EFFECTIVE EXCHANGE RATES AND THE INTERNATIONAL MONETARY FUND'S MULTILATERAL EXCHANGE RATE MODEL: A REVIEW*

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I. INTRODUCTION

Since the breakdown of the Bretton Woods System, almost all trading nations have adopted more flexible exchange rate regimes. But few have been willing simply to allow market forces to determine their rate. Hence policy makers have been increasingly concerned with effective exchange rate calculations. These calculations are supposed to contain information on the impact of the exchange rate changes of partner countries on the competitive position of the home country and they play an important role in the formulation of counter exchange rate policy.[1]

Section II of this paper reviews the economic logic of the usual effective exchange rate computations. Our conclusion is that ideally effective exchange rate calculations should be carried out with reference to a multicountry, multicommodity model of international trade. This leads us into the main part of the paper, our review of the International Monetary Fund's (IMF) multilateral exchange rate model (MERM).

The review of MERM is in Sections III and IV. Section III explains how MERM is used for effective exchange rate calculations and sets out its structural equations. Some areas for possible improvement will be apparent even before the theoretical basis for the equations is investigated. Section IV discusses the underlying theory and the procedures used by the Fund staff to obtain parameter estimates.

The paper is unavoidably technical in some parts (particularly Section IV) and it might be helpful for the reader to decide at the outset on a reading strategy. Those who are interested merely in the effective exchange rate concept may be content with Section II alone. Those readers who would like an overview of MERM and its major shortcomings should concentrate on Table I, and the last few paragraphs of Section III. Readers with more time can find the details of MERM's production and utility function specifications in Section IV. Major conclusions are listed in Section V.

It is hoped that the paper is intelligible without the reader having to refer to cited documents. In fact, a major aim of the review is to provide a unified exposition of the effective exchange rate concept and the Fund model.

* I am heavily indebted to Alan Powell for his detailed and constructive criticism of an earlier draft. Also, some of the ideas in Section II were drawn from unpublished work done jointly with Subhash Thakur. However, the views expressed here are mine alone and do not necessarily reflect the position of the Industries Assistance Commission or any other organization or person.

[1] For example, in 1974, Australia adopted a fixed effective exchange rate policy (see Section II).

II. Effective Exchange Rate Changes

Most effective exchange rate computations employ the formula

$$e_k = \overset{*}{T}_k - \sum_{j \neq k} w_{kj} \overset{*}{T}_j , \qquad \sum_{j \neq k} w_{kj} = 1 , \tag{2.1}$$

where e_k is the effective exchange rate change for country k over a specified period, say the previous year, $\overset{*}{T}_j$ is the percentage change[2] over the period in the value of the currency of the country j in terms of some numeraire currency (for example, SDR), *i.e.*, the $\overset{*}{T}_j$'s are the percentage changes in the numbers of yen, won, rupees, *etc.*, per SDR, and the w_{kj}'s are a set of weights. If e_k is positive, then country k is said to have "effectively devalued", if it is negative then it has "effectively revalued". If country k is following a fixed effective rate policy, then it continuously changes T_k so that $e_k = 0$. For example, if the Japanese weight in the Australian formula is .3, then the Australian authorities will meet a 10 per cent devaluation of the yen by a 3 per cent devaluation of the Australian dollar.

Frequently used schemes for determining the weights w_{kj} involve trade shares.[3] A popular choice is

$$w_{kj} = \frac{E_{kj} + M_{kj}}{\sum_{j \neq k} (E_{kj} + M_{kj})} \tag{2.2}$$

where E_{kj}, M_{kj} are the values in terms of the currency of country k of k's exports to and imports from country j over a particular period, usually the most recent year for which trade data are available.

Although weighting schemes such as (2.2) have some intuitive appeal, it is easy to think of situations in which they would be misleading. For example, imagine that in the base year Australia conducts all of its trade with Japan, exporting wool and importing cars, and that Argentina also exports wool and imports cars but that its trade is predominantly with the U.S.A.. When Japan revalues by 10 per cent and the U.S.A. devalues by 10 per cent, weighting scheme (2.2) used in formula (2.1) implies that Australia has effectively devalued by 10 per cent and that Argentina has effectively revalued by 10 per cent. In reality, the exchange rate changes of the U.S.A. and Japan may have very little differential impact on Australia and Argentina. Of importance to both countries is what happens to world wool and car prices. If a U.S. devaluation depresses world wool prices, not only will there be an impact in Argentina, but also in Australia. Similarly, Japanese exchange rate policy can have an impact on Argentina, even in the absence of any direct Japanese-Argentinian trade. Trade shares, by concentrating attention on bilateral relationships, are too narrow a basis for effective exchange rate calculations.

In order to develop a satisfactory alternative to (2.1) – (2.2), we must first have a rigorous definition of an effective exchange rate change. We suggest the following:

> For any particular period, the effective devaluation (revaluation) by country k with respect to an economic variable Ξ_k, is the revaluation (devaluation) which country k would need to undertake in order to neutralize the effect on Ξ_k of the exchange rate changes occurring in the world in that period.

[2] The superscript * denotes percentage change. Hence, T_j is the exchange rate for country j.
[3] See Hirsch and Higgins [7].

There are two aspects of this definition which require some explanation. First, there is the economic variable Ξ_k. We have in mind various measures of foreign trade balance—the balance of payments, the balance of trade, the balance on current account, *etc.* However, in principle, effective exchange rates could be defined with respect to any economic variable which is closely related to exchange rate changes. The only limitation is that country k's exchange rate policy should have sufficient influence on Ξ_k so that it is possible for k's exchange rate adjustments to compensate for the effects on Ξ_k of external exchange rate changes.[4]

The second part of the definition which requires elaboration is the phrase "in order to neutralize the effect on Ξ_k". It would be desirable for effective exchange rate calculations to include some indication of timing. For example, by "neutralize" we might understand that the effective exchange rate adjustment is such that if it were undertaken at time t, then the balance of trade over the next n months would be at the level it would have reached in the absence of the exchange rate changes of the last m months, *i.e.*, during the period between $t-m$ and t. In practice, the estimation of the lags involved in exchange rate effects is notoriously difficult, and the problem will be ignored in the remainder of this paper.

For our definition of an effective exchange rate change to become operational, we require a model which can be reduced to form[5]

$$\Delta\Xi_k = \Delta\Xi_k\,(\overset{*}{T}_1,\ \overset{*}{T}_2,\ \ldots,\ \overset{*}{T}_J)\,, \tag{2.3}$$

where $\Delta\Xi_k$ is the change[6] (or perhaps percentage change) in Ξ_k attributable to the direct and indirect effects of exchange rate changes. Then k's effective exchange rate change with respect to Ξ_k can be found by solving for e_k in the equation

$$0 = \Delta\Xi_k\,(\overset{*}{T}_1,\ \overset{*}{T}_2,\ \ldots,\ \overset{*}{T}_k - e_k,\ \ldots\ \overset{*}{T}_J)\,. \tag{2.4}$$

(2.4) means that if country k has devalued by 10 per cent over the relevant period (*i.e.*, $\overset{*}{T}_k = 10$), but a 12 per cent devaluation would have been required to sterilize the effects on Ξ_k of the exchange rate changes $\overset{*}{T}_1, \ldots, \overset{*}{T}_{k-1}, \overset{*}{T}_{k+1}, \ldots, \overset{*}{T}_J$ of other countries, then k has effectively revalued by 2 per cent (*i.e.*, $e_k = -2$) relative to Ξ_k.

Ideally, the structural model from which (2.3) is obtained is a multicountry, multiproduct one; multicountry and multiproduct because the influence of j's exchange rate on relevant variables in country k may be transmitted indirectly *via* its impact on *third countries* and also *via* its impact on "world" *commodity* prices. It is clear, however, that the construction of a suitable multicountry, multicommodity model is a formidable task. It can only be achieved by an institution with specialized interests in exchange rate policy and extensive experience with international data sources. Hence, the model build-

[4] For example, alterations of k's exchange rate may not influence the ratio M_{kj}/M_{kt}. Hence it would be meaningless to define an effective exchange rate with respect to $\Xi_k = M_{kj}/M_{kt}$.

[5] Economic models generally identify more variables than equations, and the form of (2.3) will depend on which subset of variables is assumed to be exogenous (*i.e.*, determined independently of exchange rate changes). In Section III we will see that a complete report on effective exchange rate calculations from any particular model must include a listing of the exogenous variables and their assumed values.

[6] It may seem more natural to have the percentage change in Ξ_k on the left side of (2.3). But if Ξ_k is k's balance of trade, then the base period value might be zero, and percentage changes will be meaningless.

ing efforts of the International Monetary Fund, which is perhaps unique in its endowment of the appropriate resources, warrant particular attention. In its latest Annual Report,[7] the IMF computes effective exchange rate changes with respect to the balance of trade (*i.e.*, Ξ_k is the balance of trade) for the major industrial countries. These calculations were made using reduced form equations, of the form (2.3), from the Fund's multicountry, multicommodity model, MERM. In the next two sections we will review MERM with the idea of (a) explaining its structure and estimation, and (b) comparing the implied MERM weighting schemes for effective exchange rate calculations with the standard trade share schemes.

III. THE MERM : ITS STRUCTURE AND ITS APPLICATION TO EFFECTIVE EXCHANGE RATE ANALYSIS

The stages in MERM's evolution are documented in several articles in the Fund's journal, *Staff Papers*,[8] the latest and most complete being J. R. Artus and R. R. Rhomberg's contribution to the November, 1973 issue. This review is based on Artus and Rhomberg's paper and for convenience we will refer to the collective Fund authorship of MERM by their initials (A & R).

It should be noted that MERM is continuously being modified. Publication lags mean that published reviews will be based on the model as it existed some time ago. However, recent information (August, 1975) indicates that the structure of the currently operational MERM is unchanged from the November, 1973, version, except that several countries, including Australia, have been added.

Table I is designed to give the reader quick access to MERM's structural equations. It might be useful to emphasize that the model is cast in percentage changes. Thus all parameters are "elasticities" or "shares". The table is to a large extent self-explanatory, and here we will make only a brief exposition.

A & R distinguish between five goods, *i.e.*, H=5. Two of these goods are classified as "intermediate goods", *i.e.*, G=2. The remaining three are "final goods".[9] In the November 1973 version there were 15 countries included in the model, *i.e.*, J=15, and the subscript (ij) is used to denote the ith good produced in the jth country, or simply the (ij)th product. For each product, there are demand equations [see equations (T1.1), (T1.2)], a supply equation (T1.3), and an equilibrium condition (T1.4).

Equation (T1.1) implies that in each country the demand for intermediate products is related to the level of real national output and the prices of all intermediate products. The demand for each final product depends on the aggregate level of "consumer"[10] expenditure

[7] IMF [10].

[8] See Armington [1, 2], Rhomberg [14], and Artus and Rhomberg [3]. Goodman *et al.* [5] present a model similar to MERM.

[9] The intermediate goods are crude materials (SITC 2 & 4) and mineral fuels (SITC 3). Manufacturers (SITC 5–9), food, beverages and tobacco (SITC 0–1) and "non-traded" (good H) are the three final goods. For notational convenience, we have not separated the non-traded good from the four traded goods. For example, putting $i = H$ in (T1.2) of Table I shows the rate of change of the demand (*i.e.*, $\mathring{x}^k_{Hj}$) by country k for the non-traded good produced in country j. To enforce the prior specification that H is non-traded we set the relevant parameters (ϵ^k_{Hj}, $\eta^k_{(Hj)(jm)}$, $\eta^k_{(jm)(Hj)}$, $\gamma_{(ik)(Hj)}$, A^k_{Hj}, C^k_{Hj} for $k \neq j$) to zero.

[10] We will use the word "consumer" to cover all users of final goods.

and the prices of final products. The supply equations (T1.3) are quite general. A & R recognize that price changes (at least initially) can alter the relative profitability of industries, and cause resource shifts and changes in the level of resource utilization. Therefore they allow for the output of (ij) to be influenced by all price changes in country j. (T1.4) are straightforward market clearing equations.[11] Equation (T1.5) implies that there is a "world price", p_{ij}, for each product. p_{ij} is in a numeraire currency, say SDR, and T_k is k's exchange rate--number of units of k's currency per SDR. (T1.6) defines the movements of the national output indexes,[12] and the last equation (T1.7) gives the changes in aggregate expenditure on final goods in each country.

Although quite large, the model presents no particular computational difficulties, each of the equations being linear.[13] It is only necessary to check that the number of endogenous variables equals the number of independent structural equations. A & R make the conventional assumption that consumers always plan their purchases so as to satisfy their budget constraints. Hence, equations (T1.7) are redundant—the parameter values in equations (T.2) are set so that (T1.7) can be deduced from (T1.2). On eliminating (T1.7), Table I indicates that $2HJ^2 + 2HJ + J$ equations remain, with $2HJ^2 + 2HJ + 3J$ variables.

TABLE I

The International Monetary Fund's Multilateral Exchange Rate Model

Identifier	Equation	Subscript Range	Number	Description
(T1.1)	$\overset{*}{x}{}^{k}_{ij} = \epsilon^k_{ij}\,\overset{*}{O}_k + \sum_{f=1}^{G}\sum_{m=1}^{J} \eta^k_{(ij)(fm)}\,\overset{*}{p}{}^{k}_{fm}$	$i = 1 \ldots G$; $j, k = 1 \ldots J$	GJ^2	Demand for intermediate products.
(T1.2)	$\overset{*}{x}{}^{k}_{ij} = \epsilon^k_{ij}\,\overset{*}{D}_k + \sum_{f=G+1}^{H}\sum_{m=1}^{J} \eta^k_{(ij)(fm)}\,\overset{*}{p}{}^{k}_{fm}$	$i = G+1 \ldots H$; $j, k = 1 \ldots J$	$(H-G)J^2$	Demand for final products.
(T1.3)	$\overset{*}{x}_{ij} = \sum_{f=1}^{H}\sum_{m=1}^{J} \gamma_{(ij)(fm)}\,\overset{*}{p}{}^{j}_{fm}$	$i = 1 \ldots H$. $j = 1 \ldots J$	HJ	Supply functions.
(T1.4)	$\overset{*}{x}_{ij} = \sum_{k=1}^{J} \overset{*}{x}{}^{k}_{ij} A^k_{ij}$	$i = 1 \ldots H$. $j = 1 \ldots J$	HJ	Market clearing equations.
(T1.5)	$\overset{*}{p}{}^{k}_{ij} = \overset{*}{p}_{ij} + \overset{*}{T}_k$	$i = 1 \ldots H$; $j, k = 1 \ldots J$	HJ^2	Relationships between numeraire and local currency prices.
(T1.6)	$\overset{*}{O}_k = \sum_{i=1}^{H} \overset{*}{x}_{ik} B_{ik}$	$k = 1 \ldots J$	J	Real national output.
(T1.7)	$\overset{*}{D}_k = \sum_{i=G+1}^{H}\sum_{j=1}^{J} (\overset{*}{x}{}^{k}_{ij} + \overset{*}{p}{}^{k}_{ij})\, C^k_{ij}$	$k = 1 \ldots J$	J	Total expenditure on final goods.

NOTATION: The MERM variables are all percentage changes, and this is denoted by *. For example, $\overset{*}{y}$ is the percentage change in y. The parameters are either elasticities (denoted by Greek letters) or shares (denoted by Roman capitals). The countries included in the model are numbered $1 \ldots J$ and the goods are $1 \ldots H$. The first G goods are intermediate goods and the rest are final goods. Good H is also a non-traded good (see footnote 9). The variables and parameters are defined in the remainder of the table.

[11] It is worth pointing out that the market clearing equations ensure that the sum across countries of the trade balances (calculated in a common currency) is zero.

[12] (T1.6) corresponds to A & R's equation (19):

In A & R's version, $B_{ik} = (p^k_{ik} x_{ik}) / \sum_{i=1}^{H} p^k_{ik} x_{ik}$. However, it is clear that they intended the B_{ik} to be defined as in Table I.

[13] Many variables can be eliminated by substitution before a matrix inversion is necessary.

TABLE I (continued) : *The International Monetary Fund's Multilateral Exchange Rate Model*

Variables Symbol	Subscript Range	Number	Definition
x_{ij}^k	$i = 1 \ldots H$; $j,k = 1 \ldots J$	HJ^2	The demand in country k for good i produced in country j, *i.e.*, k's demand for (ij).
x_{ij}	$i = 1 \ldots H$; $j = 1 \ldots J$	HJ	The supply of (ij).
0_k	$k = 1 \ldots J$	J	The real national output index in k.
D_k	$k = 1 \ldots J$	J	k's total expenditure on final goods.
p_{ij}^k	$i = 1 \ldots H$; $k,j = 1 \ldots J$	HJ^2	The price of (ij) in k's currency.
p_{ij}	$i = 1 \ldots H$; $j = 1 \ldots J$	HJ	The world price, in SDR, of (ij).
T_k	$k = 1 \ldots J$	J	k's exchange rate, units of k's currency per SDR.
Parameters (Elasticities)			
ϵ_{ij}^k	$i = 1 \ldots H$; $j,k = 1 \ldots J$	HJ^2	The definitions of elasticity parameters are clear from the structural equations.
$\eta_{(ij)(fm)}^k$	$f,i = 1 \ldots G$; $k,j,m = 1 \ldots J$	G^2J^3	For example, $\eta_{(ij)(fm)}^k$ obviously refers to the effect on k's demand for (ij) of changes in the price of (fm), *i.e.*, $\eta_{(ij)(fm)}^k$ is k's cross elasticity of demand for product (ij) with respect to changes in the price of (fm).
$\eta_{(ij)(fm)}^k$	$f, i = G+1 \ldots H$; $k,j,m = 1 \ldots J$	$(H-G)^2J^3$	
$\gamma_{(ij)(fm)}$	$f,i = 1 \ldots H$; $m,j = 1 \ldots J$	H^2J^2	

TABLE I (concluded) : *The International Monetary Fund's Multilateral Exchange Rate Model*

(Share Parameters)

A_{ij}^k is the proportion of the output of (ij) which is used in country k.

B_{ik} is the weight of (ik) in the real national output index of country k, *i.e.*, B_{ik} is the share of total value added produced in economy k which is contributed by industry (ik).

C_{ij}^k is the share of k's expenditure on final products which is devoted to product (ij).

Another share parameter which is used later in the paper is

S_{ij}^k, the share of product (ij) in k's expenditure on good i.

The system is closed by setting values for $2J$ variables. This could be done in a variety of ways, but under a regime of government-administered exchange rates, the $\overset{*}{T}_k$, $k = 1 \ldots J$ are an obvious choice as exogenous variables. In their November 1973 paper, A & R chose the $\overset{*}{0}_k$, $k = 1 \ldots J$ as the other exogenous variables. Hence, they solved the model by expressing each of the price, demand and supply variables as functions (linear in percentage changes) of exchange rates and aggregate real outputs.

In some experiments, the $\overset{*}{0}_k$ were set at zero and A & R computed the effects on trade flows and prices of a given set of exchange rate changes, under conditions in which governments do not allow aggregate real output (and presumably employment) to be affected by international developments. In particular, under the restriction $\overset{*}{O}_k = 0$, the model can be solved to generate equations of the form[14]

$$\left\{ \begin{array}{l} \overset{*}{M}_k = \sum_j u_{kj} \overset{*}{T}_j \\ \overset{*}{E}_k = \sum_j v_{kj} \overset{*}{T}_j , \qquad k = 1 \ldots J \end{array} \right. \tag{3.1}$$

where M_k and E_k are the values of country k's imports and exports (in terms of the numeraire currency, SDR) and the u_{kj} and v_{kj} are reduced from elasticity parameters. (3.1) implies that

$$100 \; \Delta B_k = \sum_j (u_{kj} M_k - v_{kj} E_k) \overset{*}{T}_j , \qquad k = 1 \ldots J$$

where ΔB_k is the change in the SDR value of the balance of trade of country k arising from the exchange rate changes, $\overset{*}{T}_j$. Finally, an effective exchange rate calculation (relative to the SDR value of the balance of trade) can be made by solving for e_k in the equation

$$0 = \sum_{j \neq k} (u_{kj} M_k - v_{kj} E_k) \overset{*}{T}_j + (u_{kk} M_k - v_{kk} E_k) (\overset{*}{T}_k - e_k) ,$$

giving

$$e_k = \overset{*}{T}_k - \sum_{j \neq k} w_{kj} \overset{*}{T}_j$$

where

$$w_{kj} = \frac{u_{kj} M_k - v_{kj} E_k}{- u_{kk} M_k + v_{kk} E_k} , \qquad j \neq k . \tag{3.2}$$

(3.2) defines a weighting scheme for a MERM calculation of effective exchange rate changes. Other MERM weighting schemes are also possible, even where "effective" means "effective" relative to the same variable, *viz.*, the SDR value of the balance of trade. For example, we might use the D_k, $k = 1 \ldots J$, as exogenous variables rather than the O_k. The model would then be solved by expressing each of the price, demand and supply variables as functions of exchange rates and aggregate final expenditures. If the $\overset{*}{D}_k$ were set to zero, we could again compute effective exchange rate changes by solving for an e_k of the correct size to sterilize the movements in B_k which would otherwise be induced by movements in exchange rates. These e_k's would measure the impact of world exchange rate changes on the home country under the assumption that levels of final expenditure (rather than outputs) were controlled independently of exchange rate changes.[15]

It is interesting to investigate the properties of a MERM weighting scheme such as (3.2). First, it is not certain that all the MERM weights are positive. We would expect the denominator of (3.2) to be positive : in the absence of highly perverse elasticities the model

[14] The model can be solved for the $\overset{*}{x}{}^k_{ij}$'s and $\overset{*}{p}_{ij}$'s. The $\overset{*}{E}_k$'s and $\overset{*}{M}_k$'s are then formed by making the obvious aggregations.

[15] There seems to be no public recognition by the IMF that several MERM weighting schemes are possible. The Annual Report [10] does not indicate which was used for the calculations reported there, and we can only assume that it was the fixed output version—the case presented by A & R.

parameters will be such that a devaluation by country k will increase the SDR value of its exports ($v_{kk} > 0$) and decrease the SDR value of its imports ($u_{kk} < 0$). In the usual case, the numerator will also be positive. A devaluation by country j will decrease k's exports ($v_{kj} < 0$) and increase its imports ($u_{kj} > 0$). However, v_{kj} could be positive if country k's exports are complementary with those of country j, whilst u_{kj} could be negative if world demand for imports from j is inelastic so that j's devaluation results in sizeable price reductions for its products.

A second feature of the MERM weights is that they may not sum to one, *i.e.*, $\sum_{j \neq k} w_{kj}$ is not necessarily equal to one. In Table I, it is clear that a 10 per cent devaluation of all currencies (*i.e.*, $\overset{*}{T}_k = 10$ for all k) implies a 10 per cent decrease in all world prices (p_{ij}). There are no changes in the p_{ij}^k or any of the real variables. Hence, MERM implies that the SDR values of imports and exports are homogeneous of degree -1 in the T_k's. Therefore, from (3.1)

$$\sum_j u_{kj} = \sum_j v_{kj} = -1$$

which leads, *via* (3.2), to

$$\sum_{j \neq k} w_{kj} = 1 - (M_k - E_k)/(-u_{kk}M_k + v_{kk}E_k). \tag{3.3}$$

On the other hand, if we compute effective exchange rate changes with Ξ_k being the \$US value of k's balance of trade rather than the SDR value, the appropriate MERM weights will satisfy $\sum_{j \neq k} w_{kj} = 1$. The reader can obtain this result by noting that the MERM implies that trade balances evaluated in any of the national currencies are homogeneous of degree zero in the T_k's where it is worth restating that the T_k's are exchange rates expressed against SDR and not a national currency.

The final noteworthy aspect of the MERM weights is that w_{kj} can be non-zero even when there is no trade between countries k and j. MERM allows j's exchange rate changes to affect k's balance of trade *via* their impact on the world prices for k's products. For example, if j's exports are similar products to k's, then a devaluation by j will reduce both the price and quantity of k's exports.

In view of the differences in the properties of the weighting schemes, it is not surprising that the results of effective exchange rate calculations using MERM can differ markedly from those based on the conventional formula (2.2). The IMF Annual Report (IMF [10] shows that the trade weights imply that the USA has effectively devalued by 5 per cent over the period June 1972 to June 1975, whereas the MERM implies an effective devaluation of nearly 13 per cent.[16]

However, we should not accept the MERM calculations uncritically. The present MERM structure contains some potentially serious shortcomings.

First, each good is classified *exclusively* as either an intermediate good or a final good. This is unrealistic, especially when we are dealing with highly aggregate commodity groups. For example, A & R classify "manufacturers" (SITC 5–9) as a final good, but it is obvious

[16] IMF [10, p. 29].

that a significant proportion of the production and trade of manufactured goods is directed toward satisfying the demands for intermediate inputs. Similarly, A & R are faced with the awkward question as to whether the "non-traded" good (good H) is an intermediate good or a final good. They choose final good, but services, construction, etc., are also used as intermediate inputs. The failure of MERM to allow goods of the same type to be classified as both intermediate and final has unsatisfactory implications. For example, according to equations (T1.1) – (T1.2), the MERM structure allows national output, 0_k, in country k to increase with no direct[17] impact on the demand for manufactured goods (domestic or imported) or non-traded goods.

A second peculiarity of the MERM structure is that it does not relate the demands for intermediate inputs to the product composition of national outputs. It is easy to visualize a situation in which this simplification would lead to serious errors. For example, a major effect of a devaluation by country k would be to change the structure of its national output in favour of export and import competing industries. (This effect is captured *via* the supply equations (T1.3).) The impact of k's devaluation on country j may depend largely on whether or not country j is a supplier of inputs to k's export industries. However, equation (T1.1) relates k's demand for inputs from j to the aggregate level of k's output and precludes MERM from reflecting the effects of changed industrial structure on the demand for intermediate inputs.

A third problem is that the model recognizes only one type of user of final products. Equation (T1.2) implies that a 5 per cent increase in D_k has the same effect on country k's demand for final products irrespective of whether it is undertaken by households, the government or investors. In fact, it appears that A & R assume that demands for final products are made as though all of D_k is used to satisfy the needs of households.[18] Notice that price changes for intermediate products are excluded from (T1.2). While it may be true that household consumption decisions are independent of movements in the prices of intermediate inputs, the same cannot be said for investment decisions. An increase in the price of (ik) or a reduction in (ik)'s material input costs will motivate investment. If capacity creation in (ik) is highly import intensive, there will be increases in country k's imports from its major suppliers of capital goods. Since for many countries, capital goods and inputs into their construction are a major component of international trade, a multilateral exchange rate model should contain an explicit theory of investment behaviour, emphasizing the effect of international price movements on the distribution of investment expenditures between industries and the resulting effects on trade flows.

IV. The Underlying Theory and the Determination of MERM'S Parameters

Despite the limitations built into its structure, MERM contains an immense number of parameters.[19] Hence, in order to simplify the estimation problems, A & R make various

[17] Some indirect effect may occur *via* the impact of changes in 0_k on prices and the level of final expenditure, D_k. But there is nothing in the MERM structure to suggest that the indirect affects adequately reflect the obvious direct effects, *i.e.*, the demand for manufactured and non-traded intermediate inputs.

[18] This view will be confirmed in Section IV. There we will see that the estimation of the parameters in (T1.2) is based on the standard model of household behaviour–utility maximization subject to budget constraints.

[19] With $G = 2$, $H = 5$, $J = 15$, the system (T1.1) – (T1.6) contains over 33,000 elasticity parameters, even after good H is declared non-traded.

assumptions about producer and consumer behaviour. Many of these assumptions are not explicit in the A & R paper. Therefore, this section presents what appear to be A & R's production, utility and behavioural assumptions. Only when these assumptions are made explicit it is possible to understand and assess the estimating procedures.

Producers in each country, k are assumed to be efficient: for any given level of national output, 0_k, the costs of intermediate inputs are minimized, *i.e.*, x_{ij}^k, $i = 1 \ldots G$, $j = 1 \ldots J$ minimize

$$\sum_{i=1}^{G} \sum_{j=1}^{J} p_{ij}^k x_{ij}^k \tag{4.1}$$

subject to $F^k(x_{11}^k, x_{21}^k, \ldots, x_{GJ}^k) \geqslant 0_k$ where F^k is the aggregate production function. In particular F^k is written as

$$F^k = \min \left(\frac{x_1^k}{a_1^k}, \frac{x_2^k}{a_2^k}, \ldots, \frac{x_G^k}{a_G^k} \right) \tag{4.2}$$

where

$$\sum_{j=1}^{J} \left(\frac{x_{ij}^k}{x_i^k} \right)^{q_{ij}^k} Q_{ij}^k = 1 , \qquad i = 1 \ldots G \tag{4.3}$$

and x_i^k is an index of the total use of good i as an intermediate input in country k. The precise form of the index is specified by (4.3)[20], where the q_{ij}^k, Q_{ij}^k are parameters. Notice that if $q_{ij}^k = 1$ for $j = 1 \ldots J$, then (4.3) implies that $x_i^k = \sum_{j=1}^{J} x_{ij}^k Q_{ij}^k$, *i.e.*, x_i^k is simply a weighted sum of the inputs of i from different sources. However, when we are using highly aggregated data, the physical composition of imports of good i from two different countries may be quite different. The general index, (4.3), allows us a flexible way in which to recognize that inputs of (ij) may not be perfect substitutes for inputs of (im), $m \neq j$, *i.e.*, as we increase the inputs of (ij) relative to inputs of i from other sources, there will be a decrease in the ability of additional units of (ij) to augment the "effective supply", x_i^k, of intermediate input i. This idea is captured by setting $q_{ij}^k < 1$ for all (ij). (For convenience, we will also be assuming that $q_{ij}^k \neq 0$.)

The a_i^k, $i = 1 \ldots G$ in (4.2) are positive parameters. Hence, (4.2) is an aggregate Leontief production function. It implies that an additional unit of output requires at least a_i^k additional "effective" units of intermediate input i for each $i = 1 \ldots G$.

On performing the optimization indicated by the model (4.1) – (4.3), it is not hard to show that

$$\epsilon_{ij}^k = 1 \, , \quad \text{for all} \;\; i \epsilon \{1 \ldots G\}; \; j \epsilon \{1 \ldots J\} \, . \tag{4.4}$$

$$\eta_{(ij)(fm)}^k = 0 \text{ for all } f \neq i; \; i, f \epsilon \{1 \ldots G\}; \; m, j \epsilon \{1 \ldots J\} \, . \tag{4.5}$$

[20] (4.3) is a CRESH (constant ratios of elasticities of substitution, homogeneous) index and is discussed by Hanoch [6]. CRESH is a generalization of the well-known CES (constant elasticity of substitution) index. If $q_{ij}^k = q_i^k$ for all j, then (4.3) defines a CES index. By using CRESH rather than CES, A & R are able to allow the elasticity of substitution between products (ij) and (im) to differ from that between (ij) and (iq), where j, m and q are distinct.

and

$$\eta^k_{(ij)(im)} = \frac{\alpha^k_{ij}\alpha^k_{im}S^k_{im}}{\sum_{q=1}^{J} \alpha^k_{iq}S^k_{iq}} - \delta_{jm}\alpha^k_{ij} \text{ for all } i\epsilon\{1\ldots G\}; j, m\epsilon\{1\ldots J\} \quad (4.6)$$

where $\alpha^k_{ij} = 1/(1 - q^k_{ij})$, $\delta_{jm} = 1$ for $j = m$ and 0 otherwise, and the S^k_{iq} are share parameters defined in Table I. It is assumed that the S^k_{iq} are directly computable from published statistics.

No mathematical analysis is necessary to see how restrictions (4.4) and (4.5) follow from the model (4.1) – (4.3). If we increase output, 0_k, by 1 per cent, the aggregate Leontief production function implies that each of the x^k_i must be increased by 1 per cent. Then in the absence of any price changes, (4.3) implies that the cost minimizing way to provide the 1 per cent increase in x^k_i is to increase each of the x^k_{ij} by 1 per cent.[21] Hence $\epsilon^k_{ij} = 1$. Restriction (4.5) can be explained as follows: with no changes in 0_k, (4.2) implies that price changes will not affect the x^k_i. Also, a change in price p^k_{fm}, $f \neq i$ will not affect the cost minimizing selection of products (ij), $j = 1 \ldots J$ required to generate x^k_i. Hence (4.5). Some formal algebra is necessary to derive (4.6). The interested reader can investigate the first-order conditions in the problem,[22]

choose $\quad x^k_{ij}, j = 1 \ldots J$

to minimize $\quad \sum_{j=1}^{J} x^k_{ij}p^k_{ij}$

subject to (4.3) where x^k_i is treated as a given constant.

The payoff from the A & R assumptions (*i.e.*, cost minimization, Leontief aggregate production function and CRESH indexing functions) can be assessed by comparing the

[21] If an equal proportionate increase in all the x^k_{ij} is not the cost minimizing method of providing an increase in x^k_i, then the initial levels of x^k_{ij} could not be the cost minimizing combination to provide the initial level of x^k_i.

[22] The i subscript and the k superscript play no role in the optimization and it is convenient to omit them. The resulting notation in this footnote should not be confused with notation in other parts of the paper. Without the i and k, and ruling out the possibility of corner solutions by assuming that $q_j < 1$ for all j, we can write the first order conditions as

(F1) $\quad p_j = \lambda q_j (x_j/x)^{q_j - 1} \; Q_j/x$, and

(F2) $\quad \sum_j (x_j/x)^{q_j} \; Q_j = 1$, where λ is the Lagrangean multiplier on (4.3). By totally differentiating (F1) with respect to λ, p_j and x_j (x is treated as a constant) and expressing the result in terms of proportionate changes, we obtain

(F3) $\quad \overset{*}{p}_j = \overset{*}{\lambda} + (q_j - 1)\overset{*}{x}_j$. Also, (F2) implies that

$\sum_j \overset{*}{x}_j q_j (x_j/x)^{q_j} \; Q_j = 0$, and via (F1) this reduces to

(F4) $\quad \sum_j \overset{*}{x}_j S_j = 0$. From (F3) and (F4), we find that

$\overset{*}{\lambda} = (\sum_t \alpha_t S_t \overset{*}{p}_t)/\sum_t \alpha_t S_t$. Hence,

(F5) $\quad \overset{*}{x}_j = -\alpha_j \overset{*}{p}_j + \alpha_j(\sum_t \alpha_t S_t \overset{*}{p}_t)/\sum_t \alpha_t S_t$. (4.6) follows from (F5).

number of unknown parameters in the unrestricted equations with the number remaining after imposing (4.4) – (4.6). For example, if $G = 2$ and $J = 15$, as in the A & R model, then for each country k, there are $G^2J^2 = 900$ elasticities of the form $\eta^k_{(ij)\ (fm)}$, $i,f \epsilon \{1 \ldots G\}; j,m \epsilon \{1 \ldots J\}$, and 30 of the form ϵ^k_{ij}, $i \epsilon \{1 \ldots G\}$; $j \epsilon \{1 \ldots J\}$. With the restrictions (4.4) – (4.6), all the elasticity parameters can be computed once values have been estimated for the $30\alpha^k_{ij}$'s, *i.e.*, the number of unknown parameters has been reduced from 930 to 30.

For the behaviour of consumers, A & R appear to assume the following: in each country k, the x^k_{ij}, $i = G+1 \ldots H$, $j = 1 \ldots J$, are chosen to maximize

$$\min \left(\frac{x^k_{G+1}}{a^k_{G+1}}, \frac{x^k_{G+2}}{a^k_{G+2}}, \ldots, \frac{x^k_H}{a^k_H} \right) \tag{4.7}$$

where

$$\sum_{j=1}^{J} \left(\frac{x^k_{ij}}{x^k_i} \right)^{q^k_{ij}} Q^k_{ij} = 1 \ , \ i = G+1 \ldots H \ , \tag{4.8}$$

and

$$\sum_{i=G+1}^{H} \sum_{j=1}^{J} p^k_{ij} x^k_{ij} = D_k \ . \tag{4.9}$$

(4.7) is a fixed coefficient utility function—the a^k_i are positive parameters. x^k_i is an index of the consumption of good i and the indexing function is given by (4.8). (4.9) is the budget constraint.

The implications of the model (4.7) – (4.9) can most easily be seen *via* the Hicks-Slutsky partition of the total effect of a price change into its income and substitution effects,[23] *i.e.*,

$$\eta^k_{(ij)\ (fm)} = -C^k_{fm}\epsilon^k_{ij} + \bar{\eta}^k_{(ij)\ (fm)} \tag{4.10}$$

where $\bar{\eta}^k_{(ij)\ (fm)}$ is country k's compensated cross elasticity of demand for (ij) with respect to changes in the price of (fm) and the remaining notation is defined in Table I.

It is clear that (4.7) – (4.9) imply

$$\epsilon^k_{ij} = 1 \quad \text{for all } i \epsilon \{G+1 \ldots H\}; j \epsilon \{1 \ldots J\} \ . \tag{4.11}$$

Also,

$$\bar{\eta}^k_{(ij)\ (fm)} = 0 \ \text{ for all } i \neq f,\ i,f \epsilon \{G+1 \ldots H\}; j,m \epsilon \{1 \ldots J\} \ . \tag{4.12}$$

[23] A short reminder on the Hicks-Slutsky partition may be useful. $\bar{\eta}^k_{(ij)\ (fm)}$ is the percentage effect on k's consumption of (ij) when there is a 1 per cent increase in p^k_{fm} and at the same time, D_k is increased sufficiently to allow consumers to maintain their initial level of utility. In textbook discussions of the Hicks-Slutsky partition, the direct utility function, unlike (4.7), is everywhere differentiable with respect to consumption quantities. However, the Hicks-Slutsky partition does not depend on the existence of marginal utilities for products. All that is required is the existence of derivatives (of the indirect utility function) of the form $\partial U/\partial D$ and $\partial U/\partial p$ where $\partial U/\partial D$ is the marginal utility of expenditure and $\partial U/\partial p$ refers to the change in utility resulting from a change in a particular price (all other prices and the expenditure level remaining fixed).

(4.12) can be explained as follows: (4.7) implies that a compensated increase in the price of (fm) will leave each of the x_i^k, $i = G+1 \ldots H$ unchanged. If there is no change in x_i^k or the price of products in the ith group, there will be no change in the x_{ij}^k, $j = 1 \ldots J$. On the other hand, compensated changes in the prices of products within the ith group will influence the x_{ij}^k's. It can be shown that

$$\bar{\eta}^k_{(ij)\,(im)} = \frac{\alpha_{ij}^k \alpha_{im}^k S_{im}^k}{\sum\limits_{q=1}^{J} \alpha_{iq}^k S_{iq}^k} - \delta_{jm}\alpha_{ij}^k \tag{4.13}$$

where $\alpha_{ij}^k = 1/(1 - q_{ij}^k)$, $\delta_{jm} = 1$ for $j = m$ and 0 otherwise, and the S_{iq}^k are share parameters defined in Table I. (4.13) is derived in the same way as (4.6), *i.e.*, we determine the effect of a change in p_{im}^k on the cost minimizing bundle of products (ij), $j\epsilon\{1 \ldots J\}$ required to generate a fixed level for x_i^k.

With the system (4.10) – (4.13), the problem of determining all the elasticities in the final demand equations (T1.2) is reduced to determining the parameters α_{ij}^k for all $i\epsilon\{G+1 \ldots H-1\}$[24], $k, j\epsilon\{1 \ldots J\}$.

Although the two models (4.1) – (4.3) and (4.7) – (4.9) dramatically reduce the number of unknown demand equation parameters, further simplification is necessary. With $J = 15$, $H - 1 = 4$, there are 900 parameters of the form α_{ij}^k, $k, j\epsilon\{1 \ldots J\}$, $i\epsilon\{1 \ldots H-1\}$. A & R make the assumption that

$$\alpha_{ij}^k = \alpha_{ij} \quad \text{for all} \quad k \neq j;\ j, k\epsilon\{1 \ldots J\};\ i\epsilon\{1 \ldots H-1\} \tag{4.14}$$

and thus reduce the estimation problem to that of determining the 120 parameters α_{ij}^j, α_{ij}, $i\epsilon\{1 \ldots 4\}$, $j\epsilon\{1 \ldots 15\}$. Unfortunately, they provide almost no economic interpretation of their assumption, and leave their readers uncertain as to what it implies in terms of familiar elasticity concepts.[25] However, on the basis of (4.14), they describe an ingenious method for determining the unknown parameters from observed import and export demand elasticities. They note that the value of k's imports of good i is $M_i^k = \sum_{j\neq k} p_{ij}^k x_{ij}^k$. If there is a 1 per cent increase in the price of all imported products of the ith type and $\mathring{0}_k = 0$ in the case where i is an intermediate good, and $\mathring{D}_k = 0$ for a final good, then the percentage change in M_i^k is given by

$$\mathring{M}_i^k = \sum_{j\neq k} \Big(1 + \sum_{m\neq k} \eta^k_{(ij)\,(im)}\Big) R_{ij}^k$$

where

$$R_{ij}^k = p_{ij}^k x_{ij}^k / M_i^k$$ Hence,

[24] Domestic sources are the only means of satisfying requirements for good H (see footnote 9). Hence we may assume that $x_{Hk}^k = x_{H}^k$ and set $q_{Hj}^k = ½$ (say) with $Q_{Hj}^k = 1$ for $j = k$ and 0 otherwise. Therefore $\alpha_{Hj}^k = 2$ for all j and k, and the unknown parameters are reduced to α_{ij}^k where $i\epsilon\{G+1 \ldots H-1\}$, $k, j\epsilon\{1 \ldots J\}$.

[25] Armington's appendix to the A & R paper provides an interesting interpretation of the α_{ij}^k. A further development of his ideas on "competing" and "non-competing" shares (A & R, p. 610) might lead to an economically more meaningful simplification than (4.14).

$$\eta_i^k = \sum_{j\neq k}\ \sum_{m\neq k} \eta^k_{(ij)\,(im)}\, R^k_{ij} \quad \text{for all } i\epsilon\{1\ldots H-1\},\ k\epsilon\{1\ldots J\}, \tag{4.15}$$

where η_i^k is country k's *volume*[26] elasticity of demand for imports of good i.

Next they note that the value of j's exports of good i in terms of j's currency is

$$E_i^j = \sum_{k\neq j} p^j_{ij} x^k_{ij} \tag{4.16^{27}}$$

Hence,

$$\eta_{ij} = \sum_{k\neq j} \eta^k_{(ij)\,(ij)}\, Z^k_{ij} \quad \text{for all } i\epsilon\{1\ldots H-1\}, j\epsilon\{1\ldots J\}, \tag{4.17^{28}}$$

where $Z^k_{ij} = p^j_{ij}x^k_{ij}/E^j_i$ and η_{ij} is the volume elasticity of demand for country j's exports of good i. Then they quote various studies to provide values for the 120 import and export demand elasticities η^j_i, η_{ij}, $i = 1\ldots 4$, $j = 1\ldots 15$, and finally, they substitute (4.6), (4.10), (4.11), (4.13) and (4.14) into (4.15) and (4.17) and solve the resulting 120 equations for the 120 unknowns, α^j_{ij}, α_{ij}, $i = 1\ldots 4$, $j = 1\ldots 15$.

In contrast to their treatment of the demand equations, A & R introduce very little theory to help them estimate the supply equation parameters [equation (T1.3)]. They appear to assume that

$$x_{ij} = f_{ij}\left(\frac{p^j_{1j}}{s_{1j}}, \frac{p^j_{2j}}{s_{2j}}, \ldots, \frac{p^j_{Hj}}{s_{Hj}}\right) \quad \text{for all } i\epsilon\{1\ldots H\}, j\epsilon\{1\ldots J\} \tag{4.18}$$

where s_{ij} is a measure of the unit wage and material costs in industry (ij). (4.18) implies that the volume of supply of product (ij) depends on the percentage return per dollar of input costs in industry (ij) compared with the returns in other industries in country j. From (4.18), we obtain

$$\overset{*}{x}_{ij} = \sum_{n=1}^{H} \beta'_{(ij)\,(nj)}(\overset{*}{p}{}^j_{nj} - \overset{*}{s}_{nj}), \quad i = 1\ldots H,\ j = 1\ldots J \tag{4.19}$$

where $\beta'_{(ij)\,(nj)}$ is the cross-supply elasticity defined by

$$\beta'_{(ij)\,(nj)} = \frac{\partial f_{ij}}{\partial(p^j_{nj}/s_{nj})}\ \frac{p^j_{nj}/s_{nj}}{f_{ij}}, \quad i.e., \ \beta'_{(ij)\,(nj)}$$

is the volume elasticity of supply of (ij) with respect to changes in p^j_{nj}/s_{nj}. To make (4.19) look more like the A & R supply equations,[29] we will define

$$\beta_{(ij)\,(nj)} = \beta'_{(ij)\,(nj)} + \delta_{ni}$$

where $\delta_{ni} = 1$ if $n = i$ and $\delta_{ni} = 0$ if $n \neq i$. The $\beta_{(ij)\,(nj)}$ are value elasticities rather than volume elasticities.[30] Then (4.19) can be rewritten in the form

[26] $\eta^k_i = \overset{*}{M}{}^k_i - 1$ where $\overset{*}{M}{}^k_i$ is the percentage change in the *value* of k's imports of i given a 1 per cent increase in the prices of i from all foreign sources.

[27] (4.16) is true on the assumption that country j's selling price for product (ij) is the same for all markets.

[28] (4.17) is derived on the assumption that $\overset{*}{p}{}^j_{ij} = \overset{*}{p}{}^k_{ij}$ for all k.

[29] A & R's equation (15).

[30] A & R's statement that "the value elasticity equals the volume elasticity plus unity" (A & R, page 598) is true only for own price elasticities.

$$\overset{*j}{p}_{ij} + \overset{*}{x}_{ij} = \sum_{n=1}^{H} \beta_{(ij)(nj)} (\overset{*j}{p}_{nj} - \overset{*}{s}_{nj}) + \overset{*}{s}_{ij} \quad \text{for all } i, j, \tag{4.20}$$

which is equivalent to the A & R equation (15).

Following the A & R approach, it seems that an appropriate specification for the unit cost changes is

$$\overset{*}{s}_{ij} = \rho_j b_{ij} \left\{ \sum_{n=G+1}^{H} \sum_{f=1}^{J} SCL^j_{nf}\, \overset{*j}{p}_{nf} \right\} + \sum_{n=1}^{G} \sum_{f=1}^{J} SPC^{ij}_{nf}\, \overset{*j}{p}_{nf} \quad \text{for all } i, j, \tag{4.21}$$

where SCL^j_{nf} is the share of product (nf) in the cost of living index in country j. Thus the term between the brackets is the percentage increase in the cost of living. ρ_j is the proportion of cost of living increases which is passed on to wages, and b_{ij} is the wage share of total costs in industry (ij). Thus the first term on the right of (4.21) is the percentage increase in unit costs in industry (ij) resulting from increased wages. SPC^{ij}_{nf} is the share of product (nf) in the unit costs of industry (ij) and the second term on the right side of (4.21) is the percentage increase in unit costs of industry (ij) resulting from changes in the prices of material inputs.[31]

The exact restrictions imposed on the γ's of equation (T1.3) by the model (4.18) and (4.21) can be deduced by substituting (4.21) into (4.20) and comparing the result with (T1.3). Rather than doing this, we will compute the payoff in terms of the reduction in the number of unknown parameters. With $H = 5$, $J = 15$, (T1.3) contains 5625 unknown γ parameters.[32] Assuming that the share parameters, b_{ij}, SCL^j_{nf} and SPC^{ij}_{nf} are readily observable, the unknown parameters in the system (4.20) – (4.21) are the $H^2J = 375$ β's plus the 15 ρ_j's. But apparently 390 supply parameters are still too many for estimation, and A & R simply assume values. In particular they use the same values for all countries, *i.e.*,

$$\rho_j = \rho \text{ and } \beta_{(ij)(nj)} = \beta_{(i^*)(n^*)} \quad \text{for all } j = 1 \ldots J.^{33}$$

A & R fail to give any indication of the basis for their supply elasticity assumptions. At least, in the absence of empirical estimates, a convincing presentation of MERM requires some sensitivity analysis. For example, the ρ_j's, relating wages to the cost of living indexes, may be key parameters in determining the effects of exchange rate changes on trade flows and balances. Therefore, it seems that either the results of an attempt to measure the ρ_j's should be presented or an analysis of the effects of varying the ρ_j's from the A & R assumed value of .75 should be made.

Our final task is to comment on the estimation procedures outlined in this section. Little more can be said about the supply equation parameters. Also, Section III contained

[31] Without justification, A & R exclude changes in the prices of domestically produced intermediate inputs (*i.e.*, produced in country j) from influencing material input costs in industry (ij). Instead of the second term on the right side of (4.21), they have

$$\sum_{n} \sum_{f \neq j} SPC^{ij}_{nf}\, \overset{*j}{p}_{nf}, \quad \text{[see their equation (16)]}.$$

[32] This number is reduced to 4575 when good H is specified as non-traded (see footnote 9).

[33] Some of the parameters for the "rest of the world" are set at different values from the equivalent parameter in other countries.

an analysis of the A & R approach to the demand for intermediate products, and therefore it is unnecessary to discuss model (4.1) – (4.3). But some constructive suggestions can be made about the parameters in the final goods equations.

The utility function (4.7) seems oversimplified. Expenditure elasticities are comparatively easy to measure, and there would seem to be little justification in simply assuming them all to be 1. For example, elasticities in the food group (SITC 1) have systematically been found to be less than 1.[34] Also, (4.7) inadequately represents substitution effects. A & R support their implied use of the fixed coefficient utility function by arguing that "if the various goods are clearly distinguished by the kinds of want or need they serve, the income compensated price elasticities . . . are likely to be quite small . . . [and] . . . will be assumed to be equal to zero for the purposes of simplification".[35] However, this argument is false. If various goods are clearly distinguished by the want or need they serve, then it is acceptable to assume that the utility function exhibits want independence or additivity.[36] A & R have given us the left and right shoe situation–one gains nothing from an additional right shoe without the left one. This is quite different from the additivity assumption where the utility derived from additional units of "food" is independent of (*e.g.*) the level of "clothing" consumption.

V. Summary and Conclusion

Effective exchange rate calculations are used in many countries to give a summary indication of the domestic impact of world exchange rate movements. An important part of the effect on country A of a devaluation by country B may come *via* the effects of B's actions on third countries and world commodity prices. Hence, if possible, effective exchange rate calculations should be made with reference to a multicommodity, multicountry model.

The International Monetary Fund has an important role as an agent for harmonizing the exchange rate policies of member countries. Also it is a repository for a vast amount of information on international trade and finance. For these reasons, it is very appropriate that a major research project of the Fund staff in recent years has been the development of the multilateral exchange rate model (MERM).

In Section III we described the MERM structure (Table I). Then we showed how MERM is used for effective exchange rate calculations and how the MERM "weights" differ from "trade-shares". In Section IV we introduced three models, (4.1) – (4.3), (4.7) – (4.9) and (4.18), to help us understand MERM's underlying theory. Our analysis in Sections III and IV pinpointed several surmountable weaknesses in the currently operational version of MERM. Our main suggestions, roughly in inverse order of research costs, are as follows:

34 For a pioneering multicountry study of expenditure elasticities, see Houthakker [8]. Recent research includes Lluch and Powell [11], and Lluch and Williams [12].

35 A & R, page 595.

36 An additive utility function has the property that all its cross second derivatives are zero, *i.e.*, the marginal utility of each good is independent of the consumption of other goods. There are numerous applications of additive utility functions in demand studies, for example, Frisch [4], Houthakker [9], Powell [13]. Generalization of the additivity assumption is discussed by Sono [15].

(a) The fixed coefficient utility functions (4.7) should be replaced with more general additive functions, thus allowing compensated price elasticities for goods to be different from zero. Also, the expenditure elasticities should be allowed to vary from 1. (b) The demands for intermediate inputs should be related to the compositions of the national outputs. (c) The exclusive classification of goods as either final goods or intermediate goods should be dropped. (d) A distinction should be made between households, investors and the government. It may be important to have a theory which relates the profitability of capacity expansion in each industry to international price movements. Then profitability should be translated into demand for capacity expansion, and demand for capacity expansion should be translated into demands for imports. (e) The supply functions should be formulated in such a way that estimation of the parameters becomes feasible. Only then will it be possible adequately to recognize variations among countries in the response of their exports and imports to international price movements. Sensitivity analysis on key supply parameters, especially those relating the cost of living indexes to wages, could be useful in isolating top priority areas for empirical research.

It is worth emphasizing that improvements in MERM's structure could be carried out one country at a time. There seems to be no compelling reason for preserving the model's "symmetry". For example, it might be a sensible research strategy to improve the specification of the supply and demand equations of the major trading countries before working on those of the smaller ones.

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