IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

Paper Presented to

SIMULATION CONFERENCE

SIMSIG - 1978

Canberra

September 4 - 6, 1978

SOLUTION TECHNIQUES FOR THE ORANI MODEL OF THE AUSTRALIAN ECONOMY

by

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General Paper No. G-11 Melbourne September 1978

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8. CONCLUDING REMARKS

The ORANI model of the Australian economy has been implemented using the method of solution described in the previous sections. The computer programs have been checked both by tests with data manufactured for 3 industries and 2 occupations (rather than 109 industries and 9 occupations), and by inspection of the 109-industry results to confirm that they are consistent with the equations of the model. The techniques described in section 7, which are desirable additions to the suite of programs but not of central importance to obtaining results, have not yet been implemented.

From a computational point of view a necessary development of the solution algorithm will be the allowance for large changes in the proportional change variables, and the associated non-linearities. One method of solution is to approximate a large change by a sequence of small changes and update the input-output table after each small change.

9. ACKNOWLEDGEMENTS

The ORANI model is basically the work of Peter Dixon, Associate Director of the IMPACT Project. The theoretical structure of the model, the linearization of the equations and the first stage of the elimination procedure are entirely his work, although he should not be held responsible for any misrepresentations of these matters in the present paper. I am grateful to Brian Parmenter for his helpful criticisms of the text of this paper.

If the vectors u and v are interchanged between the endogenous and exogenous sets the corresponding new solution is

$$\begin{pmatrix} v \\ y \end{pmatrix} = \left(\begin{array}{c|c} J_{11}^{-1} & -J_{11}^{-1} J_{12} \\ \hline J_{21}J_{11}^{-1} & J_{22} - J_{21}J_{11}^{-1} J_{12} \end{array}\right) \begin{pmatrix} u \\ z \end{pmatrix} .$$

For u and v of length a, y of length b, and z of length c, computation of J_{11}^{-1}, $K = J_{11}^{-1} J_{12}$, $J_{21}J_{11}^{-1}$ and $J_{21}K$ for this second matrix involves $a(a + b)(a + c)$ operations. This expression is to be compared with more than m^3 operations required for the usual method, i.e., m^3 operations for the matrix inversion plus further operations for $C = - A^{-1}B$ and possibly back solutions. For $(a + b) = m$ and $(a + c) = m$ the number of operations in the method just described is less than a/m times the number required for the normal method.

This is a very powerful technique for two reasons. Firstly, many of the solutions of interest will involve changing just a few variables between the endogenous and exogenous sets, and can be computed very cheaply by this method. Secondly, it provides a means of making <u>any</u> variable exogenous, even if it has previously been eliminated in the algebra as "always endogenous." Once a back solution for such a variable is available it can be swapped from the endogenous to the exogenous list in exchange for some variable which is already on the exogenous list. With this technique the distinction between the eliminated variables and the final system variables, with regard to exogeneity, no longer applies.

ABSTRACT

ORANI is a large and detailed mathematical model of the Australian economy which emphasizes the equilibirum relationships between different sectors of the economy at a 109-industry level of disaggregation. In addition to many accounting identities it also contains a large number of non-linear optimization problems. By restricting attention to small changes, the model can be approximated by a system of equations which is linear in proportional changes in the economic variables. As the number of variables exceeds the number of equations, the model can be used to examine a wide range of problems by selecting different sets of variables to be exogenous. Solutions are computed by eliminating many of the variables, representing the remaining variables in matrix form, solving this matrix system, and finally back solving for eliminated variables of interest.

operations in the method just described is $4s/m$ times the number required for direct inversion.

Usually the part of A which is to be changed, H, will not be suitably situated near the border of A. H can be moved to the border by premultiplying and postmultiplying A by appropriate transformation matrices K and L to form a new matrix A^*, i.e. $A^* = KAL$. K and L contain only zeros except for a single 1 in each row and each column. (The matrix multiplications are used only for the theory. Computationally there would simply be interchanges of rows and interchanges of columns.) As $K^{-1} = K^T$ and $L^{-1} = L^T$, where T denotes the transpose, the old A^{-1} can be converted to $(A^*)^{-1}$ by $(A^*)^{-1} = L^T A^{-1} K^T$. Next the new $(A^*)^{-1}$ is computed as a result of changes to A^*_{12}, A^*_{21} and A^*_{22} using the method described above. Finally, $(A^*)^{-1}$ is converted to a new A^{-1} by $A^{-1} = L(A^*)^{-1}K$.

7.2 Swapping variables between the endogenous and exogenous sets

Consider an ORANI solution (or part of a solution)

$$\begin{pmatrix} u \\ y \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} v \\ z \end{pmatrix},$$

where

- u and y are vectors of endogenous variables (which may include variables in the back solution), and
- v and z are vectors of exogenous variables.

u, v, y, z need not include all the relevant variables but can be restricted to the variables of interest.

7.1 Changes to the A matrix

Given A and A^{-1}, matrix partitioning can be used to compute the new A^{-1} which results from changes to parts of A. First consider changes which are restricted to a narrow strip around the border of A. A is partitioned into

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

where A_{11} and A_{22} are square and of full rank, and the changes are restricted to A_{12}, A_{21} and A_{22}. A is of dimensions $m \times m$ and A_{22} is of dimensions $s \times s$.

The inverse of A can be written as

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \left(\begin{array}{c|c} A_{11}^{-1} + GF & -G \\ \hline -D^{-1}F & D^{-1} \end{array}\right),$$

where

$$E = A_{11}^{-1} A_{12} \qquad F = A_{21} A_{11}^{-1}$$

$$D = A_{22} - A_{21}E \qquad G = ED^{-1}.$$

Calculation of the new A^{-1} involves extracting A_{11}^{-1} from the original M by means of $A_{11}^{-1} = M_{11} - M_{12} M_{22}^{-1} M_{21}$ and then computing the new E, D, D^{-1}, F, G, GF and $D^{-1}F$. The total number of operations is $4ms(m - s) + s^3$, which is to be compared with m^3 operations for direct inversion. For s small compared with m, the number of

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into column partitions of widths up to 110 columns each. This partitioning also enables the user to restrict computations to columns of interest.

The output of job (b) is a collection of matrix coefficients for a set of vector equations. The basic solution program (c) assembles these coefficients into a tableau of equations by variables, one column partition at a time. Many of the parameters of the model are input at this stage and are used to modify or form coefficients. The elimination of q_3 and removal of the associated equations is accomplished within the tableau. The remaining parts of the tableau are then split into matrices A and B according to the set of endogenous variables specified by the user.

Matrix A, of dimensions 250×250, is assembled and inverted as a single matrix in core. If A were much larger, or if the programs were run on a computer with a smaller memory, it would be necessary to use matrix partitioning. It should be mentioned that the inversion of A is performed with a standard matrix inversion package which uses the method of Gaussian elimination, including a search for maximum elements as pivots so as to minimize roundoff errors. For any meaningful economic problem, A can be transformed by simple row and column transformations and multiplications to an identity matrix plus a matrix of small perturbations. It follows that the computation of the inverse is relatively insensitive to round-off errors and that there is no need for exotic inversion techniques.

7. MODIFYING EXISTING SOLUTIONS

Once the solution to a particular problem has been computed it is often possible to efficiently derive solutions for related problems without repeating the entire solution procedure.

of the $q(= q_1 + q_2 + q_3)$ variables results in a system of approximately 250 equations in approximately 1250 variables which is solved by means of (7). Back solutions are computed only for those components of q_2 and q_3 which are of interest.

6. COMPUTATIONS

Computer programs which solve the ORANI model have been written in FORTRAN and implemented on the CSIRO's CYBER 76. There are separate jobs for (a) formation of the ORANI input-output table, (b) elimination of q_1 and q_2, (c) the basic solution which includes elimination of q_3 and formation of A, B, A^{-1} and C, (d) back solution and (e) printing results. The CPU times for these jobs are approximately 10, 125, 85, 15 (for 109 components) and 3 seconds respectively. It is not necessary to run all jobs for each simulation. To date, most computations have used the same (a) and (b), and it has been necessary to run only (c), parts of (d), and (e).

In anticipation that the model will be used on other computers, the programming is restricted to ANSI FORTRAN as far as possible, and options which are specific to the CYBER 76 or the CSIRONET system are restricted to easily accessible parts of the programs.

The programs involve operations on many relatively large matrices and require large amounts of core storage and disc storage. The CYBER 76 allows usage of nearly 150,000 60-bit words of core storage and current programs use a large proportion of this. The matrices B and C, of dimensions 250×1000, exceed the core storage capacity and are divided

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1. INTRODUCTION

The ORANI model has already been described in the First Progress Report of the IMPACT Project, Volume 2[1]. That document describes the theoretical structure and empirical implementation of the model and presents some results. The present paper commences with a brief account of the model and its potential applications in policy analysis. After a discussion of the types of equations in the model, the major part of the paper is concerned with the mathematical and computational aspects of obtaining solutions.

2. THE ORANI MODEL

ORANI is a large and detailed mathematical model of the Australian economy which has been developed, as part of the Australian Government's IMPACT Project[2], for policy analysis on time scales of 2 to 10 years. It is concerned

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1. Dixon, P. B., B. R. Parmenter, G. J. Ryland and J. Sutton, ORANI, A General Equilibrium Model of the Australian Economy : Current Specification and Illustration of Use for Policy Analysis - First Progress Report of the IMPACT Project, Volume 2 (Canberra : Australian Government Publishing Service, 1977).

2. Powell, Alan A., The IMPACT Project : An Overview - First Progress Report of the IMPACT Project, Volume 1 (Canberra : Australian Government Publishing Service, 1977).

* This paper has been published by the Convening Committee of the Simulation Special Interest Group in Simulation Modelling : Techniques and Applications, Proceedings of SIMSIG - 78 : Simulation Conference (Canberra : C/- Division of Computing Research, CSIRO, 1978), pp. 114-118.

primarily with the equilibrium relationships between the different sectors of the economy, and especially the relationships between the (109) different industries. The model ignores the disequilibrium phenomena which constitute the business cycle. Instead it emphasizes underlying structural relationships. ORANI will eventually interact with a macroeconomic model, which will allow for monetary and disequilibrium phenomena, and a demographic model which will describe the size and structure of the population and workforce, including allowance for education, immigration and family formation.

The ORANI model is appropriate for examining the detailed effects of changes which may be superimposed on the economy, whether by circumstances outside Australia, e.g., by changes in world prices, or by decisions taken within Australia, e.g., by changes in tariffs, quotas, exchange rate, wage relativities, etc.. For example, given a change in the import quota for motor vehicles, ORANI can be used to compute the output and employment effects in each industry, and the change in the CPI, etc., which can be attributed to this change in the quota. The model can also be used to determine which combinations of instruments could be used by the Australian government to achieve certain objectives such as a given level of real consumption or a given level of employment.

It should be emphasized that the model has been developed for the purpose of policy analysis rather than for forecasting. It is not intended to answer questions such as what will be the CPI in 4 years time. Rather, it is intended to answer questions of the kind, what will be the effect on the CPI of a specified change in the tariff rate for footwear, assuming that all other exogenous variables are held constant and assuming a specified economic environment. For ORANI the economic environment is defined by the choice of parameters, e.g., wage indexation as a percentage of the CPI, and the choice as to

However most of the variables are of no particular interest to the user. Providing there will never be a need to set them exogenously, q of the variables can be eliminated by "high school" algebra, leaving a system of $(m - q)$ equations in $(m + n - q)$ variables. The number of exogenous variables is still n but the number of endogenous variables is reduced to $(m - q)$. Solutions for the q eliminated variables can be obtained later by back substitution.

In practice the eliminations are performed in three stages,

(i) the elimination of q_1 (approximately 300,000) variables which are of no further interest to the user, e.g., the value of wholesale trade associated with the flow of goods from industry i to industry j ;

(ii) the elimination of q_2 (approximately 1,000) variables which are of interest but will always be endogenous, e.g., the prices of domestically produced goods ;

(iii) the elimination of a further q_3 (approximately 100) variables which are declared endogenous for the problem of interest. This elimination is performed within the solution program prior to forming A and B . It was judged that the use of $q_3 = 100$, rather than 0 or 200 , minimized the total computing cost of programming effort plus computer processing.

The eliminations require specification of some of the parameters of the model. The compositions of q_1 and q_2 are fixed, but the composition of q_3 is specified by the user. The elimination of

$$Ax + By = 0 \qquad (6)$$

where A is a square matrix of dimensions $m \times m$ and B is of dimensions $m \times n$. This has the solution

$$x = Cy \quad \text{where} \quad C = -A^{-1}B \,. \qquad (7)$$

This method of solution avoids any requirement for a fixed classification of variables into endogenous and exogenous. It recognizes that the model should be thought of as specifying a relationship between variables, and that the particular classification will depend on what the model is being used for. The user has the freedom to choose which are the endogenous variables, and the remaining variables are exogenous. This option allows the same model to be used to examine quite different policies simply by altering the set of endogenous variables.

The interpretation of the elements of matrix C is that a 1 per cent increase in an exogenous variable Y_i, i.e., $y = 0.01$, will produce a C_{ij} per cent increase in an endogenous variable X_i, all other variables being held constant. In all calculations to date it has been assumed that changes are sufficiently small that the endogenous changes are proportional to the exogenous changes and that the partial contributions from different exogenous changes are additive.

The numbers of operations in the inversion of matrix A and the matrix multiplication of A^{-1} with B are of the order of m^3 and m^2n respectively. For the ORANI system of equations m is approximately 300,000 and n is approximately 1000. Such a system is clearly too large to solve by the method just described.

which economic variables are exogenous, i.e., have pre-determined values. Whereas world prices of imported goods would always be exogenous as far as the ORANI model is concerned, other variables, such as employment, would be exogenous for some experiments and endogenous for others.

ORANI is a general equilibrium model, i.e., equilibrium conditions involving both quantities and prices must be satisfied simultaneously in a number of interacting markets. The equilibrium conditions usually require that in each market supply is equal to demand and price is equal to costs (including profits). However, for many uses of the ORANI model the labour market is not required to be in equilibrium.

The model also allows for (imperfect) substitution between alternative inputs such as between different consumer goods, or between imports and corresponding domestically produced goods. In such cases it is assumed that the actual combination of inputs is determined by optimization conditions such as the minimization of costs.

The equations of the model describe the economy in the base year in a state of equilibrium. A disturbance or shock is then applied. In response to this disturbance, the economy then changes to a new equilibrium state. This is achieved by changes to the endogenous variables, in accordance with constraints imposed by the equations of the model and the specified changes in the exogenous variables. The time taken to reach the new equillibrium state is indeterminate - the model will be given a time scale when it interacts with the macroeconomic model.

3. THE EQUATIONS OF THE ORANI MODEL

The equations can be considered in three categories, (1) accounting relationships, (2) behavioural equations and (3) macroeconomic equations.

The "accounting" relationships are derived from an input-output table, that is a table which shows the flows (measured in $) of goods and services in a given year between the different sectors of the Australian economy, and in particular the flows between different industries. ORANI uses a modified version of the most recent table produced by the Australian Bureau of Statistics. This refers to the year 1968/69 and became available in 1977.

A simplified version of the ORANI input-output table is shown in figure 1. The column for each industry (or final demand category) shows the inputs into that industry. The row for each industry (or factor) shows how its output is used in other sectors of the economy. The first column shows how intermediate usage, i.e., domestic production, uses inputs of domestically produced goods, imported goods, wages, fixed capital, agricultural land and other costs. The produced goods are sold to other industries and to the final demand categories of investment, household consumption, exports and government expenditure.

The "behavioural" equations describe the choices made by the various economic agents in the model. In the production process there is the choice between imported and domestically produced inputs (assumed to be imperfect substitutes for one another), the choice as to the combination of

where X_1 and X_2 are quantities of imperfectly substitutable inputs 1 and 2 (e.g., capital and labour) which constitute the effective input X, P_1 and P_2 are the unit prices of inputs 1 and 2, and b_1, b_2 and ρ are parameters.

Expressed in proportional change form, the solution is

$$x_i = x - \sigma(p_i - \sum_j S_j p_j) , \qquad (5)$$

where $\sigma = \frac{1}{1+\rho}$ and S_j is the share of $P_j X_j$ in costs PX. This result can be obtained directly without having to linearize a result obtained in levels[1].

5. MATRIX METHOD OF SOLUTION

The complete model consists of m equations which are linear in $m + n$ proportional change variables, where $n > 0$. The aim of the solution method is to solve the system of equations in a manner which is efficient with regard to computing costs and programming effort, and also allows flexibility in the choice as to which variables are endogenous and exogenous. It is also important that the user have freedom to change the values of the parameters.

The linear system of equations can be solved by specifying values for the n exogenous variables y and solving for the remaining m endogenous variables x. In matrix form ,

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1. See Dixon et al., op. cit., page 27.

where X_1 , X_2 and X_3 are variables, and W_1 and W_2 are coefficients formed from parameters. Conversion to proportional change form is by

$$dX_3 = W_1 dX_1 + W_2 dX_2$$

$$\frac{dX_3}{X_3} = \frac{W_1X_1}{X_3}\frac{dX_1}{X_1} + \frac{W_2X_2}{X_3}\frac{dX_2}{X_2}$$

to give

$$x_3 = S_1x_1 + S_2x_2 \quad , \qquad (3)$$

where S_1 and S_2 are the shares of the X_1 and X_2 terms in the expression for X_3 .

2. Similarly there are equations of the form $X_3 = X_1X_2$ which become

$$x_3 = x_1 + x_2 \ .$$

3. Finally there are the constrained optimization problems, most of which are non-linear. These yield equations which are linear in the proportional change variables. The constrained optimization problems in the ORANI model are of the form :

Given P_1, P_2 and X , determine X_1 and X_2 so as to minimize costs $PX = P_1X_1 + P_2X_2$ subject to the CES constraint

$$X = \left(b_1X_1^{-\rho} + b_2X_2^{-\rho}\right)^{-\frac{1}{\rho}} \quad , \qquad (4)$$

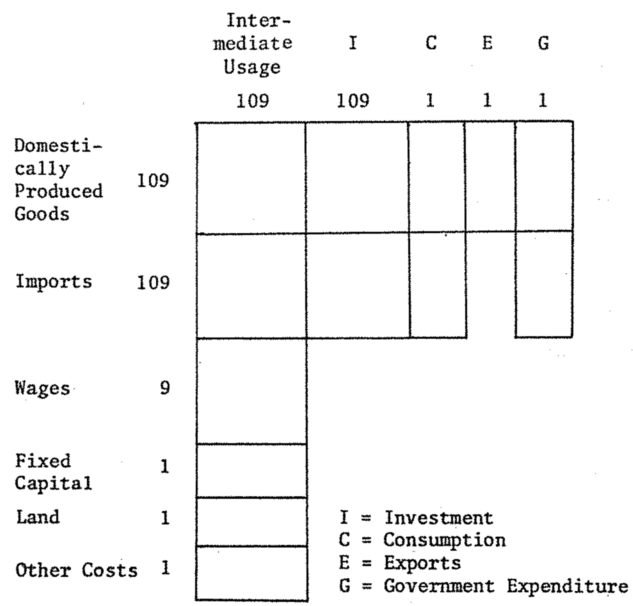

Figure 1 - ORANI Input-Output Table (simplified)

labour and fixed capital (and agricultural land), and within the labour category a choice in the composition of skills. Similarly, the entrepreneur has the choice as to which industries to invest in, and the consumer has a choice in selecting a bundle of consumption goods. The fundamental behavioural assumptions which are adopted in the ORANI model are that producers and investors minimize costs, that investors respond to relative rates of return (in such a way as to equalize expected long run rates of return on investment), and that households maximize utility (i.e., a function which reflects consumer preferences). Each of these choices involves either a budget constraint and/or a constraint on the substitution possibilities between alternative inputs. Clearly each solution depends on the relative prices of the alternative inputs.

The "macroeconomic" equations relate macroeconomic aggregates to the sums of disaggregated quantities, e.g., total employment equals the sum of employment in each occupation. They also allow for indexation of disaggregated variables to aggregate variables, e.g., the indexation of wages (by occupation) to the CPI. There are also a few equations which relate macroeconomic variables to one another. This part of the model will be expanded at a later date by addition of the macroeconomic model.

4. LINEARIZATION

Suppose that the economy can be described mathematically by the system of equations $f(X, Y) = 0$ where X is a vector of endogenous variables, Y is a vector of exogenous variables and f is a vector function. Initially the system is in equilibrium at state (X_0, Y_0). If a small disturbance dY is applied, the system will eventually reach a new equilibrium state $(X_0 + dX, Y_0 + dY)$. For sufficiently small values of dX and dY, dY is given by

$$\frac{\partial f(X_0, Y_0)}{\partial X} dX + \frac{\partial f(X_0, Y_0)}{\partial Y} dY = 0 . \qquad (1)$$

Equation (1) is linear in the increments dX and dY. Consequently, providing the number of variables exceeds the number of equations, it is possible to solve for the m endogenous variables dX in terms of the n exogenous variables dY simply by solving the appropriate set of

simultaneous equations. Such a form has the added advantage that it is possible to examine the effects of different policies simply by swapping variables between the two sets of variables dX and dY.

Although (1) is in a linear form ORANI follows a slightly different linearization which was used by Johansen[1]. Equation (1) is now written

$$\frac{\partial f(X_0, Y_0)}{\partial X} X_0 \frac{dX}{X_0} + \frac{\partial f(X_0, Y_0)}{\partial Y} Y_0 \frac{dY}{Y_0} = 0 ,$$

i.e.,

$$Ax + By = 0 ,$$

where

$$A = \frac{\partial f(X_0, Y_0)}{\partial X} X_0 , \qquad B = \frac{\partial f(X_0, Y_0)}{\partial Y} Y_0 ,$$

and lower case letters are now used to represent proportional changes in corresponding upper case variables,

i.e.,

$$x = \frac{dX}{X_0} \quad \text{and} \quad y = \frac{dY}{Y_0} .$$

For the purpose of linearization the equations of the model can be considered in three groups. (These should not be confused with the three categories used in section 3.)

1. Many of the equations are of the form

$$X_3 = W_1 X_1 + W_2 X_2 , \qquad (2)$$

1. Johansen, L., A Multisectoral Study of Economic Growth, 2nd edition, (Amsterdam : North-Holland Publishing Company, 1974). See pages 34 and 45 ff.