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AN ECONOMIC-DEMOGRAPHIC MODEL OF AUSTRALIAN POPULATION, LABOUR FORCE AND HOUSEHOLDS

by

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The views expressed in this paper do not necessarily reflect the opinions of the participating agencies, nor of the Commonwealth government

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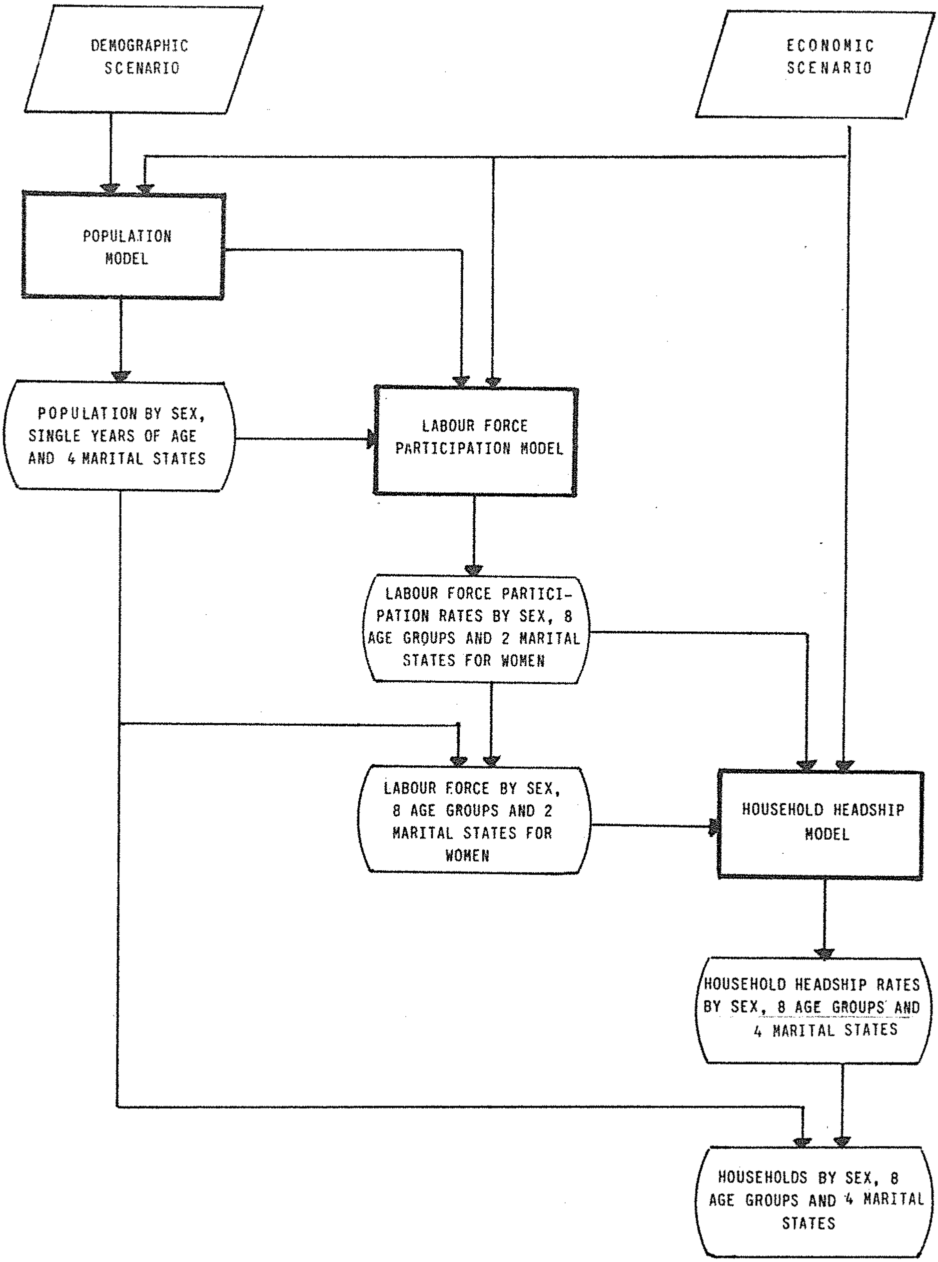
1. INTRODUCTION

Through the development of a suite of integrated economy-wide policy-analytic models, the IMPACT Project[1] has attempted to provide a systematic framework for the analysis of the impact of economic, demographic and social change on the structure of the Australian economy. One such model is BACHUROO, a large economic-demographic model of population and labour supply, which is intended to model the size and skill composition of the Australian labour force. Within BACHUROO, the Population Projection Facility has the task of tracking the evolution through time of the age, sex and marital status composition of the Australian population, labour force and number of households. The purpose of this paper is to provide a description of that Facility.

The Population Projection Facility produces conditional projections of the Australian population, labour force and number of households, subject to imposed demographic and economic scenarios. Unlike more conventional methods of producing demographic projections, the Facility explicitly incorporates the influence of economic and social factors upon demographic change, via the use of econometrically estimated models of the relationships between the economic and social environment and fertility, marriage, divorce, labour force participation and household formation behaviour. However, unlike most other economic-demographic models, which have adopted a crude approach to demographic interactions and have usually modelled only aggregate behaviour, the Facility strictly observes demographic accounting identities and maintains a high level of disaggregation in its projections.

A schematic representation of the Facility is given in Figure 1. On the basis of chosen demographic and economic scenarios, a set of projections of migration, fertility, marriage, divorce and mortality are derived, either by assumption or using econometric models. Projections of the population are then made using conventional cohort-component methods, in which the population in each year is updated by applying the projections of migration, fertility, marriage, divorce and mortality, in accordance with a strict set of demographic accounting identities. This population projection technique is essentially identical to that used by the Australian Bureau of Statistics[2], with the added innovations of using econometric models and of providing projections disaggregated not only by sex and single years of age but also by marital status. To produce labour force and household projections, rates of labour force participation and household formation for selected demographic subgroups are projected using econometric models and the chosen demographic and economic scenarios. These projected rates are then applied to the projected populations for each demographic subgroup. Given that rates of labour force participation and household headship differ substantially between marital states, the projection of population disaggregated by marital status is important in determining both the size and composition of the labour force and of households.

FIGURE 1 : SCHEMATIC REPRESENTATION OF THE IMPACT PROJECT'S FACILITY FOR THE PROJECTION OF AUSTRALIAN POPULATION, LABOUR FORCE AND HOUSEHOLDS



The philosophy motivating development of the IMPACT Population Projection Facility has been to build, for public use, an integrated model based on conventional demographic projection techniques, but with demographic behaviour controlled by econometric models which relate such behaviour to the economic and social environment. The principal benefit in using these econometric models is that they permit the generation of consistent projections of population, labour force and households, subject to chosen economic scenarios. The Facility is thus able to produce highly disaggregated projections of population, labour force and households, based on detailed representations of both demographic identities and econometrically estimated behavioural relationships. These projections are therefore mutually consistent and subject to a common economic scenario chosen by the user. To aid the user, the Facility has been encoded as a flexible suite of publicly-available computer programs.[3]

In the next section, the implementation of the Facility is discussed, although details are reserved for four Appendices. Section 3 provides information on the computing aspects of the Facility and the final section provides a summary.

2. THE IMPACT POPULATION PROJECTION FACILITY

The IMPACT Population Projection Facility is composed of three submodules, which deal with population, the labour force and household headship. The structure of the Facility is illustrated in Figure 2, and the variables included within it are listed in Appendix 1. In this section we begin with an overview of the structure and major features of the Facility, followed by more detailed discussion of the population projection algorithm, the treatment of marriage and divorce and of births, and the econometric models of fertility, marriage, divorce and female labour force participation, and of household formation.

2.0 An Overview

The population submodule provides projections of the Australian population by sex, age and marital status. The outputs of the population submodule are :

(a) the population by:

sex,

single years of age (0 to 100+),

and

four marital states; namely,

never married

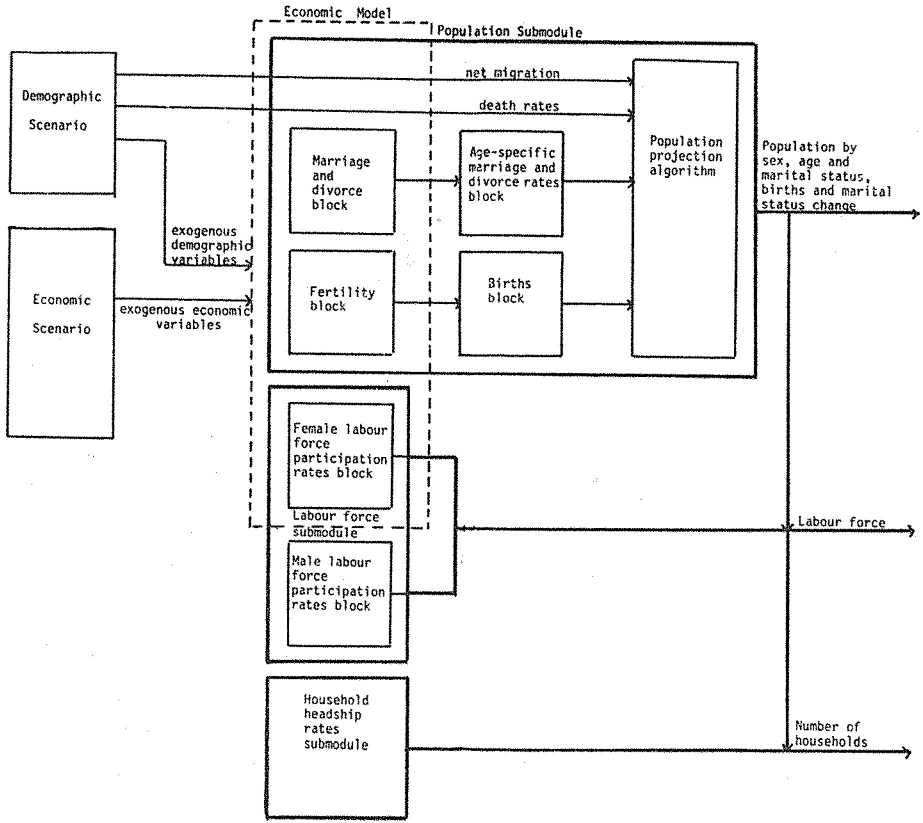
married,

divorced, and

widowed,

at the beginning of a given year;

FIGURE 2 : SCHEMATIC DIAGRAM OF THE IMPACT POPULATION PROJECTION FACILITY[1]



1. The economic model of fertility, marriage, divorce and female labour force participation is contained within the dashed lines.

(b) marital status flows by;

type of flow; namely

first marriages,

remarriages of divorcees,

remarriages of widow(er)s

divorces, and

widowings,

sex,

and

single years of age (15 to 100+)

within a given year;

(c) deaths by sex, age and marital status within a given year;

(d) net migration gain by sex, age and marital status within a given year;

and

(e) births by sex within a given year.

The labour force submodule provides projections of labour force participation rates (and, when combined with the population submodule, the labour force) by:

sex,

eight age groups; namely

15-19, 20-24, 25-34, 35-44, 45-54, 55-59, 60-64, and 65+,

and

two marital states for women; namely,

married,

and

not married.

The econometric model which forms the basis for the labour force submodule projects participation rates for women (but not men), by:

three age groups; namely,

15-24, 25-54, and 55+,

and

two marital states,

married,

and

not married,

for each given year. These are then mechanically disaggregated using historical ratios to provide projections for eight age groups. Projections of participation rates for males are supplied exogenously. It is hoped that a future development of the Facility will incorporate an econometric model of labour force participation rates directly disaggregated by eight age groups for men and women and also by two marital states for women.

The household headship submodule provides projections of household headship rates (and, when combined with the population submodule, the numbers of household heads) by:

sex,

eight age groups; namely,

15-19, 20-24, 25-34, 35-44, 45-54, 55-59, 60-64, and 65+,

and

four marital states; namely,

never married,

married,

divorced,

and

widowed.

As depicted in Figure 2, the population submodule consists of five blocks:

(i) the population projection algorithm,

(ii) a block to calculate age specific marriage and divorce rates,

(iii) a births block, and

(iv) two parts of the econometric model of fertility, marriage, divorce and female labour force participation (namely, the fertility block and the marriage and divorce block).

The population projection algorithm is implemented as a set of demographic accounting identities and is the central component of the Facility. It determines the composition of the population at the end of the period from:

(i) the initial composition of the population;

(ii) the age specific rates of first marriage, remarriage and divorce for both sexes;

(iii) the annual number of births by sex;

(iv) the net migration of men and women by sex, age, and marital status; and

(v) death rates by sex, age, and marital status.

Widowings of both men and women at each age are determined by applying the appropriate death rates to the age distribution of spouses.

It is this algorithm which enables the Facility to produce consistent population projections at the high level of disaggregation adopted. The algorithm uses the conventional demographic technique of applying age specific rates to appropriate populations at risk to allow the resultant population to be influenced by both changes in these rates and in the sex, age and marital status structure of the population. The algorithm also includes a two-sex marriage model which ensures consistency between the numbers of men and women marrying at each age and a two-sex divorce model which achieves the analogous result for divorces (see Sams (1981) for details). As well, the method adopted to calculate widowings of women (men) ensures that their number is consistent with the number of deaths of married men (women) in each year.

The high level of disaggregation maintained within the Projection Facility enables the projections to be used very flexibly, but it does impose an enormous information load. Whilst annual projections of the net numbers of international migrants are assumed to be supplied exogenously to the Facility, model schedules (that is, smooth approximations) of the age distributions of mortality and of marriages and divorces have been used to condense the required disaggregated annual projections of these demographic transitions into a few parameters for each transition in each year.

For marriages and divorces, the age specific rates of first marriage, divorce and remarriage of divorced and of widowed persons for each sex in each year are assumed to be adequately approximated by a smooth distribution (see Williams (1981) for details). Each of these eight model schedules in each year can then be fully characterized by three parameters. The model schedules for the age distributions of age specific death rates of persons of each sex and marital status in each year are based upon the distributions suggested by Heligman and Pollard (1979) (see Brooks, Sams and

Williams (1980) for details). Using these model schedules, the eight age distributions of age specific death rates in any year can each be characterized by nine or fewer parameters.

The use of these model schedules has the advantage of providing, for each demographic transition in each year, a manageable number of interpretable descriptive statistics. The time series of these statistics can capture changes in the underlying determinants of demographic transitions and thereby provide the basis for econometric estimation and projection (in the case of marriage and divorce) or allow straightforward projection of future trends (in the case of mortality).

The econometric model supplies a consistent description of a set of fertility parameters, of the parameters of the model schedules of the age specific rates of marriage and divorce for men and women, and of the labour force participation rates for women in selected demographic groups. It also models the relationship of these variables to each other and to economic and social conditions. The model is outlined briefly later in this paper and is described in detail in Brooks, Sams and Williams (1982).

In summary, the population submodule provides a framework for projecting the Australian population disaggregated by sex, age and marital status, where component flows are determined by an econometric model and are subject to economic and social influences. When combined with household headship rates and labour force participation rates (which are also determined by econometric models), the Facility as a whole provides projections of the number of households by the sex, age and marital status of the head, and the total labour force by sex, age and marital status (for women).

These outputs are then available for use;

(a) in their own right as inputs to policy discussions;
(b) as inputs to the other submodules of BACHUROO (in particular, to the submodules which determine the occupational breakdown of the labour force); and
(c) as inputs, via the BACHUROO Module as a whole, to other modules in the suite of IMPACT models.

2.1 The Population Projection Algorithm

The population projection algorithm, which is set out in more detail in Appendix 2, is straightforward; it relates the changes in the population between the beginning and the end of each year to all the vital flows which are projected to occur during that year. The algorithm maintains population stocks disaggregated by two sexes, 101 single years of age and four marital states (never married, married, divorced and widowed). The probabilities of transition from one marital state to another are modelled explicitly for each age and sex. This enables stocks to be updated through time by accounting for:

(i) changes in marital status occuring as a consequence of

first marriages,
remarriages of divorcees,
remarriages of widow(er)s,
divorces, and
widowings;

(ii) deaths of persons in each sex-age-marital status category;

(iii) net migration in each sex-age-marital status category;

and

(iv) births of each sex.

The econometric model provides estimates of the age distribution of the sex and age specific rates of first marriage, remarriage of divorcees, remarriage of widow(er)s, and divorce, in each projected year (see Appendix 3). When these rates are multiplied by the stocks of people at risk of each of these transitions, estimates are obtained (separately for males and females) of the net additions arising from marriage and divorce which must be made to stocks in each of the four marital states cross classified by single year of age. Since projections of the total numbers of men and women marrying (and divorcing) must agree, it is necessary to match these marital events which initially are estimated separately for each sex. This is achieved in the case of marriages by a procedure which takes as binding the constraint that the number of marriages cannot exceed the supply of partners available in each age and marital state.

Consider men and women who marry in any given year. From historical data are calculated initial estimates of the following (stationary) probability distributions at each age of wife and husband:

(a) the probability that a woman of given age who marries will marry a male of age i_m (i_m = 15, ..., 100+);

(b) the probability that a man of given age who marries will marry a female of age i_f (i_f = 15, ..., 100+).

Of course, many of these probabilities are very small; most of the probability density clusters around ±5 years of the age of the wife in (a) and of the husband in (b). In a given year, the total number of males of age i_m and of females of age i_f marrying, as we have seen, are projected using initial stocks and the age specific rates of marital transition projected by the econometric model. Applying the probability distributions (a) and (b), we obtain separate estimates of the number of males of age i_m who select a partner of age i_f and of females of age i_f who select a partner of age i_m (i_m, i_f = 15, ...,100+). These must be reconciled. This is done using a two-sex marriage model (see Sams (1981)), which involves averaging the two estimates of the number of couples in any female age-male age cell marrying in a given year as determined from the 'female' distribution (a) and the 'male' distribution (b). This averaging must take as binding the available supply of partners of each sex. In particular, account must be taken not only of age and sex, but of previous marital state. Any excess demand for spouses over the available supply is eliminated by distributing this excess across ages and marital states.

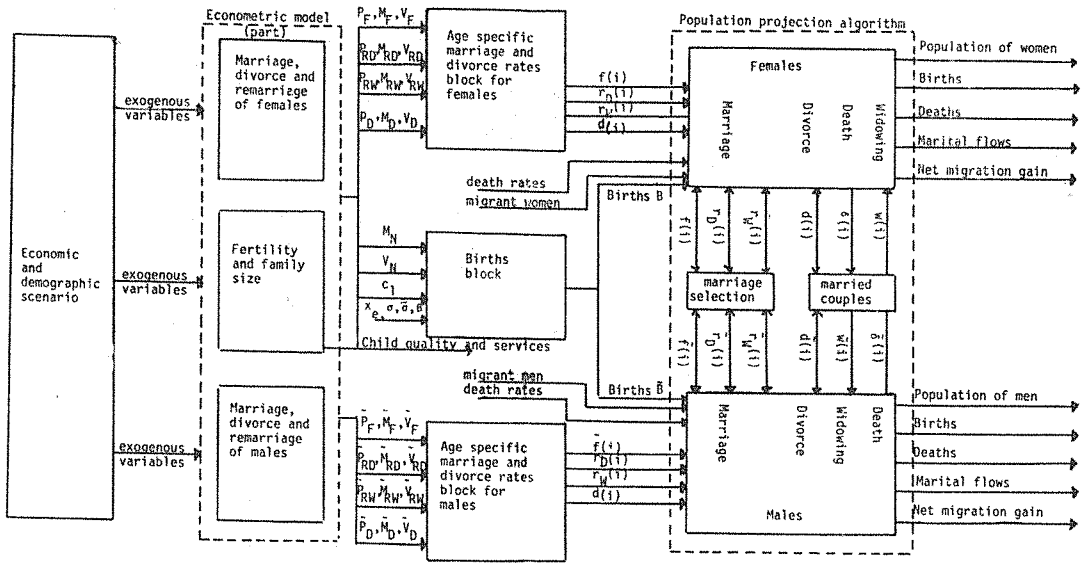
Having considered entry into marriage, we must now consider exit via divorces and widowings. To keep track of the number of persons at risk of these events, the algorithm maintains year-by-year estimates of the stocks of married couples, cross classified by the ages of husband and wife (the 'couples matrix'). Age specific divorce rates for each sex in any given year are supplied by the econometric model, and the number of divorces involving wives of age i_f and husbands of age i_m in that year is projected by multiplying the average of the divorce rates for each sex by the relevant element of the couples matrix. In the case of widowings, the couples matrix again supplies the population at risk. The probability of a woman (man) becoming widowed in a given year is taken to be the probability that a man of age equal to her (his) husband's (wife's) dies. These probabilities are independent of the age of the survivor. To calculate the probability that a representative woman (man) of age i_f (i_m) is widowed, the age-specific probabilities of male (female) death must be weighted by the age distribution, from the couples matrix, of husbands (wives) married to women (men) aged $i_f(i_m)$.

Apart from marital status transitions, stock accumulations are affected by net migration and mortality. Both migration and mortality are treated exogenously in the Facility. For mortality the schedule of sex-age-marital status specific death rates is required by the population projection algorithm, whereas for migration, sex-age-marital status specific projections of numbers of people are needed. The user of the Facility is free to specify his own scenario about changes in mortality rates and about prospective migration flows over time.

Finally, births enter the population projection algorithm at age zero and, with allowance for survival, progress to older ages over successive periods. Two options are available to the user in the projection of births. In the first, the user directly supplies age specific confinement rates disaggregated by nuptiality of mother and by birth order for nuptial confinements. In the second, the econometric model is used to supply this information (see Appendix 4).

The flows within the population projection algorithm are illustrated in Figure 3, in which the structure of, and flows in, the whole population submodule are shown in some detail. Notation used in this figure, and generally throughout the Facility, is listed in Appendix 1.

FIGURE 3 : MORE DETAILED VIEW OF INFORMATION FLOWS IN THE POPULATION SUBMODULE



2.2 Marriage and Divorce

Age specific first marriage, remarriage and divorce rates are calculated within the Facility by the age specific marriage and divorce rates block. In general, age specific rates of marriage and divorce are low at young ages, rise sharply with age to a peak and then decrease slowly with increasing age. It is not possible to econometrically model these rates of marriage and divorce for each single year of age, so a model schedule (that is, a smooth distribution across ages) is used as an approximation to the actual age specific rates. To illustrate the approach, age specific rates of first marriage for two typical years, together with the smooth approximation to their distribution, are shown in Figures 4 and 5 for women and men, respectively.

We are able to describe this smooth approximation to the age distribution of first marriage, remarriage or divorce in any one year by three parameters:

index of propensity to first marry, remarry, or divorce (that is, the area under the smooth approximation),

the mean of the smooth approximation to the age distribution of the age specific rates,

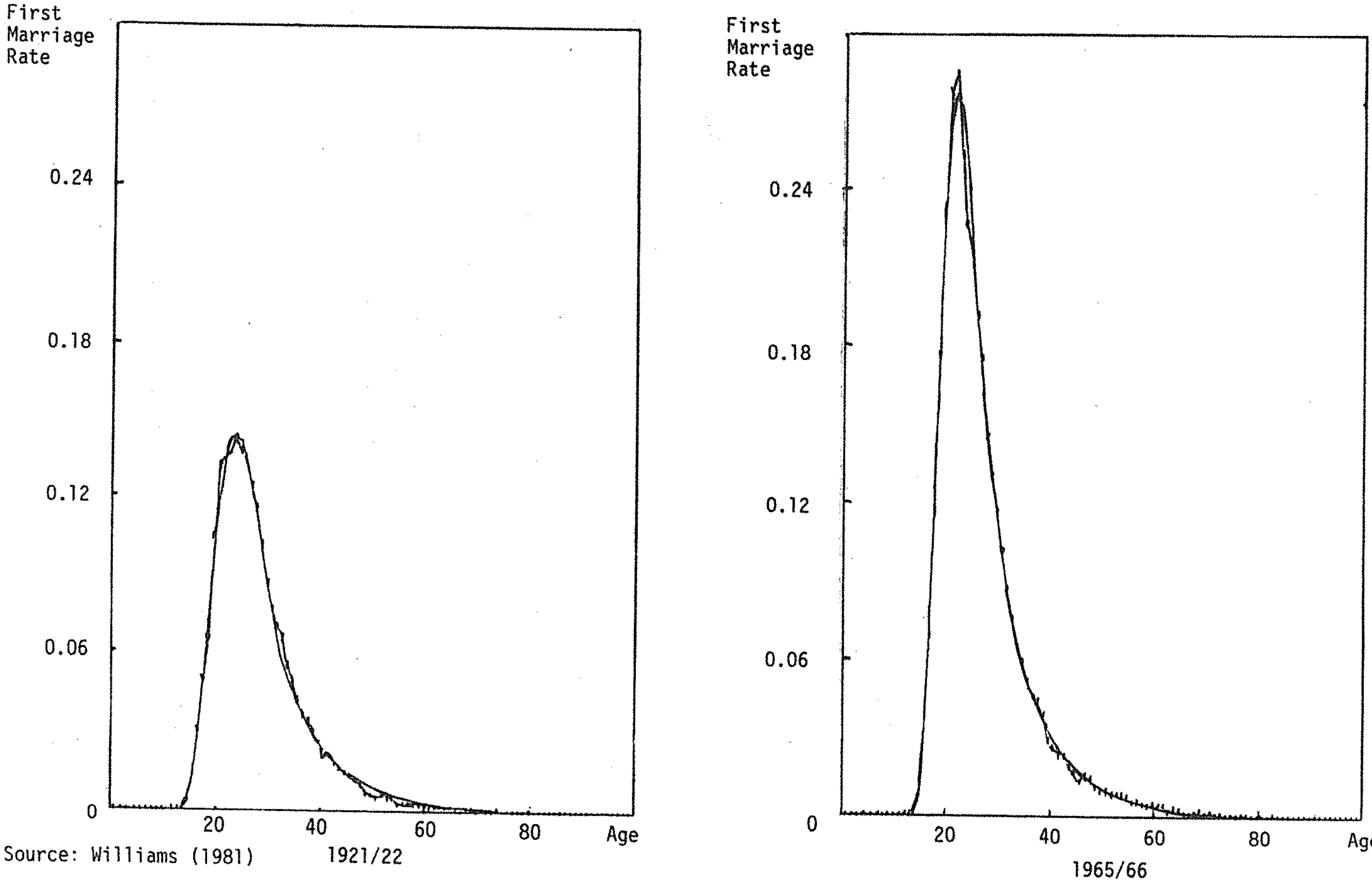
and

the variance of the smooth approximation to the age distribution of the age specific rates.

The first parameter can be thought of as providing a measure of the scale, the second a measure of the location, and the third a measure of

FIGURE 4

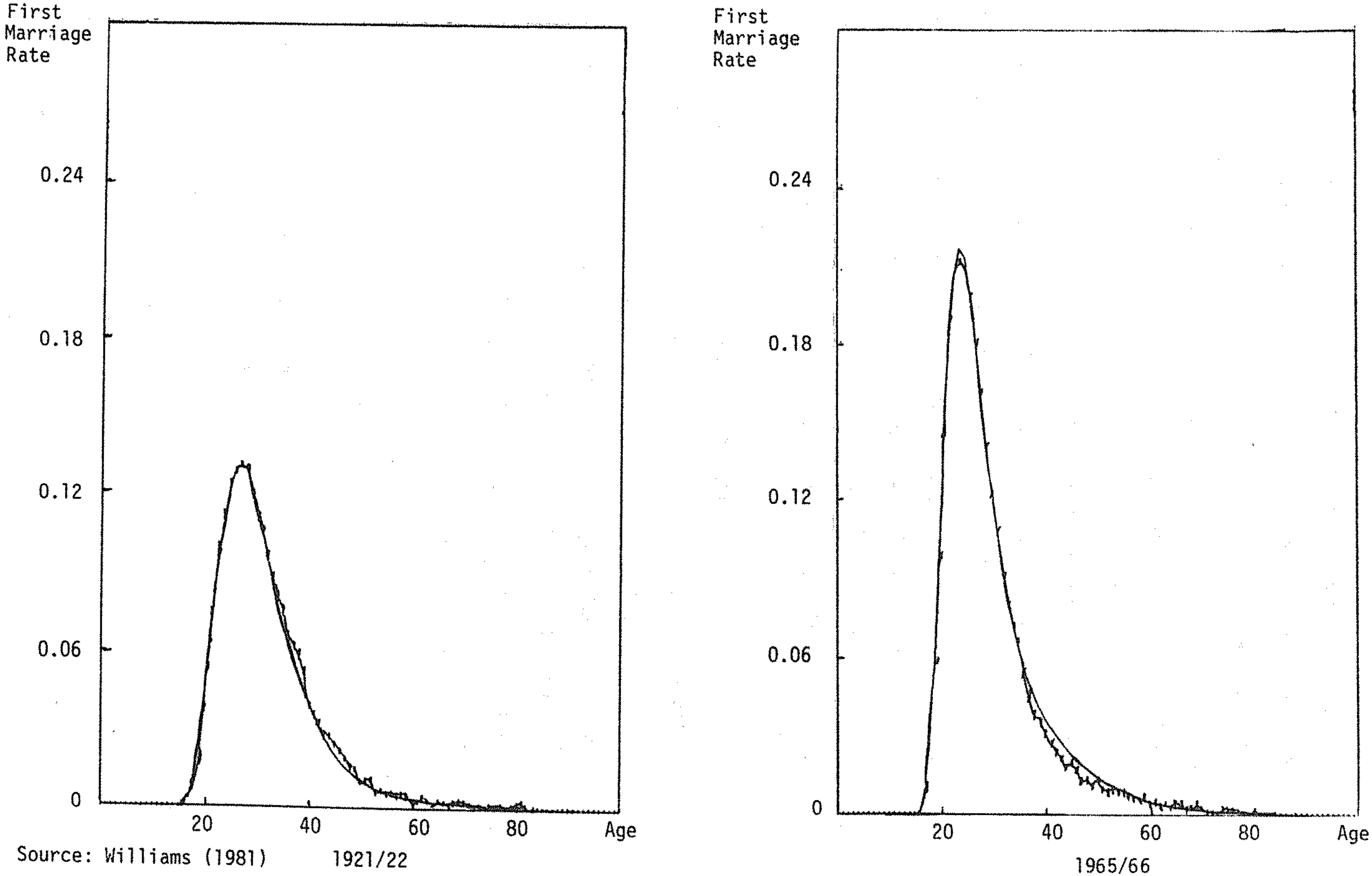
TYPICAL AGE DISTRIBUTIONS OF THE RATES OF FIRST MARRIAGE OF AUSTRALIAN FEMALES (++++) AND THEIR MODEL SCHEDULES (—), 1921/22 AND 1965/66



Source: Williams (1981)

FIGURE 5

TYPICAL AGE DISTRIBUTIONS OF THE RATES OF FIRST MARRIAGE OF AUSTRALIAN MALES (+++)
AND THEIR MODEL SCHEDULES (—), 1921/22 AND 1965/66



Source: Williams (1981)

the width, of the age distribution. The three parameters vary from year to year and capture movements in the level and shape of the age distribution. A fourth parameter, the location of the origin, is also required to determine the age distribution for each marital flow. This parameter, which is held stationary, was selected by observation. The smooth approximation used is a gamma distribution. Within the sample period (1921/22 to 1975/76), the parameters of each model schedule for each year were obtained by minimising the sum of the squares of the difference between the actual number of marriages (or divorces) at each age and that given by the model schedule, given the number of people "at risk" of marriage (or divorce) in each age group. As an example, the parameters for the period 1921/22 to 1975/76 for first marriage are illustrated in Figure 6. Further details are given in Williams (1981) and in Appendix 3.

The parameters of the model schedules for men and women are related to the economic and social environment by the econometric model. Thus the economic environment influences the projected parameters of the model schedules of age specific rates of marriage and divorce, and these rates in turn influence the marital status changes within the population projection algorithm and thereby the calculation of the succeeding populations of men and women.

2.3 Births

The births block of the population submodule supplies the number of births of each sex to the population projection algorithm on the basis of either exogenously determined confinement rates or the econometric model of fertility, marriage, divorce and female labour force participation.

FIGURE 6

THE PARAMETERS OF THE MODEL SCHEDULES OF AGE SPECIFIC RATES OF FIRST MARRIAGES FOR AUSTRALIAN FEMALES AND MALES, 1921/22 TO 1975/76

Source: Williams (1981)

FIGURE 6 a

INDEX OF PROPENSITY TO FIRST MARRY FOR AUSTRALIAN FEMALES (+++) AND MALES (—), 1921/22 TO 1975/76

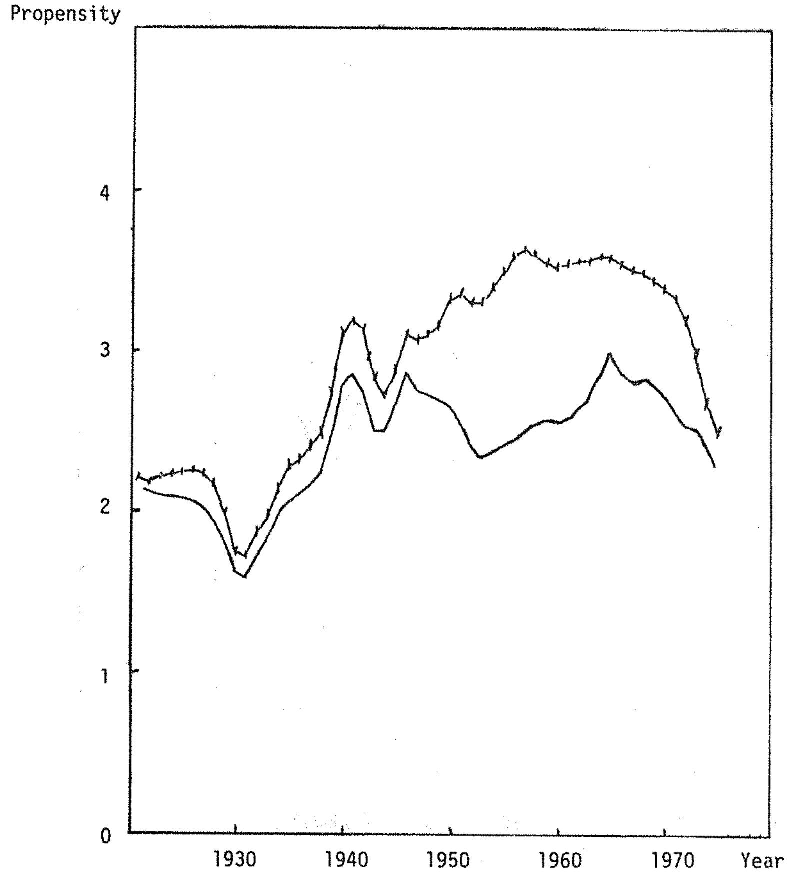


FIGURE 6b

MEAN AGE AT FIRST MARRIAGE FOR AUSTRALIAN FEMALES (+++) AND MALES (—), 1921/22 TO 1975/76

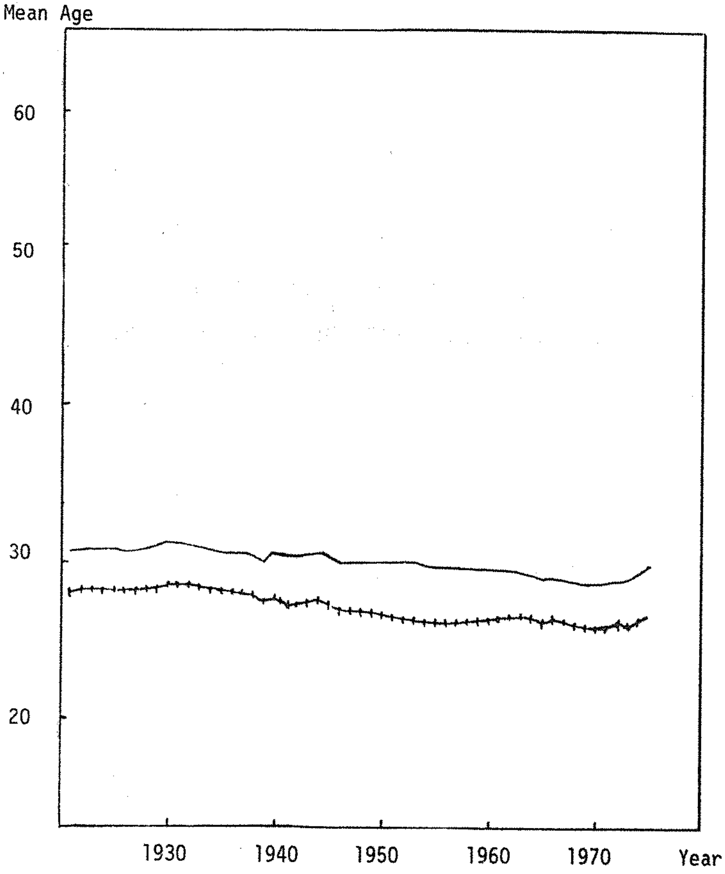
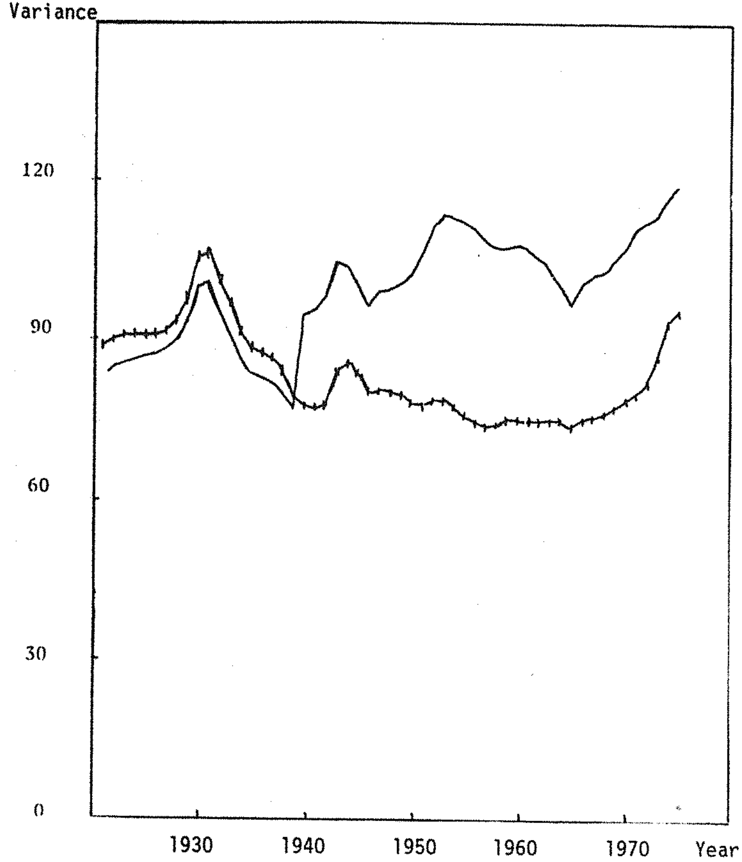


FIGURE 6c

VARIANCE OF AGE AT FIRST MARRIAGE FOR AUSTRALIAN FEMALES (+++) AND MALES (—), 1921/22 TO 1975/76



In the former case, the user specifies a set of age specific confinement rates for married and unmarried women. The nuptial confinement rates can be further disaggregated by birth order if desired. When applied to the appropriate populations of married and unmarried women at each age, these rates determine the number of confinements. In turn, the number of live births of each sex is determined by applying conversion factors to allow for multiple births and stillbirths and for the ratio of the sexes at birth.

In the latter case, fertility decisions are treated in terms of a sequential decision-making process, where the decision to have children (that is, to have a first nuptial confinement) is modelled separately from the decisions to have additional children. The econometric model supplies to the births block both the rate of first nuptial confinements and two measures of completed family size. From the former and the number of married women aged 15-44 years old, we can obtain the number of first confinements and from the latter we can derive parity progression ratios[4] for the probability of the second, third and higher order confinements. The family size distribution is approximated by a smooth distribution which is characterized by two parameters, called the "mean implied completed family size" and "the variance in implied completed family size". These time subscripted parameters are respectively the mean and the variance of the stationary distribution of family sizes that would occur if the parity progression ratios for the year in question were to hold indefinitely. The number of live nuptial births of each sex is determined as a constant multiple of the number of confinements. Ex-nuptial births are assumed to be proportional to the number of nuptial births, where that proportion is currently set exogenously. Further details of the births block are given in Appendix 4.

2.4 The Econometric Model of Fertility, Marriage, Divorce and Female Labour Force Participation

The econometric model of fertility, marriage, divorce and female labour force participation assumes that decisions concerning marriage, divorce, child bearing and labour force participation involve a strong component of economic rationality. For each year the econometric model endogenously determines for each sex the 12 parameters of the smooth approximating distributions for marriages and divorces, 3 parameters for fertility, a measure of child quality and the labour force participation rates for women in each of six groups (obtained by cross-classifying three age groups by two marital states, married and unmarried). All thirty-four equations are linear in the logarithms of the variables, with the exception of the female labour force participation rate equations in which the participation rates are transformed to ensure that they lie between zero and unity. The specification of the econometric model is detailed in Brooks, Sams and Williams (1982), as supplemented by Sams and Williams (1982a).[5]

The econometric model is inspired by the so-called "new home economics", which explores the relationships between economic variables and marriage, divorce, childbearing and labour force participation (see Becker (1960) and (1965), Lancaster (1966), and Willis (1974)). This approach (which is an extension of consumer theory to incorporate non-pecuniary aspects of consumption) treats the individual or, where appropriate, the family as a decision-making unit which maximises its utility from the consumption of "household commodities" subject to a household production function, a budget constraint and a time constraint. Thus the household is seen not only as a consumer but also as a producer; goods and services purchased in the market are combined with the time of the individual or family members to produce household commodities, which include children (where children are viewed as home-produced durable assets), home-cooked meals and the like. The inclusion of non-pecuniary aspects of household consumption, such as the utility derived from children and the role of time constraints, enables the "new home economics" to deal with demographic topics such as fertility, marriage, divorce and labour force participation.

With respect to fertility, children appear as an argument in the family utility function in the sense that parents, or would-be parents, are assumed to choose a certain volume of "child services", which is a function both of the number of children and the resource intensity or "quality" of these children. Thus, although children are not purchased in the market, inputs of market goods and services, and of time, are used by the household to "produce" child services. Children therefore have a shadow price, partly reflecting the time intensity of their production and the opportunity cost of that time. Thus, with regard to fertility, the members of a family are faced with a decision concerning the allocation of their resources of time between child-rearing, labour force participation and leisure. Since the value of the time of parents, especially of mothers, is a major cost of having and rearing children, the number of children will decline with the increasing cost of the mother's time, as measured by her wage rate in the labour force. (For further discussion of this, see Mincer (1963), Heckman (1974), and Butz and Ward (1979).) Now if, as we expect, child services are "normal goods", an increase in family income will tend to increase consumption of child services, which can imply growth in the number of children and/or in expenditure per child (that is, child quality). If an increase in family income derives from an increase in the female wage rate, the income effect will be combined with an increase in the shadow price of the woman's time, implying that a larger part of the increase in child services may be directed towards increased child quality, rather than increased numbers of children. The effect on fertility of non-economic variables, such as birth control and infant mortality rates, can also be incorporated via their effect upon the relative prices of the number and quality of children.

The "new home economics" approach has also been applied to marriage (Becker (1974)) and divorce (Becker, Landes and Michael (1977) and Hutchens (1979)) behaviour. People are assumed to marry when both parties expect to enjoy a level of utility which is greater than that attainable if they remained single. Gains from marriage are related to the complementarity of the inputs to the household of the husband and wife. This complementarity is higher for large relative wage differentials between men and women. Children provide an important source of utility to their parents, so the demand for child services, and the complementarity of inputs of males and females in producing these child services, will act as an incentive to marry and to remain married. However, the decision to marry is not costless, since a single person must spend resources searching for a spouse. According to this approach, the decision to marry, the timing of that decision and the duration of search will depend upon the gains to marriage and the costs of search (Keeley (1977) and (1979)). Since divorce and separation are the result of conscious choice on the part of at least one spouse to terminate the marriage, the reverse of the factors discussed above are assumed to apply.

The "new home economics" provides a consistent framework for dealing with female labour force participation; married females are assumed to choose to allocate their time between labour force participation, leisure and participation in household activity, whereas unmarried females are assumed to have only the former two choices open to them. Where the choice is between labour force participation and leisure, it is expected that the level of the female wage rate and the demand for female labour will act as positive inducements for women to join the labour force. The availability of alternative income, either from government transfer payments for unmarried women or from increases in the spouse's income for married women, will tend to reduce participation in the labour force. For married women of childbearing and rearing ages, the fertility decisions of earlier periods and the desired levels of child quality can also influence the level of participation in the labour force, and, in particular, rising levels of child quality can act as an inducement for married women to enter the labour force in order to supplement the family income.

In the econometric model, fertility is modelled in terms of the decision to have a first nuptial confinement and then to have higher order confinements. Higher order confinements are calculated from the mean and variance of implied completed family size, which are related to:

(i) the real female hourly wage rate;
(ii) real GDP per head;
(iii) the infant mortality rate;
(iv) the oral contraceptive usage rate;
(v) the real old age and invalid pension rate; and
(vi) two dummy variables to account for the effects of World War II.

First nuptial confinements are determined by an equation which relates the first nuptial confinement rate to the above variables plus an additional variable, the number of weighted first marriages per married woman. This variable is included to take account of the effect upon the first nuptial confinement rate of the timing decisions about children taken by recently married couples. In parallel with these equations which determine the number of children, the model also determines the desired level of child quality, which is related to:

(i) the real female hourly wage rate;
(ii) real GDP per head;
(iii) the infant mortality rate; and
(iv) the oral contraceptive usage rate.

The econometric model explains the probability of marital status change (that is, the propensity to marry, to divorce and to remarry) and the age profile of this marital status change (via the mean age and variance in age at marriage, at divorce and at remarriage) by relating the parameters of the model schedules of marriage and remarriage to:

(i) the demand for child services;
(ii) the female/male relative wage rate;
(iii) real GDP per head;
(iv) an index of female educational attainment;
(v) the oral contraceptive usage rate; and
(vi) two dummy variables to account for the effects of World War II.

The parameters of the model schedules of divorce are related to:

(i) the female/male relative wage rate;

(ii) real GDP per head;

(iii) the number of dependents per married female;

(iv) the real widows' pension; and

(v) a dummy variable for the effects of the Family Law Act.

Labour force participation rate equations are estimated for females disaggregated by two marital states (married and unmarried) and by three age groups (15 to 24 years, 25 to 54 years and 55 years and over). The labour market decisions of women will vary according to whether or not they are married and whether or not they are past child bearing and rearing ages. Consequently, the variables chosen to explain the labour force participation rates for married females 15 to 24 years and aged 25 to 54 years are:

(i) the total demand for child quality per married female;

(ii) weighted nuptial confinements per married female;

(iii) the oral contraceptive usage rate;

(vi) the real female hourly wage rate;

(v) an indicator of demand for female labour;

(vi) the unemployment rate of all persons; and

(vii) a dummy variable to account for the effects of World War II.

The variables chosen to explain the labour force participation rates for married females aged 55 years and over and for unmarried females aged 15 to 24 years, 25 to 54 years and 55 years and over are:

(i) the real female hourly wage rate;

(ii) an indicator of demand for female labour;

(iii) the unemployment rate of all persons;

(iv) the real old age and invalid pension;

(v) the real widows' pension;

(vi) a dummy variable to account for the effects of World War II;

(vii) a dummy variable to proxy for the financial assistance which was available to widows prior to the introduction of the widows' pension; and

(viii) the education participation rate of females aged 15 to 24 years, which appears only in the equation for unmarried females aged 15 to 24 years.

Unfortunately, at this stage the Population Projection Facility has no econometric model to determine male labour force participation rates, so they must be supplied exogenously (currently for eight age groups).

The thirty-four equations of the econometric model were estimated using annual time series data for the period 1921/22 to 1975/76, with the exception that the divorce equations were estimated using data from 1950/51 onwards. A full discussion of the performance of these equations is given in Brooks, Sams and Williams (1982); suffice it to say that the coefficients on the variables generally accorded with <u>a priori</u> expectations and that the model has been moderately successful in explaining Australian marriage, divorce, fertility and labour force participation over the period 1921/22 to 1975/76.

The variables which are exogenous to (strictly speaking, predetermined from the viewpoint of) the econometric model consist of three categories, those which are endogenous to some other block of the population submodule, those which are predetermined by the Facility as a whole at some earlier period of projection, and those which are fully exogenous to the Facility. As an example of the second category, consider the output 'marital flows', shown at the right of Figure 3. With a lag, these re-enter the Facility via the exogenous variables input to the econometric model, shown on the left of Figure 3.

2.5 The Econometric Model of Household Formation

The econometric model of household formation, which models household headship rates for sixty-four demographic groups, was estimated using data from the 1961, 1966, 1971 and 1976 Censuses (see Williams and Sams (1981)). The model has been able to explain successfully the evolution of household headship rates over the 1960's and 1970's using a set of economic variables which are thought to influence the decisions of people to form households.

It is assumed that the household headship rate for each demographic group is determined by the average expected income of that group. Each demographic group can receive income from five sources:

(i) wages, salaries and supplements,

(ii) the unemployment benefit,

(iii) government welfare payments, such as sickness and invalid pensions, the old age pension, supporting parent's benefit and the like, which are intended to replace labour income,

(iv) other government transfers such as child endowment, and health benefits, which are not intended to replace labour income, and

(v) unearned private income from dividends, interest and the like.

The contribution to average expected income from each of these income sources for each demographic group is determined by the average labour force attachment of each group and the state of the labour market, as encapsulated in the following variables for each group:

(i) the labour force participation rate,

(ii) the fraction of the year a typical person is in the labour force,

(iii) the unemployment rate, and

(iv) the average duration of unemployment.[6]

Average expected income is deflated by the housing and household equipment cost components of the CPI, rather than the CPI itself, since housing rather than general consumer prices are more relevant to household headship. The household formation model is designed to determine the level of demand for households and the effect of the supply of housing on household formation is only transmitted to the model to the degree that it affects the cost of occupying and maintaining a home.

Thus, the demand for household formation is modelled subject to the influence of a set of variables which determine the ability of members of each demographic group to form and maintain a household. The model suggests that, in the past, Australian household formation has been a normal good for all demographic groups, in the sense that increasing income has led to increases in household headship. However, the extent to which changes in income from any particular source lead to increased household headship for a given demographic group depends upon the importance of that income source for the group. The sensitivity of household headship to changes in average expected income has been the lowest for older age groups and the more traditional household heads, such as married males and widowed females, and greatest for young unmarried males. Young unmarried females do not respond as much as young males to changes in their expected income but they are more responsive than the traditional groups. Of all income sources, changes in wages and salaries have the greatest effect upon headship rates for all demographic groups. The exception is older people who are more greatly affected by changes in social welfare payments and unearned incomes, as these are more important sources of their income. Similarly, in the current situation of high youth unemployment, the headship rates of young unmarried persons, especially males, are sensitive to the rate of unemployment benefits.

Increases in the labour force participation rate of a demographic group increase the household headship rate for that group if wages remain the same, since this raises the average expected income of the group. The sensitivity of headship rates to changes in labour force participation is similar to that for average expected income, in that the headship rates of the traditional groups are little affected by changes in participation rates while those of young unmarried males are the most sensitive and those of young unmarried females have an intermediate response. As expected, increases in the unemployment rate and the duration of unemployment discourage household formation but the effect is quite small in both cases, except for young people.

Within the Facility, household projections follow on directly from projections of the population and labour force. Care has been taken to ensure that consistency is maintained between these projections. Thus, variables which are exogenous to the household formation model but are endogenous to other parts of the Facility (such as labour force participation rates and population in each demographic group), and exogenous variables which have been used elsewhere in the Facility (such as the total unemployment rate and real GDP per head) are directly input to the household formation model. This ensures that the projections of households, labour force and population are fully consistent with each other and with the underlying demographic and economic environment.

3. COMPUTING ASPECTS OF THE FACILITY

To facilitate its use as a tool for the analysis of Australian demographic, economic and social change, the IMPACT Population Projection Facility is implemented on the CYBER 76 computer of CSIRONET, a scientific computing service managed by the CSIRO's Division of Computing Research. The service is provided through a network of "nodes" throughout Australia and the Facility can be accessed by any authorised user from any CSIRONET node. In this section the computer implementation of the Facility will be described briefly; greater detail is given in Sams and Williams (1982b).

The Facility makes available to interested users:

(i) a set of comprehensive, consistent and easily accessible databases, in computer readable form, detailing important aspects of Australian demography and the economy (see Williams (1980) and Brooks (1981) for a discussion of the demographic and economic databases, respectively);

(ii) a suite of integrated computer programs which can be used to project and analyse Australian population, labour force and households; and

(iii) a set of econometrically estimated models, embodied within the Facility, which quantify the relationships between demographic, labour supply and household formation behaviour and the economic environment on the basis of Australian historical experience.

Whilst projections of the Australian population and, to a lesser extent, labour force and households, are available elsehwere,[7] the IMPACT Facility provides several important features not available with these projections. Firstly, the Facility is designed to be an analytical tool which can be used to produce conditional projections on the basis of user specified exogenous economic and demographic scenarios. The user is thus free to vary the assumptions incorporated in the exogenous scenarios and to test the sensitivity of projections to such variations. Secondly, the projections produced with the Facility are more disaggregated than in most other national projections and subject to control by a set of econometric models which enable the Facility to produce projections consistent with the underlying economic and demographic environment. As an analytical tool, the Facility can be used to improve our understanding of the interrelationships between economic and demographic behaviour and explicitly trace the links between economic and demographic change. Finally, the computer implementation of the Facility is designed to be used directly by demographers, economists and policy analysts with limited computing experience.

Although the Facility contains a large number of individual elements, such as programs and data files, all these elements are accessed via one interactive program run at a computer terminal using a simple set of commands. This program simplifies access to the Facility, by using a direct question and answer format to determine the details of any projections to be produced and to allow the user to analyse these projections. The Facility is supported by a set of computing manuals which provide detailed descriptions of the processes necessary to specify and generate projections.

4. SUMMARY

The IMPACT Population Projection Facility provides projections of the Australian population, labour force and households by sex, age and marital status within a tightly integrated framework which allows projections to be made on the basis of changing economic and social conditions. An econometric model is used to relate decisions concerning marriage, divorce, fertility and labour force participation to each other and to economic and social variables, while household formation behaviour is explained by a further econometric model.

The Facility makes use of the standard demographic technique of calculating demographic flows (such as deaths, confinements, marriages, divorces, labour force and households) by applying age-specific rates to the appropriate populations at risk. This approach enables the Facility to separate the effects of changes in behaviour from changes arising from the evolution of the demographic structure of the population. This simplifies the task of the econometric models of marriages, divorce, labour force participation and household headship by limiting their role to capturing behavioural changes and not those arising from mechanical changes in the population structure. The population projection algorithm within the Facility, which imposes a rigorous accounting discipline on the stocks and flows, enables the evolution of the demographic structure of the population to be reliably projected at a high level of disaggregation. A critical element in this integration of a highly disaggregated demographic algorithm with a set of econometric models is the use of module schedules to represent distributions across age or birth order. The econometric equations for fertility, marriage, remarriage and divorce project a small set of parameters which, using the model schedules, can determine disaggregated rates of fertility, marriage and divorce.

The role of the Facility is not to supply just a single forecast, or one best estimate, of the Australian population over some future period. Rather, its purpose is policy analytical. Not only does it provide specific population estimates for a range of settings of the exogenous variables, but it also provides insights into the mechanisms linking changing economic conditions to the future level and composition of the Australian population and its labour force participation and household formation. In designing the computer implementation, care has been taken to maintain flexibility so that the model can be used at various levels of integration both internally and within the BACHUROO model.

At this time, the Facility has been implemented in an initial form and has been made available for use by policy analysts outside the IMPACT Project. Several illustrative projections using the Facility have already been documented. Sams, Williams, Williams and Stevenson (1981) document a projection of the Australian population comparable to that provided by the Australian Bureau of Statistics (1980), thus illustrating the ability of the Facility to make projections using conventional demographic techniques. The value of extending these techniques to include econometric models is highlighted in various illustrative projections of Australian population and labour force produced with the full Facility (see Sams and Williams (1982a), (1983a) and (1983b)). The Facility's model of household formation has also been used to provide an illustrative projection of the number of households under varying scenarios of economic conditions and to analyse the sensitivity of household projections to changes in exogenous economic variables (see Williams and Sams (1982) for details).

The current version of the Population Projection Facility represents a first attempt to construct an integrated economic-demographic model, soundly based in demographic and economic theory. The Facility has several strengths:

Firstly, the population projection algorithm (including the two sex marriage model) provides consistent and integrated demographic projections at a high level of disaggregation.

Secondly, the use of model schedules for fertility, marriage, remarriage, divorce and death rates enables the large information requirements of the population projection algorithm to be supplied and controlled easily.

Thirdly, the econometric model of fertility, marriage, divorce and female labour force participation provides some insights into the relationship between demographic and labour force behaviour and the economic environment, although the success here is somewhat qualified.

Fourthly, the econometric model of household formation provides a detailed and tightly structured model which relates changes in the state of the labour market and the level of income from different sources to the expected income of each demographic group. Expected income is then used to determine the headship rate of the demographic group, and, when combined with the population projections, the number of households.

Fifthly, the computer implementation of the Facility provides a flexible, easy-to-use tool for the generation and analysis of projections and for the analysis of historical data.

However, the Facility has several weaknesses:

Firstly, the current treatment of fertility does not provide an integrated approach to economic-demographic modelling. The link between the econometric model and the population projection algorithm for fertility would be better achieved via the use of age and birth order specific confinement rates for married women and age specific confinement rates for unmarried women applied to the appropriate populations at risk by age, marital status and birth order. These age specific rates could be approximated by model schedules to enable the econometric model to control fertility behaviour via a small set of parameters.

Secondly, the econometric model of fertility, marriage, divorce and female labour force participation, needs improvement. The development of a more tightly integrated model of the relationships between the economic environment and demographic behaviour is essential. More explicit use of microeconomic theory, especially of constrained utility maximization, is required to improve the specification of the existing "new home economics" relationships in the model.

Thirdly, the current model of labour force participation is inadequate. It is essential to develop a behavioural model which systematically covers at least eight age groups for each of men, married women and unmarried women and provides a sound structural representation of the labour market choices faced by each demographic group.

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FOOTNOTES

1. The IMPACT Project is an inter-agency initiative of the Commonwealth Government in co-operation with the University of Melbourne and La Trobe University. For a full discussion of the IMPACT Project and the BACHUROO module see Powell (1977) and (1983).

2. For details, see Australian Bureau of Statistics (1982).

3. The IMPACT Population Projection Facility is a publicly available tool for the analysis of the effects of changes in the economic and demographic environment on the size and composition of the Australian population and labour force. A computer implementation of the model is available on the CSIRONET Computing System and can be accessed by those with appropriate computing facilities, expertise and finance. Anyone who wishes to run their own projections should contact the IMPACT Project.

4. Parity progression ratios are the probabilities that a woman with a given number of previous confinements will have at least one further confinement.

5. This model derives from an earlier model reported in Filmer and Silberberg (1977).

6. It may be helpful to consider income from each of these five sources as income rates. Weighting each income rate by the labour market attachment of each demographic group produces an expected income rate from each income source. Finally, applying these latter rates to the numbers in each group and summing over groups and income sources produces a total equal to total household disposable income.

7. For example, population projections are produced regularly by the Australian Bureau of Statistics (see Australian Bureau of Statistics (1982)), labour force projections have been produced by National Population Inquiry (1975) and household projections are produced regularly by the Indicative Planning Council (see Indicative Planning Council (1982)).

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APPENDIX 1 : NOTATION FOR THE IMPACT POPULATION PROJECTION FACILITY[1]

Submodule and Variable	Symbols	
	Females	Males
Population Submodule		
Population		
Number of persons of age i_f (females) or age i_m (males) of marital status k at the start of the year.	$X_k(i_f)$	$\tilde{X}_k(i_m)$
Alternative estimates of the number of married couples with wife of age i_f and husband of age i_m, both ages at the start of the year. When s=1, for each i_f, $\sum_{i_m} C_1(i_f,i_m) = X_2(i_f)$ serves as control total. When s=2, for each i_m $\sum_{i_f} C_2(i_f,i_m) = \tilde{X}_2(i_m)$ serves as control total.	$C_s = [C_s(i_f,i_m)]$	
'Phantom' number of married women (men) at the start of the year. [See Appendix 2, Section A2.4 for explanation.]	$X_2^P(i_f)$	$\tilde{X}_2^P(i_m)$
Migration		
Net migration during the year of persons of age i and marital status k at the start of the year.	$X_k^M(i)$	$\tilde{X}_k^M(i)$
Alternative estimates of the net migration of married couples with wife of age i_f and husband of age i_m, both ages at the start of the year. When s=1, for each i_f, $\sum_{i_m} C_1^M(i_f,i_m) = X_2^M(i_f)$ serves as control total. When s=2, for each i_m, $\sum_{i_f} C_2^M(i_f,i_m) = \tilde{X}_2^M(i_m)$ serves as control total.	$C_s^M = [C_s^M(i_f,i_m)]$	
'Phantom' net arrivals of married migrant women (men). [See Appendix 2, Section A2.4 for definitions.]	$X_2^{MP}(i_f)$	$\tilde{X}_2^{MP}(i_m)$

Submodule and Variable	Symbols	
	Females	Males
Deaths		
Number of people of age i and marital status k at the time of death who die within the year expressed as a proportion of those of age i and marital status k at that time.	$\delta_k(i)$	$\tilde{\delta}_k(i)$
Number of people of age i and marital status k at the start of the year who die within the year expressed as a proportion of those of age i and marital status k at the start of the year.	$d_k(i)$	$\tilde{d}_k(i)$
Marriage and Divorce		
Number of persons of age i and marital status ℓ at the start of the year who change to status k within the year.	$X_{k\ell}(i)$	$\tilde{X}_{k\ell}(i)$
Number of marriages between brides of age i_f and grooms of age i_m, both ages at the start of the year.	$M(i_f,i_m)$	
Alternative preliminary estimates of the number of marriages between women of age i_f and men of age i_m. [See Appendix 2, Section A2.5 for definitions.]	$M_1(i_f,i_m)$	$M_2(i_f,i_m)$
Number of divorces with wife of age i_f and husband of age i_m both ages at the start of the year.	$D(i_f,i_m)$	
Initial estimate of the number of divorces within the year involving a wife aged i_f and a husband aged i_m at the start of the year as estimated from the female (male) divorce probabilities $\omega(i_f,i_m)$ (or $\tilde{\omega}(i_f,i_m)$ for males).	$D_1(i_f,i_m)$	$D_2(i_f,i_m)$
Number of people of age i and marital status ℓ at the time of the event who change to status k within the year, expressed as a proportion of those of age i and marital status k at the time of the event.	$\alpha_{k\ell}(i)$	$\tilde{\alpha}_{k\ell}(i)$

Submodule and Variable	Symbols	
	Females	Males
<u>Marriage and Divorce</u> (Contd)		
Marital status transition probabilities. They may be interpreted as the probability that a person who is of age i and marital status ℓ at the beginning of the year will be in marital status k at the end of the year.	$a_{k\ell}(i)$	$\tilde{a}_{k\ell}(i)$
Alternative notation for $a_{k\ell}(i)$ and $\tilde{a}_{k\ell}(i)$; rates by age i at the time of the following events:		
first marriage,	$f(i)\equiv a_{21}(i)$	$\tilde{f}(i)\equiv\tilde{a}_{21}(i)$
remarriage of divorcees,	$r_D(i)\equiv a_{23}(i)$	$\tilde{r}_D(i)\equiv\tilde{a}_{23}(i)$
remarriage of widow(er)s,	$r_W(i)\equiv a_{24}(i)$	$\tilde{r}_W(i)\equiv\tilde{a}_{24}(i)$
divorce,	$d(i)\equiv a_{32}(i)$	$\tilde{d}(i)\equiv\tilde{a}_{32}(i)$
widowing.	$w(i)\equiv a_{42}(i)$	$\tilde{w}(i)\equiv\tilde{a}_{42}(i)$
Of women (men) who are marrying at age $i_f(i_m)$, the proportion who marry a man (woman) of age $i_m(i_f)$, <u>both ages at the time of the event</u>.	$\gamma(i_f,i_m)$	$\tilde{\gamma}(i_f,i_m)$
The probability that a woman who is in marital status ℓ at the beginning of the year and who (if she marries) is aged i_f at the time of marriage, within that year marries a man aged i_m.	$\epsilon_\ell(i_f,i_m)$	
Male counterpart of $\epsilon_\ell(i_f,i_m)$.		$\tilde{\epsilon}_\ell(i_f,i_m)$
The probability that a woman who is in marital status ℓ at the start of the year and who (if she marries) is aged i_f at the time of marriage, within that year marries a man who is aged i_m at the start of the year.	$\eta_\ell(i_f,i_m)$	
Male counterpart of $\eta_\ell(i_f,i_m)$.		$\tilde{\eta}_\ell(i_f,i_m)$
The probability that a woman (man) who is of marital status ℓ and age $i_f(i_m)$ at the start of the year, marries a man (woman) whose age is i_m (i_f) at the start of the year.	$\nu_\ell(i_f,i_m)$	$\tilde{\nu}_\ell(i_f,i_m)$

Submodule and Variable	Symbols	
	Females	Males
Marriage and Divorce (Contd)		
Probability that a married woman divorces within a given year, and that her age at the time of the event is i_f and the age of her spouse at the start of the year is i_m.	$\psi(i_f, i_m)$	
Male counterpart of $\psi(i_f, i_m)$.		$\tilde{\psi}(i_f, i_m)$
Probability that a married woman who is age i_f at the start of the year divorces within the year, a man aged i_m at the start of the year.	$\chi(i_f, i_m)$	
Male counterpart of $\chi(i_f, i_m)$.		$\tilde{\chi}(i_f, i_m)$
Fertility		
Estimate of the number of married females of age i_f in the middle of the financial year; i.e. at end of December	$X_2^D(i_f)$	
Proportion of married women of age i at the time of the event who have an n^{th} order nuptial confinement within the year.	$\rho_n(i)$	
Proportion of unmarried women of age i at the time of the event who have an ex-nuptial confinement within the year.	$\rho^e(i)$	
The probability that a woman, given that she has her $(n+1)^{th}$ confinement in t, will do so at an elapsed time τ since her n^{th} confinement.	$\rho_{n,t,\tau}$	
Parity progression ratio; i.e., the probability that a woman who has had an n^{th} order confinement will proceed during her lifetime to an $(n+1)^{th}$ order confinement.	p_n	

Submodule and Variable	Symbols	
	Females	Males
<u>Fertility</u> (Contd)		
The total number of nuptial confinements within the year.	$C.$	
The number of nuptial confinements of order n within the year.	C_n	
The number of ex-nuptial confinements within the year.	C^e	
The ratio of ex-nuptial to nuptial confinements within the year.	x^e	
The separation factor for infant deaths.	π	$\tilde{\pi}$
The average number of live births per confinement, allowing for stillbirths and multiple births within the year.	β	$\tilde{\beta}$
The proportion of births which are female (male) within the year where $\sigma + \tilde{\sigma} = 1.0$.	σ	$\tilde{\sigma}$
The number of births within the year.	B	$\tilde{B}$
Labour Force Submodule		
Labour force participation rates by age i and marital status p.	$\ell_p(i)$	$\tilde{\ell}_p(i)$
Labour force by age i and marital status p.	$L_p(i)$	$\tilde{L}_p(i)$
Household Submodule		
Household Headship rates by age i and marital status k of head.	$h_k(i)$	$\tilde{h}_k(i)$
Households by age i and marital status k of head.	$H_k(i)$	$\tilde{H}_k(i)$

Submodule and Variable	Symbols	
	Females	Males
Econometric Model of Fertility, Marriage, Divorce and Female Labour Force Participation		
Endogenous Variables		
Fertility		
Mean implied completed family size.	M_N	
Variance of implied completed family size.	V_N	
Child quality proxy.	Q	
First nuptial confinements per thousand married females aged 15 to 44 years.	c_1	
Marriages		
Index of propensity		
to first marry,	P_F	$\tilde{P}_F$
for divorcees to remarry,	P_{RD}	$\tilde{P}_{RD}$
for widow(er)s to remarry.	P_{RW}	$\tilde{P}_{RW}$
Mean of the age distribution of age specific rates of:		
first marriage,	M_F	$\tilde{M}_F$
remarriage of divorcees,	M_{RD}	$\tilde{M}_{RD}$
remarriage of widow(er)s.	M_{RW}	$\tilde{M}_{RW}$
Variance of the age distribution of age specific rates of:		
first marriage,	V_F	$\tilde{V}_F$
remarriage of divorcees,	V_{RD}	$\tilde{V}_{RD}$
remarriage of widow(er)s.	V_{RW}	$\tilde{V}_{RW}$
Divorces		
Index of propensity to divorce.	P_D	$\tilde{P}_D$
Mean of the age distribution of age specific rates of divorce.	M_D	$\tilde{M}_D$
Variance of the age distribution of age specific rates of divorce.	V_D	$\tilde{V}_D$

Submodule and Variable	Symbols	
	Females	Males
Female Labour Force Participation Rates:		
Married females aged 15-24 years.	$\ell_{m,15-24}$	
Married females aged 25-54 years.	$\ell_{m,25-54}$	
Married females aged 55 years and over.	$\ell_{m,55+}$	
Unmarried females aged 15-24 years.	$\ell_{u,15-24}$	
Unmarried females aged 25-54 years.	$\ell_{u,25-54}$	
Unmarried females aged 55 years and over.	$\ell_{u,55+}$	
Exogenous Variables		
Weighted first marriages of females per thousand married females aged 15 to 44 years.	$\hat{f}$	
Weighted nuptial confinements per thousand married females aged 15 to 44 years.	$\hat{c}$	
Children under 15 years per married female 15 years and over.	K	
The oral contraceptive usage rate.	Ω	
The infant mortality rate.	ϕ	$\tilde{\phi}$
Real gross domestic product per head.	Y	
The real female hourly wage rate.	W	
The female/male relative hourly wage rate.	$W/\tilde{W}$	
The real old age and invalid pension.	G_a	
The real widows' pension.	G_W	
The total unemployment rate.	u	
An indicator of demand for female labour.	L_D	
An indicator of female educational attainment.	E	
The education participation rate of other females aged 15 to 24 years.	$e_{u,15-24}$	

Submodule and Variable	Symbols	
	Females	Males
A dummy variable for World War II.	Z_2	
A dummy variable for the immediate post-war period.	Z_1	
A dummy variable for conscription.	Z_3	
A dummy variable for the Family Law Act.	Z_4	
A dummy variable for the widows' pension.	Z_5	
The Econometric Model of Household Formation		
Endogenous Variables		
Household headship rates by age i and marital status k.	$h_k(i)$	$\tilde{h}_k(i)$
Exogenous Variables		
Labour force participation rates by age i and marital status k.	$\ell_k(i)$	$\tilde{\ell}_k(i)$
Unemployment rates by age i and marital status k	$u_k(i)$	$\tilde{u}_k(i)$
Average proportion of year spent in labour force by age i and marital status k.	$\lambda_k(i)$	$\tilde{\lambda}_k(i)$
Average duration of unemployment by age i and marital status k.	$\mu_k(i)$	$\tilde{\mu}_k(i)$
Income from source j by age i and marital status k of recipient.	$I_{kj}(i)$	$\tilde{I}_{kj}(i)$

1. Note that, for notational simplicity, the time argument has been suppressed for all variables. i, i_f and i_m run over 101 single years of age: 0,1,..., 99, 100+. k and ℓ range over 1,2,3,4, indicating respectively the following marital states: never married, now married, divorced and widowed. j runs over 1,...,5, indicating the five income sources set out on p.31 of the text.

APPENDIX 2 : THE EQUATIONS OF THE POPULATION PROJECTION ALGORITHM

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APPENDIX 2 : THE EQUATIONS OF THE POPULATION PROJECTION ALGORITHM

A2.0 Introduction

The equations of the population projection algorithm allow the size and composition of the population to be adjusted from the beginning of one year of projection to the next. These equations constitute a system of linear difference equations whose general form is described in section A2.1. While this presentation is convenient for describing the basic idea behind the method, its details are better handled in terms of the specific flows and the adjustment of individual stocks.

In order to obtain the stock of persons of age i+1, sex s, and marital status k at the beginning of year t+1, the demographic flows over the year are added to (or subtracted from) the stock one year earlier of persons aged i of sex s in this status. These flows are mortality, migration, and marital transitions between the four marital status categories (k=1,...,4) listed on page 5 of the text. Each of these adjustment terms is a flow. The complete set of such flows can be subsumed under the headings:

1. Mortality,
2. Migration,
3. Marriages,
4. Divorces,
5. Widowings, and
6. Births.

These are discussed in order in sections A2.3 through A2.8. First, however, in section A2.2, the timing of events on two different definitions is reconciled. This is required because the form in which vital statistics are collected is not suitable for direct application to the stocks of persons recorded in the Australian Demographic Databank and used by the projection algorithm. The Appendix concludes with a description of the stock adjustment process and a consolidated list of the equations of the algorithm.

A2.1 Basic Difference Equations

The recursion equations used to track the male and female populations through time have the same form. Thus it will suffice here to give details of the equations for females. The equations specifying the annual adjustments to the population of women can be written in matrix notation as:

$$\text{(A2.1)}\quad X(i+1,t+1) = (I - D(i,t))\,(A(i,t)X^*(i,t) + \tfrac{1}{2}X^M(i,t)) + \tfrac{1}{2}X^M(i,t),$$

where

$$X(i,t) = (X_1(i,t), X_2(i,t), X_3(i,t), X_4(i,t))^T$$

is an age specific column vector whose components are:

(i) the number of never married women of age i at the start of year t, $X_1(i,t)$;

(ii) the number of married women of age i at the start of year t, $X_2(i,t)$;

(iii) the number of divorced women of age i at the start of year t, $X_3(i,t)$;

(iv) the number of widows of age i at the start of year t, $X_4(i,t)$.

The meaning of the asterix superscript on $X(i,t)$ on the right hand side of equation (A2.1) is explained below. For the time being it will do no harm to ignore the asterisk when reading (A2.1).

$X^M(i,t)$ is an age specific vector whose components are the net migration during year t of women of each of the four marital status categories and age i at the start of that year. If migrants arrive uniformly during the year, it can be assumed that their risk of death within Australia is equal to half that of the resident population. Hence death rates are applied to only half the migrant population.

The matrix $D(i,t)$ is a diagonal matrix whose elements $d_k(i,t)$ are the age and marital status specific annual death rates of a woman of age i at the end of year t. I is the identity matrix. The term $(I - D(i,t))$ is thus the proportion of women of each age and marital status at the beginning of t surviving until the end of that year. Hence equation (A2.1) represents a two-stage treatment of demographic change: the changes in marital status are first dealt with independently of death, and then survival rates are applied according to marital status at the end of the year.

The matrix A is a 4x4 matrix of transfer coefficients, where each element $a_{k\ell}(i,t)$ is the probability that a woman who is of marital status ℓ and age i at the start of year t is of marital status k at the end of that year. The indexes k and ℓ each range from 1 through 4 indicating, respectively, never married, married, divorced and widowed. For example, the element $a_{21}(i,t)$ is the probability that a never married ($\ell = 1$) woman of age i at the start of t will first marry in that year and end the year married (k=2). Further examples of the use of this notation are:

$a_{23}(i,t)$ = the probability of remarriage of a divorcee,

$a_{24}(i,t)$ = the probability of remarriage of a widow,

$a_{32}(i,t)$ = the probability of divorce,

and

$a_{42}(i,t)$ = the probability of being widowed.

The diagonal elements, a_{11}, a_{22}, a_{33} and a_{44}, are the marital status specific probabilities of not changing status within the period.

Several elements in the transition matrix **A** are identically zero, since we assume that, in any one year, a person will experience only one marital status change. For example, we have excluded the possibility that someone might marry and divorce in the same year, so the probability of transferring from never married to divorced, $a_{31}(i,t)$, is zero. However, there is one exception; new divorcees are permitted to remarry in the same year. To allow for this, the vector $\mathbf{X}^*(i,t)$ is defined as being equal to $\mathbf{X}(i,t)$ except that:

(A2.2) $$X_3^*(i,t) = X_3(i,t) + \tfrac{1}{2}X_{32}(i,t) ,$$

where $X_{32}(i,t)$ is the number of divorces during period t of women aged i at the start of the year. Since marital status change and death are treated as independent, it is possible, although unlikely, that someone will marry and be widowed in the same year. The calculation of these non-zero probabilities is necessary to maintain the strict identity between the number of married men who die within a year and the number of widows created within that year.

The probabilities of first marriage, remarriage of divorcees and of widow(er)s, and divorce are supplied explicitly by the age specific marriage and divorce rates block of the population submodule, and are specifically excluded before age 15. For a woman of a given age, the probability of being widowed is calculated, as indicated above, from the probability of her husband dying. To achieve this, the population projection algorithm maintains a "couples matrix" containing the numbers of married couples in which the wife is of one age and the husband of the same or another age. Thus, the probability of a randomly selected woman of age i being widowed is calculated from the proportional representation of men of each age married to women of age i and the age specific death rates applicable to these men.

Men are treated identically to women in the population projection algorithm, with variables for men being written with a tilde overmarking. Thus, $\tilde{X}(i,t)$ is the four-component vector of men of age i at the start of period t. Widowings of males parallel those of females and are calculated from the age distribution of wives and their age specific death rates. Thus widowings of both men and women are determined via the "couples matrix" and the probability of loss of the partner.

Since the number of marriages of men and women must be equal in any year, the number of marriages of each sex are reconciled via a two sex marriage model algorithm. The principal elements of this are:

(i) a "marriage selection" rule for women which defines, for women marrying at each age, the preferred age distributions of grooms. These age distributions are currently assumed to be stationary. Given these distributions and the number of female marriages at each age, the desired number of unions of brides and grooms by the ages of each partner can be calculated according to the female preference;

(ii) an analogous "marriage selection" rule for men, which enables the desired number of pairings of brides and grooms by the age of each partner to be calculated according to the male preference;

(iii) an initial estimate of the number of marriages by the age of each partner which is calculated as the arithmetic mean of the female and male preferences;

(iv) two feasibility requirements, which ensure that there are sufficient unmarried men and women of each age to satisfy the estimated number of marriages of males and females of each age;

(v) a revision rule which is invoked whenever either or both of the feasibility requirements are not satisfied. Under this rule any estimated number of marriages resulting in an infeasibility is decreased and the surplus redistributed to other age combinations until the constraints are satisfied.

Full details of the operations of the two-sex marriage model are given in Sams (1981).

Finally, births enter the population projection algorithm at age zero and, with allowance for survival, progress to older ages over successive periods.

A2.2 Timing of Events

The distinction between age at the start of the year and age at the time of the transition is important. Within the demographic accounting equations, population stocks and mortality, marital status and migration flows are all specific to the age of the person at the beginning of the projected year, whereas marriage, divorce and death rates, and the marriage selection rules, are specific to the age(s) of the person(s) at the time of the event. The necessity for this distinction can be illustrated by the case of first marriage. The vital statistics for first marriages are collected by age at the time of marriage, whereas the population of never married females is measured at a particular time, which we can define as the beginning of the year. By $\alpha_{21}(i,t)$ we denote the probability that a never married woman will first marry during year t, i being her age at the time of marriage. Now a woman who is aged i at the beginning of year t and who marries during that year may be aged i or (i+1) at the time of the marriage. Hence we specify $a_{21}(i,t)$, the probability of first marriage within year t of a never married woman who is of age i at the start of that year, to be half the sum of the probabilities of first marriage in t of never married women of ages i and (i+1) at the time of the event. More generally, we write:

(A2.3) $$a_{k\ell}(i,t) = \tfrac{1}{2}(\alpha_{k\ell}(i,t) + \alpha_{k\ell}(i+1,t)) \qquad (k,\ell = 1,2,3,4) \quad ,$$

where $\alpha_{k\ell}(i,t)$ is the probability that a woman makes a marital transition from status ℓ to status k within year t, i being her age at the time of the event. Similar considerations apply in the case of deaths, where:

(A2.4) $$d_k(i,t) = \tfrac{1}{2}(\delta_k(i,t) + \delta_k(i+1,t)) \quad ,$$

in which the left hand side is the k^{th} element of the diagonal matrix $D(i,t)$, and $\delta_k(i,t)$ is the mortality rate of women during year t who are of age i and marital status k at the time of death. Further timing corrections of this genre apply to widowings, divorces, and confinements. These are discussed in later sections.

A2.3 Deaths

Mortality rates for each year of projection are required to be supplied to the projection algorithm on a sex, age, and marital status specific basis. $d_k(i,t)$ and $\tilde{d}_k(i,t)$, respectively, denote annual mortality rates for females and males of marital status k and age i. As is clear from equation (A2.1), deaths are calculated by multiplying these rates by the corresponding numbers of persons at risk. In this and the following sections of this Appendix, we henceforth suppress the explicit time argument in the projection variables.

During their first year of life, the exact age specific mortality rates of children (expressed as a function of age in days or weeks) declines rapidly with age. This results in a sizeable difference between the following two mortality rates:

(a) the probability of dying within one year of birth, $\delta_1(0,t)$,

and

(b) the probability of dying within the (arbitrarily dated) year in which a baby is born, $d_1(-1,t)$.

The ratio of (b) to (a) is called the <u>separation factor</u> and is denoted by π for girls and $\tilde{\pi}$ for boys. Its use is explained below in section A2.7.

A2.4 Migration (and Keeping Track of Couples)

The projection algorithm, and the Facility as a whole, requires age, sex and marital status specific projections of net migration to be supplied exogenously. A complication arises because the net number of married migrant men arriving in any one year does not equal the net number of married migrant women arriving in the same year. To handle this we construct phantom populations of married men and women such that the number of married migrant women entering Australia in any year equals the number of phantom married migrant men entering Australia in the same year. Similar populations of phantom married migrant women are constructed for each year such that they balance the numbers of married migrant men arriving. Suppressing the time argument t (which does not change in this discussion), we define two married migrant couples matrices,

(A2.5) $$C_1^M = [C_1^M(i_f,i_m)] \text{ such that } \sum_{i_m} C_1^M(i_f,i_m) = X_2^M(i_f) \quad ,$$

and

(A2.6) $$C_2^M = [C_2^M(i_f,i_m)] \text{ such that } \sum_{i_f} C_2^M(i_f,i_m) = \tilde{X}_2^M(i_m) \quad ;$$

and, as well, two resident married couples matrices,

(A2.7) $$C_1 = [C_1(i_f,i_m)] \text{ such that } \sum_{i_m} C_1(i_f,i_m) = X_2(i_f) \quad ,$$

and

(A2.8) $$C_2 = [C_2(i_f,i_m)] \text{ such that } \sum_{i_f} C_2(i_f,i_m) = \tilde{X}_2(i_m) \quad .$$

Notice that (A2.5) and (A2.6) represent flows within year t, whereas (A2.7) and (A2.8) represent stocks at the start of year t. We note that the net phantom populations of married migrant women and married migrant men arriving in t are:

(A2.9) $$X_2^{MP}(i_f) = \sum_{i_m} C_2^M(i_f, i_m)$$

and

(A2.10) $$\tilde{X}_2^{MP}(i_m) = \sum_{i_f} C_1^M(i_f, i_m) \quad .$$

The differences between C_1 and C_2, and between C_1^M and C_2^M, in principle come from two sources: (a) reporting errors and (b) the existence of marriages in which spouses are located in different countries. We ignore (a) in our accounting equations. In the case of migrant couples, (b) is often a temporary circumstance. In particular, the arrivals of migrant wives tend to lag that of their husbands. Thus, equations (A2.7) and (A2.8) define, respectively, the numbers of married women and men in a way which in principle takes into account the fact that Australia is not a closed demographic system. More importantly, use of the couples matrix C_1 controls the total number of married women in line with the value of this variable in the database, while use of C_2 similarly controls the total number of married men.

The couples matrices C_1 and C_2 are updated as follows (where a prime indicates the start of (t+1), and its absence indicates the start of t):

(A.11) $$C_s'(i_f+1, i_m+1) = \left(C_s(i_f, i_m) + M(i_f, i_m) - D(i_f, i_m) + \tfrac{1}{2}C_s^M(i_f, i_m)\right)$$

$$\times \left[1 - \tfrac{1}{2}\left(\delta_2(i_f) + \delta_2(i_f+1)\right)\right]$$

$$\times \left[1 - \tfrac{1}{2}\left(\tilde{\delta}_2(i_m) + \tilde{\delta}_2(i_m+1)\right)\right]$$

$$+ \tfrac{1}{2}C_s^M(i_f, i_m) \qquad (s=1,2; i_f, i_m = 15, \ldots, 100+)$$

The timing of mortality rates in (A2.11) has been corrected along the lines of (A2.4). M and D respectively denote marriages and divorces, which are discussed in the next two sections. It can be seen that, even if initially C_1 and C_2 were equal, during the process of updating this equality would be disturbed by migration flows.

A2.5 Marriages

By $\gamma(i_f,i_m)$ and $\tilde{\gamma}(i_f,i_m)$ are denoted the stationary probabilities (a) and (b) listed in the text on page 13. They indicate, respectively, the age specific preferences of marrying females and of marrying males for spouses of various ages.

Consider a female of age i_f at the time of her marriage. For women in the ℓ^{th} marital state ($\ell \neq 2$) at the start of the year, $\alpha_{2\ell}(i_f)$ is the probability of marriage at age i_f within the year. Given that she marries at age i_f within the year, the probability that she marries a male aged i_m at the time of the marriage is $\gamma(i_f,i_m)$. Thus,

$$\text{(A2.12)} \qquad \epsilon_\ell(i_f,i_m) = [\alpha_{2\ell}(i_f)\gamma(i_f,i_m)]$$

is the probability that a woman who is of marital status ℓ at the beginning of the year and who (if she marries) is aged i_f at the time of marriage, marries within that year a man who is aged i_m at the time of the event. The probability, $\eta_\ell(i_f,i_m)$, that a woman who is of marital status ℓ at the beginning of the year and who (if she marries) is aged i_f at the time of marriage marries within that year a man aged i_m at the beginning of the year, is obtained by applying a timing correction to $\epsilon_\ell(i_f,i_m)$:

(A2.13) $$\eta_\ell(i_f,i_m) = \tfrac{1}{2}[\epsilon_\ell(i_f,i_m) + \epsilon_\ell(i_f,i_m+1)] \quad .$$

Finally, the probability, $\nu_\ell(i_f,i_m)$, that a woman who is of marital status ℓ and who is aged i_f at the beginning of the year marries within that year a man aged i_m at the beginning of the year, is obtained by applying a timing correction to $\eta(i_f,i_m)$:

(A2.14) $$\nu_\ell(i_f,i_m) = \tfrac{1}{2}[\eta_\ell(i_f,i_m) + \eta_\ell(i_f+1,i_m)] \quad .$$

The number of females of age i_f and marital status ℓ at the beginning of the year who are at risk of marriage (specifically, to a man aged i_m at the beginning of the year), is $X^*_\ell(i_f)$ (ℓ=1,3,4). Hence the projected number of marriages $M_1(i_f,i_m)$, as calculated from the female probabilities $\gamma(i_f,i_m)$, occurring within the year between women and men whose ages at the beginning of the year, are i_f and i_m respectively, is:

(A2.15) $$M_1(i_f,i_m) = \sum_{\ell=1,3,4} X^*_\ell(i_f)\nu_\ell(i_f,i_m) \quad .$$

In developing (A2.12) through (A2.15), we could have started with the 'male' probabilities, $\tilde{\gamma}(i_f,i_m)$, defined under (b) on page 13 of the text. We would then obtain an alternative estimate, $M_2(i_f,i_m)$, of the number of marriages within the year of women and men aged i_f and i_m respectively at the start of the year:

(A2.16) $$M_2(i_f,i_m) = \sum_{\ell=1,3,4} \tilde{X}^*_\ell(i_m)\tilde{\nu}_\ell(i_f,i_m) \quad .$$

In the algorithm these two estimates are averaged to obtain:

(A2.17) $$M(i_f,i_m) = \tfrac{1}{2}[\sum_{s=1,2} M_s(i_f,i_m)] \quad .$$

From (A2.17) the number of marriages of brides and grooms, by their ages at the beginning of the year, are calculated as:

(A2.18) $$M(i_f) = \sum_{i_m} M(i_f, i_m),$$

and

(A.19) $$\tilde{M}(i_m) = \sum_{i_f} M(i_f, i_m) \ .$$

If any of these exceed the available stocks of partners,

$$\sum_{\ell=1,3,4} X_\ell(i_f) \quad \text{or} \quad \sum_{\ell=1,3,4} \tilde{X}_\ell(i_m) \quad ,$$

they are set equal to the available stock and the excess distributed to other ages. These must then be allocated among the previous marital states from which they came. This is done by a simple pro-rating according to the initial proportions of the three types of marriage at each age. Thus:

(A2.20) $$X_{2\ell}(i_f) = \left[\sum_{i_m} M(i_f, i_m)\right] \frac{X^*_\ell(i_f)\ \tfrac{1}{2}\left[\alpha_{2\ell}(i_f) + \alpha_{2\ell}(i_f+1)\right]}{\sum_{\ell=1,3,4} X^*_\ell(i_f)\ \tfrac{1}{2}\left[\alpha_{2\ell}(i_f) + \alpha_{2\ell}(i_f+1)\right]} \qquad (\ell=1,3,4)$$

is implemented in the case of brides. For grooms, $\tilde{X}_{2\ell}(i_m)$ is allocated using analogous proportions (that is, using the ratios on the right of (A2.20) after each X* and α has a tilde affixed to it). If one or more values of $X_{2\ell}(i_f)$ or of $\tilde{X}_{2\ell}(i_m)$ exceed the available eligible stocks ($X^*_\ell(i_f)$ and $\tilde{X}^*_\ell(i_m)$, $\ell = 1,3,4$), they are reduced to their maximum feasible value, and the surplus redistributed over other marital states (for details see Sams (1981)).

A2.6 Divorces

The number of divorces during a year which involve a woman of age i_f and a man of age i_m at the start of the year is obtained by averaging two estimates, $D_1(i_f,i_m)$ and $D_2(i_f,i_m)$. The former (latter) is calculated by multiplying the number of couples at risk, $\bar{C}(i_f,i_m)$, by the probability that a woman (man) who is of age $i_f(i_m)$ will divorce a man (woman) aged i_m (i_f), both ages reckoned at the start of the year.

There are two respects in which the procedure is simpler for divorces than for marriages. First, the two sets of estimates of the doubly age specific stocks of couples discussed above in section 2.4 are averaged to obtain a single estimate of the number of couples at risk; that is:

(A2.21) $$\bar{C}(i_f,i_m) = \tfrac{1}{2}[C_1(i_f,i_m) + C_2(i_f,i_m)] \quad (i_f,i_m = 15,\ldots, 100+).$$

Second, only two (rather than three) nests of timing corrections are involved. By $\alpha_{32}(i_f)$ is denoted the probability that a woman who is married at the beginning of the year will divorce during the year at a time when her age is i_f. Given that a woman divorces within the year at a time when her age is i_f, $\omega(i_f,i_m)$ is the probability that she divorces a man then aged i_m. Thus the probability that a woman will divorce within the year at a time when her age is i_f and her partner's age is i_m is $[\alpha_{32}(i_f)\ \omega(i_f,i_m)]$. To obtain the probability $\psi(i_f,i_m)$ that a married woman divorces within a given year, where her age at the time of divorce is i_f, and the age of her spouse at the beginning of the year is i_m, requires that the preceding probabilities be corrected for timing as follows:

(A2.22) $$\psi(i_f,i_m) = \tfrac{1}{2}\alpha_{32}(i_f)[\omega(i_f,i_m) + \omega(i_f,i_m+1)] \quad .$$

A woman who is divorced within a given year and whose age is i_f at the start of the year may be aged i_f or $(i_f + 1)$ at the time of divorce. Hence, the probability that a married woman who is aged i_f at the start of the year divorces within the year a man aged i_m at the start of the year, is estimated by :

$$\text{(A2.23)} \qquad \chi(i_f,i_m) = \tfrac{1}{2}[\psi(i_f,i_m) + \psi(i_f+1,i_m)] \quad .$$

If we approach the question from the viewpoint of the 'female' divorce probabilities, therefore, our estimated number of doubly age specific divorces within the year is:

$$\text{(A2.24)} \qquad D_1(i_f,i_m) = \bar{C}(i_f,i_m)\,\chi(i_f,i_m) \quad .$$

Starting from the 'male' divorce probabilities $\tilde{\omega}(i_f,i_m)$, and reworking (A2.22) and (A2.23), we obtain a second estimate,

$$\text{(A2.25)} \qquad D_2(i_f,i_m) = \bar{C}(i_f,i_m)\;\tilde{\chi}(i_f,i_m) \quad .$$

These estimates are then simply averaged:

$$\text{(A2.26)} \qquad D(i_f,i_m) = \tfrac{1}{2}\sum_{s=1,2} D_s(i_f,i_m) \quad .$$

The numbers of divorces by age of wife, and by age of husband, are respectively:

$$\text{(A2.27)} \qquad X_{32}(i_f) = \sum_{i_m} D(i_f,i_m)$$

and

$$\text{(A2.28)} \qquad \tilde{X}_{32}(i_m) = \sum_{i_f} D(i_f,i_m) \quad .$$

A2.7 Widowings

The convention in the algorithm is that stocks of women at the start of the year who are at risk of widowing during the year are first adjusted to take account of the net flow of marriages and married migrants within the year. The husbands of this adjusted stock are then subjected to mortality (see section 2.1 above). Since age specific mortality rates do not depend strongly on marital status, the approximations involved in this procedure are satisfactory.

The widowing of a wife is defined as the death of her husband. The timing-corrected mortality rate $\tilde{d}_2(i_m)$ is obtained by putting k=2 in (A2.4) and affixing tildes to d and δ. When calculating the number of husbands at risk of death (and therefore the number of wives at risk of being widowed), it is necessary to observe the control totals on the stocks at the beginning of the year of married women of each age. Accordingly the C_1 and C_1^M couples matrices are used. The resulting estimates of the number of widowings by age of wife is obtained by summing deaths of husbands over their ages for a given age of wife:

(A2.29) $$X_{42}(i_f) = \sum_{i_m} [C_1(i_f,i_m) + M(i_f,i_m) - D(i_f,i_m) + \tfrac{1}{2}C_1^M(i_f,i_m)]\, \tilde{d}_2(i_m) \quad .$$

Widowings of men, by age of husband, are calculated analogously:

(A2.30) $$\tilde{X}_{42}(i_m) = \sum_{i_f} [C_2(i_f,i_m) + M(i_f,i_m) - D(i_f,i_m) + \tfrac{1}{2}\, C_2^M(i_f,i_m)]d_2(i_f) \quad .$$

A2.8 Births

Births of female and of male children in a given year are calculated as:

$$B = (C. + C^e)\ \beta\sigma\ , \qquad \text{(A2.31)}$$

and

$$\tilde{B} = (C. + C^e)\ \beta\tilde{\sigma}\ , \qquad \text{(A2.32)}$$

repectively, where C. and C^e respectively are the total number of nuptial and ex-nuptial confinements within the year, β is the average number of live births per confinement, and σ and $\tilde{\sigma}$ respectively are the femininity and masculinity proportions of newly born children. C. is the sum over birth orders of nuptial confinements:

$$C. = \sum_n C_n\ , \qquad \text{(A2.33)}$$

where C_n is the number of confinements of birth order n. C_n is either supplied from the econometric model (see Appendix 4) or is calculated from age specific confinement rates disaggregated by nuptiality of mother and by birth order in the case of nuptial confinements.

In the latter case, the number of n^{th} birth order nuptial confinements is calculated using:

$$C_n = \sum_{i_f} X_2^D(i_f)\rho_n(i_f)\ . \qquad \text{(A2.34)}$$

On the right of (A2.34), $X_2^D(i_f)$ is the mid year population of married women who were aged i_f at the start of the year, and $\rho_n(i_f)$ is the age specific nuptial confinement rate of birth order n. $X_2^D(i_f)$ itself is calculated as the average of the stocks of married women aged i_f at the beginning and the end of the year:

(A2.35) $$X_2^D(i_f) = \tfrac{1}{2}(X_2(i_f) + X'_2(i_f)) \quad ,$$

in which unprimed variables are timed at the beginning of t, and the primed variable at the end of that year.

Ex-nuptial confinements are handled analogously, except that (a) the number of women at risk is obtained by summing over the three unmarried categories and (b) ex-nuptial confinements are not diaggregated by birth order. Thus instead of (A2.34) and (A2.35), we use:

(A2.36) $$C^e = \sum_{i_f} \sum_{\ell=1,3,4} X_\ell^D(i_f)\rho^e(i_f)$$

and

(A2.37) $$X_\ell^D = \tfrac{1}{2}[X_\ell(i_f) + X'_\ell(i_f)] \qquad (\ell=1,3,4) \quad .$$

A2.9 Stock Adjustment Equations

The stock adjustment equations, which update the population at the beginning of the year to that at the start of the following year, have been given above in section A2.1 in matrix form, along with a description of their function. If we write equation A2.1 in terms of marital status flows, rather than transition coefficients, we obtain for women of marital status k and age i_f at the start of the year:

$$(A2.38) \qquad X'_k(i_f+1) = \left[X_k(i_f) + \sum_{\ell \neq k} X_{k\ell}(i_f) - \sum_{\ell \neq k} X_{\ell k}(i_f) + \tfrac{1}{2}X^M{}_k(i_f)\right]$$

$$\times \left[1 - \tfrac{1}{2}\left(\delta_k(i_f) + \delta_k(i_f+1)\right)\right] + \tfrac{1}{2}X^M_k(i_f) \qquad (i_f=0,\ldots,99) \quad ,$$

where the first summation represents marital status flows over the year into marital status k and the second summation represents marital status flows out of marital status k, and the prime on the left hand side variable indicates that it is the stock at the start of the next year.

The equation for men is similarly:

$$(A2.39) \qquad \tilde{X}'_k(i_m+1) = \left[\tilde{X}_k(i_m) + \sum_{\ell \neq k} \tilde{X}_{k\ell}(i_m) - \sum_{\ell \neq k} \tilde{X}_{\ell k}(i_m) + \tfrac{1}{2}\tilde{X}^M{}_k(i_m)\right]$$

$$\times \left[1 - \tfrac{1}{2}\left(\tilde{\delta}_k(i_m) + \tilde{\delta}_k(i_m+1)\right)\right] + \tfrac{1}{2}\tilde{X}^M_k(i_m) \qquad (i_m=0,\ldots,99)$$

At the maximum age recorded (100+), the stock is simply accumulated for each sex and marital state such that those who would have been 101 at the start of the next period are added to the population who are 100 to form those who are 100+.

At age zero, the stock of female infants at the start of the next period is the sum of the number of female births during the year and the number of female infants who are born elsewhere and who migrate to Australia during the year, corrected for survival. The stock adjustment equations can be written for female infants as:

(A2.40) $$X_1'(0) = (B + \tfrac{1}{2}X_1^M(-1))\,[1 - \pi\delta_1(0)] + \tfrac{1}{2}X_1^M(-1) \quad ,$$

and

$$X_k'(0) = 0.0 \qquad (k=2,3,4) \quad ,$$

and for male infants as:

(A2.41) $$\tilde{X}_1'(0) = (\tilde{B} + \tfrac{1}{2}\tilde{X}_1^M(-1))[1 - \tilde{\pi}\tilde{\delta}_1(-0)] + \tfrac{1}{2}\tilde{X}_1^M(-1) \quad ,$$

and

$$\tilde{X}_k'(0) = 0.0 \qquad (k=2,3,4) \quad ,$$

where

the prime indicates the stock of infants aged zero at the start of next year;

$B(\tilde{B})$ is the number of births of female (male) children in the year;

$X_1^M(-1)$ $(\tilde{X}_1^M(-1))$ is the net number of female (male) migrant arrivals who were born overseas during the year and who, being unborn at the start of the period, were thus aged -1;

$\delta_1(0)$ $(\tilde{\delta}_1(0))$ is the infant mortality rate for females (males); and

$\pi(\tilde{\pi})$ is the separation factor (explained in section A2.3).

Obviously, there are no infants of any marital status other than never married.

The stock adjustment equations for the couples matrices have been described above in section A2.4 and will not be repeated here.

A2.10 Consolidated List of Equations

The equations which form the population projection algorithm are listed below for reference. The order is that in which they are executed in each year of the projection period.

1) <u>Divorces</u>

(a) Divorces by age of wife and age of husband:

$$D(i_f,i_m) = \bar{C}(i_f,i_m)\,\tfrac{1}{2}\left\{\tfrac{1}{2}\left[\alpha_{32}(i_f)\,\tfrac{1}{2}\left[\omega(i_f,i_m) + \omega(i_f,i_m+1)\right]\right.\right.$$

$$\left. + \;\alpha_{32}(i_f+1)\tfrac{1}{2}\left[\omega(i_f+1,i_m) + \omega(i_f+1,i_m+1)\right]\right]$$

$$+ \;\tfrac{1}{2}\left[\tilde{\alpha}_{32}(i_m)\tfrac{1}{2}\left[\tilde{\omega}(i_f,i_m) + \tilde{\omega}(i_f+1,i_m)\right]\right.$$

$$\left.\left. + \;\tilde{\alpha}_{32}(i_m+1)\tfrac{1}{2}\left[\tilde{\omega}(i_f,i_m+1) + \tilde{\omega}(i_f+1,i_m+1)\right]\right]\right\},$$

where

$$\bar{C}(i_f,i_m) = \tfrac{1}{2}\sum_{s=1}^{2} C_s(i_f,i_m)$$

(b) Divorces by age of wife:

$$X_{32}(i_f) = \sum_{i_m} D(i_f,i_m)$$

(c) Divorces by age of husband:

$$\tilde{X}_{32}(i_m) = \sum_{i_f} D(i_f,i_m)$$

2) Marriages

(a) Marriages by age of bride and age of groom:

$$M(i_f,i_m) = \sum_{\ell=1,3,4} \tfrac{1}{2}\Bigg\{X^*_\ell(i_f)\ \tfrac{1}{2}\Big[\alpha_{2\ell}(i_f)\ \tfrac{1}{2}\big[\gamma(i_f,i_m) + \gamma(i_f,i_m+1)\big]$$

$$+ \alpha_{2\ell}(i_f+1)\ \tfrac{1}{2}\big[\gamma(i_f+1,i_m) + \gamma(i_f+1,i_m+1)\big]\Big]$$

$$+ \tilde{X}^*_\ell(i_m)\ \tfrac{1}{2}\Big[\tilde{\alpha}_{2\ell}(i_m)\ \tfrac{1}{2}\big[\tilde{\gamma}(i_f,i_m) + \tilde{\gamma}(i_f+1,i_m)\big]$$

$$\tilde{\alpha}_{2\ell}(i_m+1)\ \tfrac{1}{2}\big[\tilde{\gamma}(i_f,i_m+1) + \tilde{\gamma}(i_f+1,i_m+1)\big]\Big]\Bigg\}$$

(b) Marriages by age of bride:

$$X_{2\ell}(i_f) = \left[\sum_{i_m} M(i_f,i_m)\right] \frac{X^*_\ell(i_f)\ \tfrac{1}{2}\big[\alpha_{2\ell}(i_f) + \alpha_{2\ell}(i_f+1)\big]}{\sum_{\ell=1,3,4} X^*_\ell(i_f)\ \tfrac{1}{2}\big[\alpha_{2\ell}(i_f) + \alpha_{2\ell}(i_f+1)\big]} \quad (\ell=1,3,4)$$

(c) Marriages by age of groom:

$$\tilde{X}_{2\ell}(i_m) = \left[\sum_{i_f} M(i_f,i_m)\right] \frac{\tilde{X}^*_\ell(i_m)\ \tfrac{1}{2}\big[\tilde{\alpha}_{2\ell}(i_m) + \tilde{\alpha}_{2\ell}(i_m+1)\big]}{\sum_{\ell=1,3,4} \tilde{X}^*_\ell(i_m)\ \tfrac{1}{2}\big[\tilde{\alpha}_{2\ell}(i_m) + \tilde{\alpha}_{2\ell}(i_m+1)\big]}$$

(4) Stocks at beginning of next period:

(a) Married couples by age of wife and age of husband:

$$C_s'(i_f+1, i_m+1) = \left[C_s(i_f, i_m) + M(i_f, i_m) - D(i_f, i_m) + \tfrac{1}{2}C_s^M(i_f, i_m)\right]$$

$$\times \left[1-\tfrac{1}{2}\left(\delta_2(i_f) + \delta_2(i_f+1)\right)\right]$$

$$\times \left[1-\tfrac{1}{2}\left(\tilde{\delta}_2(i_m) + \tilde{\delta}_2(i_m+1)\right)\right]$$

$$+ \tfrac{1}{2}C_s^M(i_f, i_m) \qquad (s=1,2)$$

(b) Subject to the condition that:

$$C_s'(i_f, i_m) \geq 0$$

(c) Women by age and marital status:

$$X_k'(i_f+1) = \left(X_k(i_f) + \sum_{\ell \neq k} X_{k\ell}(i_f) - \sum_{\ell \neq k} X_{\ell k}(i_f) + \tfrac{1}{2}X^M{}_k(i_f)\right)$$

$$\times \left[1 - \tfrac{1}{2}\left(\delta_k(i_f) + \delta_k(i_f+1)\right)\right] + \tfrac{1}{2}X_k^M(i_f)$$

(d) Men by age and marital status:

$$\tilde{X}_k'(i_m+1) = \left(\tilde{X}_k(i_m) + \sum_{\ell \neq k} \tilde{X}_{k\ell}(i_m) - \sum_{\ell \neq k} \tilde{X}_{\ell k}(i_m) + \tfrac{1}{2}\tilde{X}^M{}_k(i_m)\right)$$

$$\times \left[1 - \tfrac{1}{2}\left(\tilde{\delta}_k(i_m) + \tilde{\delta}_k(i_m+1)\right)\right] + \tfrac{1}{2}\tilde{X}_k^M(i_m)$$

(A3.7) $p(i,t) = \frac{P_1}{\beta_1^{\alpha_1}\Gamma(\alpha_1)}(i-i^*)^{\alpha_1-1}e^{-(i-i^*)/\beta_1} + \frac{P_0}{\beta_0^{\alpha_0}\Gamma(\alpha_0)}(i-i_0)^{\alpha_0-1}e^{-(i-i_0)/\beta_0}$,

where P_0, α_0, β_0, i_0 are constants, and P_1, α_1 and β_1 are specific to each year projected. (For simplicity, the t affix has been suppressed on the right of (A3.7).)

The propensity to first marry is given by:

(A3.8) $P = P_1 + P_0$,

while the mean of the distribution is given by

(A3.9) $M = (P_1\alpha_1\beta_1 + P_0\alpha_0\beta_0)/(P_1 + P_0)$,

and the variance by

(A3.10) $V = \left\{P_1^2\alpha_1\beta_1^2 + P_0^2\alpha_0\beta_0^2 + P_1P_0[\alpha_1(\alpha_1 + 1)\beta_1^2 + \alpha_0(\alpha_0 + 1)\beta_0^2 - 2\alpha_1\beta_1\alpha_0\beta_0]\right\}/(P_1 + P_0)^2$.

Over the period from 1921/22 to 1975/76, the value of the constants in the above equations are estimated to be:

	P_0	α_0	β_0	i_0
Females				
Pre-war[(a)]	.455	2.2	6.8	33.0
Wartime and post-war	1.481	2.0	6.6	27.0
Males				
Pre-war[(a)]	.249	1.7	8.0	35.0
Wartime and post-war	.721	1.9	7.7	29.0

(a) 1921/22 to 1938/39.

propensity to first marry, remarry or divorce, as appropriate. On the right of (A3.1), each of the terms P, α and β in principle have a t affix, which here has been suppressed for simplicity.

The parameters α and β are related to the mean M, and variance V, of the distribution by:

(A3.3) $$M = \alpha\beta + i^*$$

and

(A3.4) $$V = \alpha\beta^2 ,$$

respectively. Conversely,

(A3.5) $$\alpha = (M-i^*)^2/V ,$$

while

(A3.6) $$\beta = V/(M-i^*) .$$

Hence, from a knowledge of the propensity, mean and variance of the gamma distribution of ages of each of first marriages, remarriages and divorces in any year, it is possible to construct the probability of first marriage, remarriage or divorce for each single year of age within the age specific rates block.

The gamma distribution is adequate to describe remarriage and divorce rates. However, first marriage rates rise too quickly and then fall too slowly across ages for it to provide a satisfactory approximation. The model schedule for first marriage rates is therefore specified to be the sum of two gamma distributions (one stationary and the other evolving over time):

APPENDIX 3 : THE CALCULATION OF AGE SPECIFIC RATES OF MARRIAGE AND DIVORCE

The age specific rates of first marriage, remarriage and divorce for women are, respectively, the distributions across ages of the number of first marriages of women divided by the number of never married women of the same age, the number of remarriages of women divided by the number of widowed or divorced women of the same age and the number of divorces of women divided by the number of married women of the same age. The age specific rates for men are defined analogously. The notation used in this Appendix is self-contained.

The age specific rates characteristically begin at a low value for young ages, rise rapidly with age to a peak and then decrease slowly with increasing age. These rates, p(i,t), are approximated within the age specific marriage and divorce rate block of the population submodule by a model schedule; in this case, the gamma distribution:

$$\text{(A3.1)} \qquad p(i,t) = \frac{P}{\beta^{\alpha}\Gamma(\alpha)} (i-i^*)^{(\alpha-1)} e^{-(i-i^*)/\beta} ,$$

where i is the age at the time of marriage or divorce, i* is an appropriate lower age limit for the event in question (for numerical values, see Williams (1981), p.9), and

$$\text{(A3.2)} \qquad P = \int_0^{\infty} p(i,t)di$$

is the area under the age distribution, referred to as the index of

5) Births

(a) Stocks of zero age children in next period:

$$X_1'(0) = (B + \tfrac{1}{2}X_1^M(-1))[1 - \pi\delta_1(0)] + \tfrac{1}{2}X_1^M(-1)$$

$$X_k'(0) = 0.0 \qquad (k=2,3,4)$$

$$\tilde{X}_1'(0) = (\tilde{B} + \tfrac{1}{2}\tilde{X}_1^M(-1))[1 - \tilde{\pi}\tilde{\delta}_1(0)] + \tfrac{1}{2}\tilde{X}_1^M(-1)$$

$$\tilde{X}_k'(0) = 0.0 \qquad (k=2,3,4)$$

(b) Births of male and female children:

$$B = (C_{\cdot} + C^e)\ \beta\sigma$$

$$\tilde{B} = (C_{\cdot} + C^e)\ \beta\tilde{\sigma}$$

(c) Nuptial confinements by age of mother:

$$C_{\cdot} = \sum_n C_n$$

$$C_n = \sum_{i_f} X_2^D(i_f) \quad \rho_n(i_f)\ ,$$

where

$$X_2^D(i_f) = \tfrac{1}{2}(X_2(i_f) + X_2'(i_f))$$

(d) Ex-nuptial confinements by age of mother:

$$C^e = \sum_{i_f} \sum_{\ell=1,3,4} X_\ell^D(i_f) \quad \rho^e(i_f)\ ,$$

where

$$X_k^D(i_f) = \tfrac{1}{2}(X_k^D(i_f) + X_k^{D\prime}(i_f)) \qquad (k=1,3,4)$$

By using the propensity, mean and variance of the total distribution and these constants, it is possible to reconstruct the age distributions of the rates of first marriage for males and females in any year of projection for which time specific values of the former three parameters are given.

APPENDIX 4 : THE CALCULATION OF BIRTHS

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APPENDIX 4 : THE CALCULATION OF BIRTHS

A4.0 Introduction

In the births block of the population submodule are calculated annual projections of the number of births of each sex. These are based on projections of the number of confinements in each year. There is a choice of two approaches to the projection of the latter variable:

> In the first, exogenous age specific confinement rates disaggregated by nuptiality of mother and by birth order for nuptial confinements, are supplied to the population projection algorithm. The number of confinements is then calculated by applying these rates to the projected population of women.
>
> In the second, the econometric model provides projections of the first nuptial confinement rate and the mean and variance of "implied completed family size". In the births block, the number of confinements of each birth order is then calculated from :
>
> (a) the first nuptial confinement rate and the number of married women aged 15-44 years,
>
> (b) the number of confinements of lower order in preceding periods,
>
> (c) the probabilities of women of each order progressing to a higher order,
>
> (d) the intervals between confinements of each order, and

(e) the ratio of ex-nuptial confinements to nuptial confinements in each year.

The econometric model only provides estimates of the total number of nuptial confinements at each birth order without disaggregation by age of mother. Nevertheless, the Facility can provide age specific estimates of the number of confinements if it is supplied with an exogenous set of age specific confinement rates. To achieve this, the Facility uniformly scales the confinement rates across ages for each birth order so that the total numbers of confinements implied by the adjusted rates at each birth order are equal in each year to those given by the econometric model. Similarly, the exogenously supplied age specific ex-nuptial confinement rates are scaled to give the same total number of ex-nuptial confinements as calculated from the econometric model.

In both methods, the relationship between the number of live births of each sex and the number of confinements is assumed to be determined by biological factors, whilst the decision to have a confinement is assumed to be influenced by economic and social conditions. Thus, the numbers of births of each sex are calculated by multiplying the projected number of confinements by constant factors for the average number of live births per confinement and for the sex ratios of births.

A4.1 Births

The remainder of this Appendix gives details of the second approach, in which the econometric model is used to project births via the births block. According to this approach, the number of live female births in year t, B_t, is given by:

(A4.1) $$B_t = (C_{.t} + C_t^e)\beta_t\sigma_t \quad ,$$

and the number of live male births in year t, $\tilde{B}_t$, is given by:

(A4.2) $$\tilde{B}_t = (C_{.t} + C_t^e)\beta_t\tilde{\sigma}_t \quad ,$$

where

$C_{.t}$ is the number of nuptial confinements in year t ,

C_t^e is the number of ex-nuptial confinements in year t ,

β_t is the average number of live births per confinement allowing for stillbirths and multiple births ,

σ_t is the proportion of births which are female ,

and

$\tilde{\sigma}_t$ is the proportion of births which are male.

In the current implementation of the births block, the number of ex-nuptial confinements in year t, C_t^e, is determined exogenously as a proportion of the number of nuptial confinements; that is:

(A4.3) $$C_t^e = x_t^e C_{.t} \quad .$$

A4.2 Nuptial Confinements

The number of nuptial confinements is simply the sum of nuptial confinements of all orders; that is:

(A4.4) $$C_{.t} = \sum_n C_{nt} \quad ,$$

where C_{nt} is the number of nuptial confinements of birth order n in year t .

Projections of the first nuptial confinement rate are provided directly by the econometric model, while higher order confinements are calculated from the interval distribution between confinements of each order and the probability of progressing to a higher order.

Let $\rho_{n,t,\tau}$ equal the probability that a married woman who (from the viewpoint of the beginning of the year t) had her n^{th} confinement τ years ago, has her $(n+1)^{th}$ confinement in year t. Then the number of confinements of order (n+1) in year t is:

(A4.5) $$C_{n+1,t} = \sum_{\tau=0}^{\infty} C_{n,t-\tau}\, \rho_{n,t,\tau} \quad ,$$

which represents the sum of the number of n^{th} order confinements in each previous period multiplied by the probability, $\rho_{n,t,\tau}$, that the next confinement occurs in year t.

The probability, $\rho_{n,t,\tau}$, can be separated into two multiplicative components: the parity progression ratio of order n , and the interval distribution between confinements. Ideally, the parity progression ratio of order n in year t, P_{nt} , would be defined as the probability that a married woman who has her n^{th} confinement in t will have at least one additional confinement if conditions prevailing in t persist. Pollard (1975) proxies the n^{th} parity progression ratio determined at t by:

(A4.6) $$P_{nt} = \sum_{\tau=0}^{\infty} \rho_{n,t,\tau} \quad .$$

Pollard's definition employs the interval distribution between past confinements and the confinements occurring in t as a proxy for the interval distribution between confinements in t and future confinements. The parity progression ratio is defined by:

(A4.7) $$(1 - p_{nt}) = \prod_{\tau=0}^{\infty} (1 - \rho^*_{n,t,\tau}) \quad ,$$

where $\rho_{n,t,\tau}$ equals the probability that a woman who had her n^{th} confinement in t will have her $(n+1)^{th}$ confinement τ years later. So;

(A4.8) $$p_{nt} = \sum_{\tau=0}^{\infty} \rho^*_{n,t,\tau} - \sum_{\substack{\tau,\tau' \\ \tau \neq \tau'}}^{\infty} \rho^*_{n,t,\tau}\, \rho^*_{n,t,\tau'} + \cdots\cdots \quad .$$

Thus Pollard's formula identifies $\rho^*_{n,t,\tau}$ with $\rho_{n,t,\tau}$ and omits the second and higher order terms. We proceed on the basis of this approximation. The distribution of intervals between confinements for each parity in any year has the following probability frequency function:

(A4.9) $$q_{n,t,\tau} = \rho_{n,t,\tau}/p_{nt} \quad .$$

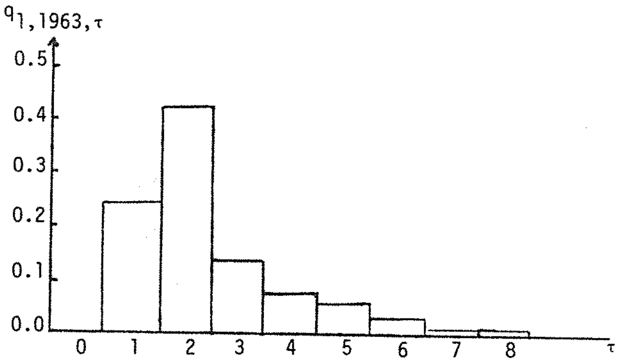
The quantity on the left is defined as the probability that a married woman having her $(n+1)^{th}$ confinement in t had her n^{th} confinement τ years earlier. Pollard (1975, Table 3), gives the interval distribution for the progression from the first to second confinement for the years 1960 to 1970. By way of example, the birth interval distribution, $q_{1,1963,\tau}$, is illustrated in Figure A4.1.

The number of nuptial confinements of order (n+1) in t is then:

(A.10) $$C_{n+1,t} = \left(\sum_{\tau=0}^{\infty} C_{n,t-\tau}\, q_{n,t,\tau} \right) p_{nt} \quad ,$$

and its calculation requires

Figure A4.1: Birth Interval Distribution between First and Second Nuptial Confinements for 1963



Source: Pollard (1975).

(i) the number of confinements of lower order in previous periods,

(ii) the parity progression ratio of order n, and

(iii) the interval distribution for n^{th} order confinements.

The number of confinements in previous periods is predetermined within the births block, while the parity progression ratios are obtained from the fertility parameters provided by the econometric model (as is now explained in section A4.3) and the distribution of intervals between confinements is modelled crudely (as explained in section A4.4).

A4.3 Parity Progression Ratios

The parity progression ratios, p_{nt}, are modelled via two parameters, the mean and the variance of the "implied completed family size" distribution. The distribution of implied completed family size is not directly observable but can be calculated from:

(A4.11) $$f_{1t} = 1 - p_{1t} ,$$

and

(A4.12) $$f_{nt} = \left[1 - p_{nt}\right] \left[\prod_{j=1}^{n-1} p_{jt}\right] \qquad (n = 2,3,....) ,$$

where

f_{nt} is the probability that a married woman who has already had her first confinement will have exactly n confinements in her lifetime, if conditions prevailing in t persist. Thus f_{nt} is an "implied completed family size" as it is dependent on conditions at time t persisting into the future. We define f_{0t} = 0.0 for later convenience;

and

p_{nt} is the parity progression ratio of order n.

In order to estimate the coefficients of the econometric model, the implied completed family size distribution and its mean and variance were calculated from values of the parity progression ratios provided by G. Spencer (see Spencer (1974) and Brooks (1981) for details).

The calculated probability density is, by definition, zero for an implied completed family size of zero, rises sharply to a modal value at a family size of two, and then falls slowly with increasing family size. This suggests that the frequency distribution of implied completed family size can be approximated by integrating a gamma density over each parity interval; that is:

(A4.13) $$f_{nt} = \int_{n-1}^{n} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} dx \qquad (n = 1,2,3 \ldots.) \quad ,$$

where Γ is the gamma function.

The mean, M_N, and the variance, V_N, of this distribution are related to the parameters α and β of the continuous gamma distribution by:

(A4.14) $$M_N = \alpha\beta + 0.5 \quad ,$$

and

(A4.15) $$V_N = \alpha\beta^2 \quad .$$

In projection, the births block translates the values of the mean and variance of the implied completed family size distribution into the parity progression ratios for each birth order. For this purpose the following are used: equations (A4.13), (A4.14), (A.4.15), and

(A4.16) $p_{it} = 1 - f_{it}$,

and

(A4.17) $$p_{nt} = \left(1 - \sum_{j=1}^{n} f_{jt}\right) \Big/ \left(1 - \sum_{j=1}^{n-1} f_{jt}\right) \qquad (n = 2,\ldots) .$$

A4.4 The Interval Distribution Between Confinements

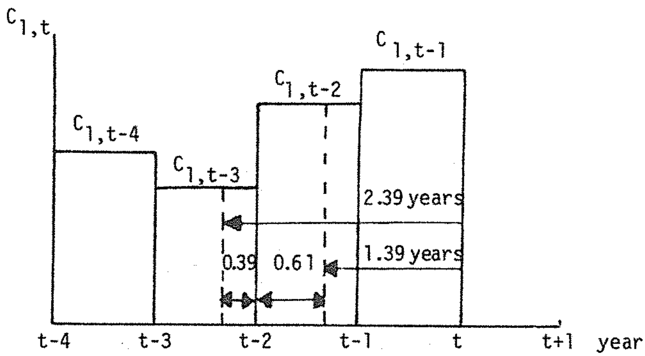
In the current implementation of the births block, the interval distribution is approximated by assuming that the interval for all confinements of order n in t is simply equal to the mean interval, $\bar{\tau}_n$, of such confinements. But, since confinements are spread over the whole of year t, the previous confinement could have occurred between $t - \bar{\tau}_n$ and $t - \bar{\tau}_n + 1$. For example, if the average interval between first and second nuptial confinements is 2.39 years as in 1965 then we would assume that all the first confinements of women having a second confinement in 1965 would have occurred (uniformly) between 1.39 and 2.39 years before the beginning of this year.

The next assumption is best explained in terms of our example and Figure A4.2. The equation for the number of second confinements in 1963 would be:

(A4.18) $$C_{2t} = (0.61C_{1,t-2} + 0.39C_{1,t-3})\, p_{1t} .$$

The assumption is that since 61 per cent of the period between 1.39 and 2.39 years prior to t covers the year (t-2) and 39 per cent covers year (t-3), then 61 per cent of the second confinements in t correspond to women who had their first confinement 2 years before and the remaining 39 per cent of the second confinements in t correspond to women who had their first confinement 3 years before.

Figure A4.2: The Overlap of the Interval $(t-\bar{\tau}_n, t-\bar{\tau}_n+1)$ with the Preceding Years



The approximation that is currently used is equivalent to replacing the interval distribution ($q_{1,1963,\tau}$) illustrated in Figure A4.1, which has a mean interval of 2.39 years, by the following interval distribution having the same mean:

$$
\begin{aligned}
q_{1,1963,1} &= 0.00 \\
q_{1,1963,2} &= 0.61 \\
q_{1,1963,3} &= 0.39 \\
q_{1,1963,\tau} &= 0.00 \quad , \quad \tau > 4 \ . \\
\text{Sum} \quad & \underline{\overline{1.0}}
\end{aligned}
$$

This interval distribution is illustrated in Figure A4.3. Table A4.1 provides the assumed interval distribution for all birth orders.

The current approach to the modelling of fertility within the births block allows the decisions to have a family and the size of the family to be dealt with separately. However, the number of births is only loosely related to the size, age structure and marital status composition of the female population. As well, the birth interval distribution is fixed and no attempt is made to capture adjustment to fertility rates arising from the timing of births over the life cycle of the mother. It is hoped that these deficiencies will be removed in future versions of the Facility.

Figure A4.3: The Approximation in the Current Implementation of the Births Block to the Birth Interval Distribution for 1963

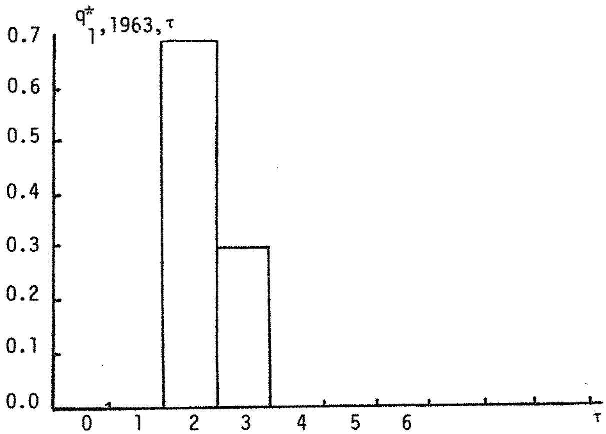


TABLE A4.1 : Birth Interval Distributions for the Calculation of Higher Order Confinements

Formula Relating 2nd and Higher Order Confinements to Earlier Confinements	Mean Interval
$C_{2,t} = (0.69C_{1,t-2} + 0.31C_{1,t-3})\ p_{1,t}$;	$\bar{\tau}_2 =$ 2.31 years
$C_{3,t} = (0.61C_{2,t-2} + 0.39C_{2,t-3})\ p_{2,t}$;	$\bar{\tau}_3 =$ 2.39 years
$C_{4,t} = (0.65C_{3,t-2} + 0.35C_{3,t-3})\ p_{3,t}$;	$\bar{\tau}_4 =$ 2.35 years
$C_{5,t} = (0.67C_{4,t-2} + 0.33C_{4,t-3})\ p_{4,t}$;	$\bar{\tau}_5 =$ 2.33 years
$C_{6,t} = (0.77C_{5,t-2} + 0.23C_{5,t-3})\ p_{5,t}$;	$\bar{\tau}_6 =$ 2.43 years
$C_{7,t} = (0.90C_{6,t-2} + 0.10C_{6,t-3})\ p_{6,t}$;	$\bar{\tau}_7 =$ 2.10 years
$C_{8,t} = (0.05C_{7,t-1} + 0.95C_{7,t-2})\ p_{7,t}$;	$\bar{\tau}_8 =$ 1.95 years
$C_{9,t} = (0.06C_{8,t-1} + 0.94C_{8,t-2})\ p_{8,t}$;	$\bar{\tau}_9 =$ 1.94 years
$C_{m,t} = (0.06C_{m-1,t-1} + 0.94C_{m-1,t-2})\ p_{m-1,t}$;	$\bar{\tau}_m =$ 1.94 years (for $m \geq 10$)