IMPACT PROJECT

A Commonwealth Government inter-agency project in co-operation with the University of Melbourne, to facilitate the analysis of the impact of economic demographic and social changes on the structure of the Australian economy

REVISED ESTIMATES OF

LABOUR SUPPLY ELASTICITIES

by

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IMPACT Research Centre

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The views expressed in this paper do not necessarily reflect the opinions of the participating agencies, nor of the Commonwealth government.

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Table A.3 Data on Other Variables

Year	After Tax Income per Working Person ($)			Hourly Wage Rate(f)	Hours per Year per Working Person(e)	Number of Working Persons(d) (Thousands)
	Labour Income(b)	Non-Labour Income(c)	Total disposable income(a)			
1964-65	2323.39	642.48	2965.87	1.1346	2048	4559
1965-66	2404.39	601.71	3006.10	1.1794	2039	4691
1966-67	2529.40	690.45	3219.85	1.2398	2046	4823
1967-68	2653.02	630.30	3283.32	1.2936	2051	4939
1968-69	2829.96	740.62	3570.58	1.3918	2033	5057
1969-70	3036.43	744.05	3780.48	1.4962	2029	5221
1970-71	3383.06	748.70	4131.76	1.6701	2026	5369
1971-72	3697.14	882.39	4579.53	1.8200	2031	5410
1972-73	4075.36	1090.51	5165.87	2.0407	1997	5538
1973-74	4761.28	1303.36	6064.64	2.3828	1998	5702
1974-75	6040.63	1426.86	7467.49	3.0868	1957	5715
1975-76	6646.53	1788.28	8434.81	3.4474	1928	5785

Notes: (a) Source: Australian National Accounts (op.cit.).
(b) Calculated from the before tax figures in ANA (op.cit.) by applying the tax rate derived from data on total before tax income excluding government transfer payments and total income tax paid.
(c) Obtained from (a) & (b).
(d) Average of four quarters derived from : Australian Bureau of Statistics, The Labour Force (Ref. No. 6.22), annual publications.
(e) Derived from average hours per week estimates from quarterly labour force surveys (ABS, Ref. No. 6.22).
(f) Derived from (b) and (e).

Table A.2 Price Indices

Year	Food	Drink and Tobacco	Clothing and Footwear	Housing	Household Durables	Private Transport	Public Transport	Other Goods and Services
	Price Indices for the Eight Commodities (1966-67 ≡ 1.00)							
1964-65	0.9501	0.8917	0.9695	0.9189	0.9877	0.9615	0.8943	0.9327
1965-66	0.9796	0.9746	0.9799	0.9549	0.9918	0.9767	0.9292	0.9639
1966-67	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1967-68	1.0336	1.0359	1.0228	1.0433	1.0045	1.0223	1.0199	1.0441
1968-69	1.0457	1.0549	1.0416	1.0943	1.0212	1.0565	1.0597	1.0814
1969-70	1.0705	1.0854	1.0727	1.1740	1.0409	1.0884	1.0940	1.1264
1970-71	1.1173	1.1515	1.1149	1.2866	1.0724	1.1487	1.1573	1.2047
1971-72	1.1521	1.2278	1.1799	1.3846	1.1250	1.2159	1.2604	1.2955
1972-73	1.2369	1.2891	1.2501	1.4781	1.1615	1.2601	1.2409	1.3766
1973-74	1.4671	1.4012	1.4170	1.6204	1.2526	1.3797	1.2629	1.5269
1974-75	1.6230	1.6271	1.7101	1.8596	1.4252	1.6499	1.4373	1.8343
1975-76	1.7875	2.0190	1.9807	2.1782	1.5348	1.9438	1.5907	2.1229

Note: Price indices are derived from estimates of consumer expenditure at current and constant (1966-67) prices given in Australian Bureau of Statistics, Australian National Accounts, National Income and Expenditure 1975-76, Canberra, 1977.

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APPENDIX

DATA SERIES USED IN THE PAPER

Table A.1 Household Consumption Expenditure per Working Person, Australia 1964-65 to 1975-76

Year	Household Consumption Expenditure by Commodity ($)								
	Food	Drink and Tobacco	Clothing and Footwear	Housing	Household Durables	Private Transport	Public Transport	Other Goods and Services	Total
1964-65	584.75	251.14	278.78	298.08	211.00	279.22	89.05	671.61	2663.63
1965-66	605.01	270.10	280.55	316.15	207.43	277.35	92.31	705.81	2754.75
1966-67	627.38	280.10	287.98	341.47	211.69	295.24	98.48	750.95	2893.29
1967-68	647.75	298.06	299.48	370.55	226.99	333.70	103.67	809.54	3089.74
1968-69	660.92	311.47	307.12	403.83	237.71	362.10	108.77	872.72	3264.64
1969-70	683.74	326.36	319.27	443.18	253.58	393.77	118.17	942.29	3480.36
1970-71	711.32	348.30	337.87	499.17	270.26	424.48	126.10	1007.28	3724.78
1971-72	766.06	376.56	367.32	564.38	302.80	460.12	134.21	1133.75	4105.20
1972-73	825.07	405.04	401.61	626.43	338.95	486.48	142.30	1253.59	4479.47
1973-74	945.74	452.62	473.13	708.12	435.61	548.01	159.23	1411.16	5133.62
1974-75	1083.21	532.16	541.95	855.54	533.73	673.02	184.62	1745.56	6149.79
1975-76	1227.05	643.26	602.63	1039.66	654.84	771.19	218.34	2024.69	7181.66

Note: Consumption expenditure figures are from Australian Bureau of Statistics, Australian National Accounts, Income and Expenditure 1975-76, Canberra 1977. Number of working persons from Table A.3.

5. CONCLUSION

In this paper the hours offered for work by a typical consumer/worker/saver have been modelled as responding to commodity prices, the hourly wage rate, and non-labour income. The twice-extended linear expenditure system has been fitted by full information methods to national time series data spanning 1964-65 through 1975-76. As expected, the results yielded the classical 'backward-bending supply curve' for hours of work offered by employees. From a comparison with comparable U.S. work, it appears that the marginal preference of Australians for leisure considerably exceeds that of their American counterparts.

ABSTRACT

Tulpulé, Ashok, Revised Estimates of Labour Supply Elasticities

In this paper a demand analysis framework has been used to study the effects of changes in wage rates, incomes and commodity prices on the supply of work hours. The supply of hours worked by a representative consumer/worker is viewed as the counterpart of the demand for leisure. Demand equations for a number of commodities and leisure are derived using Betancourt's Twice Extended Linear Expenditure System (TELES). The system is estimated empirically using Australian time-series data to yield numerical estimates of labour supply elasticities.

This value indicates that if there are no commodity price changes and the hourly wage rate goes up by one per cent the hours which the typical worker would wish to work would decrease by 0.06 per cent.

Cross price elasticities η_{hi} : The TELES model can be used to estimate the effect on hours worked due to a change in the price of a given commodity when all other prices, the wage rate and non-labour income remain constant. Thus η_{hi} gives the percentage change in hours worked if the price level of commodity i changes by one per cent. Differentiating (4.12) with respect to p_i yields

$$\eta_{hi} = \frac{\beta_L \, p_i \gamma_i}{Hw} \quad . \tag{4.18}$$

The estimated cross price elasticities are given in Table 3.

TABLE 3 : CROSS PRICE ELASTICITIES OF HOURS WORKED

Commodity whose price changes	Cross Price Elasticity η_{hi}	(and standard errors)
Food	0.0421	(0.0043)
Drink and Tobacco	0.0172	(0.0018)
Clothing and Footwear	0.0208	(0.0027)
Housing	0.0	-
Household Durables	0.0038	(0.0008)
Private Transport	0.0101	(0.0015)
Public Transport	0.0047	(0.0004)
Other Goods and Services	0.0265	(0.0078)

$$e_h^* = -\beta_L \frac{y}{wH} \quad . \qquad (4.16)$$

Evaluation of e_h^* (and the asymptotic standard error) at the mean values of y and wH yields[1]

$$\boxed{e_h^* = \begin{matrix} -\ 0.3056 \\ (0.0321) \end{matrix}} \; .$$

<u>Price elasticity</u> : The uncompensated price elasticity of hours worked, η_{hh} , gives the percentage change in hours worked due to a one per cent change in the hourly wage rate, when all commodity prices and non-labour income remain constant; i.e., from (4.12)[2]

$$\eta_{hh} = \frac{\partial H}{\partial w} \cdot \frac{w}{H} \quad ,$$

$$= \frac{\gamma_h}{H} (1 - \beta_L) - 1 \quad . \qquad (4.17)$$

The elasticity of hours supplied evaluated at the mean value of H is obtained by substituting the estimated values of β_L , γ_h and $\bar{H}$ = 2000.58 in (4.17).. The estimated elasticity value (and the standard error) is

$$\boxed{\eta_{hh} = \begin{matrix} -\ 0.0648 \\ (0.0069) \end{matrix}} \; .$$

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1. Note that e_h^* depends critically on β_L and the ratio of total income to labour income. As both these estimates have been revised downwards, the revision leads to a substantial reduction to the e_h^* estimate as compared to the previous estimate (Ashok Tulpulé, <u>op. cit.</u>, p. 22).
2. Substituting for H from (4.12) the elasticity can be expressed as

$$\eta_{hh} = -\frac{\beta_L}{wH} (\Sigma\, p_i\, \gamma_i - a) \quad . \qquad (4.17a)$$

REVISED ESTIMATES OF

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Ashok Tulpulé

1. <u>INTRODUCTION</u>

The supply of labour in an economy depends on workforce participation rates and on the number of hours which those working, or actively seeking work, are prepared to offer. Since any individual's propensity to work additional hours is the counterpart of his demand for leisure, it is natural to model the supply of hours worked using classical demand analysis. In the present paper, the hourly wage rate

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* This paper supercedes an earlier paper (Ashok Tulpulé, "Empirical Estimation of Labour Supply Elasticities," IMPACT <u>Preliminary Working Paper</u> No. BP-12, Industries Assistance Commission, <u>Melbourne, July 1978</u>). There are two major changes :

(a) In the previous paper, non-labour income included all the income from unincorporated private enterprises including farms. In this paper a very large part of this income is included in labour income and the estimates of non-labour income are reduced accordingly.

(b) In the earlier paper alternative commodity aggregation schemes that yielded different estimates of the Frisch parameter, ω , were tried and the one that gave an acceptable value was used. In this paper the value of the Frisch parameter is estimated exogenously using results from an international cross sectional study.

The reasons for making these revisions are discussed in sections 3 and 4 respectively. Data for two additional years are now available. However, the up-to-date data series includes a number of revisions to the previous series on consumption, income and population and the constant price consumption series are revalued at the 1973-74 prices. These revised data series have not been used in this paper, and as before the analysis is based on years 1964-65 to 1975-76.

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is treated as constant.[1] The supply of hours offered for work by a representative consumer/worker is derived from the maximization of a Klein-Rubin utility function subject to a budget constraint in which the hourly wage rate enters as a determinant of labour income. The resultant demand system includes a supply of work hours function whose arguments are the wage rate (which is the imputed price of leisure), the prices of commodities, and the level of non-labour income.

In Section 2 the model adopted, Betancourt's Twice Extended Linear Expenditure System[2] (TELES), is derived, using Howe's short-cut method.[3] A similar model has been implemented empirically using U.S. data by Abbott and Ashenfelter.[4] In Section 3 the Australian time series data base is described, with the empirical estimation following in Section 4. Conclusions are given in Section 5.

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1. The consequences of relaxing this assumption to allow for penalty rates of pay for overtime are explored in Alan A. Powell, Ashok Tulpule and Richard J. Filmer, "Commodity Specific Subsidies, Demand Patterns and the Incentive to Work," IMPACT Preliminary Working Paper No. BP-10, Industries Assistance Commission, Melbourne, November 1977 and in Alan A. Powell, "The Theory of Labour Supply and Commodity Demand with an Endogenous Wage Rate" IMPACT Preliminary Working Paper No. BP-19, Industries Assistance Commission, Melbourne, October 1979.

2. R. Betancourt, "Household Behaviour in a Less Developed Country : An Econometric Analysis of Cross-Sectional Data," University of Maryland, August 1973 (mimeo).

3. H. Howe, "Development of the Extended Linear Expenditure System from Simple Saving Assumptions," European Economic Review, Vol. 6 (1975).

4. M. Abbott and O. Ashenfelter, "Labour Supply Commodity Demand and the Allocation of Time," Review of Economic Studies, Vol. XLIII (3), No. 135 (October 1976), pp. 389-411.

As estimated from the TELES model the values of the parameters are

$$\beta_L = 0.2436 \quad \text{and} \quad \gamma_h = 2473.50 \ .$$

Therefore the income elasticity of hours supplied (and the asymptotic standard error) evaluated at the sample mean values of the exogenous variables is

$$\boxed{e_h = \underset{(0.0162)}{-0.1394}} \ .$$

It should be noted, however, that this result and formula (4.14) are based on a rise in income generated by equi-proportional changes in the hourly wage rate and in non-labour income. The negative sign of e_h is consistent with the famous 'backward-bending supply curve' for labour.

The income response given by (4.14) is not a pure income effect because it contains a substitution component due to the rise in the hourly wage rate. A pure income effect can be estimated by assuming that only the non-labour income, a, changes in which case all relative prices remain undisturbed.

Pure income elasticity : When a alone changes, differentiating (4.12) with respect to a yields

$$(4.15) \qquad \frac{\partial H}{\partial a} = \frac{-\beta_L}{w} \ .$$

The derivative (4.15) can be regarded as the income derivative of hours supplied after compensation for changes in relative prices of commodities and of leisure. Thus the pure income elasticity of hours worked e_h^* is given by

When y changes as a result of equi-proportional changes in a and w, H will change by δH, where

$$(4.13) \qquad \delta H = \frac{\beta_L}{w^2}\delta w \left(a - \sum_{i=1}^{8} p_i\gamma_i + w\gamma_h \right) + \frac{\beta_L}{-w}\left(\frac{a}{w} + \gamma_h\right)\delta w \quad .$$

$$= \frac{-\beta_L}{w^2}\left(\sum_{i=1}^{8} p_i\gamma_i \right)\delta w \quad .$$

Substituting from (4.11) into (4.13), taking the limit as $\delta y \to 0$, and converting into elasticity terms, one obtains[1] :

$$(4.14) \qquad e_h = \frac{\partial H}{\partial y}\cdot\frac{y}{H} = \left[\frac{-\beta_L \sum_{i=1}^{8} p_i\gamma_i \Big/ w}{(1-\beta_L)\quad(a+\gamma_h w)} \right] \frac{y}{H} \quad .$$

For 1964-65 to 1975-76 the mean values of the variables on the right hand side are :

$$\bar{y} = 4639.19$$
$$\bar{a} = 940.81$$
$$\bar{w} = 1.8486$$
$$\bar{H} = 2000.58$$

and from (4.3)

$$\sum_{i=1}^{8} \bar{p}_i\gamma_i = 1901.79$$

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1. Substituting $y = a + wH$ in (4.14) the elasticity can also be expressed as

$$(4.14a) \qquad e_h = \left[\frac{-\beta_L \sum_{i=1}^{8} p_i\,\gamma_i}{(1-\beta_L)(a+\gamma_h W)} \right]\left(1 + \frac{a}{wH}\right) \quad .$$

2. THE MODEL

The celebrated Linear Expenditure System (LES) of Richard Stone[1] was extended into an inter-temporal framework by Lluch who formulated the consumer/saver's choice as a control problem in continuous time.[2] This enabled the savings function of an individual consumer to be modelled simultaneously with his allocation of expenditure among commodities. Hadar has demonstrated that inter-temporal consumption problems under assumptions of continual replanning 'collapse' into equivalent 'as if' one period problems.[3] An example of this is Howe's short-cut for deriving Lluch's Extended Linear Expenditure System (ELES).[4] Betancourt, working in discrete time, further extended ELES to endogenize the number of hours worked.[5] In the treatment below, Betacourt's system (TELES) is derived using Howe's short-cut method. This involves treating saving as if it were just one additional commodity for which the 'minimum necessary quantity' is zero.

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1. R. Stone, "Linear Expenditure System and Demand Analysis : An Application to the Pattern of British Demand", Economic Journal, Vol. 64, No. 255 (September 1954), pp. 511-527.
2. C. Lluch, "The Extended Linear Expenditure System", European Economic Review, Vol. 4 (April 1973), pp. 21-32.
3. J. Hadar, Mathematical Theory of Economic Behaviour (Reading, Massachusetts: Addison-Wesley Publishing Company, 1971), pp. 224-228.
4. Howe, op. cit..
5. Betancourt, op. cit..

So much for background. The representative consumer/saver/worker's behaviour is generated by the maximization of the utility function[1]

$$U = \sum_{i=1}^{n-2} \beta_i \, \ell n \, (x_i - \gamma_i) + \beta_s \, \ell n \, s + \beta_L \, \ell n \, (L - \gamma_L) \quad , \tag{2.1}$$

subject to

$$\sum_{i=1}^{n-2} p_i x_i + s + (T-H)\, w = Tw + a \quad , \tag{2.2}$$

where

x_i = quantity of i^{th} good consumed ;

γ_i = socially acceptable minimum level (subsistence level) of consumption of good i ;

β_i = marginal propensity to consume good i out of 'full income', including the imputed value of voluntary leisure -- i.e., of leisure in excess of γ_L ;

β_s = marginal propensity to save out of full income ;

β_L = marginal propensity to consume leisure out of full income[2] ;

L = quantity of 'leisure' hours consumed ;

γ_L = socially acceptable minimum leisure hours ;

p_i = price of good i ;

s = savings in dollars ;

(T-H) = L = total leisure hours ;

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1. The additive utility function (2.1) rules out complementarity. Therefore the commodity classification should avoid items like "Sporting and Recreational Equipment" which would be complementary with leisure time.

2. The parameters of U may be normalized in a variety of ways which are observationally equivalent. The normalization used here, which underlies the interpretation of the $\{\beta_i\}$, is that $\sum_{i=1}^{n-2} \beta_i + \beta_s + \beta_L = 1$.

Summing over the 8 commodities in (2.11) and savings (2.12), yields

$$y = (v+s) = \sum_{i=1}^{8} p_i\gamma_i + \left[a - \sum_{i=1}^{8} p_i\gamma_i + w\gamma_h\right] \left[\beta_s + \sum_{i=1}^{8} \beta_i\right] \quad ;$$

which, since $\sum_{i=1}^{8} \beta_i + \beta_s + \beta_L = 1$, yields

$$y = \beta_L \sum_{i=1}^{8} p_i\gamma_i + (1 - \beta_L)\,(a + w\gamma_h) \quad . \tag{4.9}$$

If there is a small change in both a and w , denoted by δa and δw respectively, then assuming no price changes, y will change by δy , where

$$\delta y = (1 - \beta_L)\,(\delta a + \gamma_h \delta w) \quad . \tag{4.10}$$

From (4.5) and (4.10), the following response in total income to a change in the hourly wage rate accompanied by an equi-proportional change in non-labour income is obtained :

$$\delta y = (1 - \beta_L)\,(\frac{a}{w} + \gamma_h)\,\delta w \quad . \tag{4.11}$$

The supply of hours equation (2.15) is equivalent to

$$H = \gamma_h + \frac{\beta_L}{-w}\left[a - \sum_{i=1}^{8} p_i\gamma_i + w\gamma_h\right] \quad . \tag{4.12}$$

<u>Income elasticity</u> : The income elasticity of hours supplied measures the percentage change in this variable resulting from a one per cent change in income. In practice, it is unlikely that income will change without a change in the wage rate. Consider a situation in which both the non-labour income and the hourly wage rate change by the same proportion. That is, assume that

$$\frac{\delta a}{a} = \frac{\delta w}{w} \quad . \tag{4.5}$$

Total income y is given by

$$y = a + wH \quad (= \text{non-labour} + \text{labour income}) \ , \tag{4.6}$$

and

$$y = v + s \quad (= \text{total expenditure} + \text{savings}) \ . \tag{4.7}$$

The income elasticity of hours supplied is defined as

$$e_h = \frac{\partial H}{\partial y} \cdot \frac{y}{H} \quad . \tag{4.8}$$

The demand equations for commodities (2.11) and for savings (2.12) appropriate to an 8 commodity model are :

$$v_i = p_i\gamma_i + \beta_i \left(a - \sum_{i=1}^{8} p_i\gamma_i + w\gamma_h \right) \qquad (i=1,\ldots,8) \ ; \tag{2.11}$$

and

$$s = \beta_s \left(a - \sum_{i=1}^{8} p_i\gamma_i + w\gamma_h \right) \ ; \tag{2.12}$$

w = after tax hourly wage rate ;

$(T-H)w$ = implicit value of leisure ;

T = maximum possible hours that can be worked if no time is spent on leisure ;

H = hours worked ;

a = after tax non-labour income .

Notice that savings provides the numeraire for the system and has price indentically unity. Equation (2.2) can also be written as

$$Hw + a = s + \sum_{i=1}^{n-2} p_i x_i \quad . \tag{2.3}$$

The Lagrangean (F) is

$$F = U + \lambda \left[a + Hw - \sum_{i=1}^{n-2} p_i x_i - s \right] \ . \tag{2.4}$$

The first order conditions comprise equations (2.3) and those equations obtained by setting $\frac{\partial F}{\partial x_i}$ $(i=1,\ldots,n-2)$, $\frac{\partial F}{\partial H}$ and $\frac{\partial F}{\partial s}$ to zero; <u>viz.</u>,

$$\frac{\beta_i}{(x_i-\gamma_i)} = \lambda p_i \qquad (i=1,\ldots,n-2) \ ; \tag{2.5a}$$

$$\frac{\beta_s}{s} = \lambda.1 \ ; \tag{2.5b}$$

$$\frac{\beta_L}{(L-\gamma_L)} = \lambda w \quad . \tag{2.5c}$$

An expression for λ can be obtained by making use of the condition $\sum_{i=1}^{n} \beta_i = 1$ and summing the n equations (2.5a, b, c).

$$\Sigma \beta_i = 1 = \lambda \left\{ \sum_{i}^{n-2} p_i (x_i - \gamma_i) + s + w (L - \gamma_L) \right\} . \tag{2.6}$$

Therefore,

$$\lambda = 1 \Big/ \left\{ \sum_{i}^{n-2} p_i x_i + s + w (T-H-\gamma_L) - \sum_{i}^{n-2} p_i \gamma_i \right\} , \tag{2.7}$$

$$= 1 \Big/ \left\{ \left(\sum_{i}^{n-2} p_i x_i + s - wH \right) - \left(\sum_{i}^{n-2} p_i \gamma_i - w\gamma_h \right) \right\} , \tag{2.8}$$

where

$$\gamma_h = T - \gamma_L . \tag{2.9}$$

γ_h can be interpreted as the maximum possible hours that can be worked if only the minimum necessary leisure is consumed.

Substitution of (2.3) into (2.8) yields

$$\lambda = 1 \Big/ \left(a - \sum_{i}^{n-2} p_i \gamma_i + w\gamma_h \right) , \tag{2.10}$$

where $(a+w\gamma_h)$ is "full income"; i.e., the sum of non-wage income and maximum possible wage income that can be earned if only the minimum necessary leisure is consumed. Now by substituting for λ from (2.10)

propensity to consume leisure, $\tilde{\beta}_L$, in Australia is estimated to be 2½ times the U.S. value, indicating that the average Australian worker shows a very high marginal preference for leisure as compared to his American counterpart. The very high $\tilde{\beta}_L$ estimate for Australia, however, is not strictly comparable with the U.S. figure because one should not compare the $\tilde{\beta}_L$ values without taking into account the γ_h values. The γ_h estimate for Australia (see Table 1) is 2473 hours per year as against an estimate of 2357 hours[1] for the U.S.. Thus while the marginal propensity to consume leisure in Australia is high, the minimum leisure requirements (according to the estimates) are somewhat lower than in the U.S..

Elasticity Estimates

The estimates of the parameters of the TELES model can be used to calculate elasticities of hours supplied in response to changes in the exogenous variables; namely, non-wage income, prices of commodities, and the hourly wage rate. A given total change in income consists of two parts : that due to changes in non-wage income and that due to changes in the hourly wage rate. In measuring the response to such changes it is essential to take into account how the exogenous variables change. Elasticity formulae based on some specific assumptions are derived below.

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1. Abbott and Ashenfelter, *op. cit.*.

TABLE 2 : COMPARISON OF ESTIMATES OF MARGINAL BUDGET SHARES

Commodity	Marginal Budget Shares			
	β_i^* TELES, this study	β_i^* Other Estimates[a] ELES	Normalized Values, $\tilde{\beta}_i$ TELES, this study	Normalized Values, $\tilde{\beta}_i$ U.S. Estimates[b]
Food	0.0643	0.143 (Food and Drink & Tobacco)	0.0455	0.1630 (Food and Drink & Tobacco)
Drink & Tobacco	0.0521		0.0368	
Clothing & Footwear	0.0276	0.050	0.0195	0.1340
Housing	0.2464	0.220	0.1744	0.0755
Household Durables	0.1252	0.082	0.0886	0.2380
Private Transport	0.1311	0.224 (Private and Public Transport)	0.0927	0.0997 (Private and Public Transport)
Public Transport	0.0275		0.0194	
Other Goods & Services	0.3258	0.280	0.2306	0.1674
Leisure	--	--	0.2924	0.121
Marginal Propensity to Consume, μ	0.7795	0.796	--	--
Marginal Propensity to Save, $1 - \mu$	0.2205	0.204	--	--
Sum			1	1

(a) Lluch, Powell and Williams (Table 3.6, p. 44-45, op. cit.).

(b) Abbott and Ashenfelter, op. cit..

into (2.5a), (2.5b) and (2.5c), the following demand functions are obtained :

$$(2.11) \qquad v_i = p_i x_i = p_i \gamma_i + \beta_i \left[a - \sum_i^{n-2} p_i \gamma_i + w\gamma_h \right] \quad ;$$

$$(2.12) \qquad s = \beta_s \left[a - \sum_i^{n-2} p_i \gamma_i + w\gamma_h \right] \quad ;$$

$$(2.13) \qquad wL = w\gamma_L + \beta_L \left[a - \sum_i^{n-2} p_i \gamma_i + w\gamma_h \right] \quad .$$

Substituting $\gamma_L = (T - \gamma_h)$ into (2.13), yields

$$(2.14) \qquad wL = w(T - \gamma_h) + \beta_L \left[a - \sum_i^{n-2} p_i \gamma_i + w\gamma_h \right] \quad .$$

Given that $L \equiv T\text{-}H$, (2.14) implies

$$(2.15) \qquad -wH = -w\gamma_h + \beta_L \left[a - \sum_i^{n-2} p_i \gamma_i + w\gamma_h \right] \quad .$$

The TELES system estimated in this paper consists of equations (2.11), (2.12) and (2.15) . To keep the variance - covariance matrix of the stochastic errors which are later appended to the behavioural equations non-singular, one of the equations must be deleted before estimation. In this case, equation (2.12) has been deleted. The model is estimated via the remaining (n-1) equations only.[1] Estimates of the parameters of the above TELES model can be used to calculate estimates of own price elasticities of demand for the (n-2) commodities, the elasticity of supply of work hours with respect to the hourly wage rate, and a number of cross elasticities. The marginal propensities to consume different commodities and to save are given by the following formulae. The marginal propensity to consume good i out of actual income is

$$\beta_i^* = \frac{\beta_i}{1-\beta_L} \qquad (i=1,\ldots,n-2) \quad . \tag{2.16}$$

Summing over the (n-2) goods yields the marginal propensity to consume out of an extra $ actual income,

$$\mu = \sum_i^{n-2} \beta_i^* \quad , \tag{2.17}$$

and thus the marginal propensity to save is $(1-\mu)$. Various income and price elasticity formulae are derived in Section 4.

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1. The choice of equation eliminated is arbitrary. The maximum likelihood estimates of the parameters of the system are invariant to the equation deleted. See Anton Barten, "Maximum Likelihood Estimation of a Complete Set of Demand Equations", European Economic Review, Vol. 1 (1969), pp. 7-73.

acceptable maximum possible hours that a typical worker can work in one year. Thus if the typical worker works for the whole year, the estimate in Table 1 for γ_h of 2474 hours per year is equivalent to nearly 47½ hours per week. All the marginal budget share parameters are very similar to those obtained by using a slightly lower $-\omega$ value and previous estimates of non-labour income.[1]

The β_i's in Table 1 represent the marginal budget shares out of 'full income'. This contrasts with the usual analysis of Linear Expenditure models where the marginal budget shares are calculated in relation either to 'actual income' (ELES) or to 'total expenditure' (LES). It is possible and useful to normalize the β coefficients of the TELES model to enable comparison with the conventional estimates of marginal budget shares. The marginal budget shares out of total expenditure are given by $\beta_i^* = \frac{\beta_i}{1 - \beta_L - \beta_S}$. If the β's are normalized to add to 1 when the β for saving is excluded (i.e., $\sum_{i=1}^{8} \tilde{\beta}_i + \tilde{\beta}_L = 1$) , then the results can be compared with the U.S. results obtained by Abbott and Ashenfelter.[2] These comparisons are shown in Table 2.

It is interesting to note that both the TELES and ELES models produce similar estimates of the marginal propensity to consume, even though there is a difference in the sample period. The main differences between estimates of marginal budget shares β_i^* from the two Australian studies are as follows. In the present study the β^* values for food, drink and tobacco, clothing and footwear and for transport are substantially smaller than in the previous study; on the other hand the β^* values for household durables and other goods and services are larger than previously. The marginal

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1. Ashok Tulpulé and Alan Powell, op cit., p. 14.
2. Abbott and Ashenfelter, op cit..

The simultaneous system of nine equations (2.11) and (2.15) was estimated using Wymer's RESIMUL computer package.[1] The resulting parameter estimates are shown in Table 1.

TABLE 1 : ESTIMATES OF PARAMETERS OF THE TELES MODEL

Commodity	Parameters and (estimated Asymptotic Standard Errors)	
	Minimum Necessary Quantities, γ_i	Marginal Budget Shares, β_i
Food	530.74 (5.6562)	0.0379 (0.0021)
Drink & Tobacco	212.37 (1.6795)	0.0307 (0.0015)
Clothing & Footwear	256.57 (7.5882)	0.0163 (0.0030)
Housing	0.11 (7.3884)	0.1453 (0.0062)
Household Durables	51.19 (12.5492)	0.0738 (0.0031)
Private Transport	125.39 (9.6336)	0.0773 (0.0048)
Public Transport	61.76 (1.7193)	0.0162 (0.0006)
Other Goods & Services	311.13 (7.2857)	0.1921 (0.0073)
Savings	0[(a)]	0.1668 (0.0103)
Labour Supply/Leisure	2,473.50[(b)] (67.8353)	0.2436 (0.0256)
Sum		1.0000

(a) A priori restriction.
(b) Maximum possible work hours per year.

The first eight γ's are 'subsistence quantities.' By themselves, these are not very meaningful; however, $p_i \gamma_i$ can be interpreted as the cost at current price p_i of purchasing a socially accepted minimum quantity of good i. On the other hand γ_h can be interpreted as the socially

1. C. R. Wymer, Computer Programs: RESIMUL Manual, (Washington, D.C.: International Monetary Fund, 1977), (mimeo).

3. THE DATA

The following data series are required for estimating the TELES model described below :

(i) Consumer expenditure at current prices on (n-2) commodities $(p_i x_i)$,

(ii) (n-2) prices indices (p_i),

(iii) After tax labour income (wH),

(iv) Total disposable income (y),

(v) After tax non-labour income (a),

(vi) Total hours worked (H),

(vii) Hourly wage rate (w), and

(viii) Number of working persons .

The last series is required to calculate the various averages (expenditure, income, etc.) for a "typical" consumer/worker.[1]

(i) Consumer expenditure : Disaggregated data on private final consumption expenditure at current prices for 1964-65 to 1975-76 are given in the Australian National Accounts.[2] For the purposes of this paper the commodities are grouped into the following eight groups :

1. The increased workforce participation of married women over the period 1964-65 to 1975-76 raises the question of validity of the concept of a typical or representative consumer/worker. In a forthcoming paper separate estimates of elasticities are made for several types of workers classified by their demographic characteristics : Ashok Tulpulé, "Estimation of Elasticities of Supply of Labour Hours for Australian Workers Classified by Sex and Marital Status, IMPACT Preliminary Working Paper, University of Melbourne, Melbourne (forthcoming).

2. Australian Bureau of Statistics, Australian National Accounts, National Income and Expenditure 1975-76 (Ref. No. 7.1), Canberra, 1977.

1. Food,
2. Drink and Tobacco,
3. Clothing and Footwear,
4. Housing,
5. Household Durables,
6. Private Transport,
7. Public Transport, and
8. Other Goods and Services.

(ii) Prices indices : The Australian National Accounts[1] (ANA) also include estimates of private final consumer expenditure at average 1966-67 prices. Price indices for the eight consumption categories are obtained by dividing the current price expenditure estimates by the corresponding constant price estimates.

(iii) After tax labour income : Data on after tax labour income are not available for 1964-65 to 1975-76. The ANA[1] gives estimates of before tax income from wages, salaries and supplements, income from unincorporated enterprises, including farms, and of income from other sources. As well it provides an estimate of total income tax paid. Estimates of after tax labour income are obtained by applying the average tax rates to before-tax labour income. The average tax rate is calculated from the total income (less government transfer payments) and total tax series.

The before tax labour income is defined as the sum of income from wages, salaries and supplements and a part[2] of the income from unincorporated private enterprises including farms. The split of the income from unincorporated private enterprises into labour and non-labour income is not known. An imputed value of the labour income is

1. ABS, op. cit..
2. For 1974-75 and 1975-76, 100 per cent of the income from unincorporated private enterprises was assumed to be labour income. In other years the percentage varied from 70 to 95 per cent.

14 countries Lluch, Powell and Williams[1] arrived at the following relationship between ω and GNP per head in 1970 US dollars , X ,

$$\omega = -36\, X^{-0.36} \quad . \tag{4.2}$$

According to Williams[2], in 1974-75 Australian GNP per head was \$4078 in 1970 US dollars. The equivalent figure in 1966-67 Australian dollars is \$2425, which is 11.80 per cent higher than the average of the sample period, \$2169. The later figure would be equivalent to \$3647 (4078/1.1180) in terms of 1970 US dollars.

By substituting $X = 3647$ in (4.1) we get

$$\omega = -1.8744 \quad .$$

Following Ryan[3] and Tulpulé and Powell,[4] this estimate of ω is used to impose (4.1) on the demand equations. This is equivalent to fixing the level of subsistence expenditure at the sample mean:

$$\sum_{i=1}^{8} \bar{p}_i \gamma_i = \bar{v}\,[1 + \tfrac{1}{\omega}] = 4076.75\,[1 - \tfrac{1}{1.8744}] = 1901.79 \quad . \tag{4.3}$$

When this condition is imposed there are only 17 free parameters to be estimated from (2.11) and (2.15). Thus γ_8 is eliminated from the list of free parameters and its value is recovered from

$$\gamma_8 = \left[(1 - \tfrac{1}{1.8744})\,\bar{v} - \sum_{i=1}^{7} \bar{p}_i \gamma_i \right] \Big/ \bar{p}_8 \quad . \tag{4.4}$$

1. Op.cit..
2. R. Williams, op.cit..
3. David Ryan, "Effects of Ethnic Origin on Household Consumption Patterns in Australia," IMPACT Preliminary Working Paper No. SP-10, Industries Assistance Commission, Melbourne, June 1977.
4. Ashok Tulpulé and Alan Powell, "Estimates of Household Demand Elasticities for the ORANI Model," IMPACT Preliminary Working Paper No. OP-22, Industries Assistance Commission, Melbourne, September 1978.

The first point can be illustrated as follows: In the linear expenditure family of models including TELES, the eslasticity of the marginal utility of money with respect to total expenditure (which is Frisch's ω) is, strictly speaking, a variable. The empirical tradition is to evaluate this elasticity at sample means $(\bar{v}, \bar{p})$ as follows:

(4.1) $\omega = -\bar{v}/(\bar{v} - \Sigma\bar{p}_i \gamma_i)$. An implausibly high value of $-\omega$ is obtained when $\Sigma\bar{p}_i \gamma_i$, i.e., the subsistence expenditure at mean prices, as estimated from the γ parameters, is too high. The elasticity of labour supply η_{hh} with respect to the wage rate which is derived later (see 4.17a) is given by $\eta_{hh} = -\beta_L(\Sigma p_i \gamma_i - a)/wH$. The negative value of this elasticity produces the traditional backward-bending labour supply curve.[1] It is clear from this formula that if $-\omega$ (and hence $\Sigma p_i \gamma_i$) is overestimated, this will lead to an over-estimation of the absolute value of the elasticity of labour supply with respect to the wage rate. Thus the higher is $(\Sigma p_i \gamma_i)$ the more negative is η_{hh}.

The second point will be clear from an examination of (2.15). A high value of $(-\omega)$ leads to a high value of $\Sigma p_j\gamma_i$. Suppose that, given an initial set of estimates of the parameters, equation (2.15) holds without error at some realized set of values of the variables. If $\Sigma p_i\gamma_i$ is overestimated, to correct this without disturbing the fit of the equation it will be necessary to make compensatory changes in the estimates of parameters β_L and γ_h. Errors in the estimation of ω may therefore lead to biases in β_L, γ_h, or in both.

Given the importance of the Frisch parameter in estimating the labour supply elasticities and the difficulties encountered in estimating it, in this paper, use is made of the procedure adopted by Williams.[2] This allows one to impose an a priori value of ω on the system at the sample midpoint. Using time series data for

1. It is assumed that $\Sigma p_i\gamma_i > a$.
2. R. Williams, op cit., p. 18.

calculated by assuming that the self employed persons and farmers, that is, the recipients of income from unincorporated private enterprises, receive, on average, the annual labour income per head that is received by the average wage and salary earner. The average for the wage and salary earner is obtained by dividing the total income from wages, salaries and supplements by the number of wage and salary earners. Thus a part of the income from unincorporated private enterprises is considered to be labour income. In those years when the imputed value exceeded the total income from unincorporated private enterprises (usually the years when farm incomes were low), it is assumed that all such income is labour income.

In the previous paper (Ashok Tulpulé, op. cit.) only the income from wages, salaries and supplements was considered to be labour income. That assumption leads to an over-estimation of non-labour income and consequently yields a high value of the income elasticity of supply of labour hours [see formulae (4.14a)], and a low value of the elasticity of labour supply with respect to the wage rate [see formulae (4.17a)].

(iv) Total disposable income : Figures are published in the Australian National Accounts.[1]

(v) After tax non-labour income : The figures are obtained by subtracting (iii) from (iv).

1. ABS, op. cit..

(vi) Total hours worked : Estimates of total hours worked are obtained as a sum of four quarterly figures. For each quarter the total hours worked are estimated as the product of number of employed persons, average hours per employed person per week and number of weeks per quarter.

Estimates of the number of employed persons[1] and average hours worked per week are obtained from various labour force survey reports.[2] As the average hours figure is based on the month in which the survey was conducted, it will be biased as an estimate of the quarterly figure. This error will be most severe in the November and February quarters, as the very low average hours worked in December and January will not be reflected in the November and February figures. The above method thus gives an overestimate of total hours worked as it does not fully take into account annual holidays. It is not currently possible to make an allowance for this bias.

(vii) The hourly wage rate is estimated as the ratio of after tax labour income to total hours worked. As the hours worked are overestimated, the hourly wage rate is slightly underestimated.

(viii) Number of working persons : For this variable a simple average of the four quarterly figures is used as the annual value.

The above time-series data for the period 1964-65 to 1975-76 are given in the Appendix.

1. Employed persons comprised all those who, during the survey week did any work for pay, profit, or payment of any kind; worked 15 hours or more without pay in a family business or farm, or had a job but did not work because of illness, leave, holiday, etc..
2. Australian Bureau of Statistics, The Labour Force (Ref. No. 6.22). Figures for 1964 to 1968 are from Historical Supplement to the Labour Force and for other years from the annual reports. Estimates of average hours per week are not available for 1964-65. The figures used are derived from the distribution of number of persons by hours worked.

4. EMPIRICAL ESTIMATION

Equations (2.11) and (2.15) for an eight commodity model are :

$$(2.11) \qquad v_i = p_i x_i = p_i \gamma_i + \beta_i \left\{ a - \sum_{i=1}^{8} p_i \gamma_i + w\gamma_h \right\} , \qquad (i=1, \ldots, 8) ;$$

$$(2.15) \qquad v_9 = -wH = -w\gamma_h + \beta_L \left\{ a - \sum_{i=1}^{8} p_i \gamma_i + w\gamma_h \right\} ;$$

where v_i is the consumption expenditure per working person on the i^{th} commodity and all other variables are as defined previously. Thus a system of nine equations is estimated simultaneously.

In the previous paper (Ashok Tulpulé, op. cit.) a number of commodity aggregation schemes were tried and a five commodity aggregation that gave the lowest value of the Frisch parameter,[1] $-\omega$, viz., 2.51, was used. However, even this value is too high and not consistent with other evidence on this matter.[2,3] The consequences of accepting a wrong value of ω are twofold. Firstly, a high value of $(-\omega)$, say in excess of 2, will produce a high absolute value of the elasticity of labour supply; and secondly, an incorrect value for $(-\omega)$ may lead to poor estimates of the γ_h and β_L parameters.

1. R. Frisch, "A Complete Scheme for Computing all Direct and Cross Elasticities in a Model with Many Sectors," Econometrica, 27 (1959), pp. 177-96.
2. C. Lluch, A. Powell and R. Williams, Patterns in Household Demand and Saving (New York : Oxford University Press, 1977), pp. 74-81.
3. R. Williams, "Demographic Effects on Consumption in Australia : A Preliminary Analysis of the ABS 1974-75 Household Expenditure Survey," IMPACT Preliminary Working Paper No. SP-11, Industries Assistance Commission, Melbourne, February 1978.