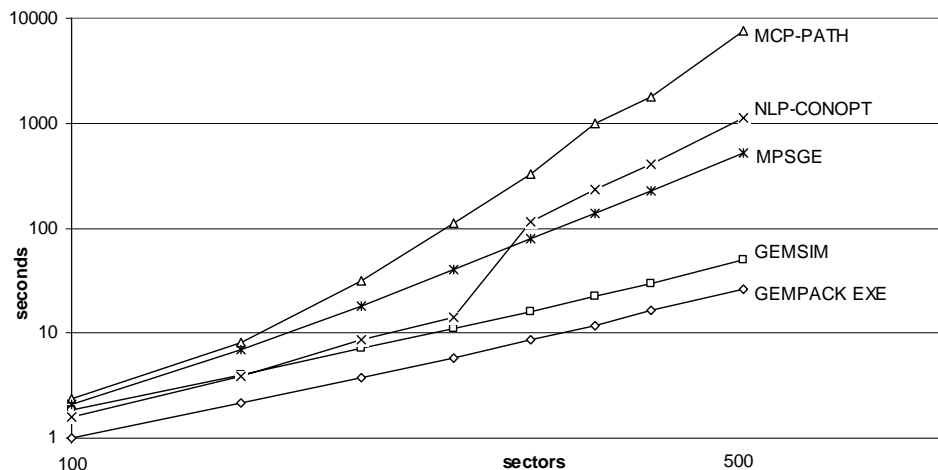


Figure 3 Solution time versus no. of sectors: Log-log scale

Very likely a skilled practitioner could 'tweak' any of these implementations to halve solution times. Nonetheless, Table 7 well illustrates how moving from, say, 400 to 500 sectors dramatically lengthens solution time. The point is even clearer in Figure 3, which graphs the same figures on a log-log scale. Each solution method yields a straight-line graph²⁹, the slope of which shows the elasticity of solution time with

²⁷ The models were solved on one core of an Intel Q9550 cpu, under 64-bit Vista. The databases were sparse: about 70% zeroes. All material needed to repeat the simulations is downloadable: item TPMH0103 at <http://www.monash.edu.au/policy/archivev.htm>.

²⁸ MPSGE times have been halved, because the GMS file specified two simulations.

²⁹ For CONOPT, there is a break in the line, between 250 and 300 sectors.

respect to number of sectors. That elasticity, measured over the 400-500 interval, is shown in the final column of Table 7, and is markedly lower for the two GEMPACK implementations.

The main reason that GEMPACK scales well in this example is that high-dimension equations of size COM*IND have been automatically substituted out. Hence, the final (after-substitution) number of equations follow the number of sectors (rather than its square)³⁰. MPSGE achieves a similar effect.

A "straight GAMS" user could enjoy a similar speedup by manually substituting out high-dimension equations. For example (see subsection 4.2.1), the equation

$$x(c, s, u) = A_ARM(c, s, u) * xcomp(c, u) * [pcomp(c, u) / p(c, s)] ** SIGMA(c);$$

could be used to substitute out the LHS variable x . That strategy could lead to complicated, hard-to-maintain, model code. Alternatively the GAMS modeler may avoid complicated mechanisms which are both commodity- and sector-specific. For example, import-domestic substitution might be modeled as commodity-specific but not user-specific—greatly simplifying the model, without, usually³¹, much affecting results.

8. Checking and debugging models

How can we be sure that a CGE model is correctly coded, or that it is using reasonable data? Careful "eye-balling" of the code is too time-consuming to do often. Can the software help us?

Standard techniques for checking models include:

- Balance and sign checks on input data.
- In GAMS only, a check that the calibrated or benchmark prices and quantities satisfy model equations (in GEMPACK there are no levels prices and quantities).
- Standard experiments, such as a numeraire shock, which should increase all prices and values without affecting quantities.
- Balance checks on updated data, or a check that a redundant accounting constraint is satisfied by the results (often termed the "Walras" check).

Both GAMS and GEMPACK have the ability to apply such checks and to abort simulations with an appropriate message if conditions are not satisfied. Problems are often identified in this way. Nonetheless, such checks do not screen out all bugs:

- Under pressure, modelers might comment out the checking lines, and forget to reinstate them later.
- Many faults might evade automatic checks, including wrongly signed elasticities, or inconsistent ordering of sectors or regions in different data sources.

To detect such problems and locate their source, there is really no substitute for detailed examination of data and results. GEMPACK attempts to make this process as easy and painless as possible. There is a specialized viewer for data (ViewHAR), another for results (ViewSOL), and a third (AnalyseGE) that allows viewing of data or results (or any function of the two) by clicking on coefficients or variables in the model code. These viewers are quite sophisticated; ViewSOL, for instance, is able to load one or two timeseries of model solutions and display these as year-on-year or cumulative changes, or to display the difference between control and perturbed scenarios.

9. Comparison of GAMS and GEMPACK

GEMPACK was designed to solve CGE models; it performs a more specialized task than GAMS. The TAB file specifying GEMPACK model describes, in differential form, the model's non-linear equations: the software uses those equations to do numerical integration and find one new levels solution. It is not designed as a general-purpose programming tool.

GAMS, as well as interfacing to various optimizers and equation solvers, provides a more feature-rich computer language. It allows loops and other control constructs, include files, macro preprocessing, and,

³⁰ For N=500, the original number of equations was just over 1 million; substitution eliminated 75% of these.

³¹ The exception would be when some users of an imported good have a markedly different import share from other users. For example, in analysing textile tariff reductions we may observe that the domestic TCF industry is a particularly intensive user of imported TCF. As production is globalized we see these cases more often. To model them properly, import/domestic substitution must be user-specific.

crucially, the ability to execute other programs from within a GAMS script. This makes GAMS immensely versatile.

For example, it would be fairly easy, within GAMS, to repeat 30 times the same simulation with different values of the capital-labor substitution elasticity, and to have another program such as Excel graph some results. The task could be completely automated from within one GMS file.

To do the same job with GEMPACK, you would use another scripting language (probably the Windows BATch language) to run 30 simulations, to call other GEMPACK programs to collate and reformat the results, and finally to invoke Excel. This means that a serious GEMPACK user needs to master another skill (batch scripting).

GAMS also allows for a user to add features not provided by, or envisaged, by the developers. For example, suppose you want to define and use repeatedly a function not explicitly provided. A programmer can do this with GAMS³². But GEMPACK users must ask the developers to add a new function.

By giving access to low-level features of the software, GAMS makes it possible and easy for power users to do things not explicitly provided for by the developers. And they can write so-called `libinclude` files which give high level functionality to other users.³³

By contrast GEMPACK makes some complex tasks easy even for computer novices. Examples are:

Recursive-Dynamic Models. While it is possible to implement such models in GAMS, GEMPACK provides the RunDynam windows interface, which does all the book-keeping to organise the runs in sequence, and to assemble and report the results. GAMS users must keep track of the sequence of runs needed to cover the years in sequence and to organise the results. Provided your model fits within the (wide) range of cases allowed, using RunDynam is the easier route. But if you want to do something outside that framework, you may find that easier with GAMS.

Systematic Sensitivity Analysis. The model must be run many different times, each time with different parameters and/or shocks and the results assembled to report means and standard deviations across the runs. A power GAMS user can write a procedure to vary the parameters or shocks in the appropriate way and keep track of all the relevant files and runs to do this. GEMPACK provides a windows interface (in RunGTAP and RunGEM) for doing all of this. The user just needs to point and click the few details and the software does the rest. But of course, there are restrictions on the sorts of variations allowed. If your task falls outside those allowed, GEMPACK users would also need to handle all the details themselves.

Decomposition of Results due to Shocks. This tells us, for example, how much of the USA GDP change due to a bilateral tariff reform arises from the USA tariff reduction, and how much from the EU tariff reduction. The theory is fairly simple - see Harrison, Horridge and Pearson (2000), and may be implemented by GEMPACK users with 2 or 3 lines of code at modest additional computational cost. Power GAMS users can carry out this decomposition in GAMS³⁴, with a bit more effort.

Other differences between GAMS and GEMPACK include:

9.1.1 Optimization

GAMS is first and foremost an optimization package, with CGE modeling arising as an unexpected use. So it provides state-of-the-art capability for optimization.

CGE modelers may want to solve optimization problems in simulations which involve choice of optimum shock (eg optimal tax). But optimization problems also arise in procedures for preparing or massaging data to use in CGE models. For example, minimum entropy methods might be used to balance a SAM.

GEMPACK offers no direct features for optimization. However, by using the first-order conditions directly, it is possible to solve simpler optimization problems.

³² See <http://www.gams.com/docs/extfunc.htm>

³³ See, for example, the GAMS2HAR routines which allow conversion of GAMS text data to GEMPACK Header Array form, at <http://www.monash.edu.au/policy/gp-g2har.htm>

³⁴ See for example http://www.mpsge.org/mainpage/hhp_gtm.htm or Böhringer and Rutherford (2000).

9.1.2 Complementarities

GAMS comes with powerful algorithms (such as PATH) for explicitly modeling complementarities. GEMPACK also has explicit capabilities for handling these, which work well for most problems arising naturally in policy models, but the algorithms used by GEMPACK make this a less natural vehicle for such problems.

9.1.3 Distributing models to non-licencees

GEMPACK allows for models to be distributed as stand-alone executable images (eg GTAP.EXE) which may be run without any licence. Thus RunGTAP, an IDE for running the GTAP model, is available without cost. Only GAMS licencees can solve GAMS models.

9.1.4 Training and documentation

Several complementary techniques are used to teach CGE modeling, either for GAMS or GEMPACK:

The apprenticeship system: Most CGE modelers would learn their craft during graduate study, or at work.

Training courses: Intensive (typically one-week) courses are offered, often for beginners though also for more advanced users. Many of the GAMS courses are offered by third parties, rather than directly by the GAMS corporation.

Supplied documentation: Both GAMS and GEMPACK have produced printed documentation (see Brooke et al, 1992, and Pearson et al, 2002-8), but it is difficult to make such documentation comprehensive, up-to-date and accessible. The printed manuals are supplemented by (indeed, mostly replaced by) online help and by much instructional material from the respective websites.

On-line forums: such as the GAMS and GEMPACK mailing lists. The GAMS list is particularly active.

10. Conclusion

10.1 What are the computational challenges ?

Our survey has been somewhat retrospective, showing how standard software packages have enabled the ideas of CGE pioneers of the 1970s and 1980s to be implemented in today's more standardized software environment. That standardization has facilitated the spread of standard CGE models within and beyond academic circles. But what are today's newer ideas and techniques, and will CGE software evolve to make them practical?

- The combination of dynamic programming with explicit representation of uncertainty is computationally challenging. The two ideas are united in the area of dynamic stochastic general equilibrium (DSGE). Currently, DSGE models contain too few sectors to qualify as CGE models—but this may change. The DYNARE software (see <http://www.dynare.org>) is being used by some modelers, especially those doing modeling for central banks (<http://www.dsge.net>). It is tailored to dynamic stochastic general equilibrium (DSGE) and overlapping generations (OLG) models, that include forward-looking behavior. Tools for carrying out global sensitivity analysis are being incorporated into DYNARE³⁵.
- Agent-based modeling is an increasingly popular paradigm, which may be amenable to a high level of parallelization in computing.
- Some theorists complain that multiple equilibria are endemic in general equilibrium models, yet CGE practitioners rarely encounter them—partly because of the usual form of CGE models. However, approaches such as game theory can easily lead to multiple local equilibria, which standard CGE software does not easily identify.

Practitioners who experiment with these ideas use a variety of software, including MATLAB and even Excel. Some DSGE modelers still write Fortran code.

³⁵ Both GAMS and GEMPACK include capabilities for solving forward-looking models but have nothing like the global sensitivity tools in DYNARE.

The Institute of Computational Economics at Argonne and Chicago carries out an active program encouraging interaction between economists and algorithm specialists. The Institute provides regular training opportunities for professionals in both areas. See <http://ice.uchicago.edu/info.htm> for more details.

Another challenge is, to link together several models, for example to link a world model such as GTAP with a more detailed model of one country; or instead to link a CGE model with a climate model and perhaps also with an agronomist's model relating farm output to climate. The collection of models is supposed to jointly determine a range of economic and other results. Practical problems include: different software and data formats for the different models; model databases which may not be mutually consistent; and the likelihood that some variables are modeled independently and differently in two or more models. The typical solution is to construct a master program which will manage communication between the models and iteratively solve them in turn until all results are consistent. The master program is likely to be a "home-brewed" one-off which only ever runs on the computer network where it was developed—rather like the first CGE models. There is scope, then, to develop both a general conceptual framework for model linking, and software which is able to coordinate simulations from a range of model types.

10.2 Transferring models between the software platforms

As described in this paper's Introduction, early CGE modelers worked alone or in small groups using customized solution routines and model-specific code. It was therefore difficult to replicate model results and model documentation had to be based on trust.

Great steps have been made since those early days. Standardized modeling packages have lowered the entry costs to CGE and enormously increased the transparency of models. The establishment of CGE modeling teams in government departments has created a job market for CGE modelers, who will have transferable skills.

Nonetheless, there is still some distance to go.

- In the Introduction, we said: "Most models implemented with one of these could, at least in principle, be implemented with either of the other two and yield the same results." However, when we tried this with the SIMPLE model in section 6, we were surprised how difficult it was in practice.
- It may take 5000 lines of code to implement a large CGE model. To translate such a model from GAMS to GEMPACK or vice versa would be a large task. To understand such a model in detail rather than in outline would similarly require some months of work, even for a very experienced modeler who normally works with the relevant software (GAMS or GEMPACK).
- Most experienced CGE modelers are comfortable and fluent with only one modeling system, in which they have invested considerable time and effort. They are naturally reluctant to contemplate changing. Modelers become attached to their accustomed software, notation and equations and closure.
- While it is easy to translate data between the HAR and GDX file formats, there is no automated way to translate model code from GAMS to GEMPACK or vice-versa. A possible solution would be to represent CGE models in a third, more abstract language from which either GAMS or GEMPACK code could be automatically generated. The MPSGE language approaches this ideal. However, such systems impose a template or pattern on the model. Trend-leading CGE modelers, who by definition experiment with model specifications outside the standard recipe, may find the template annoying or constricting.

Thus, as long as CGE modeling continues to develop, it is likely that a diversity of software approaches will be used.

11. References

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