

MMRF-GREEN:

**A DYNAMIC MULTI-REGIONAL APPLIED GENERAL
EQUILIBRIUM MODEL OF THE AUSTRALIAN ECONOMY,
BASED ON THE MMR AND MONASH MODELS**

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**Draft documentation prepared for the
Regional GE Modelling Course
25-29 November 2002**

November 2002

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1. Introduction

This report describes MMRF-GREEN, a multi-regional Computable General Equilibrium (CGE) model of Australia's regional economies. MMRF-GREEN is based on the Monash Multi-Regional (MMR) model.¹ MMR divides the Australian economy into any combination of the eight economies representing the six States and two Territories. Each region is modelled as an economy in its own right, with region-specific prices, region-specific consumers, region-specific industries, and so on. There are four types of agent: industries, households, governments and foreigners. In each region, there can be up to 113 industrial sectors, depending on the level of database aggregation. The sectors can produce a variety of products, and each creates a single type of capital. Capital is sector and region specific. In each region, there is a single household and a regional government. There is also a Federal government. Finally, there are foreigners, whose behaviour is summarised by demand curves for regional international exports and supply curves for regional international imports.

MMR is a comparative static CGE model. It shows for a single year the differences produced in regional Australia by changes in taxes, tariffs and other exogenous variables. MMRF-GREEN, on the other hand, produces sequences of annual solutions connected by dynamic relationships such as physical capital accumulation. Policy analysis with MMRF-GREEN involves the comparison of two alternative sequences of solutions, one generated without the policy change, the other with the policy change in place. The first sequence, called the basecase projection serves as a control path from which deviations are measured in assessing the effects of the policy shock.

MMRF-GREEN has been developed in four stages. In the first stage, MMR was transformed into a dynamic system by the inclusion of dynamic mechanisms taken from the MONASH model.² These were added as self-contained blocks, allowing MMRF-GREEN to include MMR as a special case. As such, MMRF-GREEN retains all of the strengths of its predecessor, including a highly disaggregated regional database. The second, third and fourth stages involved a range of developments designed to enhance the model's capacity for taxation, environmental and transport analysis.

In Chapter 2, we review the core MMR model and the additions and enhancements that transform it into MMRF-GREEN. Chapter 3 contains an overview of the method used to solve the model, and a discussion of closure options. The formal description of MMRF-GREEN is given in Chapter 4. This description is organised around the TABLO file that implements the model in GEMPACK.³ Aspects of model closure are discussed in Chapter 5. Chapter 6 contains selected summary aggregates from the model's current database.

2 From MMR to MMRF-GREEN

This chapter has five sections. In Section 2.1, we review the core MMR model, concentrating on the modelling of markets, demand, government finances and regional labour markets. The dynamic mechanisms introduced from MONASH are described in Section 2.2. Section 2.3 contains a brief reprise of the additions and enhancements to the core model's treatment of taxes. Enhancements designed to

¹ MMRF is short for Monash Multi-Regional Forecasting. The suffix –GREEN emphasises its ability to undertake economic analysis relating to environmental (i.e., green) issues. An initial progress report on the development of MMR is given in Meagher and Parmenter (1993). In 1996, MMR was adapted for forecasting by the inclusion of enough dynamics to accumulate variables such as capital stocks and foreign debt over medium-run periods. This version was called the MMR Forecasting (MMRF) model. MMRF is partly documented in Peter *et al.*, (2001).

² MONASH is a dynamic CGE model of the Australian economy built and maintained at the Centre of Policy Studies, Monash University. It is described in Dixon and Rimmer (2001).

³ GEMPACK is a flexible system for solving large economic models (see Harrison and Pearson, 1996). It automates the process of translating the model specification into a model solution program. As part of this automation, the GEMPACK user creates a text file listing the equations of the model in a language that resembles ordinary algebra. This text file is called the TABLO file.

improve the model's capability for environmental analysis are outlined in Section 2.4. Finally, in Section 2.5, we describe revisions to the core model's treatment of motor vehicles and modes of transport.

2.1 MMR: the General Equilibrium core

The nature of markets

MMR determines regional supplies and demands of commodities through optimising behaviour of agents in competitive markets. Optimising behaviour also determines industry demands for labour and capital. Labour supply at the national level is determined by demographic factors, while national capital supply responds to rates of return. Labour and capital can cross regional borders so that each region's stock of productive resources reflects regional employment opportunities and relative rates of return.

The assumption of competitive markets implies equality between the producer's price and marginal cost in each regional sector. Demand is assumed to equal supply in all markets other than the labour market (where excess supply conditions can hold). The government intervenes in markets by imposing ad valorem sales taxes on commodities. This places wedges between the prices paid by purchasers and prices received by the producers. The model recognises margin commodities (e.g., retail trade and road transport freight) which are required for each market transaction (the movement of a commodity from the producer to the purchaser). The costs of the margins are included in purchasers' prices.

Demands for inputs to be used in the production of commodities

MMR recognises two broad categories of inputs: intermediate inputs and primary factors. Firms in each regional sector are assumed to choose the mix of inputs which minimises the costs of production for their level of output. They are constrained in their choice of inputs by a three-level nested production technology. At the first level, intermediate-input bundles and primary-factor bundles are used in fixed proportions to output. These bundles are formed at the second level. Intermediate input bundles are combinations of international imported goods and domestic goods. The primary-factor bundle is a combination of labour, capital and land. At the third level, inputs of domestic goods are formed as combinations of goods from each of the eight regions, and the input of labour is formed as a combination of inputs of labour from eight different occupational categories.

Household demands

In each region, the household buys bundles of goods to maximise a utility function subject to a household expenditure constraint. The bundles are combinations of imported and domestic goods, with domestic goods being combinations of goods from each region. A Keynesian consumption function determines household expenditure as a function of household disposable income.

Demands for inputs to capital creation and the determination of investment

Capital creators for each regional sector combine inputs to form units of capital. In choosing these inputs, they cost minimise subject to technologies similar to that used for current production; the only difference being that they do not use primary factors. The use of primary factors in capital creation is recognised through inputs of the construction commodity (service).

Governments' demands for commodities

Commodities are demanded from each region by regional governments and by the Federal government. In MMR, there are several ways of handling these demands, including: (i) endogenously, by a rule such as moving government expenditures with household consumption expenditure or with domestic absorption; (ii) endogenously, as an instrument which varies to accommodate an exogenously determined target such as a required level of government deficit; and (iii) exogenously.

Foreign demand (international exports)

MMR adopts the ORANI specification of foreign demand⁴. Each export-oriented sector in each state faces its own downward-sloping foreign demand curve. Thus a shock that improves the price competitiveness of an export sector will result in increased export volume, but at a lower world price. By assuming that the foreign demand schedules are specific to product and region of production, the model allows for differential movements in world prices across domestic regions.

Government finances

For each of the eight regional governments and for the Federal government, MMR includes revenue equations for income taxes, sales taxes, excise taxes, taxes on international trade and for receipts from government-owned assets. As described already, the model accounts for public expenditures on commodities (or services). It also contains outlay equations for each government for transfer payments to households (e.g., pensions, sickness benefits and unemployment benefits). Transfers from the Federal to the regional governments are modelled, appearing on the outlay side of the Federal budget and on the revenue sides of the regional budgets.

The specification in MMR of government finances makes the model a suitable tool for (a) analysing the effects of changes in the fiscal policies of both the Federal government and of the regional governments, and (b) analysing the impact on the budgetary situation of the nine governments of a wide range of shocks.

Regional labour markets

This block of equations relates regional population and population of working age, and regional population of working age and regional labour supply. It also defines regional unemployment rates in terms of regional demands and supplies of labour.

There are three main possible treatments in MMR for regional labour markets:

1. regional labour supply and unemployment rates are exogenous and regional wage differentials are endogenous;
2. regional wage differentials and unemployment rates are exogenous and regional labour supply is endogenous; or
3. regional labour supply and wage differentials are exogenous and regional unemployment rates are endogenous.

2.2 MMR to MMRF-GREEN: Inclusion of MONASH dynamics

A number of static and dynamic mechanisms from MONASH have been incorporated into MMRF-GREEN to facilitate forecasting and dynamic policy analysis. Of these, the two most important are the inter-temporal links accommodating physical capital accumulation and lagged adjustment processes.

Physical capital accumulation

It is assumed that investment undertaken in year t becomes operational at the start of year $t+1$. Under this assumption, capital in industry j in state/territory q accumulates according to:

$$K_{j,q}(t+1) = (1 - DEP_{j,q}) \times K_{j,q}(t) + Y_{j,q}(t) \quad (2.1),$$

where:

$K_{j,q}(t)$ is the quantity of capital available in industry j located in q at the start of year t ;

$Y_{j,q}(t)$ is the quantity of new capital created for industry j in region q during year t ; and

$DEP_{j,q}$ is the rate of depreciation in industry j , treated as a fixed parameter.

⁴ ORANI is a large-scale multi-sectoral model of the Australian economy (see Dixon, *et al.*, 1992). It is the predecessor of MONASH.

Given a starting point value for capital in $t=0$, and with a mechanism for explaining investment through time, equation (2.1) can be used to trace out the time paths of industry capital stocks.

Investment in industry i in state/territory s in year t is explained via a mechanism of the form

$$\frac{K_{j,q}(t+1)}{K_{j,q}(t)} - 1 = F_{j,q}^t[EROR_{j,q}(t)] \quad (2.2),$$

where

$EROR_{j,q}(t)$ is the expected rate of return on investment in industry j in region q in year t ; and

$F_{j,q}^t[\]$ is an increasing function of the expected rate of return with a finite slope.

In MONASH, two possibilities are allowed for the formation of $EROR$, static expectations and forward-looking model-consistent expectations. Under static expectations, it is assumed that investors take account only of current rentals and asset prices when forming current expectations about rates of return. Under rational expectations the expected rate of return is set equal to the present value in year t of investing \$1 in industry i in region r , taking account of both the rental earnings and depreciated asset value of this investment in year $t+1$ as calculated in the model. In MMRF-GREEN, we allow only for static expectations.

Lagged adjustment processes

MONASH contains a number of lagged adjustment processes, but just one is included in MMRF-GREEN. This relates to the operation of the labour market in year-to-year policy simulations.

In comparative static analysis, one of the following two assumptions is made about the national real wage rate and national employment:

1. the national real wage rate adjusts so that any policy shock has no effect on aggregate employment;
- or
2. the national real wage rate is unaffected by the shock and employment adjusts.

MONASH's treatment of the labour market allows for a third, intermediate position, in which real wages can be sticky in the short run but flexible in the long-run and employment can be flexible in the short-run but sticky in the long-run. The same idea is applied in MMRF-GREEN. For year-to-year policy simulations, it is assumed that the deviation in the national real wage rate increases through time in proportion to the deviation in national employment from its base case-forecast level. The coefficient of adjustment is chosen so that the employment effects of a shock are largely eliminated after about ten years. This is consistent with macroeconomic modelling in which the NAIRU is exogenous.

2.3 MMRF-GREEN: taxation enhancements

Relative to MMR, MMRF-GREEN contains an improved treatment of indirect taxes. The most important enhancements allow for the modelling of a Goods and Services Tax (GST) of the type introduced in Australia in July 2001.

GST

In MMR, all commodity sales taxes are *ad valorem* rates of tax levied on the basic price of the underlying flow. The basic price is the price received by the producer. The GST, however, is generally applied to the price inclusive of freight and other margins such as wholesale trade. In some cases (e.g., petroleum products) the GST is imposed on the value inclusive of freight and other sales taxes (e.g., the per-litre petroleum excise). Accordingly, to accommodate the GST, the range of commodity taxes in MMRF-GREEN has been increased to accommodate specific (i.e., per unit) taxes and *ad valorem* taxes levied separately on producer prices, on producer prices plus margins, and on producer prices plus margins plus other sales taxes.

2.4 MMRF-GREEN: environmental enhancements

MMRF-GREEN has been enhanced in a number of areas to improve its capability for environmental analysis. These enhancements include:

1. an energy and gas emission accounting module, which accounts explicitly for each industry and region recognised in the model;
2. equations that allow for inter-fuel substitution in electricity generation by region; and
3. mechanisms that allow for the endogenous take-up of abatement measures in response to greenhouse policy measures.

Emissions accounting

MMRF-GREEN tracks emissions of greenhouse gases at a detailed level. It breaks down emissions according to:

1. emitting agent (the number of industries plus residential);
2. emitting state or territory (8); and
3. emitting activity (5).

Most of the emitting activities are the burning of fuels (black coal, natural gas, brown coal or petroleum products⁵). A residual category named *Activity*, covers emissions such as fugitives and agricultural emissions not arising from fuel burning. *Activity* accounts for more emissions than any of the combustion activities - but is also the most speculative. The largest single *Activity* emitter is the Agriculture industry. Emissions of this type are caused by livestock digestion, by soil disturbance, and by fertiliser use.

Inter-fuel substitution

Fuel-burning emissions are modelled as directly proportional to fuel usage. No allowance is made for any invention, which might, say, allow coal-fired electricity producers to release less CO₂ per tonne of coal burned. On the other hand, MMRF-GREEN does allow for input-saving technical progress. For example, the black coal electricity industry may reduce the amount of coal that it burns per kilowatt-hour of output. This sort of technical progress is imposed exogenously.

Other, indirect, forms of substitution offer the main scope, within MMRF-GREEN for emission reduction. Inter-fuel substitution in electricity generated is handled using the "technology bundle" approach (see Hinchy and Hanslow, 1996). In the current version of the model, five power-generating industries are distinguished in each region based on the type of fuel used. There is also a separate end-use supplier (*Electricity Supply*). The electricity generated in each region flows directly to the local end-use supplier, which then distributes electricity to local and inter-state users. The end-use supplier can substitute between the five technologies in response to changes in their production costs. For example, the Electricity supply industry in NSW might reduce the amount of power sourced from coal-using generators and increase the amount sourced from gas-fired plants. Such substitution is price-induced.

For other energy-intensive commodities used in industry, MMRF-GREEN allows for substitution possibilities by including a similar, but weaker, form of input-substitution specification. If the price of say, cement, rises by 10 per cent relative to other inputs to construction, the construction industry will use 1 per cent less cement and, to compensate, a little more of labour, capital and other materials. This input substitution is driven by price changes, and so is especially important in emission-policy scenarios, which makes outputs of emitting industries more expensive.

Endogenous take-up of abatement measures in response to greenhouse policy measures

In MMRF-GREEN, non-combustion emissions are generally modelled as directly proportional to the output of the related industries. However, in simulating the effects of a carbon tax or some other price-related penalty on gas emissions, allowance is made for abatement of non-combustion emissions. The amount of abatement is directly related to the price of carbon. The constants of proportionality are

⁵ Each of these fuels is identified as a separate commodity within the model.

derived from point estimates, from various sources, of the extent of abatement that might arise at a particular price level. It should be emphasised, however, that these estimates are quite speculative, but are only really important in the case of Agriculture, which makes such a large contribution to activity-related emissions.

2.5 MMRF-GREEN: transport enhancements

Multiproduct industries and multi-industry products

MMR follows the standard CGE convention that each industry produces just one product and that each product is produced by just one industry. This convention is particularly inappropriate in modelling the petroleum refining industry, which produces a range of fuels used for varying transport purposes. Accordingly, in MMRF-GREEN allowance has been made for multiproduct industries and multi-industry products. In the current version of the model, this allows the petroleum refinery industry to produce up to six separate products:

1. petroleum for automotive use only;
2. petroleum for aviation use only, commonly called AvGas;
3. aviation turbine fuel;
4. diesel for automotive use;
5. LPG for automotive use; and
6. other petroleum products, which include kerosene, heating oil and fuel oil.

Each product has a distinct sales pattern.

The petroleum industry decides on its output mix based on the relative prices of the products that it produces. For example, if the price received for automotive petroleum increases relative to the average price received for all petroleum products, then the petroleum industry is allowed to transform its product mix towards the production of automotive petroleum. A transformation elasticity of 1 is assumed for all petroleum products.

Improved treatment of motor vehicles in household demand system

Some versions of MMRF-GREEN include a revised treatment of motor vehicles in the model's household demand system. Under the revised treatment, cars are treated as durables, providing services to households. The revisions have been implemented via the inclusion of a dummy industry, *Private transport services* (based on Lee 2002). This industry, which is analogous to the existing industry producing housing services, sells the services of the private stock of motor vehicles used by households for transport. Its intermediate inputs are fuel and materials (repairs, tyres, etc.) used to maintain and to run the private vehicle fleet. The industry sells only to consumers.

Under the revised treatment, consumers no longer buy motor vehicles directly, nor do they buy the fuels and materials associated with vehicles. Instead, they purchase private motor vehicle services, which are combinations of the services provided by the vehicle stock, and of the fuel and materials associated with vehicles.

Modes of passenger and freight transport and inter-modal freight substitution

The primary MMR database recognises four single product industries producing transport services. The four transport modes are road, rail, water and air. Each industry provides passenger services and freight transport. Passenger services are sold directly. Freight services are sold indirectly as margins on flows of goods and services.

Combining freight and passenger services into one product restricts the model's ability to analyse many transport issues, especially those that impinge on passenger transport differently from freight transport. Accordingly, in MMRF-GREEN each of the four transport industries has been

disaggregated into two industries, one producing passenger services and one producing freight services. Each has a distinct production technology and sales pattern.

In MMR, usage of each margin (freight) service is directly proportional to the quantity of goods which is being transported from the point of production to some user. The ratio of proportionality varies according to the good and the user, but is otherwise constant. That is, the core model does not allow for substitution between freight services. In MMRF-GREEN, this treatment of freight has been modified to allow for the possibility of substitution between road and rail. Specifically, for a flow from region s to region q , substitution is allowed between road freight and rail freight provided by region q . The substitution is based on relative prices. If in region q , the price of road freight increases relative to the price of rail freight, then there will be substitution away from road freight towards rail freight in all uses of the two in region q .

3. Computational Method, Interpretation of Solutions and Closures

3.1 Overview of computational method

Many of the equations in MMRF-GREEN are non-linear, which presents computational difficulties. However, following Johansen (1960), the model is solved by representing it as a system of linear equations relating changes in the model's variables. Results are deviations from an initial solution of the underlying non-linear model.

The system of linear equations is solved using GEMPACK. GEMPACK is a suite of general-purpose programs for implementing and solving large economic models. The linear version of MMRF-GREEN is specified in the TABLO syntax, which is similar to ordinary algebra. GEMPACK solves the system by converting it to an Initial Value problem and then using one of the standard methods such as Euler. GEMPACK uses multi-step processors to generate exact solutions of the underlying, non-linear, equations, as well as to compute linear approximations to those solutions. For details of the algorithms available in GEMPACK, see Harrison and Pearson (1996). For introductions to the Johansen/Euler solution method, see Dixon and Rimmer (2001, Chapter 11) and Horridge *et al.* (1993).

Writing down the equation system of the model in a linear (change) form has advantages from computational and economic standpoints. Linear systems are easy for computers to solve. This allows for the specification of detailed models, consisting of many thousands of equations, without incurring computational constraints. Further, the size of the system can be reduced by using model equations to substitute out those variables that may be of secondary importance for any given experiment. In a linear system, it is easy to rearrange the equations to obtain explicit formulae for those variables, hence the process of substitution is straightforward.

Compared to their levels counterparts, the economic intuition of the change versions of many of the model's equations is relatively transparent. In addition, when interpreting the results of the linear system, simple share-weighted relationships between variables can be exploited to perform back-of-the-envelope calculations designed to reveal the key cause-effect relationships responsible for the results of a particular experiment.

Nature of dynamic solution

Algebraically, dynamic models like MMRF-GREEN take the form

$$F(X(t)) = 0 \quad (2.3),$$

where $X(t)$ is a vector of length n referring to variables for year t , and $F()$ is an m -length vector of differentiable functions of n variables, with 3. In simulations with (2.3), given an initial solution for the n variables that satisfies (2.3), GEMPACK compute the movements in m variables (the endogenous variables) away from their values in the initial solution caused by movements in the remaining $n - m$ variables (the exogenous variables). In year-to-year simulations, the movements in the exogenous variables are from one year to the next. If the initial solution is for year t then our first computation creates a solution for year $t+1$. This solution can in turn become an initial solution for a computation that creates a solution for

year $t+2$. In such a sequence of annual computations, links between one year and the next are recognised by ensuring, for example, that the quantities of opening capital stocks in the year t computation are the quantities of closing stocks in the year $t-1$ computation.

Deriving the linear form of the underlying non-linear equations

In deriving the linear equations from the non-linear equations, we use three basic rules:

$$\begin{array}{lll} \text{the product rule: } X = \beta YZ & \Rightarrow & x = y + z, \text{ where } \beta \text{ is a constant,} \\ \text{the power rule: } X = \beta Y^\alpha & \Rightarrow & x = \alpha y, \text{ where } \alpha \text{ and } \beta \text{ are constants, and} \\ \text{the sum rule: } X = Y + Z & \Rightarrow & Xx = Yy + Zz. \end{array}$$

The MMRF-GREEN results are reported as deviations in the model's variables from an initial solution. With reference to the above equations, the *percentage* changes x , y and z represent deviations from their levels values X , Y and Z . The levels values (X , Y and Z) are solutions to the model's underlying levels equations. Using the product-rule equation as an example, values of 100 for X , 10 for Y and 5 for Z represent an initial solution for a value of 2 for β . Now assume that we perturb our initial solution by increasing the values of Y and Z by 3 per cent and 2 per cent respectively, i.e., we set y and z at 3 and 2. The linear representation of the product-rule equation would give a value of x of 5, with the interpretation that the initial value of X has increased by 5 per cent for a 3 per cent increase in Y and a 2 per cent increase in Z . Values of 5 for x , 3 for y and 2 for z in the corresponding percentage change equation means that the levels value of X has been perturbed from 100 to 105, Y from 10 to 10.3 and Z from 5 to 5.1.

In the above example, the reader will have noticed that while satisfying the percentage-change equations, updating the levels values of X , Y and Z by their percentage changes does not satisfy the levels form of the product-rule equation i.e., $105 \neq 2 \times 10.3 \times 5.1$. Given the percentage changes to Y and Z , the solution to the non-linear equation is $X = 105.06$. Comparing the levels solution to the percentage change solution shows there is a linearisation error of 0.06 (i.e., $0.06 = 105.06 - 105$). We can eliminate the linearisation error by the application of a multistep procedure which exploits a positive relationship between the size of the perturbation from the initial solution and the size of the linearisation error. The principle of the Euler version of the multistep solution method can be illustrated using our above example. Instead of increasing the values of Y and Z by 3 per cent and 2 per cent, let us break the perturbation into two steps. First, let us increase x and y by half the desired amount, i.e., 1.5 per cent and 1.0 per cent respectively. Solving the linear equation gives a value for x of 2.5 per cent. Updating the value of X by 2.5 per cent gives an intermediate value of X of 102.5 [i.e., $100 \times (1 + 2.5/100)$]. Now apply the remainder of our desired perturbation to Y and Z . The percentage increase in y is 1.4778 per cent (i.e., $100 \times 0.15/10.15$)⁶ and the percentage change in z is 0.9901 per cent (i.e., $100 \times 0.05/5.05$), giving a value for x (in our second step) of 2.4679 per cent. Updating our intermediate value of X by 2.4679 per cent, gives a final value of X of 105.045, which is close to the solution of the non-linear equation of 105.06. We can improve the accuracy of our solution by implementing more steps and by applying an extrapolation procedure.

In the percentage-change form of the power-rule equation, a constant α appears as a coefficient. In the percentage-change form of the sum-rule equation, the levels values of the variables appear as coefficients. By dividing by X , this last equation can be rewritten so that x is a share-weighted average of y and z .

3.2 Closures of MMRF-Green

A choice of the $n-m$ variables to be made exogenous is called a closure. In MONASH, there are four basic classes of closure. For MMRF-GREEN we identify three classes:

- comparative-static closures;

⁶ Note that in our first step we have also updated the values of Y and Z , e.g., after the first step, our updated value of Y is $10.15 = 10 \times 1.5/100$.

- forecasting closures; and
- policy or deviation closures.⁷

Comparative static closures are used in single computation comparative static analyses. Forecasting and policy closures are used in year-to-year simulations.

Comparative-static closures

In a comparative-static closure, we include in the exogenous set all variables that can be regarded as naturally exogenous in a CGE model. These may be observable variables such as tax rates or unobservable variables such as technology and preference variables. We also include in the exogenous set all variables that are naturally endogenous in a dynamic model, but which are naturally exogenous in a static model. These will typically include investment by industry and one of the capital stock or rate of return for each industry.

Forecasting and policy closures

Forecasting and policy closures utilise the dynamic features of the model. Thus for both classes of closure we include in the endogenous set all variables that are naturally endogenous in a dynamic model, but naturally exogenous in a static model.

In forecasting with MONASH and MMRF-GREEN we often want to take on board forecasts and information available from outside sources. Typical examples include macro forecasts made by specialist private or public-sector groups and information about future changes in tax and benefit rates announced by the government. To accommodate this information, numerous naturally endogenous variables are typically exogenised. These might include:

- the volumes of agricultural exports; and
- most macro variables.

To allow such naturally endogenous variables to be exogenous, an equal number of naturally exogenous variables must be made endogenous. For example, to accommodate forecasts for the volumes of agricultural exports we would make endogenous variables that locate the positions of foreign demand curves. To accommodate forecasts for macro variables, we would endogenise various macro coefficients such as the average propensity to consume.

In forecasting closures, tastes and technology are exogenous and normally set to historically average values. With MONASH, these historically average values are deduced in historical simulations. At this stage, we do not plan to conduct historical simulations with MMRF-GREEN. Thus, in using MMRF-GREEN for forecasting we may draw on national-level values for taste and technology variables deduced in the MONASH work. Policy variables are also generally exogenous in forecasting closures. As indicated above, in forecasting values for these variables we can draw on information from extraneous sources such as government departments.

In policy closures naturally endogenous variables, such as the volumes of agricultural exports and macro variables are endogenous. They respond to the policy change under consideration. Correspondingly, in policy closures naturally exogenous variables, such as the positions of foreign demand curves and macro coefficients, are exogenous. They are set at the values revealed in the forecasts.

In a policy simulation, most, but not all, of the exogenous variables have the values they had in the associated forecast solution. The exceptions are the exogenous variables that are shocked. The policy simulation, therefore, generates deviations from the corresponding forecast simulation in response to the exogenously imposed change.

⁷ Comparative-static closures are also available for MONASH, but this class of closure is not explicitly discussed in Dixon and Rimmer (2000). The four classes of closure referred to in Dixon and Rimmer (2000) are: forecasting and policy closures; and historical and decomposition closures. The last pair is used in single-computation analyses of historical periods.

4 MMRF-Green Represented in the TABLO Language

In this chapter, we present a formal description of the linear form of MMRF-GREEN. Our description is organised around excerpts from the TABLO file, which implements the model in GEMPACK. The TABLO language in which the file is written is essentially conventional algebra, with names for variables and coefficients chosen to be suggestive of their economic interpretations.

We base our description on the TABLO file for a number of reasons. First, by acquiring familiarity with the TABLO code allows the reader ready access to the programs used to conduct simulations with the model and to convert the results to readable form. Both the input and the output of these programs employ the TABLO notation. Second, familiarity with the TABLO code is essential for users who may wish to change the model. Finally, by documenting the TABLO form of the model, we ensure that our description is complete and accurate.

TABLO syntax and conventions observed in the TABLO representation

Each equation in the TABLO model description is linear in the changes (percentage or absolute) of the model's variables. For example, the industry labour demand equations appear as:

```
E_xllab # Industry demands for effective labour #
(all,j,IND)(all,q,REGDST)
xllab(j,q) = x1prim(j,q) + allab(j,q)
- SIGMA1FAC(j,q)*[p1lab(j,q) + allab(j,q) - xi_fac(j,q)]
+ [CAPITAL(j,q)/[TINY+LABOUR(j,q)+CAPITAL(j,q)]] * twist_1k(j,q);
```

The first element is the identifier for the equations, which must be unique. In the MMRF-GREEN code, all equation identifiers are of the form $E_{<variable>}$, where $<variable>$ is the variable that is explained by the equation in the long-run comparative static closure of the model. The identifier is followed by descriptive text between # symbols. This is optional. The description appears in certain GEMPACK generated report files. The expression $(all,j,IND)(all,q,REGDST)$ signifies that the equations are defined over all elements of the set IND (the set of industries) and REGDST (the set of domestic regions of use).

Within the equation, we generally distinguish between change variables and coefficients by using lower-case script for variables and upper-case script for coefficients. Note, however, that the GEMPACK solution software ignores case. Thus in the excerpt above, the variables are $x1lab(j,q)$, $x1prim(j,q)$, $allab(j,q)$, $p1lab(j,q)$ and $xi_fac(j,q)$. There is only one coefficient, SIGMA1FAC, which is the fixed elasticity of substitution between labour and other primary factors. A semi-colon signals the end of the TABLO statement.

With a CGE model run using the GEMPACK software, it is possible to generate a complete list of equation names, variable names and coefficient names. We do this in TABMATE with the option **Tools ... Generate documentation**. For example, from the code MMRF-CRS.TAB, this option will create three files, *MMRF-CRS.mde*, *MMRF-CRS.mdv* and *MMRF-CRS.mdc*. *MMRF-CRS.mde* contains a list of every equation name, its dimensions and a description (taken from the TABLO code). Similarly, *MMRF-CRS.mdv* contains a list of every variable in the model, its dimensions and a description. And *MMRF-CRS.mdc* contains a list of every coefficient in the model with its dimensions and a description.

Typically, set names appear in upper-case characters in the TABLO code. The main sets defined for MMRF-Green are:

COM	Commodities
IND	Industries
MAR	Margin commodities (a subset of COM)
MARGININD	Margin industries (a subset of IND)
UPCOM	Single industry products
UPIND	Single product industries
TEXP	Traditional Exports (a subset of COM)

TOUR	Tourism Exports (a subset of COM)
NTEXP	Non traditional exports (a subset of COM)
ALLDST	Destinations of goods
DOMDST	Domestic destinations of goods (a subset of ALLDST)
REGDST	Regional destinations of goods (a subset of DOMDST)
ALLSRC	Origins of goods
REGSRC	Regional origins of goods (a subset of ALLSRC)
OCC	Occupation types.

The remainder of this chapter is devoted to the exposition of the MMRF-GREEN equation system. We begin in section 4.1 with the equations of the CGE core. The remaining equations are grouped under the following section headings.

- 4.2 Government finances
- 4.3 Regional labour markets and regional migration
- 4.4 Rates of return and investment
- 4.5 Year-to-year equations: investment and capital, investment and expected rates of return, and the labour market
- 4.6 Miscellaneous equations to facilitate year-to-year simulations
- 4.7: Greenhouse gas emissions
- 4.8 Petroleum usage and taxation
- 4.9 Equations to implement the GST.

4.1. The CGE core

The CGE core is based on ORANI, a single-region model of Australia. Each regional economy in MMRF-GREEN looks like an ORANI model. However, unlike the single-region ORANI model, MMRF-GREEN includes interregional linkages.

A schematic representation of the core

Figure 4.1 is a schematic representation of the core's input-output database. It reveals the basic structure of the core. The columns identify the following agents:

- (1) domestic producers divided into J industries in Q regions;
- (2) investors divided into J industries in Q regions;
- (3) a single representative household for each of the Q regions;
- (4) an aggregate foreign purchaser of exports;
- (5) an other demand category corresponding to Q regional governments; and
- (6) an other demand category corresponding to Federal government demands in the Q regions.

The rows show the structure of the purchases made by each of the agents identified in the columns. Each of the I commodity types identified in the model can be obtained within the region, from other regions or imported from overseas. The source-specific commodities are used by industries as inputs to current production and capital formation, are consumed by households and governments and are exported. Only domestically produced goods appear in the export column. R of the domestically produced goods are used as margin services which are required to transfer commodities from their sources to their users. Commodity taxes are payable on the purchases. As well as intermediate inputs, current production requires inputs of three categories of primary factors: labour (divided into M occupations), fixed capital and agricultural land. The other costs category covers various miscellaneous industry expenses. Each cell

in the input-output table contains the name of the corresponding matrix of the values (in some base year) of flows of commodities, indirect taxes or primary factors to a group of users. For example, MAR2 is a 5-dimensional array showing the cost of the R margins services on the flows of I goods, both domestically and imported (S), to I investors in Q regions.

Figure 4.1 is suggestive of the theoretical structure required of the CGE core. The theoretical structure includes: demand equations required for our six users; equations determining commodity and factor prices; market clearing equations; definitions of commodity tax rates. In common with ORANI, the equations of MMRF's CGE core can be grouped according to the following classification:

- *producer's demands for produced inputs and primary factors;*
- *demands for inputs to capital creation;*
- *household demands;*
- *export demands;*
- *government demands;*
- *demands for margins;*
- *zero pure profits in production and distribution;*
- *indirect taxes;*
- *market-clearing conditions for commodities and primary factors; and*
- *regional and national macroeconomic variables and price indices.*

		ABSORPTION MATRIX					
		1	2	3	4	5	6
		Producers	Investors	Household	Export	Regional Govt.	Federal Govt.
	Size	$\leftarrow J \times Q \rightarrow$	$\leftarrow J \times Q \rightarrow$	$\leftarrow Q \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow Q \rightarrow$	$\leftarrow Q \rightarrow$
Basic Flows	$\begin{matrix} \uparrow \\ I \times S \\ \downarrow \end{matrix}$	BAS1	BAS2	BAS3	BAS4	BAS5	BAS6
Margins	$\begin{matrix} \uparrow \\ I \times S \times R \\ \downarrow \end{matrix}$	MAR1	MAR2	MAR3	MAR4		
Taxes	$\begin{matrix} \uparrow \\ I \times S \\ \downarrow \end{matrix}$	TAX1	TAX2	TAX3	TAX4		
Labour	$\begin{matrix} \uparrow \\ M \\ \downarrow \end{matrix}$	LABR	I = Number of commodities J = Number of industries M = Number of occupation types R = Number of commodities used as margins Q = Number of Regions S = 9: 8 \times Domestic Regions plus 1 \times Foreign Import				
Capital	$\begin{matrix} \uparrow \\ 1 \\ \downarrow \end{matrix}$	CPTL					
Land	$\begin{matrix} \uparrow \\ 1 \\ \downarrow \end{matrix}$	LAND					
Other Costs	$\begin{matrix} \uparrow \\ 1 \\ \downarrow \end{matrix}$	OCTS					

Figure 4.1. The CGE core input-output database

Naming system for variables of the CGE core

The following conventions are used (as far as possible) in naming variables of the CGE core. Names consist of a prefix, a main user number and a source dimension. The prefixes are:

- a \Leftrightarrow technological change, change in preferences;
- del \Leftrightarrow ordinary (rather than percentage) change;
- f \Leftrightarrow shift variable;
- nat \Leftrightarrow a national aggregate of the corresponding regional variable;
- p \Leftrightarrow prices;
- x \Leftrightarrow quantity demanded;
- z \Leftrightarrow quantity supplied.

The main user numbers are:

- 1 \Leftrightarrow firms, current production;
- 2 \Leftrightarrow firms, capital creation;
- 3 \Leftrightarrow households;
- 4 \Leftrightarrow foreign exports;
- 5 \Leftrightarrow regional government;
- 6 \Leftrightarrow Federal government

The number 0 is also used to denote basic prices and values. The source dimensions are:

- a \Leftrightarrow all sources, i.e., 8 (maximum) regional sources and 1 foreign;

- r \Leftrightarrow regional sources only;
- t \Leftrightarrow two sources, i.e., a domestic composite source and foreign;
- c \Leftrightarrow domestic composite source only;
- o \Leftrightarrow domestic-foreign composite source only.

The following are examples of the above notational conventions:

- p1a \Leftrightarrow price (p) of commodities from all nine sources (a) for use by firms in production (1);
- x2c \Leftrightarrow demand (x) for the domestic composite commodity (c) for use in capital creation (2).

Variable names may also include an (optional) suffix description. These are:

- cap \Leftrightarrow capital;
- imp \Leftrightarrow imports;
- lab \Leftrightarrow labour;
- land \Leftrightarrow agricultural land;
- lux \Leftrightarrow linear expenditure system (supernumerary part);
- marg \Leftrightarrow margins;
- oct \Leftrightarrow other cost tickets;
- prim \Leftrightarrow all primary factors (land, labour or capital);
- sub \Leftrightarrow linear expenditure system (subsistence part).

4.1.1. Production: demand and prices for inputs to the production process

MMRF-GREEN recognises two broad categories of inputs: intermediate inputs and primary factors. Industries in each region are assumed to choose the mix of inputs which minimises the costs of production for their level of output. They are constrained in their choice of inputs by a three-level nested production technology (Figure 4.2). At the first level, the intermediate-input bundles and the primary-factor bundles are used in fixed proportions to output. These bundles are formed at the second level. Intermediate input bundles are constant-elasticity-of-substitution (CES) combinations of international imported goods and domestic goods. The primary-factor bundle is a CES combination of labour, capital and land. At the third level, inputs of domestic goods are formed as CES combinations of goods from each of the eight regions, and the input of labour is formed as a CES combination of inputs of labour from eight different occupational categories. We describe the derivation of the input demand functions working upwards from the bottom of the tree in Figure 4.2. We begin with the intermediate-input branch.

Demands and prices for domestic and imported intermediate inputs, user 1 (E_x1a to E_x1c)

At the bottom of the nest, industry j in region q chooses intermediate input type i from domestic region s ($X1A(i,s,j,q)$) to minimise the cost

$$\sum_{r \in \text{regsrc}} P1A(i, s, j, q) \times X1A(i, s, j, q) \quad i \in \text{COM} \quad j \in \text{IND} \quad q \in \text{REGDST} \quad (4.1.1)$$

of a composite domestic bundle

$$X1C(i, j, q) = \text{CES}_{s \in \text{regsrc}} \{X1A(i, s, j, q)\} \quad i \in \text{COM} \quad j \in \text{IND} \quad q \in \text{REGDST} \quad (4.1.2)$$

where the composite domestic bundle ($X1C(i,j,q)$) is exogenous at this level of the nest. The notation $\text{CES}\{\}$ represents a CES function defined over the set of variables enclosed in the curly brackets. The subscript indicates that the CES aggregation is over all elements s of the set of regional sources (REGSRC). The CES specification means that inputs of the same commodity type produced in different regions are not perfect substitutes for one another. This is an application of the so-called Armington (1969 1970)

specification typically imposed on the use of domestically produced commodities and foreign-imported commodities in national CGE models such as ORANI.

By solving the above problem, we generate the industries' demand equations for domestically produced intermediate inputs to production.⁸ The percentage-change forms of these demand equations are given by equation E_{x1a} . On the RHS of E_{x1a} , the coefficient $IS_DOM(s)$ has the value 1 when s is a domestic region and 0 otherwise. Conversely, the coefficient $IS_IMP(s)$ has the value 1 when s is the foreign source and 0 otherwise. Here we concentrate on the case where $IS_DOM = 1$ and $IS_IMP = 0$.

⁸ For details on the solution of input demands given a CES production function, and the linearisation of the resulting levels equation, see Dixon, Bowles and Kendrick (1980), and Horridge, Parmenter and Pearson (1993).

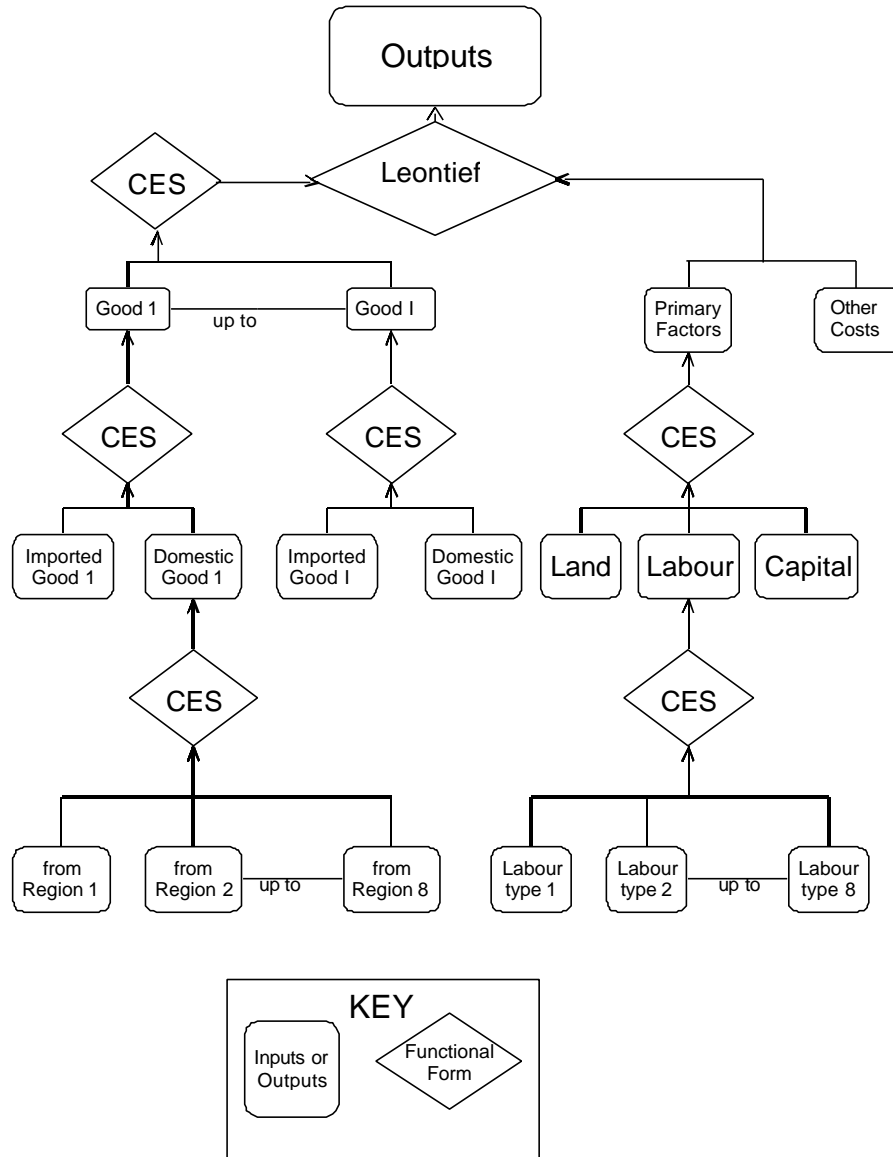


Figure 4.2. Production technology for regional sectors in MMRF-GREEN

On the RHS of E_{x1a} , the first term is the percentage change in the demand for the domestic composite ($x1c(i,j,q)$). In the absence of changes in prices and technology, it is assumed that the use of input i from all domestic sources expands proportionately with industry (j,q) 's overall usage of domestic i . The second term on the RHS of E_{x1a} allows for price substitution. The percentage-change form of the price term is an elasticity of substitution, $SIGMA1C(i)$, multiplied by the percentage change in a price ratio representing the price from the regional source relative to the cost of the regional composite, i.e., an average price of the commodity across all regional sources. Lowering of a source-specific price, relative to the average, induces substitution in favour of that source. The final term on the RHS of E_{x1a} allows for technological change. If $a1a(i,s,j,q)$ is set to -1 , then we are allowing for a 1 per cent input- (i,s) saving technical change by industry (j,q) .

The percentage change in the average price, $p1c_{i,j,q}$, is given by equation E_p1c . In E_p1c , the coefficient $PVAL1T(i,"domestic",j,q)$ is the total purchases value of commodity i from all domestic sources used by industry j in region q , and $PVAL1A(i,s,j,q)$ is the cost of commodity i region regional source s used by industry j in region q . Hence, $p1c(i,j,q)$ is a cost-weighted Divisia index of individual prices from the regional sources. Note that in cases where $PVAL1T(i,s,j,q)$ equals zero, E_p1c would leave the corresponding $p1c$ undefined. To avoid this problem, the TABLO file adds the coefficient $TINY$, which is set to a very small number.

At the next level of the production nest, firms decide on their demands for the domestic-composite commodities and the foreign imported commodities following a pattern similar to the previous nest. Here, the firm chooses a cost-minimising mix of the domestic-composite commodity and the foreign imported commodity

$$P1A(i,"imp",j,q)X1A(i,"imp",j,q) + P1C(i,j,q)X1C(i,j,q) \quad i \in COM \quad j \in IND \quad q \in REGDST \quad (4.1.3)$$

where the subscript 'imp' refers to the foreign import, subject to the production function

$$X1O(i,j,q) = CES\{X1A(i,"imp",j,q), X1C(i,j,q)\} \quad i \in COM \quad j \in IND \quad q \in REGDST \quad (4.1.4).$$

As with the problem of choosing the domestic-composite, the Armington assumption is imposed on the domestic-composite and the foreign import by the CES specification in (4.1.4).

The solution to the problem specified by (2.3) and (2.4) yields the input demand functions for the domestic-composite and the foreign import; represented in their percentage-change form by equations E_x1c and E_x1a (for $IS_DOM=0$, $IS_IMP=1$). These equations show, respectively, that the demands for the domestic-composite commodity ($X1C(i,j,q)$) and for the foreign import ($X1A(i,"imp",j,q)$) are proportional to demand for the domestic-composite/foreign-import aggregate ($X1O(i,j,q)$) and to a price term. The $X1O(i,j,q)$ are exogenous to the producer's problem at this level of the nest. Common with the previous nest, the percentage-change form of the price term is an elasticity of substitution, $SIGMA1O(i)$ multiplied by a price ratio representing the percentage change in the price of the domestic-composite ($p1c(i,j,q)$ in equation E_x1c) or of the foreign import ($p1a(i,"imp",j,q)$ in equation E_x1a) relative to the price of the domestic-composite/foreign-import aggregate ($p1o(i,j,q)$ in equations E_x1c and E_x1a).

On the RHSs of E_x1a and E_x1c are terms involving the variable $twist_src(i,q)$. This allows for cost-neutral twists in import/domestic preferences for commodity i used by industries in region q . To see how this mechanism works, assume that the technological and price terms on the RHSs of E_x1a and E_x1c are set at zero. Taking account of the fact that

$$\frac{PVAL1T(i,"domestic",j,q)}{PVAL1O(i,j,q)} + \frac{PVAL1T(i,"imp",j,q)}{PVAL1O(i,j,q)} = 1,$$

we see that

$$x1c(i,j,q) - x1a(i,"imp",j,q) = -twist_src(i,q) \quad (4.1.5).$$

Hence, in the absence of changes in prices and "a" terms, if $twist_src(i,q)$ were set at -10, then industry j would increase the ratio of its domestic to imported inputs of commodity i by 10 per cent. In other words, there is a 10 per cent twist by industry j in favour of the use of domestic good i relative to imported good i .

To see how the cost-neutral aspect works, continue to assume zero values for the "a" terms and no changes in prices. Also, for this particular example, assume $x1o(i,j,q) = 0$. Under these assumptions:

$$x1c(i, j, q) = - \frac{PVAL1T(i, "imp", j, q)}{PVAL1O(i, j, q)} \times twist_src(i, q)$$

and

$$x1a(i, "imp", j, q) = \frac{PVAL1T(i, "imp", j, q)}{PVAL1O(i, j, q)} \times twist_src(i, q) .$$

By making a weighted sum of these percentage changes in industry (j,q)'s inputs of domestic and imported good i, using as weights the shares of the two inputs in industry (j,q)'s total costs, we find that the effect on (j,q)'s costs is zero. In other words, shocks to twist_src(i,q) are cost-neutral.

The percentage change in the price of the domestic-composite/foreign-import aggregate, p1o(i,j,q) is defined in equation E_p1o. As in equation E_p1c, the composite price is a Divisia index of the individual prices.

We now turn our attention to the primary-factor branch of the input-demand tree of Figure 4.2.

Demand for labour by occupation (E_x1laboi to E_labind)

At the lowest-level nest in the primary-factor branch of the production tree in Figure 4.2, producers choose a composition of m occupation-specific labour inputs to minimise the costs of a given composite labour aggregate input. The demand equations for labour of the various occupation types are derived from the following optimisation problem for the jth industry in the qth region.

Choose inputs of occupation-specific labour type m, X1LABOI(j,q,m), to minimise total labour cost

$$\sum_{m \in occ} P1LABOI(j, q, m) X1LABOI(j, q, m) \quad j \in IND \quad q \in REGDST \quad (4.1.6)$$

subject to,

$$X1LAB(j, q) = CES_{m \in occ} \{ X1LABOI(j, q, m) \} \quad j \in IND \quad q \in REGDST \quad (4.1.7).$$

Exogenous to this problem are the price paid by the jth regional industry for each occupation-specific labour type (P1LABOI(j,q,m)) and the regional industries' demands for the effective labour input (X1LAB(j,q)).

The solution to this problem, in percentage-change form, is given by equations E_x1laboi and E_p1lab. Equation E_x1laboi indicates that the demand for labour type m is proportional to the demand for the effective composite labour demand and to a price term. The price term consists of an elasticity of substitution, SIGMA1LAB(j,q), multiplied by the percentage change in a price ratio representing the wage of occupation m (p1laboi(j,q,m)) relative to the average wage for labour in industry j of region q (p1lab(j,q)). Changes in the relative wages of the occupations induce substitution in favour of relatively cheapening occupations. The percentage change in the average wage is given by equation E_p1lab. The coefficient LAB_OCC_IND(j,q,m) is the wage bill for occupation m employed by industry j in region q. The coefficient LABOUR(j,q) is the total wage bill of industry j in region q. Thus, p1lab(j,q) is a Divisia index of the p1laboi(j,q,m).

Summing the percentage changes in occupation-specific labour demands across occupations, using appropriate occupational shares, for each industry gives the percentage change in industry labour demand (labind(j,q)) in equation E_labind.

Demand for primary factors (E_{x1lab} to E_{xifac})

At the next level of the primary-factor branch of the production nest, we determine the composition of demand for primary factors. Their derivation follows the same CES pattern as the previous nests. Here, total primary factor costs

$$P1LAB(j, q)X1LAB(j, q) + P1CAP(j, q)CAP_T(j, q) + P1LAND(j, q)X1LAND(j, q) \\ j \in IND \quad q \in REGDST,$$

where $P1CAP(j, q)$ and $P1LAND(j, q)$ are the unit costs of capital and agricultural land and $CAP_T(j, q)$ and $N(j, q)$ are industry's demands for capital and agricultural land, are minimised subject to the production function

$$X1PRIM(j, q) = CES \left[\frac{X1LAB(j, q)}{A1LAB(j, q)}, \frac{X1CAP(j, q)}{A1CAP(j, q)}, \frac{X1LAND(j, q)}{A1LAND(j, q)} \right] \\ j \in IND \quad q \in REGDST,$$

where $X1PRIM(j, q)$ is the industry's overall demand for primary factors. The above production function allows us to impose factor-specific technological change via the variables $A1LAB(j, q)$, $A1CAP(j, q)$ and $A1LAND(j, q)$.

The solution to the problem, in percentage-change form is given by equations E_{x1lab} , E_{x1cap} , E_{p1land} and E_{xi_fac} . From these equations, we see that for a given level of technical change, industries' factor demands are proportional to overall factor demand ($X1PRIM(j, q)$) and a relative price term. In change form, the price term is an elasticity of substitution ($SIGMA1FAC(j, q)$) multiplied by the percentage change in a price ratio representing the unit cost of the factor relative to the overall effective cost of primary factor inputs to the j th industry in region q . Changes in the relative prices of the primary factors induce substitution in favour of relatively cheapening factors. The percentage change in the average effective cost ($xi_fac(j, q)$), given by equation E_{xi_fac} , is again a cost-weighted Divisia index of individual prices and technical changes.

Another twist term, $twist_lk(j, q)$, appears in equations E_{x1lab} and E_{x1cap} . A positive value for $twist_lk(j, q)$ causes a cost-neutral twist {towards labour, away from capital} in industry j . A negative value for $twist_lk$ causes a twist {towards capital, away from labour}. The coefficient attached to $twist_lk(j, q)$ in E_{x1lab} is the share of the cost of capital in the total cost of labour and capital for industry j in region q . The coefficient attached to $twist_lk(j, q)$ is the share of the cost of labour in the total cost of capital and labour for industry (j, q) .

Demands for commodity composites (E_{x1o} and E_{p1com})

As indicated near the top on the LHS of Figure 4.2, producers choose a composition of I commodity composites to minimise the total cost of purchasing intermediate inputs. The demand equations for commodity composites are derived from the following optimisation problem for the j th industry in the q th region.

Choose composite commodity i , $X1O(i, j, q)$, to minimise total labour cost

$$\sum_{i \in com} P1O(i, j, q)X1O(i, j, q) \quad j \in IND \quad q \in REGDST \quad (4.1.8)$$

subject to,

$$X1COM(j, q) = CES_{i \in com} \left\{ \frac{X1O(i, j, q)}{A1O(i, j, q)} \right\} \quad j \in IND \quad q \in REGDST \quad (4.1.9),$$

where $A1O(i, j, q)$ is a technological term reflecting the usage of commodity composite i by regional industry j per unit of overall intermediate inputs used by j . Exogenous to this problem are the price paid by the j th regional industry for commodity composite i ($P1O(i, j, q)$) and the regional industries' total demand for intermediate inputs ($X1COM(j, q)$).

The solution to this problem, in percentage-change form, is given by equations E_x1o and E_p1com . Equation E_x1o indicates that the demand for commodity composite i is proportional to total demand for intermediate inputs to a price-substitution term. The price term consists of an elasticity of substitution, $SIGMA1COM(j, q)$, multiplied by the percentage change in a price ratio representing the price of commodity composite i ($p1o(i, j, q)$) relative to the average price of intermediate inputs for j in region q ($p1com(j, q)$). The percentage change in the average price of intermediate inputs is given by equation E_p1com .

Demands for composite primary factors, for total intermediate inputs, and for Other costs

We have now arrived at the topmost input-demand nest of Figure 4.2. Total intermediate inputs, the primary-factor composite and 'other costs' are combined using a Leontief production function, $MIN()$, given by

$$Z(j, q) = \frac{1}{A1(j, q)} \times MIN \left(X1COM, \frac{X1PRIM(j, q)}{A1PRIM(j, q) \times A1PRIMGEN(q)}, \frac{X1OCT(j, q)}{A1OCT(j, q)} \right) \quad j \in IND \quad q \in REGDST.$$

In the above production function, $Z(j, q)$ is the output of the j th industry in region q , the A variables are Hicks-neutral technical change terms and $X1OCT(j, q)$ is the demand by industry j in region q for 'other cost tickets'. Other cost tickets allow for costs not explicitly identified in MMRF-GREEN such as working capital and the costs of holding inventories

As a consequence of the Leontief specification of the production function, each of the three categories of inputs identified at the top level of the nest are demanded in direct proportion to $Z(j, q)$, as indicated in equations E_x1com , E_x1prim and E_x1oct .

The final equation in this section (E_p1oct) specifies the movements in the price of other cost tickets ($p1oct(j, q)$). The price of other cost tickets to industry j in region q is specified as

$$P1OCT(j, q) = P3TOT(q) \times F1OCT(j, q) \quad j \in IND \quad q \in REGDST \quad (4.1.10),$$

where $P3TOT(q)$ is the level of the consumer price index (CPI) in region q , and $F1OCT(j, q)$ is a shift variable. If $F1OCT(j, q)$ is constant, then the price of other costs tickets for industry j in region q moves with the CPI in q . Changes in $F1OCT(j, q)$ cause changes in the price of other costs tickets relative to the CPI. E_p1oct is the percentage change form of (4.1.10).

4.1.2. Demands for investment goods (E_x2a to E_x2o)

Capital creators for each regional sector combine inputs to form units of capital. In choosing these inputs they cost minimise subject to technologies similar to that in Figure 4.2. Figure 4.3 shows the nesting structure for the production of new units of fixed capital. Capital is produced with inputs of domestically produced and imported commodities. No primary factors are used directly as inputs to capital formation. The use of primary factors in capital creation is recognised through inputs of the commodity, construction services.

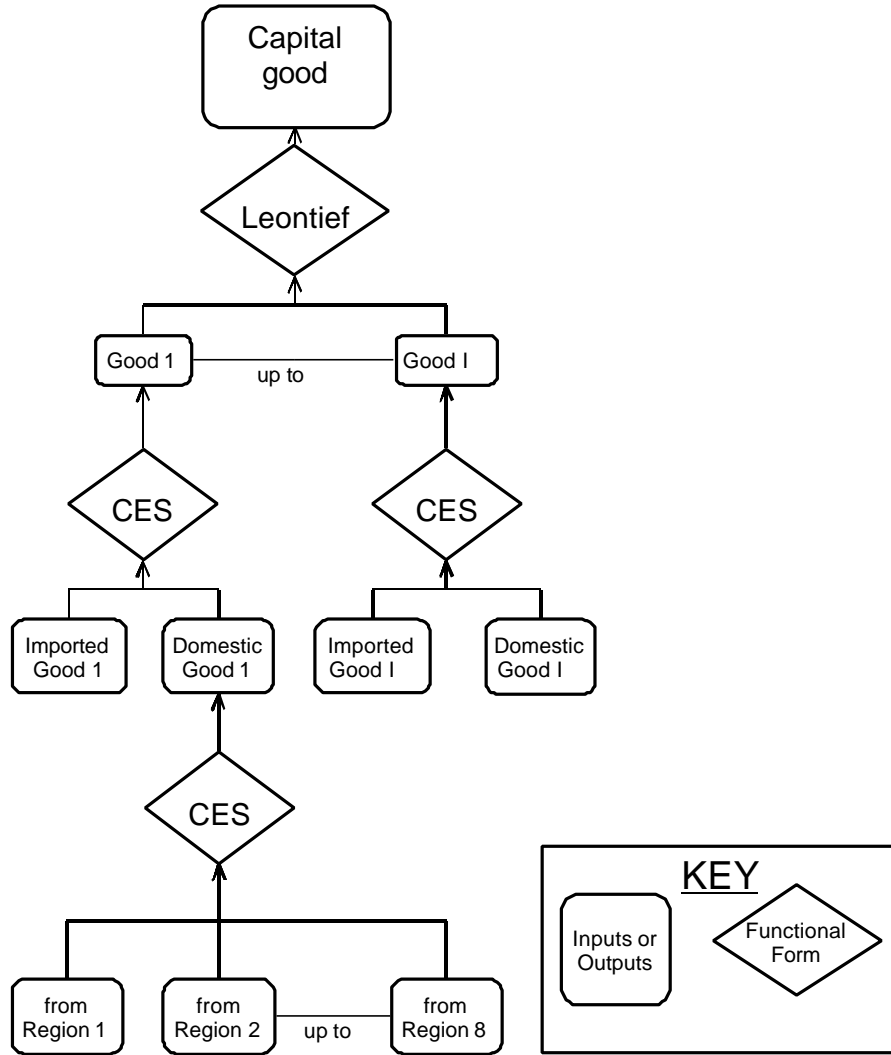


Figure 4.3. Structure of investment demand

The model's investment equations are derived from the solutions to the investor's three-part cost-minimisation problem. At the bottom level, the total cost of domestic-commodity composites of good i ($X2C(i,q)$) is minimised subject to the CES production function

$$X2C(i,q) = \text{CES}_{s \in \text{regsrc}} \{X2A(i,s,q)\} \quad i \in \text{COM} \quad q \in \text{REGDST},$$

where the $X2A(i,s,q)$ are the demands in the q th region for the i th commodity from the s th domestic region for use in the creation of capital. Similarly, at the second level of the nest, the total cost of the domestic/foreign-import composite ($X2O(i,q)$) is minimised subject to the CES production function

$$X2O(i,q) = \text{CES} \{X2A(i, \text{"imp"}, q), X2C(i, j)\} \quad i \in \text{COM} \quad q \in \text{REGDST},$$

where the $X2A(i, \text{"imp"}, q)$ are demands for the foreign imports.

Notice that $X2C$, $X2A$ and $X2O$ do not have industry indexes, i.e., they are not distinguished by industry. We assume that the domestic-source composition and the import/domestic composition of commodity i used for investment in region q is identical across all investing industries in region q .

The equations describing the demand for the source-specific inputs (E_{x2a} , E_{x2c} , E_{p2c} and E_{p2o}) are similar to the corresponding equations describing the demand for intermediate inputs to current production (i.e., E_{x1a} , E_{x1c} , E_{p1c} and E_{p1o}). The main difference is the lack of industry index in the investment equations.

At the top level of the nest, the total cost of commodity composites is minimised subject to the Leontief function

$$Y(j, q) = \min_{i \in \text{com}} \left\{ \frac{X2(i, j, q)}{A2IND(i, j, q)} \right\} \quad j \in \text{IND} \quad q \in \text{REGDST},$$

where the total amount of investment in each industry ($Y(j, q)$) is exogenous to the cost-minimisation problem and the $A2IND(i, j, q)$ are technological-change variables in the use of inputs in capital creation. The variable $X2(i, j, q)$ represents the demand for commodity composite i used for investment by industry j in region q . The resulting demand equations for the composite inputs to capital creation (E_{x2}) correspond to the demand equations for the composite input to current production (i.e., E_{x1o}). Equation E_{x2o} is the percentage change form of the relationship between $X2(i, j, q)$ and $X2O(i, q)$, i.e.,

$$X2O(i, q) = \sum_{j \in \text{ind}} X2(i, j, q) \quad i \in \text{COM} \quad q \in \text{REGDST}.$$

Determination of the number of units of capital to be formed for each regional industry (i.e., determination of $Y(j, q)$) depends on the nature of the experiment being undertaken. For comparative-static experiments, a distinction is drawn between the short run and long run. In short-run experiments (where the year of interest is one or two years after the shock to the economy), capital stocks in regional industries are exogenously determined.

In long-run comparative-static experiments (where the year of interest is five or more years after the shock), it is assumed that the aggregate capital stock adjusts to preserve an exogenously determined economy-wide rate of return, and that the allocation of capital across regional industries adjusts to satisfy exogenously specified relationships between relative rates of return and relative capital growth. Industries' demands for investment goods are determined by exogenously specified investment/capital ratios.

In year-to-year dynamic experiments, regional industry demand for investment is determined via dynamic equations like (2.1) and (2.2). Details of the determination of investment and capital, when MMRF-GREEN is run in dynamic mode, are provided in a later section.

4.1.3. Household demands (E_{x3o} to E_{x3c})

Each regional household determines the optimal composition of its consumption bundle by choosing commodities to maximise a Stone-Geary utility function subject to a household budget constraint. A *Keynesian* consumption function determines aggregate regional household expenditure as a function of household disposable income.

Figure 4.4 reveals that the structure of household demand follows nearly the same nesting pattern as that of investment demand. The only difference is that commodity composites are aggregated by a Stone-Geary, rather than a Leontief function, leading to the linear expenditure system (LES).

The equations for the two lower nests (E_{x3a} , E_{p3o} , E_{p3c} and E_{x3c}) are similar to the corresponding equations for intermediate and investment demands.

The equations determining the commodity composition of household demand, which is determined by the Stone-Geary nest of the structure, differ from the CES pattern established in sections

4.1.1 and 4.1.2.⁹ To analyse the Stone-Geary utility function, it is helpful to divide total consumption of each commodity composite ($X3O(i,q)$) into two components: a subsistence (or minimum) part ($X3SUB(i,q)$) and a luxury (or supernumerary) part ($X3LUX(i,q)$)

$$X3O(i,q) = X3SUB(i,q) + X3LUX(i,q) \quad i \in \text{COM} \quad q \in \text{REGDST} \quad (4.1.11).$$

A feature of the Stone-Geary function is that only the luxury components affect per-household utility (UTILITY), which has the Cobb-Douglas form

$$\text{UTILITY}(q) = \frac{1}{QHOUS(q)} \times \sum_{i \in \text{com}} X3LUX(i,q)^{A3LUX(i,q)} \quad q \in \text{REGDST} \quad (4.1.12),$$

where

$$\sum_{i \in \text{com}} A3LUX(i,q) = 1 \quad q \in \text{REGDST}.$$

⁹ For details on the derivation of demands in the LES, see Dixon, Bowles and Kendrick (1980) and Horridge *et al.* (1993).

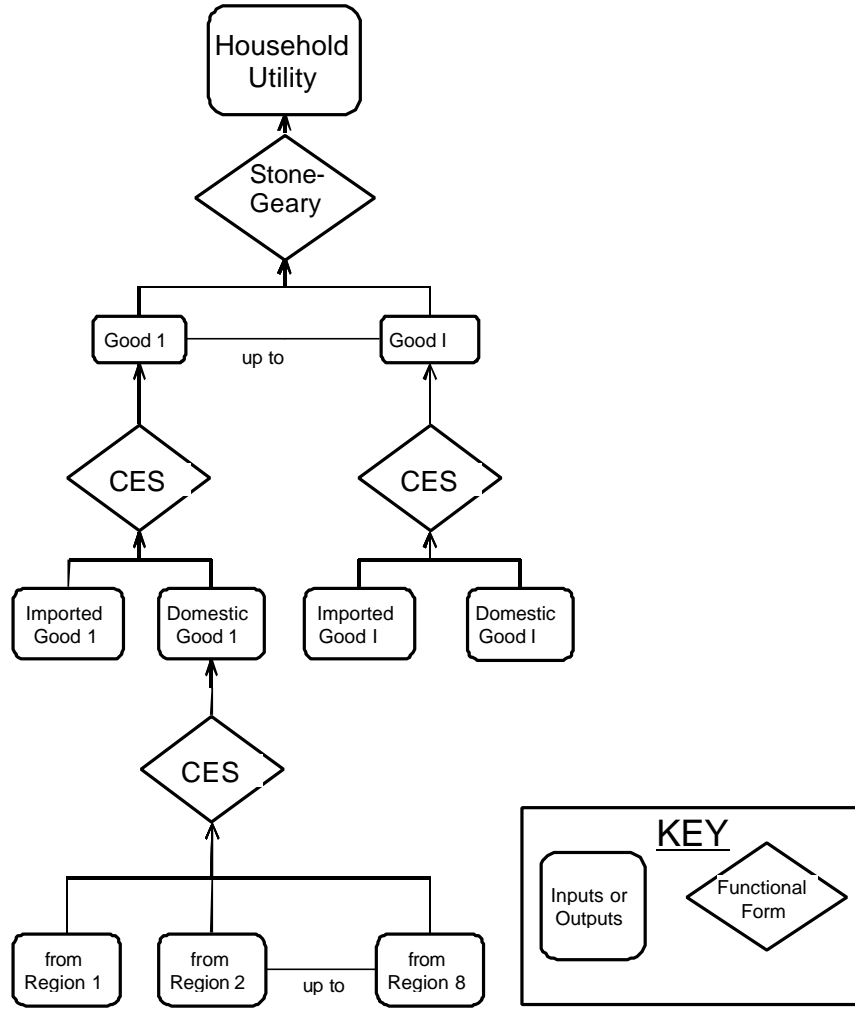


Figure 4.4. Structure of household demand

Because the Cobb-Douglas form gives rise to exogenous budget shares for spending on luxuries

$$P3O(i,q)X3LUX(i,q) = A3LUX(i,q)LUXEXP(q) \quad i \in COM \quad q \in REGDST, \quad (4.1.13)$$

$A3LUX(i,q)$ may be interpreted as the marginal budget share of total spending on luxuries ($LUXEXP(q)$). Rearranging (4.1.13), substituting into (4.1.11) and linearising gives equation E_{x3o} , where the subsistence component is proportional to the number of households and to a taste-change variable ($a3sub(i,q)$), and $ALPHA_I(i,q)$ is the share of supernumerary expenditure on commodity i in total expenditure on commodity i . Equation $E_{utility}$ is the percentage-change form of (4.1.12); the utility function.

Equations E_{a3sub} and E_{a3lux} provide default settings for the taste-change variables ($a3sub(i,q)$ and $a3lux(i,q)$), which allow for the average budget shares to be shocked, via the $a3com(i,q)$, in a way that preserves the pattern of expenditure elasticities.

The equations just described determine the composition of regional household demands, but do not determine total regional consumption. As mentioned, total household consumption is determined by regional household disposable income. The determination of regional household disposable income and regional total household consumption is described in a later section.

4.1.4. Foreign export demands

To model export demands, commodities in MMRF-GREEN are divided into four groups: the traditional exports, which comprise the bulk of exports; non-traditional exports, which are mainly manufactured goods; tourism (travel and hospitality services); and special, which consist of water transport, other transport services and communications. Exports account for relatively large shares in total sales of traditional export commodities. For each category, the model allows a different treatment of export demand.

Traditional exports (E_{x4r})

The traditional-export commodities (i.e., commodities in the set TEXP) are modelled as facing downward-sloping foreign-export demand functions

$$X4R(i,s) = FEQ(i,s) \times NATFEQ \times \left(\frac{P4R(i,s)}{FEP(i) \times NATFEP} \right)^{EXP_ELAST(i)} \quad i \in TEXP \quad s \in REGSOURCE \quad (4.1.14).$$

$X4R(i,s)$ is the export volume of commodity i from region s . The coefficient $EXP_ELAST(i)$ is the (constant) own-price elasticity of foreign-export demand. As $EXP_ELAST(i)$ is negative, (4.1.14) says that traditional exports are a negative function of their foreign-currency prices on world markets ($P4R(i,s)$). The variables $FEQ(i)$ and $FEP(i)$ allow for horizontal (quantity) and vertical (price) shifts in the demand schedules. The variables $NATFEQ$ and $NATFEP$ allow for economy-wide horizontal and vertical shifts in the demand schedules. E_{x4r} is the percentage-change form of (4.1.14).

Non- traditional exports (E_{aggnt_x4r} to E_{aggnt_p4r})

E_{nt_x4r} specifies the export demand for the non-traditional export commodities (i.e., commodities in the set NTEXP). The set NTEXP consists of all commodities except the three for which special modelling of export demands is provided and the commodities classified as traditional and tourism exporters. In MMRF-GREEN the commodity composition of aggregate non-traditional exports is exogenised by treating non-traditional exports as a Leontief aggregate. Thus, as shown in E_{nt_x4r} , with the shift variables $faggnt_i(i)$, $faggnt_s(s)$, $faggnt_is(s)$ and $natfep$ set to zero, the export demands for all non-traditional exports from source-region s move by the same percentage, $aggnt_x4r(s)$. The common percentage change is explained by equation E_{aggnt_x4r} . This equation relates movements in demand for non-traditional exports from region s to movements in the average foreign currency price of those exports via a constant-elasticity demand curve, similar to those for traditional exports. The elasticity of substitution is given by the coefficient $EXP_ELAST(\text{"cement"})$: in the current version of the model "cement" identifies the cement commodity, a typical non-traditional export commodity. Under this treatment, non-traditional exports respond as a group to changes in the group's international competitiveness (determined mainly by changes in costs in manufacturing industries expressed in foreign currency).

We use the shift variables in equations E_{aggnt_x4r} to simulate various types of vertical and horizontal shifts in the export demand schedule for non-traditional exports from region s . For example, if $aggnt_feq(s)$ has a non-zero value, then we impose a horizontal shift on the group's export demand curve.

To simulate changes in the commodity composition of non-traditional exports, we can use non-zero settings for the shift variables in E_{nt_x4r} . For example, to cause the export volume of non-traditional component C7 in region s to change by a given percentage amount, we can make $x4r("C7",s)$ exogenous by freeing up $fagnt_is("C7",s)$. In this case, the model would endogenously determine the value for $fagnt_is("C7",s)$ which would reconcile the exogenously imposed setting of $x4r("C7",s)$ with the simulated value for $aggnt_x4r(s)$.

Movements in the average foreign-currency price of non-traditional exports from region s ($aggnt_p4r(s)$) are determined via equation E_{aggnt_p4r} . The coefficient $AGGEXPNT(s)$ is the aggregate purchases value of non-traditional exports from region s .

Tourism exports (E_{tour_x4r} to $E_{nattour}$)

These equations specify demands by foreign visitors in region s for tourism services, i.e., for commodities in the set TOUR. The the foreign elasticity of demand for tourism services is set in the code at -4 .

Equations E_{tour_x4r} to E_{tour_p4r} are similar to E_{aggnt_x4r} to E_{aggnt_p4r} . As with non-traditional exports, we adopt a “bundle” approach to explaining exports of tourism services. Foreigners are viewed as buying a bundle of tourism services. The price of the tourism bundle is a Divisia index of the prices of all tourism exports.

The bundle-specification for tourism exports, which is also adopted in MONASH, is theoretically attractive. It is reasonable to think of foreign tourists as buying service bundles consisting of a fixed combination of commodities, with the number of bundles purchased being sensitive to the cost of a bundle”. In other words, it is reasonable to think of the export demands for tourism commodities being tightly linked, each being determined not by movements in their individual price, but by movements in their overall average price.

Exports of communications services (E_{x4rC})

This equation explains exports of communication services, the only element in the set COMMUNIC. Following the treatment in MONASH, exports of communications services from source s are driven by the volume of foreign imports of communications services into s ($XOIMP(i,s)$, for $i \in \text{COMMUNIC}$). This is based on the observation that communication exports consist mainly of charges by Australian telephone companies for distributing incoming phone calls, and of charges by Australian post for delivering foreign mail within Australia. Accordingly, on the assumption that outgoing communications generate incoming communications, the volume of communications imports drives the volume of communications exports. The variable $fcommunic_s(i,s)$ for $i \in \text{COMMUNIC}$ allows for shifts in the ratio of communication exports to imports in region s .

Exports of water transport services ($E_{tradexpvol}$ and E_{x4rF})

E_{x4rF} when activated deals with exports of commodities in the set WATTRANS. This equation is not activated unless the database is appropriately disaggregated. This set contains a single element, water transport freight services. Following the treatment in MONASH, exports of water transport freight in region s are assumed to move in line with the aggregate volume of traditional exports ($TRADEXPVOL(s)$). The rationale is that the main use of water transport services outside Australia is for the shipment of bulk traditional exports, especially, iron ore, coal, wool and grain. The variable $fwattrans_s(i,s)$ for $i \in \text{WATERTRAN}$ allows for shifts in the ratio of water transport exports to $TRADEXPVOL$.

$E_{tradexpvol}$ explains movements in the aggregate volume of traditional exports.

Exports of other transport services (E_{x4rD})

E_{x4rD} deals with exports of other transport services, the single element in the set OTHTRANS. Again, we follow the MONASH treatment. Exports of other transport services consist mainly of harbour

and airport services provided to foreign ships and planes in Australia. The MONASH treatment recognises three reasons for these trips to Australia: (a) to carry Australian passengers to and from Australia; (b) to carry foreign passengers to and from Australia; and (c) to facilitate commodity trade. Imports of air transport services and tourism exports are used to proxy (a) and (b), while the aggregate volume of traditional exports is used as a proxy for (c). The weights shown in E_{x4rD} are essentially guesses. The shift variable $fothtrans_s(i,s)$ allows for extraneous shifts.

4.1.5. Government consumption demands (E_{x5a} and E_{x6a})

Equations E_{x5a} and E_{x6a} determine State/territory government and Federal government demands (respectively) for commodities for current consumption. Neither of these sources of demand is modelled explicitly in MMRF-GREEN.

However, it is convenient to include equations E_{x5a} and E_{x6a} . In E_{x5a} , State/territory government consumption is constrained to preserve a constant ratio with State private consumption expenditure ($CR(q)$). The shift variables $f5a(i,s,q)$ and $f5gen(q)$ allow for shifts in the ratio of $X5A(i,s,q)$ to $CR(q)$. To impose a non-uniform change in the ratio, we can use non-zero settings for the 'f5a' variables. To impose a uniform change, we can use non-zero settings for the 'f5gen' variables. Likewise, Federal government consumption expenditure can be set to preserve a constant ratio with national private consumption expenditure ($NATX3TOT(q)$), with changes in the ratio made possible by non-zero settings for the 'f6a' and 'f6gen' variables.

4.1.6. Demands for margin services (E_{x1marg} to $E_{p4_roadrail}$)

Commodities in the set MARGCOM can be used as margins. Typical elements of MARGCOM are wholesale and retail trade, road freight, rail freight, water freight and air freight. These commodities, in addition to being consumed directly by the users (e.g., consumption of transport when taking holidays or commuting to work), are also consumed to facilitate trade (e.g., the use of transport to ship commodities from point of production to point of consumption). The latter type of demand for transport is a so-called demand for margins.

Equations E_{x1marg} , E_{x2marg} , E_{x3marg} , E_{x4marg} , E_{x5marg} and E_{x6marg} give the demands by users 1, 2, 3, 4, 5 and 6 for margins. For margins other than road and rail freight, equations E_{x1marg} to E_{x4marg} indicate that the demands are proportional to the commodity flows with which the margins are associated. For example, the demand for margin type $r = \text{'wholesale trade'}$ on the flow of good s from source s to industry j in region q for use in current production ($X1MARG(i,s,j,q,r)$) moves with the underlying demand ($X1A(i,s,j,q)$). In each equation there is a technological variable ($a1marg(q,r)$, etc.) representing the use of margin service r per unit of demand in region q .

In MMRF-GREEN, this treatment has been modified to allow for substitution possibilities between road and rail freight. Specifically, for a flow from region s to region q , substitution is allowed between road freight and rail freight provided by region q . The substitution is based on relative prices. If in region q , the price of road freight increases relative to the price of rail freight, then there will be substitution away from road freight towards rail freight in all uses of the two in region q .

The substitution effects are modelled by introducing into equations E_{x1marg} to E_{x4marg} a relative price term involving the price of margin r ($r = \text{'road freight'}$ or 'rail freight') in region q relative to the average price of road and rail transport. The coefficient $IsRoadRail(r)$ equals one when r is 'road freight' or 'rail freight', and is zero otherwise. $SIGRoadRail$ is the inter-modal substitution elasticity for road and rail.

Movements in the average prices of road and rail freight for each user of freight ($P1_ROADRAIL(i,s,j,q)$, $P2_ROADRAIL(i,s,j,q)$, $P3_ROADRAIL(i,s,q)$ and $P4_ROADRAIL(i,q)$) are explained by equations $E_{p1_roadrail}$ to $E_{p4_roadrail}$. In these equations, the 'MAR' coefficients are matrices of data showing the cost of the margins services on the flows of goods, both domestically produced and imported, to users.

4.1.7. Zero pure profits and basic prices (E_{p1cap} to E_{p0c})

As is typical of ORANI-style models, the price system underlying MMRF-GREEN is based on two assumptions: (i) that there are no pure profits in the production or distribution of commodities, and (ii) that the price received by the producer is uniform across all customers.

Also in the tradition of ORANI, is the presence of two types of price equations: (i) zero pure profits in current production, capital creation and importing and (ii) zero pure profits in the distribution of commodities to users. Zero pure profits in current production, capital creation and importing is imposed by setting unit prices received by producers of commodities (i.e., the commodities' basic values) equal to unit costs. Zero pure profits in the distribution of commodities is imposed by setting the prices paid by users equal to the commodities' basic value plus commodity taxes and the cost of margins.

Equations E_{p1cap} and E_a impose the zero pure profits condition in current production. Given the constant returns to scale which characterise the model's production technology, equation E_{p1cap} defines the percentage change in the price received per unit of output by industry j of region q ($p1tot(j,q)$) as a cost-weighted average of the percentage changes in effective input prices. The percentage changes in the effective input prices represent (i) the percentage change in the cost per unit of input and (ii) the percentage change in the use of the input per unit of output (i.e., the percentage change in the technology variable). These cost-share-weighted averages define percentage changes in average costs. Setting output prices equal to average costs imposes the competitive zero pure profits condition.

In equation E_a we define $a(j,q)$ as an aggregation of all the different types of technological change that affect the costs of industry j in region q . The different types of technological change appear on the RHS of the equation with weights reflecting their influence on industry j 's unit costs. Equation E_{pi} imposes zero pure profits in capital creation. E_{pi} determines the percentage change in the price of new units of capital ($pi(j,q)$) as the percentage change in the effective average cost of producing the unit.

Zero pure profits in imports of foreign-produced commodities is imposed by equation E_{p0c} . The price received by the importer for the i th commodity ($p0a(i,"imp")$) is given as the product of the foreign price of the import ($PM(i)$), the exchange rate ($NATPHI$) and one plus the rate of tariff (the so-called power of the tariff: $POWTAXM(i)$)¹⁰.

4.1.8. Zero pure profits in distribution and purchasers' prices (E_{p1a} to E_{p6a})

The remaining zero-pure-profits equations relate the price paid by purchasers to the producer's price, the cost of margins and commodity taxes. Six users are recognised in MMRF-GREEN and zero pure profits in the distribution of commodities to the users are imposed by the equations E_{p1a} , E_{p2a} , E_{p3a} , E_{p4r} , E_{p5a} and E_{p6a} .

The tax variables appearing on the RHS of each equation are change variables. Specifically, they are percentage-point changes in rates of *ad valorem* sales taxes. For example, $deltax3(i,s,q)$ is the percentage point change in the *ad valorem* rate of tax imposed in region q on sales to consumption of commodity i from source s .

4.1.9. Indirect taxes ($E_{deltax1}$ to $E_{deltax6}$)

Equations $E_{deltax1}$ to $E_{deltax6}$ contain the default rules for setting sales-tax rates for producers ($E_{deltax1}$), investors ($E_{deltax2}$), households ($E_{deltax3}$), exports ($E_{deltax4}$), and government ($E_{deltax5}$ and $E_{deltax6}$). At this point in the code, all sales taxes are treated as *ad valorem* on the price received by the producer, with the sales-tax variables (the *deltax*) being the ordinary change in the percentage tax rate, i.e., the percentage-point change in the tax rate. Thus a value of $deltax1$ of 20 means the percentage tax rate on commodities used as inputs to current production increased from, say, 20 percent to 40 percent, or from, say, 24 to 44 percent.

For each user, the sales-tax equations allow for variations in tax rates across commodities, their sources and their destinations via changes to a wide range of shift variables. In equation $E_{deltax1}$, the

¹⁰ If the tariff rate is 20 percent, the power of tariff is 1.20. If the tariff rate is increased from 20 percent to 25 percent, the percentage change in the power of the tariff is 4, i.e., $100(1.25-1.20)/1.20 = 4$.

coefficient $ISPETPROD(i)$ equals one when i is a petroleum production, and is zero otherwise. Thus the RHS-terms enclosed by the curly brackets $\{ \}$ only affect taxes rated on sales of non-petroleum products. One of these terms, $deltax1_gst(i,s,j,q)$ is the change in the *ad valorem* tax rate arising from the imposition of the GST. The modelling of the GST is discussed in a later section. Changes in taxes on petroleum products used by industries are modelled via non-zero values for $fueltax1$. The modelling of these changes is discussed later. The variable $fueltax1$ is used in the modelling of carbon taxes. $ISFUEL(i)$ equals one when i is a carbon-emitting fuel (e.g., black coal), and is zero otherwise. Greenhouse taxes are modelled in a later section.

4.1.10. Indirect tax revenues ($E_taxrev1$ to $E_nattaxrev6$)

Equations $E_taxrev1$ to $E_taxrev6$ compute the percentage changes in regional aggregate revenue raised from indirect taxes. The bases for the regional sales taxes are the regional basic values of the corresponding commodity flows. Hence, for any component of sales tax, we can express revenue (TAXREV), in levels, as the product of the base (BAS) and the tax rate (T), i.e.,

$$TAXREV = BAS \times T.$$

Hence,

$$\Delta TAXREV = T \Delta BAS + BAS \Delta T \quad (4.1.15)$$

The basic value of the commodity can be written as the product the producer's price ($P0$) and the output (XA)

$$BAS = P0 \times XA. \quad (4.1.16)$$

Using (4.1.15) and (4.1.16), we can derive equations $E_taxrev1$ to $E_taxrev6$ by taking the percentage changes of the tax revenue (TAXREV) and basic value (BAS) variables, and the ordinary change in the tax rate (T) variable multiplied by 100

$$TAXREV \times taxrev = TAX \times (p0 + xa) + BAS \times deltax$$

where

$$taxrev = 100 \left(\frac{\Delta TAXREV}{TAXREV} \right)$$

$$TAX = BAS \times T$$

$$p0 = 100 \left(\frac{\Delta P0}{P0} \right)$$

$$xa = 100 \left(\frac{\Delta XA}{XA} \right)$$

and

$$deltax = 100 \times \Delta T.$$

4.1.11. Market-clearing equations for commodities

Market clearing for commodities (E_x0comA to E_x0impa)

Equations E_x0comA , E_x0comB and E_x0impa impose the condition that demand equals supply for domestically produced margin and non-margin commodities and for imported commodities.

The output of regional industries producing margin commodities must equal the direct demands by the model's six users and their demands for the commodity as a margin. Note that the specification of equation E_x0comA imposes the assumption that margins are produced in the destination region, with the exception that margins on exports are produced in the source region. We write the market-clearing

equations in terms of basic values. On the LHS of E_{x0comA} , the coefficient $SALES(r,s)$ is the basic value of the output of domestic margin good r produced in region s . On the RHS, the coefficients are the basic values of intermediate (BAS1), investment (BAS2), household (BAS3), export (BAS4) and government (BAS5 and BAS6) demands plus the basic values of margin (MAR1, MAR2, MAR3, MAR4, MAR5 and MAR6) demands.

In equation E_{x0comB} , changes in the outputs of the non-margin regional industries are set equal to the changes in direct demands of the model's six users. The equation is similar to E_{x0comA} , except that it excludes the margin demands.

Equation E_{x0impa} imposed supply/demand balance for imported commodities. Import supplies are equal to the demands of the users excluding foreigners, i.e., all exports involve some domestic value added.

Commodity supply and other market clearing related equations (E_{x1tot} to E_{p0aB})

Equation E_{x1tot} relates movements in the average price received by industry j in region q to movements in the prices of products produced by industry j . On the RHS, the coefficient $MAKESHR_COM(i,j,q)$ is the share of commodity i in total production of industry j in region q . In the current version of the model, there is only one multiproduct industry, petroleum products. Thus for all industries other than petroleum products, $MAKESHR_COM(i,j,q)$ equals zero for all but one commodity.

Equation E_{q1} explains the commodity composition of the multiproduct industries. The set $JPCOM$ identifies the commodities that are produced by the multiproduct industries identified in the set $JPIND$. Equation E_{q1} specifies that the percentage change in the supply of commodity i by multiproduct industry j is made up of two parts. The first is $x1tot(j)$, the percentage change in the overall level of output of industry j . The second is a price-transformation term. This compares the percentage change in the price received by industry j for product i with the weighted average of the percentage changes in the prices of all industry j 's products. The derivation of equation E_{q1} is detailed in Section 11 of Dixon *et al.* (1982).

Equations E_{p0aA} and E_{p0aB} explain the percentage change in overall output of commodity i in region in terms of the industry-specific outputs of i in q . E_{p0aB} deals with the unique-product industries, i.e., all industries other than the multiproduct industries in the set $JPIND$.

4.1.12. Regional incomes and expenditures

In this section, we outline the derivation of the income and expenditure components of regional gross product. We begin with the nominal income components.

Income-side aggregates of regional gross product (E_{caprev} to E_{x1tot_agg})

The income-side components of regional gross product include regional totals of factor payments, other costs and the total yield from commodity taxes. Nominal regional factor income payments are given in equations E_{caprev} , E_{labrev} and E_{lndrev} for payments to capital, labour and agricultural land, respectively. The regional nominal payments to other costs are given in equation E_{octrev} .

The derivation of the factor income and other cost regional aggregates are straightforward. Equation E_{caprev} , for example is derived as follows. The total value of payments to capital in region q ($AGGCAP(q)$) is the sum of the payments of the j industries in region q ($CAPITAL(j,q)$), where the industry payments are a product of the unit rental value of capital ($P1CAP(j,q)$) and the number of units of capital employed ($X1CAP(j,q)$)

$$AGGCAP(q) = \sum_{j \in ind} P1CAP(j,q) X1CAP(j,q) \quad q \in REGDST \quad (4.1.17).$$

Equation (4.1.17) can be written in percentage changes as

$$caprev(q) = \frac{1}{AGGCAP(q)} \times \sum_{j \in ind} CAPITAL(j, q) \times (p1cap(j, q) + x1cap(j, q))$$

$q \in REGDST,$

giving equation E_caprev , where the variable $caprev(q)$, is the percentage change in rentals to capital in region q and has the definition,

$$caprev(q) = 100 \left(\frac{\Delta AGGCAP(q)}{AGGCAP(q)} \right)$$

$q \in REGDST.$

The regional tax-revenue aggregates are given by equations E_taxind and $E_taxrevm$. E_taxind determines the variable $taxind(q)$, which is the weighted average of the percentage changes in the commodity-tax revenues raised from the purchases of producers, investors, households, foreign exports and the regional government. Equation $E_taxrevm$ determines tariff revenue on imports absorbed in region q ($taxrevm(q)$). Equation $E_taxrevm$ is similar in form to equations $E_taxrev1$ to $E_taxrev6$ discussed in Section 4.1.10. However, the tax-rate term in equation $E_taxrevm$, $powtaxm_q$, refers to the percentage change in the power of the tariff rather than the percentage-point change in the tax rate (as is the tax-rate term in the commodity-tax equations of section 4.1.10).

The remaining equations in this section define movements in miscellaneous supply-side aggregates. E_l explains movements in aggregate employment, E_kt explains movements in the aggregate stock of capital, and E_x1tot_agg explains movements in aggregate output.

Expenditure-side aggregates of regional gross product (E_luxexp to E_x3tot_shr)

For each region, MMRF-GREEN contains equations determining aggregate expenditure by households, investors, regional government, the Federal government and the interregional and foreign trade balances. For each expenditure component (with the exception of the interregional trade flows), we define a quantity index and a price index and a nominal value of the aggregate. For interregional exports and imports, we define an aggregate price index and quantity index only.

As with the income-side components, each expenditure-side component is a definition. As with all definitions within the model, the defined variable and its associated equation could be deleted without affecting the rest of the model. The exception is regional household consumption expenditure (see equations E_luxexp , E_x3tot and E_p3tot). It may seem that the variable $w3tot(q)$ is determined by the equation E_luxexp . This is not the case. Nominal household consumption is determined either by a macro-style consumption function or, say, by a constraint on the regional trade balance. Equation E_luxexp plays the role of a budget constraint on household expenditure.

The equations defining the remaining aggregate regional real expenditures, nominal expenditures and related price indices are listed below in the order: investment, regional government, Federal government, interregional exports, interregional imports, international exports and international imports. The equations defining quantities are E_x2tot , E_x5tot , E_x6tot , E_xsexp , E_xsimp , E_x4tot and E_x0imp . The definitions of nominal values are given by equations E_w2tot , E_w5tot , E_w6tot , E_w4tot and E_wmtot .

The derivation of the quantity and price aggregates for the interregional trade flows involves an intermediate step represented by equations E_xsflo and E_psflo . These equations determine inter- and intra- regional nominal trade flows in basic values.¹¹ In equation E_xsflo , inter-regional exports are calculated as the difference between total inter-regional sales by source and local sales.

Equation E_x3tot_shr allows for the indexing of regional real household consumption with national real household consumption in the case where the percentage change in the regional-to-national consumption variable, $x3tot_shr(q)$ is exogenous and set to zero. Otherwise, $x3tot_shr(q)$ is endogenous

¹¹ The determination in basic values reflects the convention in MMRF-GREEN that all margins and commodity taxes are paid in the region which absorbs the commodity.

and regional consumption is determined elsewhere in the model (say, by the regional consumption function).

4.1.13. National aggregates (E_natcaprev to E_natrealdev)

This set of equations in the CGE core of MMRF-GREEN define economy-wide variables as aggregates of regional variables. As MMRF-GREEN is a bottoms-up regional model, all behavioural relationships are specified at the regional level. Hence, national variables are simply add-ups of their regional counterparts.

4.1.14. Regional and national price indexes (E_p3tot to E_nattot)

The final set of equations regional and economy-wide aggregate price indexes.

4.1.15. Regional wages (E_fwagei to E_p1laboi)

The equations in this section have been designed to provide flexibility in the setting of regional wages. Equation E_{fwagei} allows for the indexing of the workers wage to the national consumer price index (natp3tot). The workers wage is defined in a later section as the wage paid by industry (P1LABOI(j,q,m) averaged over all occupations) after payroll tax. The 'fwage' variables in E_{fwagei} allow for deviations in the growth of wages relative to the growth of the national consumer price index.

Equations E_{p_wage} and E_{pwage_p} define movements in the region-wide nominal wage received by workers, and the region-wide nominal wage paid by producers.

Equation E_{wage_diff} allows flexibility in setting movements in regional wage differentials. The percentage change in the wage differential in region q (wage_diff(q)) is defined as the difference between the aggregate regional real wage received by workers (pwage(q)- natxi3) and the average real wage received by workers across all regions (natrealwage).

Equation E_{rwage_diff} defines movements in the real wage rate differential in region q (rwage_diff(q)) as the difference between movements in the real producer wage rate in region q (realwage_p(q)) and movements in the national real producer wage rate (natrwage_p). The producer real wage rate is defined as the producer wage rate deflated by the price of GDP.

The final equation in this section, $E_{p1laboi}$ ties the price of labour for occupation m in industry j of region q to the general price of labour in industry j of q. An additional (endogenous) variable (f_p1lab(j,q)) is required on the right hand side to reconcile this equation with the existing core equation (E_{p1lab}) that sets p1lab equal to the weighted sum of percentage changes in occupational wage rates (p1laboi(j,q,m)).

4.1.16. Miscellaneous definitions of factor prices (E_natp1cap to E_realwage_p)

The equations in this section define aggregate national and regional quantities prices in the labour and capital markets. Two concepts of the real wage rate are used here. The consumer real wage rate is the nominal wage rate deflated by the consumer price index. The producer real wage rate is the nominal wage rate deflated by the producer price deflator, i.e., deflated by the GDP deflator.

4.1.17. Employment aggregates (E_lambda and E_natlambda)

E_{lambda} defines regional employment of each of the eight occupational skill groups. $E_{natlamda}$ defines national employment of each of the eight occupations.

4.2. Government finances and regional aggregates

In this block of equations, we determine the financial positions of regional and federal governments. The core model provides us with commodity tax revenues and current government expenditures. To provide more detailed government finances on income and outlays, we need to append these core data with additional government accounts data, mostly from published sources (e.g., ABS catalogue no. 5506.0).

4.2.1. Government revenues

Commodity taxes are split between Commonwealth and state revenues. MMRF-GREEN contains core equations for commodity tax revenues that need to be split between the different governments, at least to represent historical data. In the base year 1996-97, state governments collected franchise fees on gas, petroleum products, liquor and tobacco. In addition, they collected revenues from motor vehicles through stamp duty on registration and other taxes, and from gambling. Each of these revenues is a tax on a commodity usage. MMRF-GREEN contains equations linking commodity-specific sales with these revenues. The Commonwealth's portion of each tax is calculated as the total in the core database minus the state commodity tax revenues.¹²

Before proceeding with details of the revenue side of government accounts, we describe the notation system used within the model. Percentage changes in tax revenues are denoted by tr_*** . Percentage changes in government outlays are denoted by g_*** . Most variables within the state and national accounting equations of MMRF-GREEN are given names that bear some resemblance to what they represent.

The various state and local (SL) commodity taxes read into the government accounts database of MMRF-GREEN include SLTX_LIQ (liquor), SLTX_TOB (tobacco), SLTX_MV (motor vehicles), GAMBLE (gambling and lotteries) and INDR_OTH (other). The respective percentage change variables are tr_liq , tr_tov , tr_mv , tr_gamb and $tr_othindr$. Only "other" is linked to nominal gross state product in each region rather than commodity-specific sales. The equations E_tr_tovB , E_tr_liqB , E_tr_mvB , E_tr_gambB and $E_tr_othindrB$ calculate the percentage change in Commonwealth commodity tax revenues as residuals.

Next, we examine taxes on primary factor incomes. There are three such taxes in MMRF-GREEN that accrue to the Commonwealth, PAYE (PAYETOT) taxes, taxes on GOS (GOSTAXTOT) and other direct taxes (OTHDIRTX). PAYE taxes are linked to salaried labour income, GOS taxes to imputed wages and non-labour income, and other direct taxes to gross domestic product. PAYE and GOS taxes are calculated in a bottom-up manner, that is, as the sum of the relevant factor incomes at the sectoral level. In addition, the Commonwealth gathers revenues from agricultural production taxes (VLOCTX1) that are linked to output.

The constitution limits the spread of primary factor taxes charged by state governments. The sources of revenue are agricultural land taxes (LNDTX), rental property taxes and municipal rates (PROPTX_R), financial institutions' duties (abolished in 2001) and stamp duties on property purchases (allocated specifically to dwellings in PROPTX_R), mortgages and other transactions. The various duties are allocated as commodity rather than primary taxes. States also impose levies on statutory corporations and, in Queensland, on agricultural production (though at \$13 million in 1996-97, this was relatively trivial and for simplicity has been added to levy revenues).

Commonwealth to state government transfers are a major source of regional government revenue. The default equation sets these transfers proportional to nominal gross state product. Additional revenues accruing to the states include interest receipts (INT_RCPT) and other financing transactions (OTHREVN), as listed in ABS catalogue no. 5501.0.

¹² There have been important changes in legislation concerning state commodity taxes since 1996-97. A high court challenge in 1999 resulted in franchise fees being deemed unconstitutional. These were replaced by commodity taxes collected by the Commonwealth and redistributed to the states. The GST package introduced in July 2000 effectively removed the distinction between Commonwealth and state commodity taxes on the group of former "franchise fee" commodities.

Equation E_{tr_totA} (with some substitution for exposition) calculates state government revenue in percentage change terms as:

$$\begin{aligned} VOGOVREV * tr_tot = \\ PROLTX * tr_prol + PROPTX_R * tr_prop + LNDTX * tr_land + VIOCTX * tr_oct + \\ VOINDIR * tr_indir + INT_RCPT * r_int_rst + COMGRNT * trg_com_st + \\ OTHREVN * tr_oth_rev \end{aligned}$$

The corresponding equation for the percentage change in Commonwealth revenues is given by equation E_{tr_totB} .

Equations E_{wIpaye} and E_{tr_gos} show that Federal government income tax collections move in line with nation-wide movements in wages, salaries and supplements, the PAYE tax rate (t_{wagess}), the nation-wide GOS and the tax rate in non-wage income ($t_{nonwage}$); i.e., income tax collections move in line with the percentage changes in the tax base and the tax rates. Equations $E_{tr_dirothA}$ and $E_{tr_dirothB}$ impose the assumption that regional government and Federal government collections of 'other direct taxes' move in proportion to nominal gross regional product and nominal national GDP respectively.

Federal government tariff revenue is computed in equation E_{tr_tarif} . Other commodity taxes are calculated as the difference between the tax aggregates within the input-output structure of the model and state commodity taxes. For example, Federal liquor tax revenues in percentage change terms are

$$\begin{aligned} VOFEDLIQ * tr_liq("Federal") = \\ \sum_{q \in REGDST} \sum_{c \in liquor} [TAX_C(c, q) * tr_taxc(c, q) - SLTX(LIQ(q) * tr_liq(q))] \end{aligned}$$

where $TAX_C(c, q) * tr_taxc(c, q)$ is 100 multiplied by the percentage change in all liquor tax collections in region q , and Liquor is the subset of commodities to which such taxes apply. The variable tr_taxc is calculated using commodity-specific changes in prices, volumes and tax rates.

'Other commodity taxes' refer to taxes on all commodities other than the subsets Liquor, Gambling, Smokes, Petrol and MV (motor vehicles). We assume that no state taxes are collected on purchases of these commodities, so that all revenues accrue to the Commonwealth. Hence, the equation ($E_{tr_othindrB}$) is:

$$VOFEDOTH * tr_othindr("Federal") = \sum_{q \in REGDST} \sum_{c \in REST} TAX_C(c, q) * tr_taxc(c, q)$$

Other indirect taxes collected by the states, which are not commodity taxes, are tied to nominal GSP in equation $E_{tr_othindr}$.

Interest received by governments is determined in equation E_{tr_intrst} . We assume that interest received and interest paid by the government move in proportion to the size of the economy as measured by nominal gross regional product for regional governments, and nominal national GDP for the Federal government.

The final item of government revenue, 'other revenue', is described in equation $E_{tr_othrevA}$ (for regional governments) and equation $E_{tr_othrevB}$ (for the Federal government). As with Commonwealth grants, the default option is that 'other revenue' of regional governments is proportional to nominal gross regional product and for the Federal government it is proportional to national nominal GDP. Also in common with the 'grants' equations is the presence of shifters that allow the default option to be overridden.

4.2.2 Government expenditure and budgetary balance

We now summarise the expenditure-side of the government accounts by looking at the equations for the total outlays of the respective governments. For regional governments, the equation (E_g_totA) is:

$$C_EXP \times g_tot = G_GDS_SRV \times g_totgs + SUBSIDY \times g_gsubsidy + GINTRST \times g_int_rst + GOTHEXP \times g_othexp$$

We assume that subsidies are proportional to CPI, interest payments are proportional to nominal GSP and other expenditures proportional to total government expenditure. Equation E_g_totgs calculates total expenditure on goods and services as the sum of total current ($w56tot$) plus total investment ($w2totgov$) expenditures. The variable $w56tot$ for each regional government equals $w5tot$ (total current spending).

Next, we turn to the Commonwealth's total outlays equation (E_g_totB):

$$C_EXP \times g_tot = G_GDS_SRV \times g_totgs + SUBSIDY \times g_gsubdy + GINTRST \times g_int_rst + CGOTHEXP \times cg_OTHEXP + \sum_{t \in TYPE} COMGRTCUR(t) \times g_com_cur(t) + \sum_{q \in REGDST} PERSBEN_P(q) \times g_persbn(q)$$

At the Federal level, g_totgs is again calculated from weighted sums of $w56tot$ and $w2totgov$, but this time, $w56tot$ equals the weighted sum of $w6tot$ across all regions. Equation E_g_totB shows us that the Federal budget includes two groups of expenditures not incurred by the states. The first is Commonwealth grants (COMGRTCUR). There are four recipients identified in MMRF-GREEN: state governments, universities, private companies and local governments. Within the model, only grants to state governments are identified by destination. These are proportional to regional GSP, while the remaining grants are set proportional to national GDP.

The Commonwealth also disburses personal benefits to individuals, through unemployment benefits and other personal benefits. The variable g_persbn also appears in the equation E_w1hinc in which household disposable income is calculated. The consumption function within MMRF-GREEN (E_w3tot) includes the percentage change in household disposable income. In equation E_w1hinc , disposable household income is defined as the wages bill net of PAYE and payroll taxes, plus after-tax returns to land and capital, personal benefits and other transfers from governments.

Equation E_budg_def computes percentage changes in government outlays minus revenues. From a modelling perspective, there are a number of closure choices concerning the budgetary position. The modeller may wish to keep the budget balance of each government exogenous by making a particular tax shifter endogenous. There are a number of such shifters written in the code of the model, but more could be added if required.¹³

4.2.3 Disaggregation of value added

Figure 4.5 shows the disaggregation of the four elements of value added determined in CGE core (i.e., the wage bill, the rental cost of capital and land, and other costs) into ten components in order to separate production taxes from payments to factors. In addition, the block prepares national values as

¹³ From a practical modelling perspective, the modeller must be wary in choosing a suitable tax shifter to be endogenous, if the budget deficit is to be exogenous. If the revenue base of a particular tax is small, moderate changes in government outlays or revenues elsewhere could lead to a change in the sign of the level of the revenue assigned an endogenous tax shifter.

aggregates of regional values. We now turn to the details of the value-added equation block as presented in section 3.8.1 of MMRFCRS.TAB.

Cost of labour and payroll taxes ($E_w1wagess$, E_wag_impute , E_prol_base , E_w1lab_lge , E_w1prol , E_t_prol , E_f_p1lab , $E_p1laboi$, E_tr_prol and $E_tr_prolnat$)

In equations $E_w1wagess$ and E_wag_impute , we assume that the wages, salaries and supplements and the imputed wage bill vary in direct proportion to the pre-payroll-tax wage bill, which is determined in the CGE core. The other equations in this section relate to the modelling of payroll taxes.

MMRF-GREEN's modelling of payroll tax allows for a tax to be levied only on large firms within each industry. A firm is defined as large if the sum of wages, salaries and supplements and of imputed wages for that firm is above the payroll tax threshold (T). Each large firm pays a fraction (M) of the amount by which its wage bill exceeds the threshold. Given the number of large firms (L) in an industry, the payroll tax revenue for that industry is given by

$$R = M \times (B - LT) \quad (4.2.1),$$

where B is the total wage cost of large firms. The payroll tax base is given by

$$P = (B - LT) \quad (4.2.2).$$

The model explains changes in R, P and B for given changes in T and in the total number of firms for an industry (N). Data are required for P and B, while data for R come from the primary database of the model.

In terms of the TABLO code:

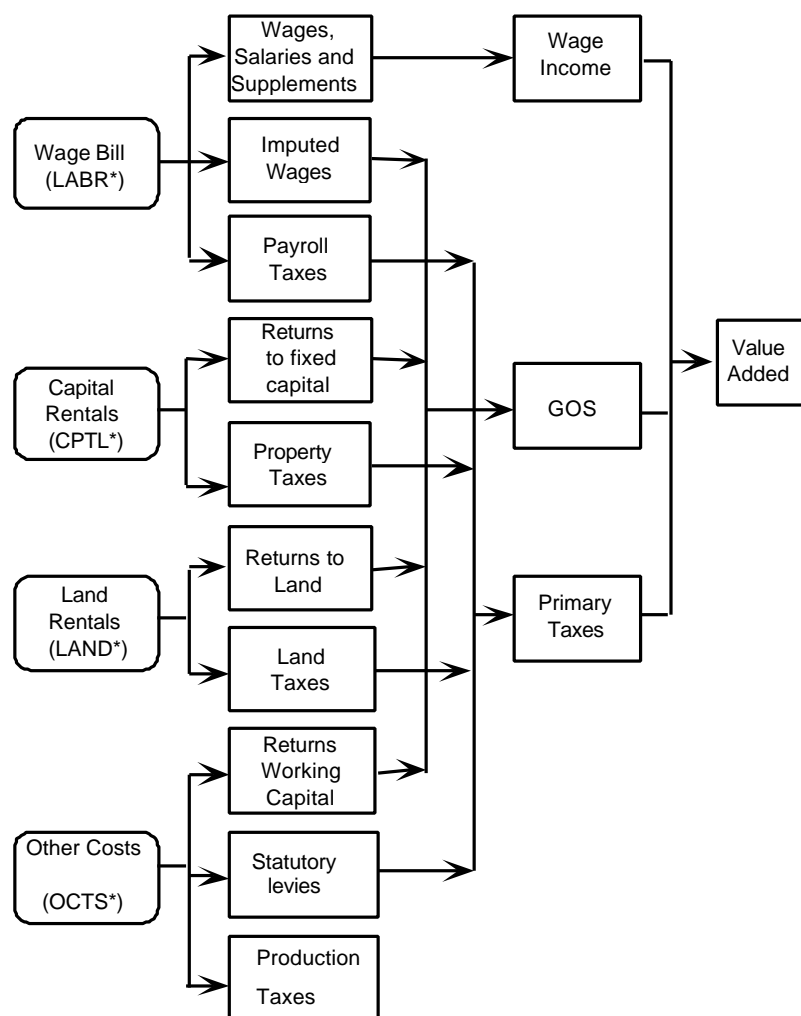
- PROLTX_IR(j,q) is the collection of payroll tax (R) from industry j in region q,
- T_PROL(j,q) is the marginal rate of payroll tax (M),
- W1LAB_LGE is the total wage cost of large firms (B),
- FIRMS is the number of large firms (L),
- PROL_THRESH is the payroll tax threshold (T), and
- PROL_BASE is the payroll tax base (B).

Equation E_prol_base relates movements in the payroll tax base for industry j in region q to movements in the before-tax value of employment ($pwagei(j,q) + labind(j,q)$), to movements in the tax threshold ($prol_thresh(q)$), and to movements in the number of large firms ($firms(j,q)$). According to E_prol_base , if there were no change in the tax threshold ($PROL_THRESH(q)$) and in the total number of firms ($FIRMS(j,q)$), then the tax base in industry j of region q will change in line with the percentage change in the before-tax cost of labour ($p1wagei(j,q) + labind(j,q)$). The coefficient $(1 + ALPHA_Z03(j,q))$ is equal to the ratio of the wage cost of large firms to the tax base. Holding the before-tax wage bill unchanged, a one per cent increase in the threshold, or a one per cent increase in the number of firms, increases the tax base by $(-ALPHA_Z03(j,q))$ per cent.

The percentage change in the wage cost of large firms is explained by equation E_w1lab_lge . This has the same general form as E_prol_base , allowing $w1lab_lge(j,q)$ to respond positively to changes in the before-tax wage bill and negatively to changes in the tax threshold and in the number of firms in the industry. The coefficient $BETA_Z03(j,q)$ is equal to minus the elasticity of the tax base with respect to the threshold value. The elasticity is evaluated by assuming that the relationship between the tax base and the threshold value has a cumulative Weibull form (see Appendix A).

Equation E_w1prol is the percentage change form of equation (4.2.1), after substitution of equation (4.2.2) and re-labelling. E_w1prol allows for flexibility in the setting of the marginal payroll tax

rate. A change in the payroll tax rate drives a wedge between the wage rate received by the workers and the cost to the producer of employing labour. The change in the cost of employing labour for a given change in the payroll tax rate depends on the share of the payroll taxes in total wages. Equation E_{f_pllab} adjusts the payroll tax rate to compute the wedge between the wage rate and labour employment costs. The wedge is used to define the before-payroll-tax wage rate ($PWAGEI(j,q)$) in the CGE core.



KEY

* The name in parenthesis indicates the corresponding array name in Figure 2.1

 Elements from the CGE Core.

 Elements in the disaggregation-of-value-added

Figure 4.5. Components of regional value added

Equations E_{tr_prol} and $E_{nat_tr_prol}$ are aggregations of industry-specific payroll tax collections.

Cost of capital (E_w1cap_at , $E_delt_prop_ir$, E_p1cap_pre , E_tr_prop and $E_nat_tr_prop$)

The return to fixed capital and property taxes are assumed to vary in proportion to the before-tax rental cost of capital. As indicated by equation E_w1cap_at , the after-tax rental cost ($KRNT_AT(q)$) equals the before-tax rental cost ($CAPREV(q)$), which is determined in the CGE core, less property tax ($PROPTX_R(q)$).

Equation $E_delt_prop_ir$ allows flexibility in the setting of the property tax rate. Note, that unlike the payroll tax rate, the property tax rate variable is an absolute-change variable. Equation E_p1cap_pre defines the pre-tax rental on capital in terms of the after-tax rental ($P1CAP(j,q)$), which is a variable of the core model, and the property tax rate. Equations E_tr_prop and $E_tr_propnat$ are aggregations of industry-specific property tax collections.

Cost of land (E_w1land , $E_delt_lnd_ir$, E_p1land_pre , E_tr_land and $E_nat_tr_land$)

Returns to agricultural land and land taxes are determined by equations E_w1land and E_tr_land respectively and vary in proportion to the total rental cost of land. Equations $E_delt_lnd_ir$ allows for the setting of the land tax rate for industry j in region q , and E_p1land_pre explains changes in the pre-tax price of land in terms of changes in the after-tax price ($P1LAND(j,q)$) and the land tax rate.

Equations E_tr_land and $E_nat_tr_land$ are aggregations of industry-specific land tax collections.

Other cost tickets (E_w1oct_at , $E_del_oct_sl$, $E_del_oct_pt$, E_p1oct_pre , E_tr_oct and E_tr_oct1)

These equations are analogous to the equation-groups applying capital and land. Taxes assigned to other costs including statutory levies collected by state governments and production taxes collected by the Commonwealth, both in agricultural sectors.

4.2.3. Gross regional domestic product and its components

This block of equations defines the regional gross products from the income and expenditure sides using variables from the CGE core.

Figure 4.6 shows gross regional product at market prices from the income side as the sum of wage income, non-wage income and indirect tax revenues.

Income side gross state product is given by $v0GSPINC(q)$. This equals the sum of returns to labour ($AGGLAB(q)$), returns to non-labour primary factors ($NLPRTOT(q)$) and indirect taxes ($INDRTXTOT(q)$). In turn, $NLPRTOT(q)$ equals the sum of returns to capital ($AGGCAP(q)$), returns to land ($AGGLND(q)$) and other costs ($AGGOCT(q)$). $INDRTXTOT(q)$ is the sum of tariff revenues ($AGGTAXM(q)$), intermediate taxes on production ($AGGTAX1(q)$) and investment ($AGGTAX2(q)$), taxes on households ($AGGTAX3(q)$) and export taxes ($AGGTAX4(q)$).

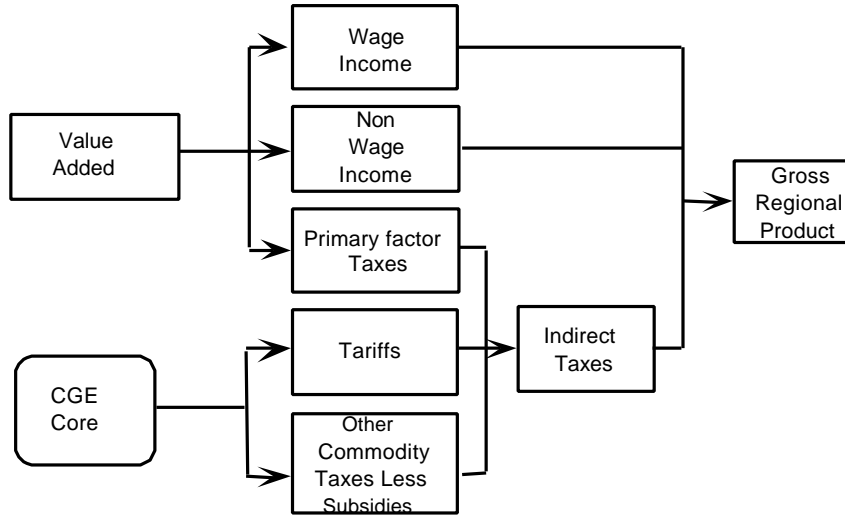


Figure 4.6. Income-side components of gross regional product

Equation E_{gos} defines regional gross operating surplus (GOS) as the sum of imputed wages and returns to fixed capital, working capital and agricultural land.

In equation E_{wIpaye} , PAYE taxes are assumed to be proportional to wage income and the PAYE tax rate. Equation E_{trgos} sets non-wage income tax proportional to non-wage income and the non-wage income tax rate.

The gross regional product from the expenditure side is the sum of domestic absorption and the interregional and international trade balances. This definition is reflected in equation E_{gspnom} . Domestic absorption is defined in equation E_{gner} as the sum of private and public consumption and investment expenditures. Equations E_{psexp} , E_{xsexp} , E_{psimp} and E_{xsimp} compute regional exports and imports. The difference between regional exports and imports forms the regional trade balance (E_{bot_inter}). The international trade variables for each region (E_{w4tot} and E_{wmtot} for international exports and imports respectively) are used to define the international trade balance for each region in equation E_{trad_bal} .

In order to calculate real GSP in each region, we need a price deflator which is calculated in equation E_{gspdef} using the same value weights as in E_{gspnom} . This in turn allows us to calculate real GSP in equation $E_{gspreal}$.

National GDP at factor cost is defined in equation $E_{natgdpfc}$ as the sum of the regional wage and non-wage incomes. Equation $E_{botstar}$ defines the percentage-point change in the national balance of trade surplus to national GDP ratio. Note that $E_{botstar}$ defines a percentage-point change in a ratio, rather than a percentage change. The underlying levels equation of $E_{botstar}$ is

$$BOTSTAR = \frac{\sum_{q \in regdst} \text{TRAD_BAL}(q)}{V0GDPEXP},$$

where the upper-case represents the levels values of the corresponding percentage-point change and percentage change variables. Taking the first difference of BOTSTAR and multiplying by 100 gives the percentage-point change (i.e., the variable *botstar* on the LHS of $E_{botstar}$)

$$100.\Delta\text{BOTSTAR} = \frac{\sum_{q \in \text{regdst}} \text{TRAD_BAL}(q) \text{tradbal}(q)}{\text{V0GDPEXP}} - \frac{\text{NATBT}}{\text{V0GDPEXP}} \times w0gdp \exp ,$$

where

$$\text{NATBT} = \sum_{q \in \text{regdst}} \text{TRAD_BAL}(q) .$$

Aggregate national income taxes are calculated in equation E_{tr_income} . They are the sum of regional PAYE taxes and regional taxes on non-wage income. Pre-tax national wage income is calculated in equation $E_{w1labnat}$ by summing pre-tax regional wage incomes. Equation $E_{p1labnat}$ defines the nominal pre-tax national wage rate as the ratio of the pre-tax wage income to national employment. The nominal post-tax national wage income is calculated in equation $E_{w1labpost}$ by summing the nominal post-tax regional wage incomes. The nominal post-tax national wage rate is defined in equation E_{wnstar} as the ratio of the nominal post-tax wage income to national employment. The real post-tax national wage rate is defined in $E_{postwage}$ by deflating the nominal post-tax national wage rate by the national CPI.

Equations $E_{w56totA}$ and $E_{w56totB}$ define the regional government and Federal government consumption expenditures respectively. The variable determined in these equations, $w56tot$, drives government consumption expenditures (see Section 4.2.4 below).

Equations $E_{natw2priv}$, $E_{w2totgovA}$, $E_{w2totgovB}$ and $E_{natw2gov}$ disaggregate investment expenditure into private and public components. The resulting values for public investment expenditure are used to drive government capital expenditures (see section 4.2.4 below). Equation $E_{natw2priv}$ imposes the assumption that aggregate private investment expenditure moves proportionally with a weighted average of total (private and public) regional investment. Aggregate public investment expenditure is then determined as the difference between total aggregate total investment and aggregate private investment ($E_{w2totgovA}$ and $E_{w2totgovB}$).

The equation block is completed with equation E_{w3tot} defining the regional household consumption function and equation E_{t_wagess} relating the PAYE tax rate to the tax rate on non-wage income.

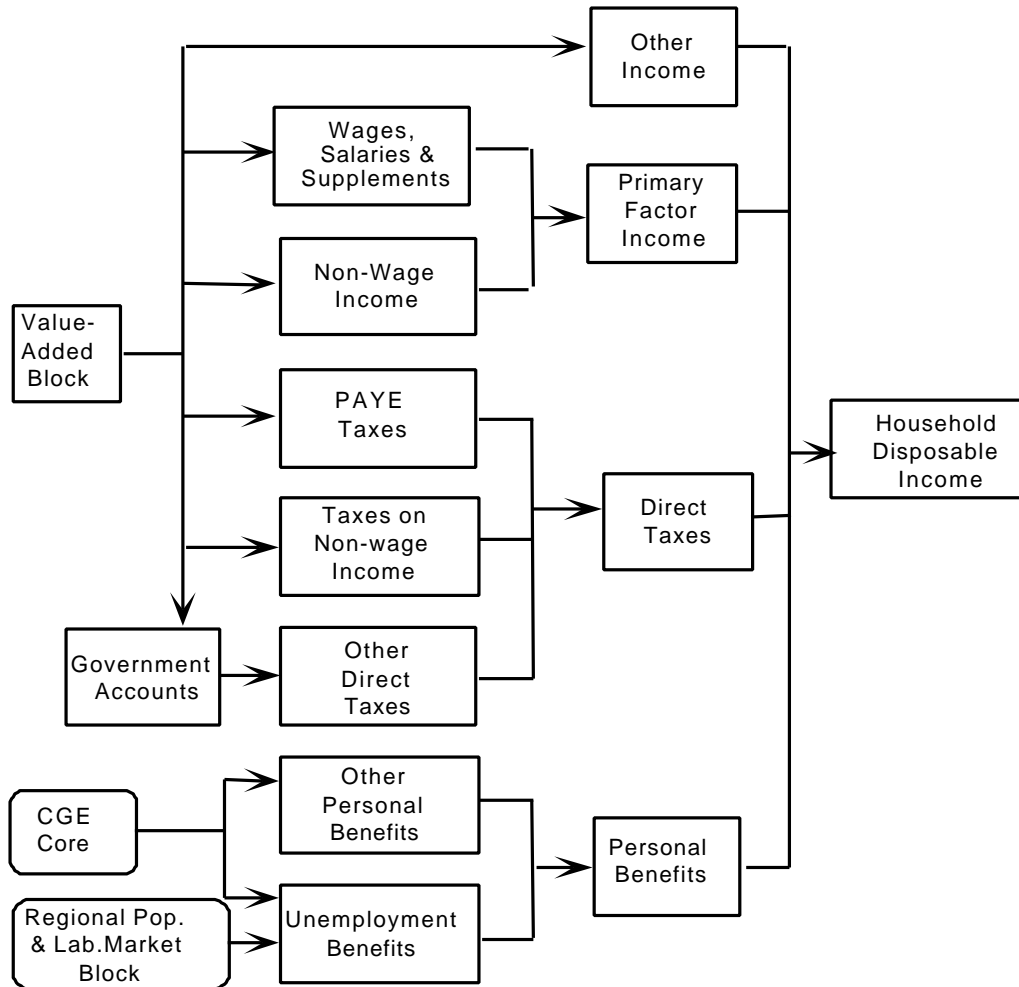


Figure 4.11. Household disposable income

4.2.4 Household disposable income

Regional household disposable income consists of four broad components: primary factor income, personal payments from the Federal government, 'other income' and direct taxes, as in equation E_{wlhinc} . The equation E_{wlprim} calculates primary factor income from changes in primary factor prices and quantities. Equation $E_{gpersbn}$ computes personal benefit payments as the weighted sum of changes in unemployment benefits and other personal benefits. In turn, unemployment benefits depend on CPI and regional population and labour market variables, as shown in equation $E_{ghunemp}$. In equation $E_{wllothinc}$, other income is proportional to regional GSP. In equation $E_{hwldirtx}$, direct taxes payable depend on primary factor incomes and direct tax rates.

4.3. Regional labour markets and labour migration

This block of equations computes regional population from natural growth, foreign migration and interregional migration. The block also includes various regional labour market relationships. For each region, the system is designed to allow for either:

- (i) an exogenous determination of regional population, with an endogenous determination of at least one variable of the regional labour market, chosen from regional unemployment, regional participation rates or regional wage relativities, or
- (ii) an exogenous determination of all the previously mentioned variables of the regional labour market and an endogenous determination of regional migration, and hence, of regional population.

In case (i), the user can take on board the forecasts of the three population flows (natural growth, regional migration and foreign migration) from a demographic model thereby exogenously determining regional populations. For example, the ABS (cat. no. 3222.0) makes forecasts of these flows and of regional population. The labour market & migration block of equations can then be configured to determine regional labour supply from the exogenously specified regional population and given settings of regional participation rates and movements in the ratios of population to population of working age. With labour supply determined, the labour market and regional migration block will determine either interregional wage differentials, (given regional unemployment rates) or regional unemployment rates (given regional wage differentials). With given regional unemployment rates and regional labour supply, regional employment is determined as a residual and wage differentials adjust to accommodate the labour market outcome. Fixing wage differentials determines the demand for labour so that with regional labour supply given, the model will determine regional unemployment rates as a residual.

In alternative (ii), interregional wage differentials and regional unemployment rates are exogenously specified. The labour market and regional migration block then determines regional labour supply and regional population for given settings of regional participation rates and ratios of population to population of working age.

The equations of this block have been designed with sufficient flexibility to allow variations on the two general methods described above. Importantly, the block allows for some regions to be subject to method (i) and other regions to be subject to method (ii) in the same simulation.

The equations can be grouped into the following categories: definitions; equations imposing arbitrary assumptions; equations imposing adding-up constraints and; national aggregates based on summing regional variables.

The definitional equations are E_del_labsup and E_wpop . The former equation defines the percentage-point change in the regional unemployment ($del_unr(q)$) in terms of the percentage changes in regional labour supply ($labsup(q)$) and persons employed ($employ(q)$). The latter equation defines the percentage change in regional labour supply in terms of the percentage changes in the regional participation rate ($pr(q)$) and the regional population of working age ($wpop(q)$).

In equation E_pop , the assumption that the regional population of working age is proportional to the regional (total) population is imposed. The default setting can be overridden by endogenising the shift variable ($f_wpop(q)$).

Equation E_rm_0 , allows for the imposition of either the assumption that the change in net regional migration ($del_rm(q)$) is equal to a forecast change in regional migration ($del_rm_0(q)$), or that $del_rm(q)$ is equal to $del_rm_0(q)$ plus a common (to all q regions) constant. We can interpret equation E_rm_0 as imposing the former assumption when $del_rm_0(q)$ are set exogenously (equal to, say, an ABS forecast) and when the shift variable, $delf_rm_0$, is set exogenously at zero change. The latter assumption is imposed when all but one of the $del_rm_0(q)$ are set exogenously and $delf_rm_0$ is set endogenously. The purpose of the second assumption is as follows. We may wish to believe some, *but not all*, of the ABS forecasts of net regional migration. We may wish to determine one of the region's net regional migration from economic factors within MMRF-GREEN. However, we may still wish that the remaining net regional migration flows to be approximately equal to the ABS forecasts. To the extent that the region's net regional migration determined by MMRF-GREEN deviates from that forecast by the ABS means that the sum of the regional net migration will not equal zero if the remaining regional net migration flows are set equal to their ABS forecasts. To overcome this problem, we distribute the positive/negative amount of net migration evenly across the regions. If it is desired that all regional net migration flows are determined by economic factors, rather than exogenously, then all elements of $del_rm_0(q)$ are set endogenously and swapped with a relevant labour market variable such as regional relative wage rates ($wage_diff(q)$, see equation E_wage_diff , Section 4.1.15 above and, in the TABLO code, section 2.8.15).

Equation E_employ imposes the assumption that regional employment in wage-bill weights is proportional to regional employment in person weights by setting the percentage change in regional wage-bill weighted employment ($l(q)$) equal to the percentage change in regional person-weighted employment ($employ(q)$) when the shift variable $f_l(q)$ is exogenous and set to zero change. The default option can be overridden by setting $f_l(q)$ to non-zero values.

Equation E_qhous imposes the assumption that regional household formation is proportional to regional population by setting the percentage change in regional household formation ($qhous(q)$) equal to the percentage change in regional population ($pop(q)$) when the shift variable $f_qhous(q)$ is exogenous and set to zero change. The default option can be overridden by setting $f_qhous(q)$ to non-zero values.

An adding-up constraint is imposed in equation E_delf_rm . If the variable $delf_rm$ is exogenous (and set to zero), then at least one of the $del_rm(q)$ must be endogenous. If all the $del_rm(q)$ are endogenous, then $delf_rm$ must endogenously equal zero for the simulation to be valid.

The remaining equations of this section, E_delnat_fm , E_del_natg , $E_natlabsup$, $E_natemploy$ and E_natunr determine national aggregate variables by summing the corresponding regional variables.

4.4. Rates of return and investment

MMRF can be run in two modes:

- comparative static; and
- year-to-year dynamic.

These modes require alternative treatments of capital formation.

In comparative-static mode, there is no fixed relationship between capital and investment. The user decides the required relationship on the basis of the requirements of the specific simulation. For example, it is often assumed that the percentage changes in capital and investment are equal, implying that

$$x1cap(j, q) = y(j, q) \quad j \in IND \quad q \in REGDST \quad (4.4.1).$$

In year-to-year dynamics, we interpret a model solution as a vector of changes in the values of variables between two adjacent years. Thus there is a fixed relationship between capital and investment. As outlined in Section 2.2, capital available for production in the solution year t is the stock of capital at the start of year t . This is determined by investment in year $t-1$ and by capital at the start of year $t-1$ after depreciation. Growth in capital between the start of t and the end of t is determined in the model via an expected rate of return mechanism. This puts in place investment for year t . Capital at the end of t is the capital available for production in year $t+1$.

Part 4.4 of the TABLO code contains equations that define actual rates of return under static expectations, and equations that explain investment in comparative static simulations.

After-tax rates of return (E_del_ror)

Equation E_del_ror explains the ordinary change in the actual rate of return on capital in industry j formed under the assumption of static expectation (del_ror). To explain this equation we start with an expression for the present value (PV) of purchasing in the current solution year, year t , a unit of physical capital for use in industry j in region q :

$$PV_t(j, q) = -\Pi_t(j, q) + \frac{[(1 - T_{t+1}) \times Q_{t+q}(j, q)] + [(1 - D(j, q)) \times \Pi_{t+1}(j, q)] + \{T_{t+q} \times D(j, q) \times \Pi_{t+1}(j, q)\}}{(1 + INT_t \times (1 - T_{t+1}))}$$

$$j \in IND \quad q \in REGDST \quad (4.4.2),$$

where:

- Π_t is the cost of buying or constructing a unit of industry j 's capital in year t ;
- T_t is the company-income tax rate in year t ;
- Q_t is the rental rate for the industry's capital in year t (equivalent to the cost of using a unit of capital in year t);
- D is the rate of depreciation (a number like 0.05); and
- INT_t is the nominal rate of interest in year t .

It is assumed that units of capital in year t yield to their owner three benefits in year $t+1$: an after-tax rental (the term in square brackets); a depreciated re-sale value (the term in round brackets) and a tax-deduction (the term in curly brackets). These benefits are converted to a present value in year t by discounting using the after-tax nominal interest rate.

The present-value sum of benefits as defined above is converted to a rate of return by dividing through by the cost of buying capital in year t . Thus we define

$$ROR_ACT_t(j, q) = -1 + \frac{[(1 - T_{t+1}) \times Q_{t+1}(j, q)] + ((1 - D(j, q)) \times \Pi_{t+1}(j, q)) + \{T_{t+1} \times D(j, q) \times \Pi_{t+1}(j, q)\}}{\Pi_t(j, q)(1 + INT_t \times (1 - T_{t+1}))} \quad j \in IND \quad q \in REGDST \quad (4.4.3),$$

to be the "actual" rate of return in year t for units of capital invested in industry j .

In MMRF-GREEN we make allowance only for static expectations.¹⁴ This means that investors expect no change in the tax rate, and expect that rental rates and asset prices will increase uniformly by the current rate of inflation (INF). It is also assumed that investors expect the real after-tax rate of interest to be zero, leaving the nominal after-tax interest rate equal to INF. Under these assumptions, the static expectation of ROR_ACT is

$$ROR_t(j, q) = (1 - T_t) \times \left\{ \frac{Q_t(j, q)}{\Pi_t(j, q)} - D(j, q) \right\} \quad j \in IND \quad q \in REGDST \quad (4.4.4).$$

E_del_ror is the change form of (4.4.4). The mapping between the TABLO notation and the notation used in (4.4.4) is as follows.

- del_ror is the ordinary change in ROR;
- $p1cap$ is the percentage change in Q ;
- pi is the percentage change in Π ;
- rk is the percentage change in T ;
- $CTAXR$ is equivalent to T ;
- $CAPITAL/VCAP$ is equivalent to Q/Π ; and
- $DEPR$ is equivalent to D .

Regional average after-tax rate of return ($E_del_ror_tot$)

This equation defines the ordinary change in the average rate of return for region q as a weighted average of the ordinary changes in rates of return for industries in q .

¹⁴ MONASH allows for two possibilities - static expectations and forward-looking expectations.

Distribution of after-tax rates of return in long-run comparative-static simulations

($E_del_f_ror$)

This equation has no role in year-to-year simulations, and can be turned off by endogenising the shift variable del_f_ror . When turned on in long-run comparative-static simulations the equation allows changes in industry rates of return to be positively correlated with percentage changes in industry capital stocks. Industries experiencing relatively strong growth in capital (indicated by a large positive value for $x1cap(j,q) - kt(q)$) will require relatively large increases in rates of return (indicated by a large positive value for $del_ror(j,q) - del_nat_ror$). Conversely, industries experiencing relatively weak capital growth will require relatively small increases or decreases in rates of return. The equation could be interpreted as a risk-related relationship with relatively fast/slow growing industries requiring premia/accepting discounts on their rates of return. The parameter $BETA_R$ specifies the strength of this relationship.

Total real investment (E_naty) and ratios of investment to capital (E_y)

The last two equations in Part 5.4 of the TABLO code define the percentage change in national investment for industry j and the percentage change in the ratio of investment to capital for industry j in region q . Several shift variables are included in E_y to allow for shifts in investment/capital ratios. These can be specific to the industry and region ($r_inv_cap_jq$), or specific to the region only ($r_inv_cap_q$), or specific to the industry only ($r_inv_cap_j$), or not specific to any industry or region (r_inv_cap).

E_y is especially useful in long-run comparative-static experiments, where we typically assume that:

1. the aggregate capital stock adjusts to preserve an exogenously determined economy-wide rate of return (ROR_TOT);
2. the allocation of capital across regional industries adjusts to satisfy exogenously specified relationships between relative rates of return and relative capital growth (see equation $E_del_f_ror$); and
3. industry demands for investment goods are determined by exogenously specified investment/capital ratios (see equation (4.4.1)).

4.5. Year-to-year equations: investment and capital, investment and expected rates of return, and the labour market

Part 6 of the TABLO code contains equations that are essential for in year-to-year simulations (i.e. dynamic simulations tracing out the paths for variables for successive years). There are two such sets of equations:

- (a) equations describing the relationship between capital and investment, and between capital growth and expected rates of return (we assume static expectations); and
- (b) equations that allow, in year-to-year policy simulations, for real wages to be sticky in the short run and flexible in the long run.

In year-to-year simulations, capital available for production in the current forecast year (year t) is given by initial conditions, with the rate of return in year t adjusting to accommodate the given stock of capital. We introduce equations to allow the percentage change in capital available for production in year t (i.e. the percentage change in capital at the start of year t) to be determined inside the model. We also specify capital supply functions that determine industries' capital growth rates through year t (and thus investment in year t). The functions specify that investors are willing to supply increased funds to industry j in response to increases in j 's expected rate of return (we assume static expectations). However, investors are cautious. In any year, the capital supply functions limit the growth in industry j 's capital stock so that disturbances in j 's rate of return are eliminated only gradually.

In most AGE analyses, one of the following two assumptions is made:

- (1) real wages adjust to a shock so there is no effect on (national) employment; or

(2) real wages remain unaffected and (national) employment adjusts).

We introduce equations that allow for a third, intermediate position. We assume that the deviation in the real wage rate from its basecase forecast level increase in proportion to the deviation in national employment from its basecase-forecast level. The coefficient of proportionality is chosen so that the employment effects of a shock are largely eliminated after about 5 years.

4.5.1. Capital, investment and expected rates of return in year-to-year simulations

Shocks to starting capital in year-to-year simulations ($E_del_f_x1cap$)

In year-to-year simulations, capital available for production in the solution year, year t , is the capital stock existing at the start of the year, or the end of the previous year, year $t-1$. We denote this stock as $QCAP$. The corresponding percentage-change variable is $x1cap$.

The appropriate value for $x1cap$ in a year-to-year computation is the growth rate of capital between the start of year $t-1$ and the start of year t . Algebraically, using a notation that emphasises the timing of each variable, we want

$$x1cap_t(j, q) = 100 \times \left(\frac{QCAP_t(j, q)}{QCAP_{t-1}(j, q)} - 1 \right) \quad j \in IND \quad q \in REGDST \quad (4.5.1),$$

where $QCAP_t(j, q)$ is the quantity of capital available for production in industry j in region q at the start of the current solution year t . Equation (4.5.1) can be rewritten as

$$x1cap_t(j, q) = 100 \times \left(\frac{QINV_{t-1}(j, q) - DEPR(j, q) \times QCAP_{t-1}(j, q)}{QCAP_{t-1}(j, q)} \right) \quad j \in IND \quad q \in REGDST \quad (4.5.2),$$

where $QINV_{t-1}(j, q)$ is the quantity of investment in industry j in year $t-1$ and $DEPR$ is a fixed parameter representing the rate of capital depreciation for industry i .

In making the computation for year t , we could treat $x1cap_t(i)$ as an exogenous variable and compute its value outside the model in accordance with equation (4.5.2). It is more convenient, however, to compute values for $x1cap_t(i)$ inside the model. This is done using equation $E_del_f_x1cap$.

To understand the levels form of $E_del_f_x1cap$, we start by re-writing (4.5.2) as:

$$QCAP_t(j, q) - QCAP_{t-1}(j, q) = 100 \times (QINV_{t-1}(j, q) - DEPR(j, q) \times QCAP_{t-1}(j, q)) \quad j \in IND \quad q \in REGDST \quad (4.5.3).$$

In year-to-year simulations we want the initial solution for year t to reflect values for year $t-1$, since the changes we are simulating are from year $t-1$ to year t . If this is the case then the initial value of $QCAP_t(i)$ is $QCAP_{t-1}(i)$. The Euler solution method requires that the initial (database) values for variables form a solution to the underlying levels form of the model. Equation (4.5.3) makes it clear that unless net investment in year $t-1$ is zero in industry j , then the initial data for a year- t computation will not be a solution to (4.5.3).

We solve this problem of initial-value by the purely technical device of augmenting equation (4.5.3) with an additional exogenous variable $UNITY$ as follows:

$$\begin{aligned}
QCAP_t(j, q) - QCAP_{t-1}(j, q) = \\
UNITY \times 100 \times (QINV_{t-1}(j, q) - DEPR(j, q) \times QCAP_{t-1}(j, q)) \\
j \in IND \quad q \in REGDST \quad (4.5.4).
\end{aligned}$$

We choose the initial value of UNITY to be 0, so that (4.5.3) is satisfied when $QCAP_t(i)$ takes its initial value regardless of the initial value of net investment in industry j . UNITY is often referred to as a “fudge factor”, while others call it a “homotopy parameter”. By moving UNITY to one, we cause the correct deviation in the opening capital stock for year t from its value in the initial solution (i.e., from its value in year $t-1$).

Equation $E_del_f_x1cap$ is the change form of (4.5.4), after changes in notation. On the right hand side of the TABLO equation, the coefficients $QINV @ 1(j, q)$ and $QCAP @ 1(j, q)$ are the levels of $QINV(j, q)$ and $QCAP(j, q)$ in the initial solution for year t . Provided that the initial solution is drawn from values for year $t-1$, then $QINV @ 1(j, q)$ corresponds to $QINV_{t-1}(j, q)$ in (4.5.4) and $QCAP @ 1(j, q)$ corresponds to $QCAP_{t-1}(j, q)$. The variable del_unity is the ordinary change in UNITY. In year-to-year simulations, del_unity is always set to 1. The variable $del_f_x1cap(j, q)$ is an on/off shift variable. In year-to-year simulations this will be exogenous, causing $x1cap(j, q)$ to be set according to (4.5.4). In comparative static simulations, $del_f_x1cap(j, q)$ will be endogenous, effectively turning off $E_del_f_x1cap$ for industry j . In making $del_f_x1cap(j, q)$ endogenous we will make one of the rate of return or capital stock in industry j exogenous.

Capital available in year $t+1$ related to investment in year t (E_cap_t1)

Equation E_cap_t1 explains the percentage change in the capital stock of industry j in region q at the end of the solution year. The levels form of this equation (with time made explicit) is

$$QCAP_T1_t(j, q) = QINV_t(j, q) + (1 - DEPR(j, q)) \times QCAP_T_t(j, q) \quad j \in IND \quad q \in REGDST \quad (4.5.5).$$

where $QCAP_T1_t(j, q)$ is the stock of capital in industry j in region q at the end of year t (or the start of year $t+1$). Note that equation (4.5.5) is satisfied by the initial solution for year t , and so there is no need to introduce the homotopy variable.

Taking ordinary changes of the left-hand side and the right hand side of (4.5.5) gives, after dropping the time index, E_cap_t1 .

Capital growth between the start and end of the solution year ($E_del_k_gr$)

In year-to-year simulations, growth in capital between the start and end of year t is determined by the expected rate of return on capital (see equation (2.2)). This relationship is modelled below. Here we define the level of the growth rate in capital for industry i ,

$$K_GR_t(j, q) = \frac{QCAP_T1_t(j, q)}{QCAP_T_t(j, q)} - 1 \quad j \in IND \quad q \in REGDST \quad (4.5.6),$$

and in equation $E_del_k_gr$ explain the change in that growth rate in terms of the percentage-change variables $x1cap(j, q)$ and $cap_t1(j, q)$.

Expected rate of return equals expected equilibrium rate of return plus expected disequilibrium rate of return (E_del_eeqror)

It is assumed that the expectation held in period t by owners of capital in industry j for industry j 's rate of return in period $t+1$ can be separated into two parts. One part is called the expected equilibrium rate of return. This is the expected rate of return required to sustain indefinitely the current rate of capital growth in industry j . The second part is a measure of the disequilibrium in j 's current expected rate of return. In terms of the notation in the TABLO code,

$$EROR(j, q) = EEQROR(j, q) + DISEQROR(j, q) \quad j \in IND \quad q \in REGDST \quad (4.5.7),$$

where $EROR(j, q)$, $EEQROR(j, q)$ and $DISEQROR(j, q)$ are the levels in year t of the expected rate of return, the expected equilibrium rate of the return and the disequilibrium in the expected rate of return. E_del_eeqror is the change form of (4.5.7).

$E_f_eeqror_jq$

The theory of investment in year-to-year simulations relates the expected equilibrium rate of return for industry j ($EEQROR(j, q)$) to the current rate of growth of capital in industry j ($K_GR(j, q)$). As shown in Figure 4.12, the relationship has an inverse logistic form, which has the algebraic form:

$$EEQROR(j, q) = RORN(j, q) + F_EEQROR(j, q) + \frac{1}{CAP_SLOPE(j, q)} \times \{ [Log(K_GR(j, q) - K_GR_MIN(j, q)) - Log(K_GR_MAX(j, q) - K_GR(j, q))] - [Log(TREND_K(j, q) - K_GR_MIN(j, q)) - Log(K_GR_MAX(j, q) - TREND_K(j, q))] \}$$

$$j \in IND \quad q \in REGDST \quad (4.5.8),$$

where:

$RORN$ is a coefficient representing the industry's historically normal rate of return;

F_EEQROR allows for vertical shifts in the capital supply curves (in the TABLO code there are three shift variables allowing for a nation-wide shift, region-wide shifts and shifts that are industry and region specific);

CAP_SLOPE is a coefficient which is correlated with the inverse of the slope of the capital supply curve (Figure 4.12) in the region of $K_GR = TREND_K$ (for further details see Dixon and Rimmer, 2001);

K_GR_MIN is a coefficient, which sets the minimum possible rate of growth of capital;

K_GR_MAX is a coefficient, set to the maximum possible rate of growth of capital; and

$TREND_K$ is a coefficient, set to the industry's historical normal rate of capital growth.

Equation (4.5.8) is explained in Dixon and Rimmer (2001) as follows. Suppose that F_EEQROR and $DISEQROR$ are initially zero. Then according to (4.5.8) and (4.5.7), for an industry to attract sufficient investment in year t to achieve a capital growth rate of $TREND_K$ it must have an expected rate of return equal to its long-term average ($RORN$) For the industry to attract sufficient investment in year t for its capital growth to exceed its long-term average ($TREND_K$), its expected rate of return must be greater than $RORN$. Conversely, if the expected rate of return on the industry's capital falls below $RORN$, then investors will restrict their supply of capital to the industry to a level below that required to sustain capital growth at the rate of $TREND_K$.

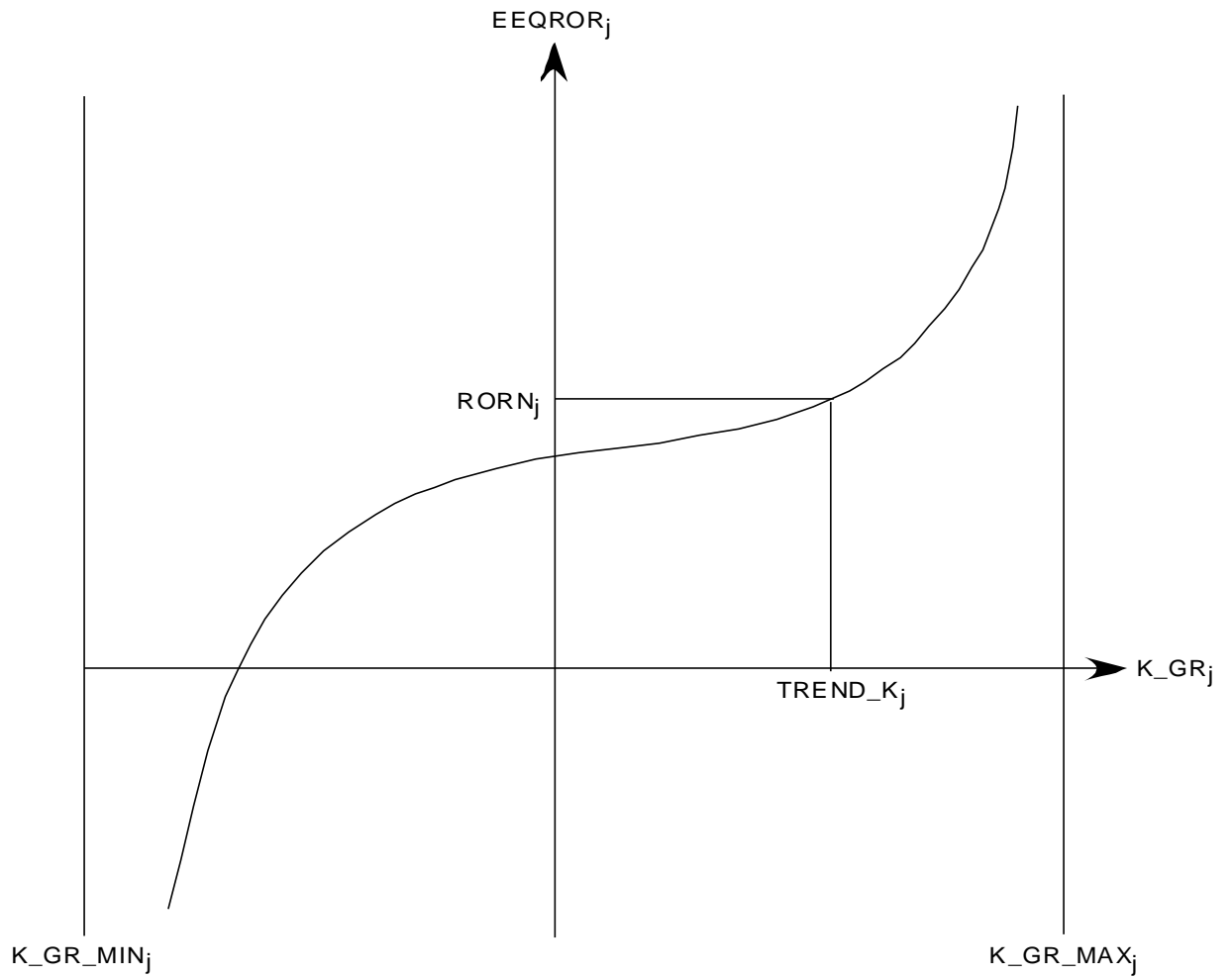


Figure 1. The equilibrium expected rate of return schedule for and industry

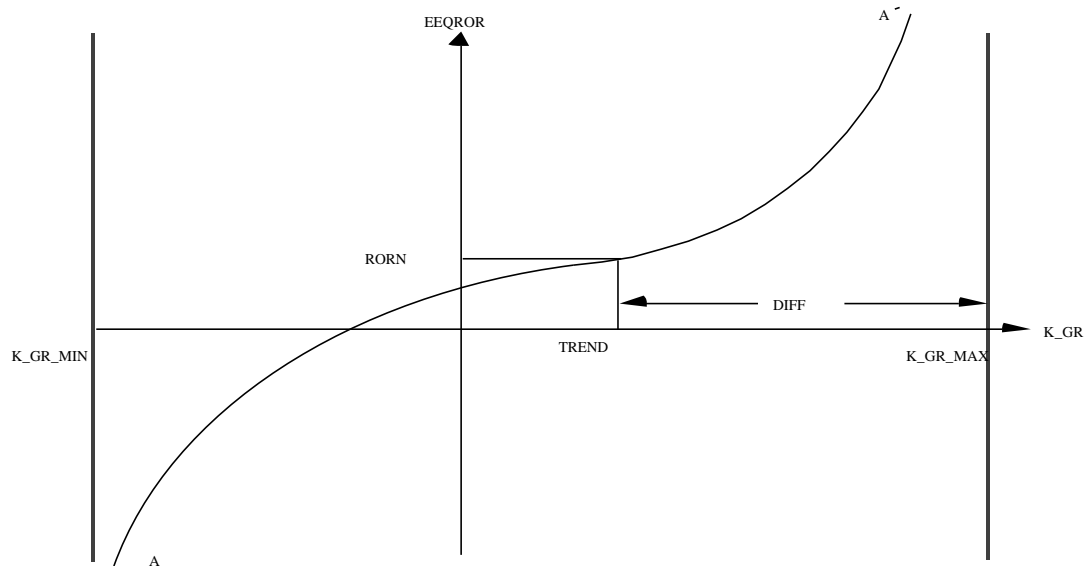


Figure 4.12: The equilibrium expected rate of return schedule for industry j

The change version of (4.5.8) is $E_f_eeqr_jq$.

Adjustment of Disequilibrium in expected rate of return towards zero (E_del_diseqr)

The initial disequilibrium in the expected rate of return (DISEQROR) is gradually eliminated over time according to the rule:

$$DISEQROR(j,q) - DISEQROR@1(j,q) = -ADJ_COEFF(j,q) \times DISEQROR@1(j,q) \times UNITY^{j \in IND \ q \in REGDST} \quad (4.5.9),$$

where DISEQROR@1 is the initial value of DISEQROR in a simulation for year t; and ADJ_COEFF is a positive parameter (less than one) determining the speed at which DISEQROR moves towards zero. E_del_diseqr is the change form of (4.5.9).

Expected rate of return equals actual rate of return under static expectations (E_del_ror)

This equation enforces the rule that the expected rate of return on capital in industry j in region q in year t equals industry j's actual rate of return in year t under static expectations (see equation (4.4.4)).

4.5.2. National employment and real wage in dynamic policy simulations

Consumer's real wage rate, after tax (E_realw_pt)

This section begins with an equation that explains movements in the national real after-tax wage rate received by consumers. On the right hand side of E_realw_pt , $natrealwage$ is the percentage change in the national real before-tax wage rate received by consumers and rl is the percentage change in the tax rate applied to labour income. The level of the tax rate is denoted LTAXR.

Deviations in the consumer's real wage rate, after tax related to deviations in employment ($E_del_f_realw_pt$)

Algebraically, the national employment/wage trade-off equation in MMRF-GREEN is written as:

$$\left\{ \frac{RWAGE_PT}{RWAGE_PT_F} - 1 \right\} = \left\{ \frac{RWAGE@1_PT}{RWAGE@1_PT_F} - 1 \right\} + LAB_SLOPE \times \left\{ \frac{EMPL}{EMPL_F} - 1 \right\} \quad (4.5.10),$$

where:

$RWAGE_PT$ is the level of the consumer's real after-tax wage rate in the deviation simulation;

$RWAGE_PT_F$ is the level of the consumer's real after-tax wage rate in the basecase-forecast simulation;

$RWAGE@1_PT$ is the initial level of the consumer's real after-tax wage rate in the deviation simulation;

$RWAGE@1_PT_F$ is the initial level of the consumer's real after-tax wage rate in the basecase-forecast simulation;

LAB_SLOPE is a positive coefficient;

$EMPL$ is the level of aggregate employment in the deviation simulation; and ;

$EMPL_F$ is the level of aggregate employment in the basecase-forecast simulation.

Equation (4.5.10) says that the percentage deviation in the real wage rate in year t from its basecase forecast value equals the percentage deviation in the real wage rate in year t-1 plus a coefficient (LAB_SLOPE) times the percentage deviation in employment in year t. The coefficient LAB_SLOPE is chosen so that the employment effects of a shock to the economy are largely eliminated after 5 years. In

other words, after about 5 years, the benefits of favourable shocks, such as outward shifts in export demand curves, are realised almost entirely as increases in real wage rates.

The change form of (4.5.10) is

$$\begin{aligned} \frac{RWAGE_PT}{RWAGE_PT_F}(realw_pt - realw_pt_f) = \\ \frac{RWAGE@1_PT}{RWAGE@1PT_F}(realw@1_pt - realw@1_pt_f) + \\ LAB_SLOPE \times \frac{EMPL}{EMPL_F}(natl - natl_f) \end{aligned} \quad (4.5.11),$$

where:

$realw_pt$ is the percentage change in the consumer's real after-tax wage rate between years t-1 and t with the policy shock in place;

$realw_pt_f$ is the percentage change in the consumer's real after-tax wage rate between years t-1 and t in the basecase forecast simulation;

$realw@1_pt$ is the percentage change in the consumer's real after-tax wage rate between years t-2 and t-1 with the policy shock in place;

$realw@1_pt_f$ is the percentage change in the consumer's real after-tax wage rate between years t-2 and t-1 in the basecase forecast simulation;

$natl$ is the percentage change in national employment between years t-1 and t with the policy shock in place; and

$natl_f$ is the percentage change in national employment between years t-1 and t in the basecase forecast simulation.

$E_del_f_wagew_pt$ is based on (4.5.11), with the first term on the RHS of (4.5.11) (i.e. the change in the real wage deviation in year t-1) rewritten as:

$$100 \times \left\{ \frac{RWAGE@1_PT}{RWAGE@1PT_F} - \frac{RWAGE@2_PT}{RWAGE@2_PT_F} \right\} \times del_unity \quad (4.5.12),$$

where

$RWAGE@2_PT$ is the initial level in year t-1 of the consumer's real after-tax wage rate in the deviation simulation; and

$RWAGE@2_PT_F$ is the initial level in year t-1 of the consumer's real after-tax wage rate in the basecase-forecast simulation.

Using (4.5.12) eliminates the need for the lagged variables $realw@1_pt$ and $realw@1_pt_f$.

Equations to transfer forecast simulation results to a policy simulation results

$(E_f_natl_f, E_f_realw_f, E_f_realw_pt_f, E_f_rl_f)$

These equations allow us to transfer basecase forecast values of variables through to the policy simulation. They are of the form

$$r_x = x - bf_x \quad (4.5.13).$$

where:

x is a variable (e.g., real wage growth) for which we need forecast results in policy simulations;

bf_x is the variable which, in policy simulations, is given the forecast simulation value of x ; and
 r_x is the difference between x and bf_x .

These equations operate in conjunction with the GEMPACK software used to run year-to-year simulations with MMRF-GREEN. With this software, a policy simulation for year t is run by:

- (1) reading the results generated in the forecast simulation for year t ;
- (2) reading the list of exogenous variables to be used in a policy simulation for year t ;
- (3) forming a CMF file in which exogenous variable r in the policy simulation is assigned the value that it had (either endogenously or exogenously) in the forecast simulation; and
- (4) adding a supplementary file of shocks for variables that are exogenous and shocked in the policy simulation.

Aided by this software we use (4.5.13) to transfer basecase forecast values for x to the policy simulation. In the forecasting simulation we set the shift variable r_x exogenously at zero change, resulting in the forecast result for bf_x being the same as that for x . In the policy simulation we change the closure by making bf_x exogenous and r_x endogenous. Thus, the forecast result for x is correctly passed to bf_x in the policy simulation.

4.6. Miscellaneous equations to facilitate year-to-year simulations

Part 6 of the TABLO code contains equations that are used to facilitate year-to-year simulations.

Flexible handling of twist variables (E_ftwist_src and E_ftwist_lk)

These equations allow for the flexible handling of the twist variables, $twist_src(i,q)$ and $twist_lk(j,q)$. Using equation E_ftwist_src , for example, we can impose a uniform all-commodity twist in the import/domestic preferences of the economy. This is done by making $ftwist_src(i,q)$ exogenous and $twist_src(i,q)$ endogenous for all i and q and shocking the uniform shifter $nattwist_src$.

Equations of this type are very useful in forecasting simulations. In forecasting, we need instruments that allow us to exogenise the aggregate volume of imports and both of aggregate employment and the real wage rate. E_ftwist_src and E_ftwist_lk give us those instruments in the form of $nattwist_src$ and $nattwist_lk$. To fix, say, the real wage rate when employment is also exogenous, we turn on the $ftwist_lk$ mechanism and then swap $nattwist_lk$ for the national real wage rate. With this closure, the model determines the general twist in labour/capital requirements that is necessary to reconcile the national-level exogenous settings for both employment and the real wage rate.

Flexible handling of technological change and the average propensity to consume out of household disposable income (E_a1prim_elim , $E_f_a1primgen$ and E_f_f3tot)

These equations are also useful in forecasting simulations. The first is used to insulate certain industries from the industry-general shift in primary factor technological change, $a1primgen(q)$. To insulate industry j in region q , we make exogenous $a1prim_elim(j,q)$ by making endogenous $a1prim(j,q)$. This ensures that $a1prim(j,q)$ equals minus $a1primgen(q)$, thus insulating industry j from $a1primgen$.

The second and third equations allow us to impose national-level shifts in the regional variables $a1primgen$ and $f3totnat$.

General commodity i -using changes in industry technologies (E_a1o to E_a4marg)

The commodity-dimension variable $ac(i,q)$ allows for general commodity i -using changes in industry technologies for current production, and (if $i = r$ is a margins commodities) for commodity r -using changes in margins usage.

As can be seen in equations E_a1o to E_a4marg , with the shift variables $f_a1o(i,j,q)$ to $fa4marg(s,r)$ and $agreen(i,j,q)$ set exogenously at zero, a value of 1.0 for $ac(i,q)$ and for $ac(r,q)$ ($r = i$)

imparts a one per cent increase in the existing technological change variables $a1o(i,j,q)$ to $a4marg(s,r)$ across all s , j and q . We explain the use of the variable $agreen$ Section 4.7.4.

4.7. Greenhouse gas emissions

The equations in this section with the accounting and taxation of greenhouse gas emissions. Gas emitters are identified in the set FUELUSER. They consist of firms in each of the industries recognised by the model, and households.

Each of the emitters pollute by using fuels in the set FUEL; and/or by releasing gas from non-combustion sources, linked to activity¹⁵. The polluting activities are identified in the set FUELX. FUELX contains the main fuels such as coal and gas, and one element for non-combustion emissions labelled “activity”. These categories cover all known gas sources except land clearing.

Combining this level of detail with the regional aspect of the model yields a series of coefficient and variable matrices with four dimensions: FUELX, ALLSOURCE, FUELUSER and REGDST. The high dimensionality of these matrices allows for considerable flexibility in setting emission taxes; taxes can vary according to fuel, region of source, user and regional location of user.

4.7.1. Tax rates and revenues

The model allows for specific gas taxes levied on units of CO₂-equivalent gas. These taxes are linked to a single price index (gastaxindex), thus preserving the model’s homogeneity. The emission taxes feed into the core model via the existing *ad valorem* tax rates, *deltax1* and *deltax3*.

Greenhouse tax revenue by state (*E_deletaxrv*)

This equation is derived as follows. Suppressing subscripts and using a simplified notation, an item of gas tax revenue may be written as:

$$REV = S \times E \times I \quad (4.7.1),$$

where:

S is the specific tax on a tonne of CO₂-equivalent gas

E is the quantity (tonnes) of CO₂-equivalent emissions; and

I is a single price index (e.g., the consumer price index) to which the tax is indexed.

The change form of (4.7.1) is:

$$del_REV = (E \times I) \times del_S + REV \times (e + i) \quad (4.7.2),$$

where *del_* indicates ordinary change and lowercase letter indicate percentage changes. In our modelling, we use the ordinary change in *S*, not the percentage change, to avoid problems when the initial value (*S*) is zero.

Equation *E_deletaxrv* is based on (4.7.2). Revenue changes are added up over all elements of FUELX, ALLSOURCE and FUELUSER to give the change in tax revenue by region of fuel user. The mapping between the TABLO notation and the notation used in (4.7.2) is as follows:

- *deletaxrv* is *del_REV* defined over REGDST;

¹⁵ Non-combustion emissions consist of fugitive emissions from agriculture, mining, metal refining, cement manufacture, chemical processes and waste dumps. Activity emissions also include the forestry sink, with a negative sign.

We treat the sink potential of Forestry conservatively. One might expect that the carbon sequestered by Forestry should be related to the rate of planting. Instead, we connect it to Forestry activity as a whole, which includes logging. If Forestry is growing rapidly we should expect the industry to be devoting an abnormally high fraction of effort to planting, rather than felling. This would, at least temporarily, somewhat increase the sink effect beyond what we model.

- del_{gastax} is del_S defined over FUELX, ALLSOURCE, FUELUSER and REGDST;
- x_{gas} is e defined over FUELX, ALLSOURCE, FUELUSER and REGDST;
- $gastaxindex$ is i ;
- $ENERINDEX$ is equivalent to I ;
- Q_{GAS} is E defined over FUELX, ALLSOURCE, FUELUSER and REGDST; and
- $ETAX$ is equivalent to REV defined over FUELX, ALLSOURCE, FUELUSER and REGDST.

Total greenhouse tax revenue ($E_{deletaxrev}$)

The change in total tax revenue is measured as the sum of changes in tax revenue by state.

Greenhouse tax rate ($E_{delgastax}$)

This equation is added for convenience. It allows for changes in the greenhouse tax rate that are uniform across regions.

Ad valorem equivalent of the specific greenhouse tax rate imposed on fuels used for current production ($E_{fueltax1}$)

This equation translates the specific emissions tax into an *ad valorem* rate imposed on the basic values of fuels used in current production. The *ad valorem* emissions taxes appear in the core part of the model in equation $E_{deltax1}$.

$E_{fueltax1}$ is derived as follows. *Ad valorem* taxes in MMRF-GREEN raise revenue according to:

$$REV = V \times P \times Q / 100 \quad (4.7.3),$$

where:

- V is the *ad valorem* rate of tax (%);
- P is the basic price per unit fuel; and
- Q is the quantity of fuel.

Combining (4.7.1) and (4.7.3) yields:

$$V = S \times \frac{100 \times E \times I}{P \times Q} \quad (4.7.4).$$

Equation (4.7.4) shows that when converting specific gas taxes to conventional *ad valorem* taxes frequent use is made of the ratio of the indexed value of emissions (EI) to the value of the *ad valorem* tax base (PQ). Indeed, this ratio and the matrix of specific tax rates are the primary additional data items read in and updated for this section of the code. The core of MMRF-GREEN supplies P, Q values. Multiplying these by $[E.I/P.Q]$ and dividing by the price index I gives us the physical emissions matrix. Multiplying the latter by the matrix of specific tax rates and by the price index I gives us the matrix of gas tax revenues.

The change form of (4.7.4) is:

$$del_V = \frac{100V}{S} \times V \times (e + i - p - q) \quad (4.7.5).$$

Equation $E_fueltax1$ is based on (4.7.5), with $fueltax1$ equivalent to del_V defined over FUEL¹⁶, ALLSOURCE, IND and REGDST. The two coefficients in $E_fueltax1$ are:

- $EIoverPQ$ which is the ratio $[EI/PQ]$ defined over FUEL, ALLSOURCE, IND and REGDST; and
- $ETAXRATE$ which is S defined over FUELX, ALLSOURCE, FUELUSER and REGDST.

Ad valorem equivalent of the specific greenhouse tax rate imposed on fuels used for consumption ($E_fueltax3$)

Equation $E_fueltax3$ relates to taxes on fuels used for consumption, i.e., by the “residential” component of the set FUELUSER. It is analogous to $E_fueltax1$.

Ad valorem equivalent of the specific greenhouse tax rate imposed on activity emissions ($E_delindtax$ and $E_delcomtax$)

These two equations deal with the taxation of activity emissions. $E_delindtax$ is similar to $E_fueltax1$ and $E_fueltax3$. On the left hand side, the variable $delindtax(j,q)$ is the ordinary change in the percentage *ad valorem* rate of tax on activity emissions from industry j in region q . On the right hand side, the coefficient $EIoverPZ(j,q)$ is the ratio of the indexed value of emissions (EI) to the value of the basic value of output (i.e., to the *ad valorem* tax base (PZ)). Values for this ratio, like values for $EIoverPQ$, are read directly from the model’s database.

The second equation, $E_delcomtax$ maps the industry-based tax, $delindtax$, to a commodity-based tax, $delcomtax$. The equations ensures that the tax on commodity i is equivalent to the value of the tax on the output of industry j which produces i . This mapping is necessary because all indirect taxes in the core part of the code are taxes on the flows of commodities.

On the right hand side of $E_delcomtax$, $IS_DOM(s)$ is a dummy coefficient. It equals one when s is a domestic source, and zero otherwise. The coefficient $MAKESHR_COM$ is the share of commodity i produced by industry j in the region corresponding to the value of the index s . The operator $SOURCE2DEST(s)$ maps the value of s , which is part of the set ALLSOURCE, to the corresponding element of REGDST.

4.7.2. Handing back of tax revenue

The equations in this section facilitate simulations in which the revenue from a carbon tax is handed back either as a lump sum to owners of the emitting firms (grandfathering), or as a subsidy (negative tax) on consumer purchases.

Diversion of tax from government revenue ($E_TAXDivert$)

[NB: this feature of the model has been turned in the version used in the July 2003 MMRF course]

The variable $deletaxrv(q)$ represents the revenue raised from a carbon tax in region q . This is defined in equation $E_deletaxrv$ (see Section 4.7.1) as the sum of revenues collected from all gas emitters in each region. If there were no intervention in the code, then this tax revenue would be counted as tax revenue of the federal government via the variables $fueltax1$ and $fueltax3$ (see Section 4.7.1). However, this is unrealistic. Typically, the carbon tax scheme is designed to return the revenue to consumers as a subsidy for purchases of goods and services, or to the owners of the emitting firms as a lump sum.¹⁷

The variable $TAXDivert(q)$ has been included to insulate government revenue from carbon tax receipts. As can be seen from equation $E_TAXDivert$, when the shift variable $fTAXDivert$ is exogenously

¹⁶ The set FUEL contains all elements of the set FUELX except “activity”. In other words its elements are the emitting fuels only.

¹⁷ Ultimately, the owners of emitting firms are consumers in Australia and overseas (reflecting the degree of foreign ownership in emitting firms).

set to zero, TAXDivert(q) takes the value of deletaxrev(q). The variable TAXDivert appears with a negative sign in equation E_dompy320, which explains government collections of indirect taxes.

Handing back as a consumption subsidy or as a lump sum to consumers (E_delcomp3 and E_fcomp3)

Equations $E_delcomp3$ and E_fcomp3 are used to hand back the carbon tax revenue as a subsidy on consumer purchases. The hand back is activated by making:

- delcomp3 endogenous and the shift variable fcomp3 exogenous and equal to zero, and
- fTAXDivert endogenous and TAXDivert exogenous and equal to zero.

The first choice causes the variable delcomp3 to take the value of deletaxrev. Equation $E_delcomp3$ is then used to infer a value for deltax3comp, which is the absolute change in the general tax rate on consumer purchases necessary to return to consumers the full amount of the carbon tax revenue (delcomp3) as a consumption subsidy. The variable deltax3comp appears in the core equation $E_deltax3$.

To hand back the carbon tax revenue as a lump sum to owners of emitting firms, we make:

- fcomp3 endogenous and delcomp3 exogenous and equal to zero, and
- TAXDivert endogenous and fTAXDivert exogenous and equal to zero.

Note that in an earlier equation, $E_Taxrefund$ (Section 4.2.5), the value of TAXDivert is equated to the variable taxrefund which is added to household disposable income.

4.7.3. Fugitive-reducing technological change

We model non-combustion greenhouse gas emissions as directly proportional to the output of the related industries, with an allowance for abatement in response to a carbon tax. The amount of abatement is directly related to the level of the carbon penalty. The constants of proportionality are derived from point estimates, from various sources, of the extent of abatement that might arise at a particular tax level. In particular, we assume that if the tax reached \$100 (93-4 dollars) per tonne CO₂-e, non-fuel-burning emissions from:

- Agriculture would drop by 60 per cent,
- Black coal mining would drop by 70 per cent,
- Crude oil extraction would drop by 40 per cent,
- Alumina/aluminium smelting and refining would drop by 25 per cent, and
- Natural gas, Brown coal, Chemicals (excl. petrol), Cement and Other services would all drop by 10 per cent.

We should emphasise that the estimates above are quite speculative, but are only really important in the case of Agriculture, which makes a very large contribution to activity-related emissions.

Equation $E_agasAct$ activates our theory of fugitive-reducing technological change. The left hand side variable, agasAct(j), is the percentage change in the quantity of fugitive emissions per unit of output in industry j. On the right hand side, there is a shift variable f_agasAct(j), which is typically exogenous and set to zero. The other term on the right hand side is the product of the change in carbon tax, delgastax_sq applied to “activity” (i.e., fugitive) emissions and of a coefficient (GASACT(j)). GASACT(j) is the value of agasAct(j) per dollar of carbon tax.

The abatement that is endogenous generated imparts a cost saving to the industry equivalent to the quantity of gas abated times the rate of carbon tax. Abatement, however, is costly. It involves additional investment in new technology and/or changes to existing technologies and work practices. We account for this by forcing a cost-increasing technological change on the abating industries which exactly

offsets the tax savings arising from the abatement. This is achieved via equation E_del_fa1 (see the next section).

4.7.4. Substitution between effective units of intermediate inputs

Substitution between effective units of intermediate inputs ($E_agreeen$)

Equation $E_agreeen$ allows for price-induced substitution between effective units of intermediate inputs, especially those inputs that are energy intensive (e.g., gas and coal). The left hand variable ($agreeen(i,j,q)$) appears on the right hand side of the equation explaining the intermediate-input technological change variable $alo(i,j,q)$ (see equation (E_alo)).

On the right hand side of $E_agreeen$ there is a shift variable, $f_agreeen(i,j,q)$. When this is exogenously set to zero, $agreeen(i,j,q)$ is equated to the percentage change in the price paid by industry j in region q for input i ($P1O(i,j,q)$) relative to the average price paid by industry j in region q for all intermediate inputs ($P1TOT(j,q)$). The constant price elasticity is given by the coefficient $SIGMAGREEN(i,j)$. A typical value for $SIGMAGREEN(i,j)$ is 0.25. The coefficient $DUMMY(i,j,q)$ is one, when the underlying flow (i,j,q) is non zero, or zero when the underlying flow is zero. According to equation $E_agreeen$, if the price paid by industry j in region q for input i increases relative to the average price paid by industry j in region q for intermediate inputs, then there will be a technological change in industry j away from effective inputs of commodity i . Conversely, if the price of effective inputs of i falls relative to the weighted average of all intermediate inputs, then there will be a technological change that favours the use of i by industry j . These technological changes are made cost neutral by the mechanism explained below.

All input-saving technological change to offset cost effects of ac , $agasAct$ and $agreeen$ (E_del_fa1)

With the shift variable $del_fa1(j,q)$ set exogenously at zero, E_del_fa1 generates an all-input-using technological change ($al(j,q)$) which offset the effects on industry j 's unit costs of values for the $ac(i,q)$, $fa1o(i,j,q)$, $agreeen(i,j,q)$ and $agasAct(j)$ variables. Without such an offset, positive values for, say, $ac(i,q)$ would represent technological deterioration. To avoid such unrealistic implications we allow E_del_fa1 to generated cost-neutralising reductions in all of industry j 's inputs per unit of output.

4.7.5. Emissions related to core model variables and aggregate emissions

There are two groups of equations in this section. The first group relates the percentage change in emissions from emissions-generating activities to percentage change variables from the core part of the model. The second group defines convenient emission aggregates.

Emissions related to core model variables (E_xgasA to E_xgasD)

Equation E_xgasA relates the percentage change in combustion emissions from the use of fuel f from source s by industry j in region j to the volume of usage ($X1A(f,s,j,q)$). Non-combustion emissions from industries are dealt with in equation E_xgasB . This relates "activity" emissions to the output of the emitting industry with an allowance for changes in the amount of emissions per unit of output ($agasAct(j)$). The coefficient $HomeSource(s,q)$ equals one when $s = q$, and is zero otherwise. Equations E_xgasC and E_xgasD relate to household emissions.

Emission aggregates (E_xgas_s to $E_xgasTot$)

The remaining equations in this section define convenient totals of emissions. These are weighted averages of the $xgas(f,s,j,q)$ variables.

4.8. Petroleum usage and taxation

The equations in this section of the TABLO code deal with the usage and taxation of petroleum products. Taxes on petroleum products fall on both the household and business sectors of the economy. Households pay taxes on petrol and diesel in the forms of fuel excise, franchise fees (now embodied in fuel excise, with the exception of QLD where no franchise fees are levied) and GST. For LPG/CNG, consumers only have to pay GST.

The business sector generally pays lower taxes on petroleum products than the household sector because of GST exemption. Furthermore, there is a concessional system that allows some business users to pay no or lower taxes on some fuels. The Diesel Fuel Rebate Scheme (DFRS) and Diesel and Alternative Fuels Grants Scheme (DAFGS) are two important elements of such scheme. DFRS offsets the excise on diesel and like fuels used in specific off-road activities. The precise rate of the rebate depends on the nature of the activity in which fuel is used. Export industries such as primary and mining are the main beneficiaries of the scheme. Water transport and rail transport have also benefited from DFRS because of the recent extension of the scheme.

4.8.1. Tax rates, usage and revenues

In modelling the scheme of petroleum taxes we make allowance for three basic types of taxation: excise net of subsidies provided under the DFRS and the DAFGS, franchise fees and the GST¹⁸. The first two are imposed on the volume (litres) of petroleum usage. The GST is imposed on the retail value, inclusive of excise and franchise fees.

Ad valorem equivalent of the tax rate imposed on petroleum products used for current production (E_pettax1)

This equation is analogous to *E_fueltax1* (see Section 4.7.1). It translates petroleum taxes into *ad valorem* taxes imposed on the basic values of petroleum products used in current production. The *ad valorem* petroleum taxes appear in the core equation *E_deltax1*. For the taxes rated on volume, the basic equation is:

$$P \times Q \times V / 100 = Q \times I \times S / 1000000 \quad (4.7.6)$$

where:

P is the basic price per litre (\$million/litre);

Q is the quantity (litre) of petroleum product sold

V is the percentage *ad valorem* tax rate on the petroleum product (a number like 10);

S is the specific tax (\$/litre); and

I is the general price index to which the specific tax is indexed.

In change form, we write

$$(100/V) \times \text{del_V} = (100/S) \times \text{del_S} + i - p \quad (4.7.7).$$

V is computed as $100 \times \text{TAX} / \text{BAS}$, where TAX and BAS refer to the tax and basic flow components of the core GE data. S is computed as $1000000 \times \text{TAX} / (Q \times I)$. The initial value for index price is one. Q is read from the GASDATA database.

Equation *E_pettax1* is based on equation (4.7.7), with *pettax1* being equivalent to *del_V* defined over PETPROD, ALLSOURCE, IND and REGDST. On the right hand side of *E_pettax1*, the variables *del_petexcise1* and *del_petfran1* are, respectively, the change in the excise tax rate (\$/litre) and the

¹⁸ Revenue from excise fees accrues to the federal government. Revenue from franchise fees accrues to the state governments. Revenue from the GST accrues, in the first instance, to the federal government. We ensure that the revenue from each tax is returned appropriately via equations *E_softy122a*, *E_softy122b* and *E_softy122c*.

change in the franchise licence fee (\$/litre) applied to the usage of petroleum product p from source s by industry j in region q . The variable pettaxindex is the percentage change in I . The variable del_petgst1 is the ordinary change in the percentage *ad valorem* rate of GST applied to p from source s used by industry j in region q . The coefficient $\text{REVGST1}(p,s,j,q)$ is the GST collection on p from s used by j in q .

Ad valorem equivalent of the tax rate imposed on petroleum products used for consumption (E_{pettax3})

Equation E_{pettax3} relates to taxes on petroleum products used for consumption. It is analogous to E_{pettax1} .

Petroleum usage ($E_{\text{del_qpetsu}}$ to $E_{\text{v_qpettot}}$)

These equations define ordinary change and percentage change variables for the usage (litres) of petroleum products.

Petroleum tax revenues ($E_{\text{del_revfran}}$ to $E_{\text{del_revgst}}$)

These equations define ordinary changes in petroleum tax collections. There are separate equations for excise, franchise fees and the GST. The equations for the per-unit taxes are analogous to the corresponding equation for greenhouse tax revenue, $E_{\text{deletaxrv}}$ (see Section 4.7.1). The equation for GST revenue differs, reflecting the different tax base (retail value including non-GST taxes).

4.9. The GST

The equations in this section provide the flexibility necessary to model the introduction of the GST. The GST is generally applied to the price inclusive of freight and other margins such as wholesale trade. In some cases (e.g., petroleum products) the GST is imposed on the value inclusive of freight and other sales taxes (e.g., the per-litre petroleum excise). Accordingly, to accommodate the GST, the range of commodity taxes in MMRF-GREEN must be increased to accommodate *ad valorem* taxes levied separately on producer prices, on producer prices plus margins, and on producer prices plus margins plus other sales taxes.

GST tax rates ($E_{\text{deltax1_GST}}$ to $E_{\text{deltax3_GST}}$)

The left hand variables in these equations are ordinary changes in percentage tax rates levied on basic-value purchases (B) by industries (for current production and capital creation) and consumers. They appear in the core equations E_{deltax1} , E_{deltax2} and E_{deltax3} .

The right hand variables consist of:

- percentage changes in tax rates (%) levied on producer prices plus existing taxes (tax1_gst , tax2_gst and tax3_gst);
- ordinary changes in tax rates (%) levied on producer prices plus margins (deltax1M_gst , deltax2M_gst and deltax3M_gst); and
- an ordinary-change dummy variable (dummy_gst).

Setting the dummy variable to one has the effect of replacing the existing tax with a tax set at 10 per cent of the value of the underlying flow (10 per cent is the initial rate of GST).

5 Closing the Model

As explained in Chapter 3, in the model specified in Chapter 4 the number of variables (n) exceeds the number of equations (m). Thus to solve the model, $(n-m)$ variables must be made exogenous. This choice determines the model's closure.

Table 5 lists the exogenous variables in the model's standard closure. This is a long-run comparative-static closure (see Section 3.2), with national aggregate employment and national rates of return on capital fixed exogenously. By swapping variables between exogenous and endogenous categories using the standard closure as a starting point we develop a comparative static short-run closure. Forecasting and policy closures are developed in a similar way, by a series of swaps applied to the short-run comparative static closure.

5.1 The standard (long-run) comparative static closure

Technological change and other exogenous variables constraining real Gross State Product from the supply side

The first group of exogenous variables listed in Table 5 are concerned with the supply side of the regional economies. Most of the variables are technological-change terms relating to primary factors and intermediate inputs at the regional and national levels. MMRF-GREEN does not explain changes in technology. However, the inclusion of these terms allows the user to simulate the effects of a wide variety of exogenously given changes in technology.

Included in this first group of variables is national employment ($natl$). In this long-run closure we assume that the exogenous shock under investigation does not affect aggregate employment in Australia. In the long-run, demographic variables, participation rates and the natural rate of employment determine aggregate employment. The national real wage rate is assumed to vary to accommodate this assumption. Note that although the shock may not affect national employment, it does affect the regional distribution of employment (see below).

Because this closure is a long-run closure, we allow for capital reallocation effects. For example, in simulating the effects of increased government expenditure in Victoria, we allow Victorian industries' capital stocks to deviate from their basecase levels. We assume that in the long run the average rate of return on capital over all regional industries will be same with and without the shock under investigation. Thus the variable del_nat_ror is exogenous. We do, however, allow increase rates of return to persist in regional industries experiencing relatively large stimuli to activity relative to those industries experiencing relatively small stimuli. Making exogenous the variable del_f_ror , and thus activating equation $E_del_f_ror$, achieves this.

Exogenous settings of real Gross State Product from the expenditure side

The variables listed under this heading relate to the size and composition of aggregate domestic absorption in each region. The first variable, $f3tot$, is the average propensity to consumer out of household disposable income. Setting this exogenously to zero activates equation E_w3tot , linking changes in nominal private consumption ($w3tot$) to changes in household disposable income ($w1hinc$) in each region.

We assume that in the long-run year, investment in each regional industry will deviate from the basecase in line with deviations in the industry's capital stock. Thus the ratio variables, $r_inv_cap_jq$, $r_inv_cap_j$, $r_inv_cap_q$ and r_inv_cap are exogenous and typically set to zero. The implication of this assumption is that the *rates of growth of industries' capital stocks* do not deviate from basecase rates of growth (see Section 4.4).

We assume that the ratio of stage government consumption to private consumption is fixed. Hence $natf5gen$ and $f5gen$ appear in the exogenous list. Similarly, for the federal government, we assume that the deviation in their expenditure from the basecase is in line with nation-wide deviation in private consumption expenditure. Hence, $natf6gen$ and $f6gen$ appear in the exogenous list. The composition of state and federal consumption expenditure is fixed by the inclusion of $f5a$ and $f6a$ as exogenous variables.

Foreign conditions: import prices fixed; export demand curves fixed in quantity and price axes

MMRF-GREEN contains no equations describing movements in foreign demand schedules and changes in foreign supply conditions. Thus the various export demand shifters and the foreign currency prices of imports (pm) are treated as exogenous variables.

Tax rates are exogenous

The variables in this section are tax terms on commodity sales and factor incomes. Although these variables are naturally exogenous to a model like MMRF-GREEN, one can imagine a computation in which some tax rates are treated endogenously. For example, in many simulations it is appropriate to assume that the federal government's budget balance is unaffected by the shock. To achieve this, we could fix the budget balance exogenously at zero change and allow the tax rate on non-wage primary factor income (rk) to be endogenous. With f_rl_f set exogenously at zero, this would cause taxes on labour and capital income to change endogenously to maintain the budget balance at its basecase value.

Regional population, labour market and wages: exogenous wage differentials and exogenous unemployment rates

We assume that interregional differentials in wages and unemployment rates are fixed at their basecase values. This is achieved by fixing exogenously wage_diff and del_unr. Interregional population movements occur in order to accommodate the labour market assumptions. With unemployment rates fixed, employment (l) is indexed to the labour supply (labsup) in each region. With participation rates (pr) exogenous, labour supply is indexed to the working population (wpop) in each region. Finally, with f_wpop fixed, the working age population moves with total population (pop). This sequence of causality implies that employment and population move together at the regional level.

From a demographic point of view, the changes in regional populations are accommodated by changes in regional migration; natural changes in population (del_g) and international migration (del_fm) are exogenous. Note that in this closure the sum of changes in regional migration (delf_rm) is endogenous. This could be avoided by making delf_rm exogenous and natl endogenous. With regional employment and regional population moving together, this would imply a value of zero for natl (as required in a long-run simulation). However, it is more transparent to have natl exogenous. Non-zero values for delf_rm can be interpreted as reflecting changes in the natural growth of Australia's population or as changes in international migration, even though variables representing these factors are formally exogenous.

Government and household accounts

The variables listed in this section are naturally exogenous.

Numeraire assumption

We have to exogenously set a price. In this closure we have chosen the national CPI (natp3tot). An obvious alternative is the exchange rate (natphi). Note that the results for real variables are unaffected by this decision.

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APPENDIX A: Proposed Revised Treatment of Payroll Taxes

Background

A payroll tax is levied only on larger firms -- those with wage bill $> T$. There are L of such firms and each pays a fraction M of the amount by which its wage bill exceeds T . The total wage bill of the L larger firms is B , so the total tax revenue $R = M(B-LT)$. We call the term $(B-LT)$, the tax base, or P : it plays a central role.

The problem is to find how tax revenue R is affected by a change in T . To begin with we assume that the number of firms and the wage bill of each firm is fixed (these assumptions will be relaxed later).

Notation and definitions

W	total wage bill in industry
M	marginal tax rate
T	threshold value
L	No of larger firms -- those paying tax
B	wage cost of firms with wage bill $> T$
R	tax revenue in industry = $M.[B - LT]$
P	Tax base = $R/M = B - LT$

If W B R M and T are known, the rest of the variables above could be deduced.

The Tax Base Function

Plainly, the tax base declines as the threshold value increases. We can define a function:

$P = \Pi(x)$ = tax base if threshold is set to x ,
which we could sketch as follows.

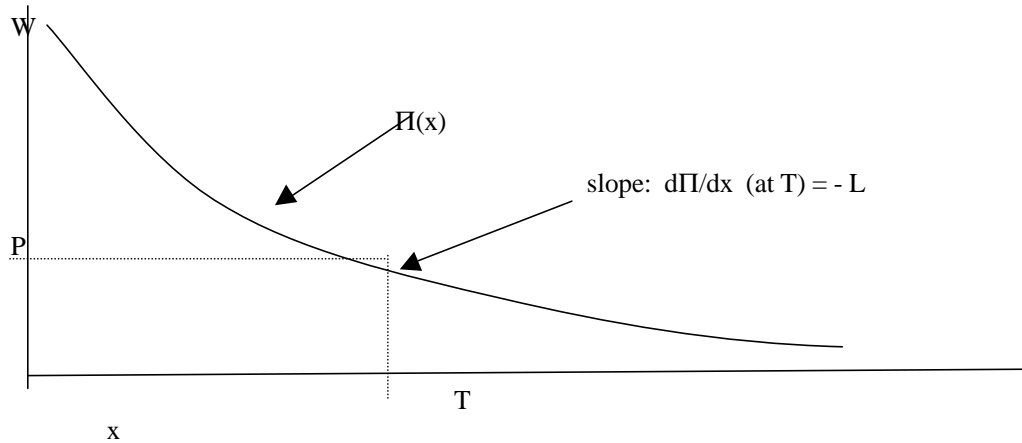


Diagram A: the tax base function

In Diagram A, note that:

$\Pi(0) = W$ since if the threshold x were zero, all wages would be taxable.

$\Pi(\infty) = 0$ since if the threshold x were high enough, no wages would be taxable.

$\Pi(T) = P$ observed in initial equilibrium

Now imagine the threshold was increased by a little bit dx . This has two effects:

- (a) a few marginal firms fall below the threshold, and
- (b) the L large firms each pay $M \cdot dx$ less tax, and P falls by $L \cdot dx$.

However, the effect of (a) on either R or P may be ignored since marginal firms pay no tax anyway. This argument is sufficient to give the slope of Π at T :

$$d\Pi/dx \text{ (at } T) = -L \quad \text{more formally proved in Appendix A}$$

Since L must be a declining function of T , Π must be convex to the origin, as drawn.

Initial Johansen Percent Change Implementation

From above we have:

$$dP = -L \cdot dT$$

or $P \cdot p = -L \cdot T \cdot t$, adopting % change notation

or $P \cdot p = - (B - P) \cdot t$,

or $p = -\alpha t$ where $\alpha = [B - P]/P$ which can be calculated from observed shares.

Tax revenue is simply given by:

$$r = m + p$$

Interestingly, we can work out these first-order effects without knowing more about the shape of the Π function. For large change, however, we need a way to update B :

$$B = LT + \Pi$$

but we have seen that $L = -d\Pi/dx = -\Pi'$

$$\text{so } B = \Pi - \Pi' \cdot T$$

$$\text{so } B' = \Pi' - \Pi'' \cdot T - \Pi' = -\Pi'' \cdot T$$

$$\text{or } dB = -\Pi'' \cdot T \cdot dT$$

$$\text{or } B \cdot b = -\Pi'' \cdot T \cdot T \cdot t \quad \text{adopting \% change notation}$$

$$\text{or } b = -\beta \cdot t \text{ where } \beta = \Pi'' \cdot T \cdot T / B$$

To evaluate Π'' we need to hypothesize a functional form for Π that is consistent with the initial observed facts.

Fitting a functional form to the Tax Base function

We assume Π has the cumulative Weibull form:

$$\Pi(x) = W \cdot \exp[-(x/a)^c]$$

Note that $\Pi(0)=W$ and $\Pi(\infty)=0$, as required. a and c are parameters which could be inferred from observed values of B and P . Our direct need is to evaluate $\beta = \Pi'' \cdot T \cdot T/B$.

We differentiate twice:

$$\begin{aligned}\Pi' &= d\Pi/dx = -\Pi(x) \cdot [c/a](x/a)^{c-1} = -\Pi(x) \cdot [c/x](x/a)^c \\ \text{so } \Pi'' &= -\Pi'(x) \cdot [c/x](x/a)^c - \Pi(x) \cdot D\{[c/a](x/a)^{c-1}\} \\ &= -\Pi'(x) \cdot [c/x](x/a)^c - \Pi(x) \cdot [c/a][(c-1)/a](x/a)^{c-2} \\ &= -\Pi'(x) \cdot [c/x](x/a)^c - \Pi(x) \cdot [c(c-1)/x^2](x/a)^c\end{aligned}$$

$$\text{At } T \quad \Pi'' = -\Pi' \cdot [c/T](T/a)^c - \Pi \cdot [c(c-1)/T^2](T/a)^c$$

We can make use of observed values of W , B , and P :

$$P = \Pi(T) = W \cdot \exp[-(T/a)^c] \quad \text{so} \quad \log(P/W) = -(T/a)^c$$

$$\Pi'(T) = -L = -\Pi(T) \cdot [c/T](T/a)^c = P \cdot [c/T] \cdot \log(P/W)$$

$$\text{so} \quad LT = B - P = P \cdot c \cdot \log(W/P)$$

$$\text{So} \quad \Pi''(T) = L \cdot [c/T] \log(W/P) - P \cdot c \cdot \log(W/P) \cdot (c-1)/T^2$$

$$\text{Use} \quad LT = P \cdot c \cdot \log(W/P)$$

$$\begin{aligned}\Pi''(T) &= L \cdot L \cdot T[1/T]/P - LT(c-1)/T^2 \\ &= L \cdot L/P - L(c-1)/T\end{aligned}$$

$$\text{We want } \beta = \Pi'' \cdot T \cdot T/B$$

$$= T \cdot T \cdot L \cdot L/[P \cdot B] - T \cdot T \cdot L(c-1)/[T \cdot B]$$

$$\text{Define symbol } A = LT = B - P \quad \text{or total tax-free allowance}$$

$$\beta = A \cdot A/[P \cdot B] - A(c-1)/B$$

$$= A \cdot A/[P \cdot B] + A/B - A \cdot c/B$$

$$= [A/B] \cdot \{1 + [A/P] - c\}$$

$$\text{Recall that} \quad LT = P \cdot c \cdot \log(W/P) \quad \text{so} \quad c = [A/P]/\log(W/P)$$

$$\begin{aligned}\text{giving} \quad \beta &= [A/B] \cdot \{1 + [A/P] - [A/P]/\log(W/P)\} \\ &= [A/B] \cdot \{1 + [A/P][1 - 1/\log(W/P)]\} \quad \text{where } A = B - P\end{aligned}$$

Summary of Progress

We have deduced:

$$p = -\alpha t$$

$$r = m + p$$

$$b = -\beta \cdot t$$

where α and β can each be calculated from observed shares as follows:

$$\alpha = [B-P]/P = A/P \quad \text{where } A = B - P$$

$$\beta = [A/B] \cdot \{1 + \alpha[1 - 1/\log(W/P)]\}$$

We assumed that all firms' wage bills were fixed.

Allowing for changes in firm size

We now allow for changes in the overall wage bill W . We assume still that the total number of firms and their *relative* sizes are fixed, so that if W increases by 1% each individual firm wage bill increases by 1%.

Our equations become:

$$p = w - \alpha[t-w] \quad \text{was} \quad -\alpha t$$

$$r = m + p \quad \text{unchanged}$$

$$b = w - \beta[t-w] \quad \text{was} \quad -\beta t$$

Thus, if both the threshold T and W increase by 10%, the wage bill of the larger firms, B , and the tax base, P , also increase by 10%.

We can use similar reasoning to find the effect of a change in the total number of firms, N . We still assume that their relative sizes are fixed. Suppose each firm split into 2 halves (ie, N doubled) and the threshold was reduced by half -- B and P would be unchanged. Alternatively if both W and N increased by 1%, with T unchanged, B and P would increase by 1%. So:

$$p = w - \alpha[n+t-w]$$

$$r = m + p$$

$$b = w - \beta[n + t - w]$$

Obvious ways to model firm numbers would be to assume that N was fixed or that N followed real industry output:

$$n = fn + z.$$

T, which is a nominal, might either be exogenous, or follow a CPI-linked rule:

$$t = ft + cpi$$

GEMPACK Implementation

Add equations for each industry:

$$p = w - \alpha[n + t - w]$$

$$r = m + p$$

$$b = w - \beta[n + t - w]$$

$$n = fn + z.$$

$$t = ft + cpi$$

and define new variables p, r, b, n, t, fn and ft.

On file store new data:

B	wage cost of firms with wage bill > T	updated by	b
R	tax revenue in industry = M.[B - LT]	updated by	r
P	Tax base = R/M = B - LT	updated by	p

Define new coefficients α and β :

$$\alpha = [B - P]/P$$

$$\beta = [(B - P)/B].\{1 + \alpha[1 - 1/\log(W/P)]\}$$

Relating the Tax Base function to firm size distribution

In this section we relate the Tax Base function, Π , to some other concepts. The intention is to provide both some background for the results above, and a starting point for attacking similar problems.

We start by defining a density function f for the number of firms of a certain size. Let the number of firms with wage bill between A and A+dA be f(A).dA; equivalently, let the number of firms with wage bill between X and Y (Y>X) be:

$$\int_{A=X}^Y f(A).dA$$

It is convenient to define two additional functions:

$$\text{let } Z(x) = \int_{A=x}^{\infty} f(A).dA \quad = \text{no of firms with wage bill} > x = L$$

$$\text{and } Q(x) = \int_{A=x}^{\infty} A.f(A).dA \quad = \text{wage bill of firms with wage bill} > x = B$$

At the initial equilibrium we have:

$$dZ/dx = dL/dT = -f(T)$$

$$dQ/dx = dB/dT = -T.f(T) \quad (\text{used to update } B)$$

$$\text{then}^{19} \quad Q(x) = x.Z(x) + \int_{y=x}^{\infty} Z(y).dy$$

¹⁹ Proof that $Q(x) = x.Z(x) + \int_{y=x}^{\infty} Z(y).dy$ [a case of definite integration by parts]

$$d(\text{LHS})/dx = -x.f(x) = d(\text{RHS})/dx = -x.f(x) + Z(x) - Z(x) \quad \text{So LHS - RHS} = \text{a constant } K$$

So $\Pi(x) = \int_{y=x}^{\infty} Z(y).dy = Q(x) - x.Z(x) = \text{tax base}$

We can define Z, Q and f in terms of Π as follows:

$$\begin{aligned} Z(x) &= -d\Pi/dx && \text{(giving change in P)} \\ Q(x) &= x.Z(x) + \Pi(x) \\ f(x) &= -dZ/dx = d^2\Pi/dx^2 \end{aligned}$$

In other words, Π is equivalent to an assumption about firm size distribution. Because we can derive Z, Q and f by differentiating Π , it turns out to be convenient to specify a form for Π . Had we specified a form for f or Z directly, some difficult integration might have been needed to find Π .

Sample computation results

Below are shown the results of adjusting T by either plus or minus 1% 325 times in succession, using the formulae of Section 6. The column headed "term" shows $\alpha[1 - 1/\log(W/P)]$.

$$\begin{aligned} \text{let } X = \infty \quad Q(\infty) &= \infty.Z(\infty) + \int_{y=\infty}^{\infty} Z(y).dy + K \\ 0 &= 0 + 0 + K \quad \text{so } K = 0 \end{aligned}$$

Note that as x grows huge, x.Z(x) must tend to 0, if wage bill is to be finite.

Increasing threshold:

step	B	P	L	T	term	alpha	beta
1	80.00	50.00	30.00	1.000	-0.266	0.600	0.275
3	79.56	49.40	29.56	1.020	-0.255	0.610	0.282
6	78.88	48.49	28.91	1.051	-0.239	0.627	0.293
10	77.94	47.27	28.04	1.094	-0.217	0.649	0.308
15	76.72	45.73	26.95	1.149	-0.188	0.677	0.328
21	75.17	43.87	25.66	1.220	-0.152	0.714	0.353
28	73.27	41.66	24.16	1.308	-0.108	0.758	0.385
36	70.94	39.13	22.46	1.417	-0.053	0.813	0.424
45	68.15	36.26	20.58	1.549	0.013	0.879	0.474
55	64.81	33.08	18.54	1.711	0.092	0.959	0.535
66	60.87	29.61	16.37	1.909	0.188	1.056	0.610
78	56.27	25.90	14.12	2.152	0.305	1.173	0.704
91	50.98	22.03	11.82	2.449	0.445	1.314	0.821
105	45.01	18.11	9.56	2.815	0.616	1.486	0.966
120	38.42	14.26	7.40	3.268	0.825	1.695	1.148
136	31.41	10.64	5.42	3.832	1.081	1.953	1.376
153	24.26	7.42	3.71	4.538	1.398	2.271	1.665
171	17.39	4.74	2.33	5.428	1.791	2.666	2.030
190	11.30	2.72	1.31	6.558	2.285	3.162	2.495
210	6.45	1.35	0.64	8.001	2.910	3.789	3.093
231	3.08	0.55	0.26	9.861	3.709	4.592	3.867
253	1.16	0.18	0.08	12.274	4.744	5.632	4.878
276	0.32	0.04	0.02	15.430	6.106	7.000	6.218
300	0.06	0.01	0.00	19.593	7.932	8.834	8.024
325	0.00	0.00	0.00	25.126	10.438	11.352	10.512

Decreasing threshold:

step	B	P	L	T	term	alpha	beta
1	80.00	50.00	30.00	1.000	-0.266	0.600	0.275
3	80.44	50.60	30.45	0.980	-0.276	0.590	0.269
6	81.08	51.49	31.11	0.951	-0.291	0.575	0.259
10	81.91	52.67	32.00	0.914	-0.311	0.555	0.246
15	82.89	54.12	33.12	0.869	-0.334	0.532	0.231
21	84.01	55.83	34.45	0.818	-0.361	0.505	0.214
28	85.24	57.79	36.01	0.762	-0.391	0.475	0.196
36	86.52	59.95	37.77	0.703	-0.423	0.443	0.177
45	87.84	62.30	39.75	0.643	-0.456	0.410	0.158
55	89.17	64.80	41.92	0.581	-0.491	0.376	0.139
66	90.46	67.42	44.28	0.520	-0.525	0.342	0.121
78	91.69	70.10	46.80	0.461	-0.559	0.308	0.104
91	92.84	72.81	49.49	0.405	-0.592	0.275	0.088
105	93.90	75.51	52.31	0.352	-0.624	0.244	0.074
120	94.86	78.15	55.26	0.302	-0.653	0.214	0.061
136	95.71	80.70	58.31	0.257	-0.681	0.186	0.050
153	96.45	83.11	61.45	0.217	-0.707	0.160	0.041
171	97.09	85.37	64.67	0.181	-0.730	0.137	0.033
190	97.63	87.46	67.97	0.150	-0.751	0.116	0.026
210	98.08	89.35	71.33	0.122	-0.770	0.098	0.020
231	98.46	91.05	74.77	0.099	-0.787	0.081	0.016
253	98.77	92.55	78.27	0.079	-0.801	0.067	0.013
276	99.02	93.86	81.86	0.063	-0.813	0.055	0.010
300	99.23	94.99	85.55	0.050	-0.823	0.045	0.008
325	99.39	95.95	89.35	0.039	-0.832	0.036	0.006