Reconciling Armington and Melitz Trade Responses: A Parameterisation Study of U.S. Motor Vehicle Tariffs

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Reconciling Armington and Melitz Trade Responses:
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Abstract

This paper examines the consistency between Armington and Melitz trade specifications in computable general equilibrium (CGE) models, focusing on the parameterisation of Melitz elasticities for the motor vehicle sector. Using the framework developed by Dixon, Jerie and Rimmer (2018), we first restrict the Melitz parameter space by imposing bounds derived from theoretical constraints and potentially observable industry characteristics. We then assess whether any admissible Melitz parameterisation can reproduce the import response generated by a standard Armington specification in GTAP-FIN for a 25 per cent U.S. tariff on motor vehicle imports. We find that no such parameter combination exists. This incompatibility raises a parameterisation dilemma: when Melitz parameters are constrained to generate empirically plausible industry characteristics, the implied import responses substantially exceed those produced under conventional Armington elasticities. We therefore reverse the calibration exercise and identify the range of Armington elasticities required to match the import responses generated by the restricted Melitz model. The implied Armington elasticities are considerably larger than values identified in the literature. One interpretation is that conventional Armington parameterisations understate trade responsiveness in sectors characterised by firm-level heterogeneity. Another is that alternative representations of heterogeneity, such as small group monopolistic competition, may warrant further investigation.

JEL codes: F12, C68, F17, F47, D58.

Keywords: Melitz model, Armington elasticities, tariffs, motor vehicles, computable general equilibrium.

Contents

1 Introduction

Quantitative trade policy analysis using computable general equilibrium (CGE) models typically relies on the Armington assumption, in which goods are differentiated by region of origin. In recent years there has been growing interest in the Melitz framework, in which firm-level heterogeneity governs export participation and trade volumes.

Dixon, Jerie and Rimmer (2018, hereafter DJR) propose a systematic approach to parameterising Melitz trade models for CGE applications. Their framework connects the key Melitz parameters (the elasticity of substitution between varieties and the Pareto shape parameter governing the distribution of firm productivities) to observable industry characteristics, including cost structures, productivity dispersion, and export concentration. DJR further argue that, where Armington elasticities have been chosen or estimated to deliver empirically plausible trade responses, Melitz parameterisations should be selected to replicate those same responses.[1]

This paper evaluates that proposition in the context of the motor vehicle sector using the GTAP-FIN model. We begin by applying the DJR restrictions to identify a range of Melitz parameter values that are both theoretically admissible and empirically plausible for motor vehicles. We then ask whether any Melitz parameterisation of the motor vehicles sector within this restricted Melitz space can reproduce the U.S. motor vehicle import response generated by a standard Armington specification in response to a 25 per cent U.S. tariff.

We find that no such parameter combination exists. The Melitz model, when constrained to generate plausible industry characteristics, produces import responses that are substantially larger than those implied by standard Armington elasticities. One interpretation of this finding is that it presents a calibration dilemma: either the Melitz specification is too restrictive, or the Armington elasticities commonly used in applied work understate trade responsiveness in sectors characterised by firm-level heterogeneity.

To resolve this dilemma, we invert the calibration exercise. Rather than forcing Melitz to match Armington outcomes, we identify the range of Armington elasticities required to replicate the import responses generated by the restricted Melitz model. We then consider whether these implied elasticities are supported by independent empirical evidence.

2 Parameterising Melitz Trade Models

2.1 Cost shares and fixed costs in the DJR framework

In their implementation of Melitz trade theory, Dixon, Jerie and Rimmer (2018) (DJR) show that the division of the total costs for an exporting firm between variable costs (v), trade

[1] Like DJR, this work uses GEMPACK software (Horridge et al. 2018) to solve the CGE model equations in percentage change form. As a result, values of Melitz variables like the number of operating firms and the mass of potential firms do not need to be specified to calibrate the model.

related fixed costs (t), and general fixed costs (g), under the assumption of a Pareto distribution of firm productivities, depends on the variety substitution parameter (σ) and the Pareto parameter (α) only (DJR 2018: 54-57). The DJR relationships between cost shares and Melitz parameters are set out in Table 1.

Table 1: Relationship between Melitz parameters and cost shares

DJR label	**Economic meaning**	**Algebraic share**
Variable cost share (*v)*	Marginal labour, materials, components.	$v = \dfrac{\sigma - 1}{\sigma}$
Trade link fixed cost share (*t*)	Overhead to keep a sales link open (sales office, homologation, local warehousing).	$t = \dfrac{\alpha - \sigma + 1}{\alpha\,\sigma}$
General fixed cost share (*g)*	Corporate R&D, platform engineering, head-office expenses.	$g = \dfrac{\sigma - 1}{\alpha\,\sigma}$

Note that in the above expression for the variable-cost share, only the elasticity of substitution between varieties (σ) appears. Hence, once a plausible value for the variety substitution parameter is decided upon, the proportion of total costs that is variable is pinned down, independent of any other parameter. The remainder of costs (1 – *v*) must be allocated between the two fixed cost components. This allocation is controlled by the Pareto-shape parameter α. Hence, once σ fixes how much of the motor vehicle sector's cost base moves proportionally with output, choosing α calibrates how the non-variable component is split between firm-wide overhead and the extra costs of maintaining each export market. The larger is α, the smaller is the general fixed share and the larger is the trade link overhead, and vice versa. The Pareto-shape parameter α also dictates how similar firms are. In the extreme, as $\alpha \to \infty$, all firms have the same productivity.[2]

2.2 Productivity and export dispersion constraints

In addition to the relationship between Melitz parameters and cost shares described in Table 1, DJR note that σ and α values also determine two potentially observable economic ratios in the Melitz system: the ratio of the productivity of the average firm ($\Phi_{\cdot sd}$) to the minimum productivity firm ($\Phi_{\min(s,d)}$) on an s,d link; and the ratio of the exports of the average firm ($Q_{\cdot sd}$) to the exports of the minimum exporting firm ($Q_{\min(s,d)}$) on the s,d link. Via equations (2.26) and (2.27) on p.21 of DJR, we have:

[2] In this case, as $\alpha \to \infty$, $g \to 0$. That is, if all firms have the same productivity, there cannot be any general set up costs.

$$\frac{\Phi_{\cdot sd}}{\Phi_{\min(s,d)}} = \beta = \left(\frac{\alpha}{\alpha - (\sigma - 1)}\right)^{\frac{1}{\sigma-1}} \quad (1)$$

where

σ is the elasticity of substitution between varieties (restricted to be >1).

α is a (positive) parameter governing the Pareto distribution describing productivity values in country s,

which is equation (9) in Melitz (2003:1702) after assuming that the firm's productivity is drawn from a Pareto distribution. That is, the ratio of the productivity of the average firm on the s,d link to the productivity of the minimum productivity firm operating on the s,d link. A reasonable bound for the value for β might be:

$$1.1 < \beta < 2 \quad (2)$$

That is, the productivity of the average productivity firm on the s,d link is between 10 per cent and 100 per cent higher than that of the minimum productivity firm on the link.

Note also that, because β must be positive and real-valued, we have the additional constraint from equation (2.27) above that:

$$\alpha > \sigma - 1, \qquad \text{since } \alpha > 0 \text{ and } \sigma > 1 \quad (3)$$

Next, we note that, via (T2.12) (see p.16 of DJR) we have:

$$\beta^{\sigma} = \frac{\mathrm{Q}_{\cdot sd}}{\mathrm{Q}_{\min(s,d)}} \quad (4)$$

That is, β^{σ} is the ratio of the quantity sent from s to d by the typical firm on the s,d link to the output sent by the minimum output and productivity firm on the s,d link. A reasonable range for this might be:

$$2 < \beta^{\sigma} < 10 \quad (5)$$

That is, the exports to region d by the average firm located in region s are between two and ten times higher than those of the minimum output and productivity firm in region s that is exporting to region d.

Note that from the definition of *v* (the variable cost share), we can calculate *f*, the total (trade link plus general) fixed cost share as:

$$f = t + g = \frac{1}{\sigma} \quad (6)$$

Hence, the share of general fixed costs in total fixed costs, *g/f*, is:

$$g/f = \frac{\sigma - 1}{\alpha} \quad (7)$$

2.3 Permissible and plausible Melitz parameter ranges

Via the above, we have five conditions, in addition to the requirements that α > 0 and σ > 1, that constrain the range of permissible and plausible values for σ and α under Melitz. These are summarised in Table 2.

Table 2: Constraints on Melitz parameter values

Condition	Range	Meaning
(A) $\alpha > \sigma - 1$, α > 0, and σ > 1		We restrict attention to real-valued β. See DJR, p. 12 and p. 21.
$(B)\ \beta = \frac{\Phi_{\cdot sd}}{\Phi_{\min\,(s,d)}}$	$1.1 < \beta < 2$	Ratio of the productivity of the average firm on the s,d link to the productivity of the minimum productivity firm operating on the s,d link.
(C) $\beta^{\sigma} = \frac{\mathrm{Q}_{\cdot sd}}{\mathrm{Q}_{\min\,(s,d)}}$	$2 < \beta^{\sigma} < 10$	Ratio of the quantity sent from s to d by the typical firm on the s,d link to the output sent by the minimum firm on the s,d link. See (T4.10) p. 42 DJR.
(D) $v = \frac{\sigma-1}{\sigma}$	$0.65 < v < 0.85$	Variable cost share. Hence, 1-v is the fixed cost share.
(E) $g/f = \frac{\sigma-1}{\alpha}$	$0.50 < g/f < 0.95$	Share of general fixed costs in total fixed costs.

We explore the implications of conditions (A) – (E) detailed in Table 2 for the range of permissible and plausible values for σ and α. Deriving the bounds for α is involved, because multiple boundary conditions are involved (see Table 2, conditions (A), (B), (C) and (E)), and each depends on σ. In this regard, we note that condition (D) uniquely determines a candidate interval for σ. Hence, we begin by defining the bounds for σ. Then, for each value of σ, we calculate the range of α values allowed by the β-type constraints (i.e., β directly, and β^{σ}) and g/f. Finally, these constraints can further restrict the final σ interval by eliminating values for σ for which the implied α constraints are infeasible.

Setting $\underline{v}$ and $\overline{v}$ as the lower and upper bounds for v, via condition (D) in Table 2 we have the following condition for the σ range:

$$\frac{1}{(1-\underline{v})} < \sigma < \frac{1}{(1-\overline{v})} \tag{8}$$

Turning to α, our first step is to combine the two β-type constraints into one interval for β. Via conditions (B) and (C) in Table 2, we require:

$$\underline{\beta} < \beta < \overline{\beta}$$

and

$$\underline{B} < \beta^{\sigma} < \overline{B}$$

where $\underline{\beta}$ and $\overline{\beta}$ are the lower and upper bounds for β , and $\underline{B}$ and $\overline{B}$ are the lower and upper bounds for β^{σ}.

Because σ > 0 and β > 0, we can take the σ root of the second inequality :

$$\underline{B}^{1/\sigma} < \beta < \overline{B}^{1/\sigma}$$

Hence, we now have two separate intervals that β must lie within. To satisfy both, β must lie within the intersection of these two intervals. The intersection of two intervals is an interval whose lower endpoint is the larger of the two lower endpoints, and whose upper endpoint is the smaller of the two upper endpoints. That is,

$$\max\,(\underline{\beta}, \underline{B}^{1/\sigma}) < \beta < \min\,(\overline{\beta}, \overline{B}^{1/\sigma})$$

or more compactly, given that both β-type constraints depend on σ:

$$\beta_L(\sigma) < \beta < \beta_U(\sigma)$$

where

$$\beta_L(\sigma) = \max\,(\underline{\beta}, \underline{B}^{1/\sigma})$$

$$\beta_U(\sigma) = \min\,(\overline{\beta}, \overline{B}^{1/\sigma})$$

Next, we rely on equation (1), which relates α and β, to solve for α explicitly. This is required, because our constraint is stated as bounds on β, so once α is written in terms of β, we can convert β bounds into α bounds by substituting in the β endpoints.

Solving equation (1) for α in terms of β we have:

$$\alpha = f(\sigma, \beta) = \frac{(\sigma - 1)\beta^{\sigma-1}}{\beta^{\sigma-1} - 1}$$

Re-arranging this equation, and examining the resulting expression, we can see that as β increases, the value of α decreases:

$$\alpha = f(\sigma, \beta) = (\sigma - 1)\left[1 + \frac{1}{\beta^{\sigma-1} - 1}\right]$$

Or more formally:

$$\frac{\partial \alpha}{\partial \beta} = -\frac{(\sigma - 1)^2 \beta^{\sigma-2}}{(\beta^{\sigma-1} - 1)^2} < 0$$

Hence, the largest allowable β values correspond to the minimum allowable α values, and vice versa. Hence, we can write the interval for α thus:

$$f\big(\sigma, \beta_U(\sigma)\big) < \alpha < f\big(\sigma, \beta_L(\sigma)\big)$$

Writing this out fully, we have:

$$\frac{(\sigma - 1)\beta_U(\sigma)^{\sigma-1}}{\beta_U(\sigma)^{\sigma-1} - 1} < \alpha < \frac{(\sigma - 1)\beta_L(\sigma)^{\sigma-1}}{\beta_L(\sigma)^{\sigma-1} - 1}$$

Finally, we must intersect this combination of the two β-type constraints with the *g/f* constraint. Let the lower and upper bounds for *g/f* be $\underline{G}$ and $\overline{G}$. Then:

$$\underline{G} < \frac{\sigma - 1}{\alpha} < \overline{G}$$

Taking the left inequality in isolation, we note:

$$\underline{G} < \frac{\sigma - 1}{\alpha} \Rightarrow \underline{G}\alpha < \sigma - 1 \Rightarrow \alpha < \frac{\sigma - 1}{\underline{G}}$$

Taking the right inequality in isolation, we note:

$$\frac{\sigma - 1}{\alpha} < \overline{G} \Rightarrow \sigma - 1 < \overline{G}\alpha \Rightarrow \alpha > \frac{\sigma - 1}{\overline{G}}$$

Hence:

$$\frac{\sigma - 1}{\overline{G}} < \alpha < \frac{\sigma - 1}{\underline{G}}$$

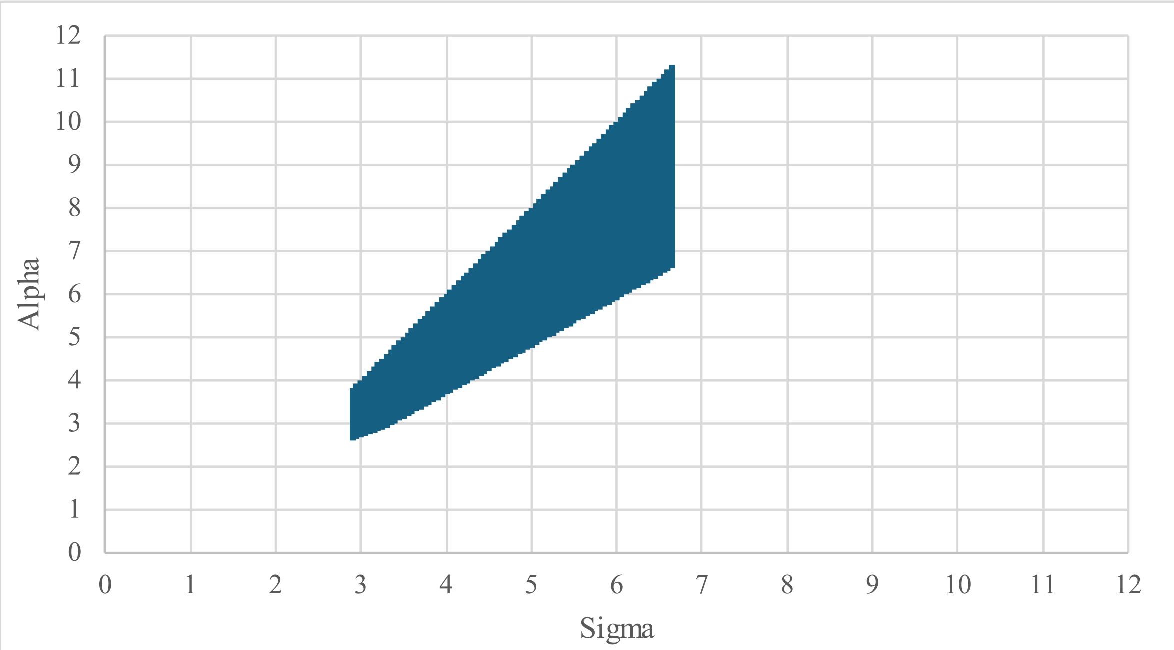
Combining the *g/f* constraint with the joint β-type constraints provides:

$$max\left\{\frac{(\sigma - 1)\beta_U(\sigma)^{\sigma-1}}{\beta_U(\sigma)^{\sigma-1} - 1}, \frac{\sigma - 1}{\overline{G}}\right\} < \alpha < min\left\{\frac{(\sigma - 1)\beta_L(\sigma)^{\sigma-1}}{\beta_L(\sigma)^{\sigma-1} - 1}, \frac{\sigma - 1}{\underline{G}}\right\} \qquad (9)$$

What does this equation mean for the α boundaries at the minimum and maximum values for σ? Via condition (D) in Table 2 and equation (8), we see that the minimum and maximum candidate values for σ (σ_{Min} and σ_{Max}) when the range for v is $0.65 < v < 0.85$ are 2.857 and 6.667. At σ_{Min} = 2.857, the β-type constraints imply $\alpha \in (2.565, 5.120)$, while the *g/f* constraint implies $\alpha \in (1.955, 3.714)$; hence $\alpha \in (2.565, 3.714)$. At σ_{Max}= 6.667, the β-type constraints imply $\alpha \in (6.599, 12.728)$ while the *g/f* constraint implies $\alpha \in (5.965, 11.333)$; hence $\alpha \in (6.599, 11.333)$.

Graphically, all combinations of α and σ such that all of conditions (A)-(E) are satisfied within the bands given in column 2 of Table 2 are reported in Figure 1.

Figure 1: Parameter combinations jointly satisfying conditions (A) - (E) given parameter ranges in Table 2.



How do the bounds in equations (8) and (9) depend upon the limiting values in Table 2? As already discussed, the admissible set of (σ, α) pairs is constructed in two stages. First, the variable cost share restriction determines the feasible interval for σ. Second, conditional on each feasible σ, admissible values for α lie on the intersection of: (i) the α interval implied by the β-type constraints (β and β^{σ}); and, (ii) the α interval implied by the fixed cost share restriction *g/f*. In terms of Figure 1, changes in the limiting values in column 2 of Table 2 therefore affect the admissible (σ, α) region either by shifting the σ window horizontally (via either v, or by creating infeasible implied α constraints) or by shifting the conditional α-band vertically at each σ (via β, β^{σ}, and *g/f*). We discuss below how varying the bounds in column 2 of Table 2 affects Figure 1. We first discuss this conceptually, before exploring examples on the perimeter of the feasible area plotted in Figure 1.

<u>Varying the bounds on β</u>. What would happen to the area in Figure 1 if we changed the limiting values assumed for the ratio of productivities of the average to minimum-

productivity firm? The upper bound for this range ($\overline{\beta}$) enters only through the upper endpoint of the feasible β-interval, $\beta_U(\sigma) = \min\,(\overline{\beta}, \overline{B}^{1/\sigma})$. Lowering $\overline{\beta}$ can therefore only lower $\beta_U(\sigma)$ (and only over the σ range where $\overline{\beta}$ is actually the minimum). Because $f(\sigma, \beta)$ is decreasing in β, lowering $\beta_U(\sigma)$ raises the β-implied lower bound on α. Graphically, this lifts the bottom edge of the admissible band (an "upward" contraction from below). If $\overline{\beta}$ is reduced far enough, it can also eliminate feasibility for smaller σ by making the intersected β-interval empty (i.e. $\beta_L(\sigma) \geq \beta_U(\sigma)$). This removes part of the band at the left (i.e. a rightward shift of the left edge). Conversely, increasing $\overline{\beta}$ has no effect whenever $\beta_U(\sigma)$ is instead determined by $\overline{B}^{1/\sigma}$ (so that $\overline{\beta}$ is slack).

The lower bound $\underline{\beta}$ enters only through the lower endpoint:

$$\beta_L(\sigma) = \max\,(\underline{\beta}, \underline{B}^{1/\sigma})$$

Hence, raising $\underline{\beta}$ raises $\beta_L(\sigma)$ (where it binds). Since the β-implied upper bound on α is $f\big(\sigma, \beta_L(\sigma)\big)$, and f decreases in β, an increase in $\beta_L(\sigma)$ lowers the top edge of the admissible bound (i.e. a downward contraction from above), and may also eliminate feasibility at high σ if it causes $\beta_L(\sigma) \geq \beta_U(\sigma)$. Lowering $\underline{\beta}$ has no effect whenever $\underline{\beta}$ is slack relative to $\underline{B}^{1/\sigma}$.

Varying the bounds on β^{σ}. What would happen to the area in Figure 1 if we changed the limiting values assumed for the ratio of the exports of the average firm to the minimum firm? The bounds for this ratio, given by $\underline{B}$ and $\overline{B}$, affect feasibility through $\underline{B}^{1/\sigma}$ and $\overline{B}^{1/\sigma}$, which enter $\beta_L(\sigma)$ and $\beta_U(\sigma)$ respectively. Increasing $\underline{B}$ raises $\underline{B}^{1/\sigma}$, which, when it binds, raises $\beta_L(\sigma)$ and therefore lowers $f\big(\sigma, \beta_L(\sigma)\big)$. Graphically, this pushes the top boundary of the admissible band downward and can also remove feasibility at larger σ if the β-interval becomes empty. Decreasing $\underline{B}$ works in the opposite direction, but only over the σ-range where $\underline{B}^{1/\sigma}$ is the binding contributor to $\beta_L(\sigma)$.

Increasing $\overline{B}$ raises $\overline{B}^{1/\sigma}$, which, when it binds, raises $\beta_U(\sigma)$ and therefore lowers $f\big(\sigma, \beta_U(\sigma)\big)$. Graphically, this pushes the bottom boundary downward, expanding the admissible band downward. Decreasing $\overline{B}$ has the opposite effect, lifting the bottom boundary and potentially removing α feasibility at smaller σ.

Varying the bounds on *v*. What would happen to the area in Figure 1 if we changed the limiting values assumed for the variable cost share, *v*? The limiting values $v \in (\underline{v}, \overline{v})$ determine the candidate σ range via $\sigma = 1/(1 - v)$, which is increasing in *v*. Hence, lowering $\underline{v}$ lowers σ_{Min} (expanding the admissible region leftward), while raising $\underline{v}$ raises σ_{Min} (contracting the candidate region from the left). Similarly, raising $\overline{v}$ raises σ_{Max} (expanding rightward), while lowering $\overline{v}$ lowers σ_{Max} (contracting from the right). Because α-feasibility is evaluated pointwise in σ, changing the *v* bound primarily changes the horizontal span of the admissible band, and secondarily changes the overall α envelope by adding/removing the band at low or high σ.

<u>Varying the bounds on *g/f*.</u> What would happen to the area in Figure 1 if we changed the limiting values associated with the share of general fixed costs in total fixed costs? As discussed earlier, the restriction $\underline{G} < (\sigma - 1)/\alpha < \overline{G}$ is equivalent (for $\sigma > 1$, $\alpha > 0$) to $(\sigma - 1)/\overline{G} < \alpha < (\sigma - 1)/\underline{G}$. Lowering $\underline{G}$ increases the upper bound $(\sigma - 1)/\underline{G}$, lifting the ceiling of the admissible band (an upward expansion in Figure 1), without changing the admissible σ range. Raising $\underline{G}$ lowers that ceiling (a downward contraction from above in Figure 1). Changes in $\overline{G}$ operate on the lower bound: raising $\overline{G}$ lowers $(\sigma - 1)/\overline{G}$ (a downward expansion from below in Figure 1), while lowering $\overline{G}$ raises it (an upward contraction from below). Whether this shifts the lower bound on α will depend on whether the β-type constraint or the $(\sigma - 1)/\overline{G}$ constraint is binding. If the former, increases in $\overline{G}$ may have little effect until the *g/f*-implied lower bound on α becomes binding.

The above discussion demonstrates that the location of the admissible (σ, α) band in Figure 1 reflects the way the constraints interact across the σ-range. For each feasible σ, the lower and upper bounds of α are determined by the intersection of the β-type constraints and the *g/f* constraint. Because each constraint may only be active over part of the σ-range, changing a particular bound affects the band only where that constraint is binding Outside that range, the band remains unchanged. Hence, in certain portions of Figure 1, modifications of the bounds can produce no visible movement. Similarly, tightening a bound can shrink the band only where it is active, leaving other portions intact. This binding-constraint logic explains the overall shape of the feasible region, and why the different edges respond selectively when the limits on β, β^{σ}, v or *g/f* are varied. With this framework in mind, we now describe the specific effects on Figure 1 of changing the bounds described in Table 2.

Increasing the upper bound on β above 2 has no impact on the admissible area in Figure 1. Decreasing the upper bound on β below 2 decreases the height of the north-west corner in Figure 1. For example, setting β = 1.9 removes the pair (α, σ) = (2.75, 3) from the admissible area. Decreasing the lower bound on β below 1.1 has no impact on the admissible area in Figure 1. Increasing the lower bound on β above 1.1 restricts the admissible area from the south-east. For example, setting β = 1.2 reduces the highest admissible α from 11 to 8.5.

In row (3), decreasing the lower bound on β^{σ} below 2 has no impact on the admissible area in Figure 1, but increasing the lower bound above 2 restricts the admissible area from the south. For example, increasing β^{σ} from 2 to 2.5 has no effect on the maximum admissible value for σ but reduces the maximum admissible value for α from 11 to 10. Increasing (decreasing) the upper bound above 10 expands (restricts) the admissible area to the north-east (south-west). For example, increasing the upper bound on β^{σ} from 10 to 11 admits the pairs (α, σ) = (4.75, 5), (5, 5.25) and (5.25, 5.5) to the admissible area.

Increasing (decreasing) the minimum variable cost share contracts (expands) the admissible area to the south-east (north-west). For example, decreasing v from 0.65 to 0.6 decreases the minimum admissible value for σ from 3 to 2.5, and the minimum admissible value for α from 2.75 to 2.5. But changing the maximum value for v has a more dramatic impact on the admissible area. Increasing from 0.85 to 0.9 increases the maximum admissible value for σ from 6.5 to beyond 10, and the maximum admissible value for α from 11 to beyond 12.

Decreasing (increasing) the lower bound on the share of general fixed costs in total fixed costs expands (contracts) the admissible area to the south (north). For example, decreasing g/f from 0.5 to 0.45 increases the admissible value for α by 0.25-1.0 for all admissible σ, but does not change the range of admissible values for σ. Increasing the upper bound on g/f has no effect on the admissible area, and the admissible area contracts from the north only after decreasing g/f from 0.95 to 0.8 or lower.

Can we find any support for the values adopted for the parameters in Table 2? There is a literature that has looked at productivity in the motor vehicles industry. For example, Pavlínek and Ženka (2011) and Grodzicki and Skrzypek (2020) each used firm-level data from the European automotive industry to consider questions including industrial upgrading, cost competitiveness and structural change. Pavlínek and Ženka (2011) report trends in apparent and wage-adjusted labour productivity in 2001 and 2006 for ten European countries for firms in the European automotive value chain. From their Table 7, the ratio of average to minimum labour productivity ranges from 1.48 to 12.18. Grodzicki and Skrzypek (2020) report unit labour costs in the automotive industry for a range of European countries and country-groupings in 2000, 2004, 2009 and 2014. Based upon their data, the ratio of average to minimum unit labour costs ranges from 1.65 to 2.68. While these observations suggest that our upper-bound for β in Table 2 of 2 might be too low, recall that we noted earlier that increasing this upper bound beyond 2 introduces only a tiny south-westerly expansion in the admissible area in Figure 1.

3 Implications of Melitz parameters for trade policy shocks

What do the values for σ and α mean for simulation results when a tariff is imposed on the imports into a given region? That is, consider a given tariff increase in destination markets: what is the effect of varying σ and α for a given tariff shock? The discussion in Table 4 anticipates the possible outcomes from varying these parameters given the theory of Melitz. To quantify the effects, we run numerous Melitz simulations under alternative parameterisations.

In order to understand how changes in these parameters impact results, we use the GTAP-FIN model to simulate the impacts of a particular tariff shock, in this case a 25 percentage point increase in the tariff charged by the US on all imports of autos and parts (GTAP-FIN commodity *mvh*) from any trading partner. First we simulate this tariff shock in a version of GTAP-FIN where all industries including *mvh* are modelled as Armington industries, where products are differentiated by country or region. Since virtually all CGE simulations of tariff shocks use this Armington approach, this gives us a well-understood model to which our Melitz results can be compared. Then, we explore the effects of the 25 percentage point increase in the US *mvh* tariff under a GTAP-FIN specification in which *mvh* is modelled as a Melitz industry.

Table 4: Anticipated effects of Melitz parameters on response to a tariff shock in export market

Parameter	What it governs in a Melitz model	First-order impact of an ad-valorem tariff τ on the industry
σ (elasticity of substitution across varieties)	1. Trade (inverse) elasticity = σ – 1 2. Mark-up = σ ⁄ (σ–1) 3. Variable-cost share v = (σ–1)⁄σ	**Two opposing channels:** **(i) Affects the *intensive*-margin response**. A higher σ implies larger substitution away from taxed varieties, and thus bigger falls in import quantities. **(ii) Affects the *extensive*-margin response**. A higher σ implies a lower fixed cost share, reducing the hurdle to exporting. Fewer firms are pushed below the cut-off for a given tariff shock, reducing the import response.
α (Pareto shape of firm productivity dispersion)	1. Thickness of the right tail (low α implies fatter tail). 2. How many firms sit "just above" the export cut-off. 3. Fixed-cost shares g and t via g = (σ–1) / (α σ)	**Affects the *extensive*-margin response**. Higher α implies more firms clustered near the cut-off; a tariff that raises unit trade costs now pushes many exporters below breakeven, thereby encouraging higher firm exit and a sharper fall in variety and industry output. Lower α (fatter tails) cushions the shock because a bigger pool of highly productive firms can absorb the tariff and stay in the export market.

3.1 Benchmark Armington results for U.S. motor vehicle tariffs

GTAP-FIN, like GTAP, uses a two-stage nested Armington structure, in which importers first choose between domestic and imported varieties of each good, before choosing between regional sources of the imported variety. Hereafter, we follow the GTAP nomenclature for the two elasticities governing substitution possibilities in each of these nests: "ESBD" for the domestic / import substitution elasticity; and "ESBM" for the substitution elasticity across alternative sources of import supply.

Under the Armington assumption, using standard GTAP Armington parameters for *mvh* (ESBD = 2.8, ESBM = 5.6) the 2025 GTAP-FIN result for U.S. imports of *mvh* is -26.1: that is, an increase of 25 percentage points in the tariff charged by the US on all imports of *mvh* causes the volume of *mvh* imports to fall by 26.1 per cent.

DJR (2018:105-7) argue that when "comparing unobservable implications from competing models (ie: characterizing *mvh* trade as either Melitz or Armington), we should parameterize the models so that they give the same results for observable outcomes." In Section 3.2, we simulate GTAP-FIN under a Melitz specification for the *mvh* industry to investigate whether we can find a Melitz parameterisation that reproduces the Armington import volume result.

3.2 Can Melitz replicate Armington import responses?

Table 5 reports the year 1 impacts on U.S. motor vehicle import volumes of a 25% U.S. tariff on motor vehicle imports under various Melitz parameterisation assumptions for σ and α. The aim of the 20 simulations reported in Table 5 is to search for a Melitz parameterisation for motor vehicles that simultaneously:

(a) generates an import volume response like that generated by Armington (consistent with the recommendation in DJR); and,

(b) represents a σ, α combination that falls within the subset of allowable (α > 0, σ > 1, $\alpha > \sigma - 1$) and plausible (in the sense that the implicit cost shares and productivity and export ratios are within the bounds laid out in Section 2.3) combinations generated by the search discussed in Section 2.3.[3] That is, each (α, σ) pair in Table 5 is an element of the admissible area identified in Figure 1 in the previous section.

We proceed by running a multiple regression of US imports of *mvh* against values for σ and α using all results reported in Table 5. This produces:

$$import\ volume\ response = -61.6 + 3.67\,\sigma - 4.05\,\alpha \qquad (10)$$

This regression closely fits the model results for the import response ($R^2 = 0.89$) and the t-statistics on the estimated coefficients are high (-29.6, 5.96, and -11.2 for the constant, σ and α terms respectively). The α sign being negative is consistent with our expectations from Melitz. A higher value for α implies a thinner productivity tail, meaning fewer high productivity exporters able to absorb the tariff increase. This suggests a stronger extensive margin contraction, causing imports to fall by more. The positive coefficient on σ suggests that the import response from the tariff increase gets smaller as the value of σ rises.[4] From our summary in Table 4, it seems that the impact of the change in σ through the intensive margin is dominated by the impact through the extensive margin. As the value of σ rises, a given tariff shock results in fewer firms exiting, and this dominates the larger substitution away from taxed varieties to result in a smaller import response.

We proceed to use equation (10) to evaluate whether a permissible and plausible combination of σ and α can produce an import volume response commensurate with that of Armington

[3] See p. 12 of DJR for the requirement that σ > 1. See p. 21 for the requirements that α > 0 and $\alpha > \sigma - 1$.
[4] For example, Table 5 shows that for α = 4.25, an increase in σ from 4 to 4.75 decreases the (absolute value of the) import response from -63.84 to -59.16.

(-26.1). We begin by targeting the Armington import volume response from Section 3.1 by substituting the import volume response (-26.1) into equation (10):

$$-26.1 = -61.6 + 3.67\,\sigma - 4.05\,\alpha \tag{11}$$

Rearrange equation (11) for σ:

$$\sigma = 9.67 + 1.10\,\alpha \tag{12}$$

Now, substitute equation (12) into the constraint from equation (3): $\alpha > \sigma - 1$

$$\alpha > 9.67 + 1.10\,\alpha - 1 \tag{13}$$

Solving equation (13) for α yields:

$$\alpha < -86.7$$

Because we require $\alpha > 0$, this result suggests there is no feasible (α, σ) pair that can generate the Armington result under the default GTAP elasticities ESBD(*mvh*) = 2.8 and ESBM(*mvh*) = 5.6.

Table 5: Results under Melitz parameterisation for U.S. mvh imports in 2025 after a 25% increase in the U.S. tariff on mvh

Sim result for U.S. *mvh* imports	Melitz parameters		Implications of Melitz parameters				
			Cost shares			Ratio of average / min firm	
	Sigma	Alpha	Variable	Link-fixed	Entry-fixed	Productivity	Exports
-61.75	3.00	2.75	0.67	0.0909	0.24	1.91	7.02
-73.29	3.00	4.00	0.67	0.1667	0.17	1.41	2.83
-62.65	3.50	3.50	0.71	0.0816	0.20	1.65	5.78
-58.22	4.00	3.50	0.75	0.0357	0.21	1.91	13.39
-60.21	4.00	3.75	0.75	0.0500	0.20	1.71	8.55
-63.84	4.00	4.25	0.75	0.0735	0.18	1.50	5.11
-59.16	4.75	4.25	0.79	0.0248	0.19	1.77	15.04
-59.58	5.00	4.50	0.80	0.0222	0.18	1.73	15.59
-64.90	6.50	6.50	0.85	0.0237	0.13	1.41	9.14
-79.57	6.50	11.00	0.85	0.0769	0.08	1.13	2.27
-68.72	8.50	8.50	0.88	0.0138	0.10	1.33	11.31
-73.72	4.00	6.00	0.75	0.1250	0.13	1.26	2.52
-75.27	4.75	7.50	0.79	0.1053	0.11	1.20	2.41
-62.88	5.25	5.25	0.81	0.0363	0.15	1.48	7.76
-76.47	5.25	8.50	0.81	0.0952	0.10	1.18	2.35
-63.61	5.75	5.75	0.83	0.0302	0.14	1.45	8.31
-77.71	5.75	9.50	0.83	0.0870	0.09	1.16	2.31
-69.01	4.75	6.00	0.79	0.0789	0.13	1.30	3.46
-71.18	5.75	7.50	0.83	0.0638	0.11	1.24	3.37
-72.53	6.50	8.50	0.85	0.0543	0.10	1.21	3.42

Our inability to find an (α , σ) pair that reproduces the Armington result for U.S. motor vehicle imports presents a difficulty. If we accept that the standard GTAP Armington elasticity and theoretical specification determines an empirically-plausible response for U.S. motor vehicle imports, we face the problem that there is no (α, σ) pair, either within the permissible and plausible range or outside it, that can reproduce the result when the motor vehicle sector is modelled as a Melitz-type industry. One way out of this difficulty is to question whether our Armington parameterisation (ESBD = 2.8 and ESBM = 5.6) is a suitable benchmark.

Our Melitz parameterisation attempts to restrict the parameter space to a range that implies empirically plausible values for observable features of the industry relating to cost shares, productivity ratios and export ratios. This suggests another approach to parameterisation: find the range of Armington elasticities under an Armington specification that generate import responses within the bounds produced by a Melitz model under a parameterisation that is permissible and plausible. Having found these elasticities, we then ask the question: is there independent evidence on the value of the *mvh* Armington elasticity that would justify moving away from the standard value, and towards that implied by Melitz.

3.3 Re-parameterising Armington elasticities to match Melitz outcomes

Our aim is to derive a back-of-the-envelope equation linking the import response from a 25% *mvh* tariff increase to the size of the import/domestic Armington elasticity (ESBD). We will then use this equation to derive the value of ESBD necessary to reproduce the range of Melitz results we found in Section 3.2. Via Table 5, we see that the import response range lies approximately between -60 and -80 per cent. We then ask whether there is empirical literature to support the required values for ESBD.

We begin by running an Armington simulation in which we simply double the standard *mvh* Armington parameters (hence, ESBD = 5.6 and ESBM = 11.2). This generates a reduction in U.S. *mvh* imports of -42.64%. Fitting a straight line through the results from the two Armington simulations for import volumes and ESBD (i.e, (2.8, -26.12) and (5.6,-42.64)) suggests that under Armington:

$$import\ volume\ response = -9.6 - 5.9\ ESBD$$

This equation implies ESUBD values of 8.5 and 11.9 to achieve *mvh* import responses of -60 and -80 respectively. Hence, we run two further simulations with ESBD set at these values. In each simulation, we retain the GTAP assumption that ESBM = 2·ESBD. The results from these simulations are reported in rows 3 and 4 of Table 6.

Table 6: Impact on U.S. imports of motor vehicles in 2025 of a 25% U.S. import tariff on motor vehicles under alternative Armington specifications for motor vehicles

Simulation name	**Sim result for U.S. *mvh* imports in 2025 (% deviation relative to baseline)**	**Value for *mvh* ESUBD (ESUBM = 2*ESUBD)**
Armington sim. 1	-26.12	2.8
Armington sim. 2	-42.64	5.6
Armington sim. 3	-55.46	8.5
Armington sim. 4	-66.25	11.9
Armington sim. 5	-72.39	14.6
Armington sim. 6	-80.23	19.7
Armington sim. 7	-59.70	9.69
Armington sim. 8	-80.14	19.63

The ESBD values applied to *mvh* import determination in Armington simulations 3 and 4 bring us close to, but fall somewhat short of, the target range of -60 to -80 per cent. From the results of simulations 3 and 4, we can see that an ESBD value somewhere between 8.5 and 11.9 can achieve an import response at the top of our target range (-60 per cent). In continuing the search for the ESUBD value that brings the import response closer to -80 per cent, we fit a regression line through the results of Armington simulations 1-4, which produces the following relationship:

$$import\ volume\ response = -16.07 - 4.38\ ESBD$$

This suggests an ESUBD value of 14.6 to generate an import response of -80 per cent. The *mvh* import result from running this simulation is reported as simulation 5 in Table 6. Noting that again the CGE model result falls short of the anticipated result from the linear regression equation, we next fit a non-linear equation to the five CGE results, generating:

$$import\ volume\ response = 4.226 - 28.258 * \ln{(ESBD)}$$

Solving for ESBD we have:

$$ESBD = e^{\frac{4.226-y}{28.258}}$$

This suggests an ESBD value of 19.7 to generate an import response of -80 per cent. The import response result from the Armington simulation (Simulation 6 in Table 6) is -80.23. This suggests that the non-linear form is a good fit for the observations, so we re-estimate the equation using all six observations. This produces:

$$import\ volume\ response = 4.347 - 28.331 * \ln{(ESBD)}$$

This implies that:

$$ESBD = e^{\frac{4.347-y}{28.331}}$$

This suggests that suitable values for ESBD to generate import responses of -60 and -80 are 9.69 and 19.63. We conclude our Armington simulations by running two final simulations with ESBD = 9.69 (simulation 7) and ESBD = 19.63 (simulation 8). For reporting purposes, we include the import response of these two equations in Table 6, even though the results of these simulations do not enter into the estimation of target ESBD values as described above. What these last two simulations provide is the range of Armington elasticities that produce Melitz-compatible results for the *mvh* sector: that is, 9.69 to 19.63.

4 Assessing the empirical plausibility of the Armington elasticities implied by Melitz

Specification or parametrization of import demand elasticities has long been recognized as an important task in the calibration of CGE models. Hillberry and Hummels (2013) provide a review of the literature estimating trade elasticities, including work that identifies demand elasticities from trade costs. They note that values of the Armington elasticity used to calibrate the GTAP model have been drawn from a time-series econometrics literature that dates to at least 1977. For our purposes, we noted that the default value for the Armington elasticity σ in the GTAP model for the motor vehicles industry is ESBM(MVH) = 5.6. A more recent example is provided by Fontagné *et al.* (2022) who estimated trade elasticities at the product level. Their estimate of the trade elasticity for motor vehicles, trailers and semi-trailers is reported as -8.92 (see Table 8 on p.18 of Fontagné *et al.* (2022)).

How does this parameter translate from an Armington model to a Melitz model? As in Armington, the elasticity σ is still the substitution elasticity between varieties in Melitz. But in Armington, varieties are unique to countries or regions, since goods are differentiated by country or region. In Melitz, each individual country will have many firms producing many different varieties of a single Melitz good.

Problems associated with the reconciliation of results between Armington and Melitz models have been identified in the literature. Balistreri and Rutherford (2013:1536) observed that "the Armington and Melitz models generate different marginal trade responses regardless of how we set the elasticities". As noted earlier, DJR argue that the Armington elasticity σ that produces the same impact on trade flows for a given tariff shock will need to be different from the Melitz substitution elasticity between varieties. For example, in their illustration of an Armington vs. Melitz specification of the "wearing apparel" (WAP) industry in GTAP (see Chapter 7 of DJR), for a given increase in the tariff on WAP imports into North America, an Armington elasticity of $\sigma = 7$ is needed to produce the same import demand response as a Melitz elasticity of $\sigma = 2.5$ and alpha = 2.5. It is worth perhaps comparing this (α, σ) choice with Figure 1, where we see that it does not lie within the admissible space. $\sigma = \alpha = 2.5$ implies $\beta = 1.84$; $\beta^{\sigma} = 4.61$; $v = 0.60$; and, $g/f = 0.60$. The admissible space can be expanded to accommodate $\sigma = \alpha = 2.5$ by lowering $\underline{v}$ to 0.60.

In pursuing this method of calibration, we have identified another problem. If the GTAP Armington elasticity is adopted for the motor vehicles industry, we find there is no way to specify the Melitz elasticity between varieties to satisfy the constraints on Melitz parameters.

If we adopt a value for the Melitz elasticity σ that is consistent with the admissible area that we identified earlier in Figure 1, the corresponding Armington elasticity for the *mvh* sector would need to be set between 9.7 and 19.6. Both the GTAP Armington elasticity of 5.6 and the trade elasticity for motor vehicles, trailers and semi-trailers of 8.9 in Fontagné *et al.* (2022) lie outside this range.

5 Concluding remarks

This paper has examined the relationship between Armington and Melitz trade specifications in a CGE setting, using the motor vehicle sector as a case study. Applying the parameter restrictions proposed by Dixon, Jerie and Rimmer (2018), we identify a range of Melitz parameterisations that imply empirically plausible cost structures and dispersions of productivity and exports. Within this restricted parameter space, the Melitz model produces substantially larger import responses to a U.S. tariff shock than those implied by a standard Armington specification in GTAP-FIN.

We find that, for the tariff experiment considered here, no parameterisation within the permissible and plausible set of Melitz parameters reproduces the Armington import response generated using conventional GTAP elasticities. This finding raises a doubt over the practicality of treating Armington outcomes as a calibration benchmark for Melitz models. This leads us to reverse the calibration exercise and identify the Armington elasticities required to replicate the range of import responses generated by Melitz models with parameterisations restricted to the permissible and plausible range. The implied Armington elasticities are markedly higher than standard values.

The interpretation of these findings remains open. We are hesitant to suggest that conventional Armington parameterisations may understate trade responsiveness in industries characterised by significant firm-level heterogeneity. Experiments with other industries, beyond motor vehicles, should be undertaken. Future work should also assess whether independent empirical estimates of trade elasticities support the higher Armington values implied by Melitz-consistent calibration, and examine whether similar divergences arise under alternative models of industry heterogeneity, such as small-group monopolistic competition.

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