IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

THE ESTIMATION OF SUPPLY RESPONSE IN AUSTRALIAN AGRICULTURE : THE CRESH/CRETH PRODUCTION SYSTEM

by

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The views expressed in this paper do not necessarily reflect the opinions of the participating agencies, nor of the Australian government.

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This suggested that our model was incapable of revealing the input substitution parameter. To investigate the influence of the substitution parameter on the product transformation parameters, we have respecified the model treating h as a known constant with a 'neutral' value (0.5). The resultant k_i parameters were : k_1 (4.30), k_2 (4.68), k_3 (3.73), k_4 (1.90), k_5 (2.80), k_6 (1.78). A likelihood ratio test was carried out on the two results. The test involves calculating $-2(L_1 - L_2)$ where L_1 is the log likelihood value from the result in which the h parameter is estimated and L_2 the log likelihood value when h is treated as a known constant. The test value of 0.22 ($\chi^2_1 = 6.63$ at 1 per cent) indicates that our estimates of the transformation parameters are fairly robust despite the instability in the input substitution parameter.

III High Rainfall Zone

Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
k_1 wool	16.86	0.97
k_2 sheep	9.67	0.94
k_3 cattle	3.67	1.94
k_4 grains, etc.	1.26	1.06

II Wheat/Sheep Zone (cont'd)

	Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
W_1	wheat quota dummy on wool output 1968/69 → 1969/70	0.033	0.74
W_2	wheat quota dummy on sheep output 1968/69 → 1969/70	0.096	0.96
W_3	wheat quota dummy on cattle output 1968/69 → 1969/70	0.262	0.97
W_4	wheat quota dummy on wheat output 1968/69 → 1969/70	-0.183	1.06
W_5	wheat quota dummy on barley output 1968/69 → 1969/70	0.215	0.86
W_6	wheat quota dummy on 'other' output 1968/69 → 1969/70	-0.155	0.66
W_7	wheat quota dummy on wool output 1972/73 → 1973/74	-0.145	1.66
W_8	wheat quota dummy on sheep output 1972/73 → 1973/74	0.075	0.49
W_9	wheat quota dummy on cattle output 1972/73 → 1973/74	-0.006	0.02
W_{10}	wheat quota dummy on wheat output 1972/73 → 1973/74	0.376	1.81
W_{11}	wheat quota dummy on barley output 1972/73 → 1973/74	-0.249	0.98
W_{12}	wheat quota dummy on 'other' output 1972/73 → 1973/74	0.098	0.39
C_1	constant in equation 1	-0.016	0.37
C_2	constant in equation 2	-0.013	0.14
C_3	constant in equation 3	-0.070	0.27
C_4	constant in equation 4	-0.062	0.27
C_5	constant in equation 5	-0.044	0.26
C_6	constant in equation 6	-0.104	0.43

1. In the result reported, the estimate of h has an implausibly large negative value and an extremely high standard error.

(footnote continued)

1. Introduction[1]

Various types of separability have been postulated in the literature of production relations.[2] One possible type of separability occurs when multi-product production processes do not involve jointness. Such a lack of jointness in (say) a two product production process with three factors would mean that the single product production functions would be well defined and could be written

(1.1) $$Y_1 = f_1(\alpha_1 X_1, \alpha_2 X_2, \alpha_3 X_3) ,$$

(1.2) $$Y_2 = f_2((1 - \alpha_1)X_1 , (1 - \alpha_2)X_2 , (1 - \alpha_3)X_3) ,$$

in which α_j is the share of total usage of factor j which is allocated to the first product. Systems of this type, however, involve a degree of separability which is incompatible with the jointness known to exist between important agricultural products in Australia (e.g., between beef and wool, or between wool and wheat).[3] Indeed, from a technical (and even from an accounting) viewpoint, the shares $\{\alpha_j\}$ are often impossible to identify in

1. An account of the research reported here is available in less condensed form in Peter B. Dixon, David P. Vincent and Alan A. Powell, "Factor Demand and Product Supply Relations in Australian Agriculture : The CRESH/CRETH Production System," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. OP-08, Industries Assistance Commission, Melbourne, November, 1976, and David P. Vincent, Peter B. Dixon and Alan A. Powell, "Estimates of the CRETH Supply System in Australian Agriculture," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. OP-17, Industries Assistance Commission, Melbourne, October, 1977. Readers may prefer to write for copies of these papers than to verify step by step all of our mathematics.

2. Ragnar Frisch (Theory of Production (Dordrecht & Chicago : D. Reidel & Rand McNally, 1965), Chapter 14) devotes a chapter to "multi-ware production" in which various separability concepts are put forward. For a recent example which takes the separability idea from the consumer literature and applies it to factor demands in the context of a one-product production function, see Henri Theil, "The Independent Inputs of Production," University of Chicago, Center for Mathematical Studies in Business and Economics, Report 7535, September, 1975, pp. 80 (mimeo).

3. See, e.g., I. R. Wills & A.G. Lloyd, "Economic Theory and Sheep Cattle Combinations," Australian Journal of Agricultural Economics, Vol. 17, No. 1 (April, 1973), pp. 58-67.

any meaningful way.[1] It is more attractive to think of inputs of a non-specific nature (e.g., of fertilizer, as distinct from shearing) as determining the location of the short-run product transformation curve. If all inputs are non-specific, then the simplest option is to define a production function whose left hand variable is a scalar index Z defining generalized capacity to produce

$$Z = f(X_1, \ldots, X_M) \quad , \tag{2.1}$$

where the $\{X_j\}$ are the total levels of factors (including both primary factors and intermediate inputs) which are input into the multi-product production activity. This capacity index then serves to locate the product transformation schedule; namely,[2]

$$g(Y_1, \ldots, Y_N) = Z \; . \tag{2.2}$$

In section 2 we postulate functional forms for f and g (namely CRESH[3] and CRETH[4]. On the assumption that certain factors are fixed (e.g., land), the optimal behaviour of a representative producer is modelled within a one period setting. This generates factor demand and product supply relations which recognize the essential jointness of production relations in Australian agriculture.

1. Consider, for example, a farmer who sows a grain crop in autumn, grazes it lightly during the winter, and harvests grain from it in the early summer. What fraction of fertilizer, ploughing, weed killing, sowing and similar costs, should be attributed to the production of wool, and what fraction to grain?
2. If (2.1) and (2.2) hold (with Z a scalar) then the production function is defined by Hasenkamp to be "separable between inputs and outputs" - - see Georg Hasenkamp, Specification and Estimation of Multiple-Ouput Production Functions (Berlin, Heidelberg & New York : Springer Verlag, 1976), p. 19.
3. Giora Hanoch, "CRESH Production Functions," Econometrica, Vol. 39, No. 5 (September, 1971), pp. 695-712.
4. CRETH (to be discussed below) is in relation to CET as CRESH is to CES. The CRETH formulation was suggested by Peter B. Dixon in "The Costs of Protection : The Old and New Arguments," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. IP-02, Industries Assistance Commission, Melbourne, June, 1976.

Appendix

CRETH Supply System : Parameter Estimates

I Pastoral Zone

	Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
k_1	wool/sheep	10.61	1.10
k_2	cattle	1.62	4.36
k_3	grains, etc.	1.22	5.78
C_1	constant in equation 1	0.024	0.37
C_2	constant in equation 2	0.155	0.42

II Wheat/Sheep Zone

	Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
k_1	wool	4.36	3.13
k_2	sheep	5.27	1.11
k_3	cattle	2.93	1.71
k_4	wheat	1.62	6.74
k_5	barley	2.92	2.39
k_6	'other' products	1.76	6.60
h	input substitution parameter[1]	-88.1	0.10
D_1	coefficient on drought variable in equation 2 (sheep)	-0.345	2.81
D_2	coefficient on drought variable in equation 3 (cattle)	-0.321	1.40
D_3	coefficient on drought variable in equation 6 ('other')	0.145	0.48

1. Footnote 1 is at the end of the table.

with product supplies and in the degree of consistency in the theoretical specification of the model. In particular, whereas under the CET specification adopted by Powell and Gruen, transformation elasticities τ_{ij} would, under a strict interpretation of the CET framework, be constrained to equality for all pairs of products, under CRETH this difficulty is avoided. Moreover, our regional disaggregation allows a more convincing application of the joint production paradigm than that afforded to Powell and Gruen by their national aggregate data. Although we failed to estimate the entire CRESH/CRETH system we have been able to estimate product supply systems in each of the three zones.

Our results suggest that :

(i) Farming in the three zones, and especially in the Wheat/Sheep Zone, is characterised by multi-enterprise production possibilities reflecting generally high technical prospects for farmers to change their enterprise mix in response to changes in the expected prices of products ;

(ii) Unless agricultural supply analysis is carried out within an economic framework sufficiently comprehensive to allow modelling the joint production features of such regions, the resultant estimates of the responsiveness of various products to changes in own prices and prices of competing products is likely to be considerably understated. This has important implications for agricultural policy analysis which often involves situations in which product prices are manipulated for a range of policy purposes.

In section 3 we develop a suitable econometric specification in the light of the model and salient features of the data which we briefly discuss in section 4. In the event we had to abandon estimation of input substitution and in section 5 we report results for the product transformation frontiers (i.e., CRETH) alone. These are given as separate estimates for the three regions comprising the cereals-livestock complex of Australian agriculture.[1] We attribute our inability to estimate the isoquant maps (i.e., CRESH) to the lack of sufficient independent variation in inputs and input prices over the sample (annual time-series data, fiscal years 1952-53 through 1973-74).

Our conclusions are given in section 6.

2. Specification of the Model

The approach embodied in (2.1) and (2.2) is that suggested by Jorgenson, Christensen and Lau,[2] and recently taken up by Hasenkamp.[3] Among the alternatives considered by these authors was the case in which f took the CES form[4] and g took the CET form.[5] For situations in which $N = M = 2$ (two factors and two products), such a specification is sufficiently flexible

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1. In the context of Australia-wide aggregate data (rather than the zonal data considered here) and a three input split, Ryland and Vincent have successfully estimated CRESH. See G. J. Ryland and D. P. Vincent, "Empirical Estimation of the CRESH Production Function," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. OP-12, Industries Assistance Commission, Melbourne, June, 1977 (mimeo), pp. 14. In that paper, however, all factors were treated as variable in the short run, whereas in the current paper land and capital were treated as fixed in the short run.

2. Dale W. Jorgenson, Laurits R. Christensen and Lawrence J. Lau, "Transcendental Logarithmic Production Frontiers," Review of Economics & Statistics, Vol. 55, No. 1 (February, 1973), pp. 28-45.

3. Georg Hasenkamp, "A Study of Multiple-Output Production Functions : Klein's Railroad Study Revisited," Journal of Econometrics, Vol. 4, No. 3 (August, 1976), pp. 253-262; and Hasenkamp, op. cit..

4. K. J. Arrow, H. B. Chenery, B. S. Minhas and R. M. Solow, "Capital-Labour Substitution and Economic Efficiency," Review of Economics & Statistics, Vol. 43, No. 3 (August, 1961), pp. 225-250.

5. Alan A. Powell and F. H. Gruen, "The Constant Elasticity of Transformation Production Frontier and Linear Supply System," International Economic Review, Vol. 9, No. 3 (October, 1968), pp. 315-328.

to accommodate most empirically interesting cases. When both the number of factors and the number of products exceeds two, the CES/CET specification suffers from the very serious weakness that each of the [½(M - 1)M] pairwise partial substitution elasticities are constrained to equality, whilst the analogous restriction applies to the partial transformation elasticities.

2.1 The CRESH/CRETH Production System

Hanoch has suggested a generalization of CES (namely, CRESH : constant ratio of elasticities of substitution, homothetic) which permits substitution elasticities to differ among different pairs of factors, without, however, introducing a large number of additional parameters.[1] The latter feature makes CRESH attractive for empirical work; a slight drawback is that the functional form of the CRESH production function, f , cannot be obtained explicitly. An implicit representation for the constant returns to scale case is given by[2]

$$\sum_{\substack{j \\ h_j \neq 0}} \frac{t_j}{h_j} \left(\frac{X_j}{Z}\right)^{h_j} + \sum_{\substack{j \\ h_j = 0}} t_j \, \ell n \left(\frac{X_j}{Z}\right) = \kappa_1 \; , \qquad (3.1)$$

in which t_j , h_j and κ_1 are parameters with $t_j \geq 0$ and $h_j < 1$ for all j . We can also assume that t_j and κ_1 are normalized so that

$$\sum_{j=1}^{M} t_j \equiv 1 \, .$$

1. Hanoch, op. cit..
2. Hanoch's (op. cit.) representation of CRESH differs slightly from ours in that he adopted an alternative normalization of the parameters. Also, he allowed for non-constant returns to scale by replacing Z in (3.1) by a general function $\psi(Z)$. In this paper, we are assuming constant returns to scale, and therefore only the special case $Z = \psi(Z)$ is required.

high rainfall zone, they are relatively high in all zones when compared with the aggregate Powell/Gruen results.

Because in our structural form the aggregate short run capacity to produce z is exogenous, we cannot obtain any information from our estimates for the Pastoral or High Rainfall Zones about supply responses to changes in absolute (as distinct from relative) product prices. This weakness is shared with the earlier (Powell/Gruen) methodology. The reduced form system fitted to the Wheat/Sheep Zone, however, does allow computation of short run expansion effects due to changes in the product price level. For example, according to Table 5, a 10 per cent increase in the expected price of all products would result one year later in the following planned output changes; wool + 0.14 per cent; sheep + 0.16 per cent; cattle + 0.11 per cent; wheat - 0.02 per cent; barley + 0.12 per cent; 'other' products + 0.03 per cent; in view of approximations inherent in the method, these estimates should be rounded to the first significant digit. Using the product shares set out in Table 1, these figures imply that the short run total agricultural supply elasticity in the Wheat/Sheep Zone with respect to the overall level of expected product prices (but with input prices and fixed inputs held constant), is about + 0.01 (which is one tenth of the weighted sum of the above responses using the value shares given in Table 1 as weights).

6. Conclusion

In this paper we have developed a theory of input demand and product supply relationships flexible enough to accommodate realistically joint production in the overwhelmingly multi-product environment of Australian agriculture. Our CRESH/CRETH framework constitutes an improvement over the earlier work of Powell and Gruen in that input demands are treated simultaneously

5.5 Some Comments on the Results

The homogeneity of degree zero of the structural form supply system in relative prices, implies that, for any product, the sum of all of its cross price elasticities is equal to the negative of its own price elasticity; that is, $\sum_{j \neq i} \xi_{ij} = - \xi_{ii}$. In the reduced form system this condition does not apply. The estimated (short run) expansion effects were sufficiently small, however, that ex post this equation applied as a good first approximation among the estimated reduced form elasticities (see Table 5).

We note that our estimated transformation elasticities are considerably higher than those reported by Powell and Gruen from national time series aggregate data using their more restrictive (and theoretically less well articulated) CET model.[1] Furthermore, our estimates of one year own and cross price elasticities of supply are considerably higher in absolute value than the estimates obtained from previous Australian agricultural supply studies which generally have employed conventional single equation techniques with the parameter space largely unconstrained.

There are a number of broad similarities across the three zones in the ease of product transformation and in own and cross price responsiveness of products. Transformation elasticities are lowest between pairs of livestock products and highest between crop products. The grain categories apparently add considerably to the flexibility of the production response in all three zones. Although transformation elasticities are lowest in the

1. In a six product analysis of aggregate Australian agricultural data, Powell and Gruen reported (after constraining a number of pairwise elasticities to zero) the following transformation elasticities; wool/wheat - 0.16; wheat/coarse grains - 0.29; beef/dairy - 0.29; lamb/dairy - 0.26; and lamb/wool - 0.13. The treatment of price expectations and other variables in the present analysis (but not the sample period) sufficiently replicates Powell and Gruen's as to allow direct comparison of estimates.

From (3.1) we find that in writing the marginal products, $\partial Z/\partial X_j$, the need for the partition vanishes, as the same algebraic expression, namely

$$(3.2) \qquad \frac{\partial Z}{\partial X_j} = t_j \left(\frac{X_j}{Z}\right)^{h_j - 1} \Bigg/ \sum_{j=1}^{M} t_j \left(\frac{X_j}{Z}\right)^{h_j}$$

accommodates both the case $h_j = 0$ and $h_j \neq 0$. (Indeed, it is this feature which justifies the form of the second term on the left of (3.1)[1]).

Finally, we note that the marginal rate of substitution between any pair of factors ℓ and k is given by

$$(3.3) \qquad MRS_{k\ell} \equiv \left.\frac{\partial X_\ell}{\partial X_k}\right|_{Z, X_j (j \neq k, \ell)\text{ constant}} \equiv - \frac{\partial Z/\partial X_k}{\partial Z/\partial X_\ell}$$

$$= - t_k \left(\frac{X_k}{Z}\right)^{h_k - 1} \Bigg/ t_\ell \left(\frac{X_\ell}{Z}\right)^{h_\ell - 1}$$

and that the restriction on the h_j's ensures that the (absolute value of the) $MRS_{k\ell}$ falls as we increase X_k/X_ℓ, holding all other inputs and Z constant.[2]

1. The addition of the logarithmic term on the left of (3.1) to cover the case of a zero exponent is in the same spirit as the suggestion made by Johansen in the context of his utility function. See Leif Johansen, "On the Relationships Between Some Systems of the Demand Functions," University of Oslo, Institute of Economics, Reprint Series No. 47, Oslo, 1969; reprinted from Liiketaloudellinen Aikakauskirja.

2. The restriction that $h_j < 1$ for all j is necessary and sufficient to ensure that CRESH is globally strictly-quasi-concave. On the other hand, if we are content with local strict quasi concavity, then one (but only one) of the h_j may exceed 1. However, if $h_1 > 1$ and $h_i < 1$, $i = 2, \ldots, M$, then it can be shown that factor 1 is a substitute for all other factors while the factors 2, ..., M form a group of complements, with negative elasticity of substitution for any pair in the group - - see Hanoch, op. cit. p. 700. It seems to us that in a study which deals with "general factors," as this one does, such as land, labour, capital, etc., little is lost by ruling out the sort of local complementarity which is potentially allowable in CRESH.

Turning now to the product-product space, Dixon has recently suggested extending the CET formulation in an analogous way; the resulting product transformation frontier (CRETH - - constant ratio of elasticities of transformation, homothetic) is[1]

$$\sum_{i=1}^{N} \frac{r_i}{k_i} \left(\frac{Y_i}{Z}\right)^{k_i} = \kappa_2 \quad , \tag{4.1}$$

where Z, as before, has the interpretation of a scalar measure of total capacity which may be thought of as either an input index or an output index.[2] The r_i and κ_2 are non-negative parameters and may be normalized so that

$$\sum_{i=1}^{N} r_i \equiv 1 \quad . \tag{4.2}$$

Restrictions on the k_i's ensure that for any pair of outputs w and v, the marginal rate of transformation (defined analogously to (3.3)) increases (in absolute value) as we increase Y_v/Y_w , holding all other outputs and Z constant.[3]

The CRESH/CRETH production system is then defined by (3.1) and (4.1) together. In the case where the h_j share a common value $h, (h < 1)$ for all M factors and the k_i share a common value $k, (k > 1)$ for all products, this becomes the CES/CET system, with substitution elasticity $1/(1 - h)$ and with elasticity of transformation equal to $1/(1 - k)$. In the

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1. Peter B. Dixon, "The Costs of Protection," op. cit..
2. As with CRESH, it is possible to introduce non-constant returns to scale into CRETH by replacing Z with a function $\theta(Z)$. In this paper we are concerned only with the constant returns case.
3. More formally, $k_i > 1$ for all i is necessary and sufficient to ensure that CRETH is globally strictly quasi-convex. It will also be noticed that since zero is not in the domain of the k_i's , the need for a logarithmic term, such as that in (3.1) , does not arise in (4.1).

5.4 Non-Parametric Testing for Serial Correlation

A "Runs Test" was carried out on the residuals of each of the simultaneous equations.[1] For all stochastic equations in the three zones, the number of runs observed was considerably below the critical number. That is, in all equations we could not reject the hypothesis that the residuals are independent. Needless to say, we harbour no illusions about the power of this test; the volume of computing involved in a more suitable test,[2] and the poor prospect of being able to make meaningful corrections for autocorrelation within such a tightly constrained system to be estimated by full information methods, led us to settle for the simple non-parametric test.

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1. For a description of the test used see E.J. Kane, Economic Statistics: Econometrics: an Introduction to Quantitative Economics (New York: Harper and Row, 1968, p. 364). Briefly, the test involved computing $Pr(n_r)$, the probability of observing n_r , the actually observed number of runs on the null hypothesis that the sequence of residuals is completely random. Whenever $Pr(n_r)$ is less than or equal to one minus the confidence coefficient, we reject the null hypothesis. $Pr(n_r)$ itself is calculated as $M(n_r)/M(S)$, where $M(n_r)$ equals the number of distinct ways in which n_r runs can occur in a sequence of N items in which n+ and n- are alike, and M(S) is the number of possible outcomes.
2. D.F. Hendry, "Maximum Likelihood Estimation of Systems of Simultaneous Regression Equations with Errors Generated by a Vector Autoregressive Process," International Economic Review, Vol. 12, No. 2 (June 1971), pp. 257-72.

Own and cross product price elasticities for the high rainfall zone are shown in Table 7.

Table 7

Estimated Own and Cross Price Responsiveness[(a)] :
High Rainfall Zone

Per cent response one year later in the planned output of (b)	Product whose expected price changes :			
	Wool	Sheep	Cattle	Grains, etc.
Wool	0.060 (1.07)	- 0.002 (0.20)	- 0.005 (0.21)	- 0.053 (1.20)
Sheep	- 0.006 (0.20)	0.112 (0.90)	- 0.010 (0.21)	- 0.097 (0.91)
Cattle	- 0.019 (0.21)	- 0.010 (0.21)	0.343 (1.64)	- 0.314 (1.25)
Grains, etc.	- 0.196 (1.20)	-0.104 (0.91)	- 0.320 (1.25)	0.620 (1.67)

(a) Ratio of estimated coefficient to asymptotic standard error in brackets.
(b) Actual output may differ from planned output due to droughts, etc..

Once again, in a predominantly woolgrowing region, own supply elasticities for wool and sheep are considerably less than those of the two other product categories distinguished. Similarly, cross price effects are lower for wool output with respect to changes in the price of competing products than for other output and price combinations. Nevertheless, estimated cross price responsiveness in the high rainfall zone is still quite large.

case where the common values of these elasticities are plus and minus one respectively, the system collapses further to a Cobb-Douglas/Ellipse pair of functional forms.

2.2 Specific versus Non-Specific Inputs

All the cases discussed so far, being based on a particularization of (2.1) and (2.2), provide a simplification of the relationship between inputs and outputs. In the general case where we write the multi-product multi-factor production function as

$$H(X_1, \ldots, X_M ; \quad Y_1, \ldots, Y_N) = 0 ,$$

there are, at every point in the input-output space, MN "free" elasticities E_{ij}, where E_{ij} is the elasticity of output of product i with respect to factor j, i.e., E_{ij} measures the effect on i of increasing j while holding all other outputs and inputs constant. Under (2.1) - (2.2), the number of free elasticities is reduced to M + N. We notice that the E_{ij} may be written as

(5) $$E_{ij} = A_i B_j \qquad (i = 1, \ldots, N ; \; j = 1, \ldots, M) ,$$

where A_i and B_j are respectively the elasticity of Y_i with respect to Z and the elasticity of Z with respect to X_j. A_i is computed from (2.2) by allowing Z and Y_i to vary while fixing all other Y's , and B_j is obtained from (2.1) by computing the effect of a change in X_j on the level of Z, with all other X's unchanged.[1]

1. In the case of CRESH/CRETH

$$A_i = \sum_{\ell=1}^{N} r_\ell \left(\frac{Y_\ell}{Z}\right)^{k_\ell} \Bigg/ \left[r_i \left(\frac{Y_i}{Z}\right)^{k_i - 1} \right]$$

and

$$B_j = t_j \left(\frac{X_j}{Z}\right)^{h_j - 1} \Bigg/ \sum_{\ell=1}^{M} t_\ell \left(\frac{X_\ell}{Z}\right)^{h_\ell} \quad \text{-- see (3.2) .}$$

The simplification (5) implies that if the elasticity of wool production with respect to labour inputs is twice that of wheat production with respect to labour inputs, then the elasticity of wool production with respect to capital inputs is also twice that of wheat production with respect to capital inputs. This means that factors are completely non-specific. If it is relatively easy to expand wool output by applying more of factor 1, then it is relatively easy to expand wool output by applying more of factor 2. In summary, no factor has comparative advantage in increasing the output of any particular product.

It is clear that in applications of models based on (2.1) - (2.2), the inputs $X_1, \ldots, X_M$ should be broadly defined (e.g., labour, capital, land, intermediate inputs). Simplification (5) would not be appropriate if the factor list included, for example, "contract shearing." Obviously, this input is quite specific to a particular output, wool. Fortunately, however, fully specific inputs often have virtually zero substitution elasticity with respect to every other factor involved in the production of their specific output. For the firm's decision making, the costs of such factors can be deducted from the product price (i.e., treated as an excise tax). Other specific factors in this category are commission and freight charges involved in getting products to market. Likewise, these factors do not possess identifiable substitutes. Thus in adopting (2.1) - (2.2) we are implicitly making the assumption that any factors which are product-specific do not have substitutes (or at least that product-specific factors with substitutes can be ignored).

2.3 Derivation of the CRESH/CRETH Behavioural Equations

We assume that the representative farmer is efficient and seeks to minimize the cost of producing any given multi-product output bundle chosen as optimal in the light of relative product prices. The common resource base consists of fixed and variable inputs. All products are assumed to have the same

Although all parameters have the correct sign, standard errors are very high. As was the case in the other zones, the estimated ease of transformation is lowest between wool and sheep and highest when the grains, etc. category is involved. Values for wool/cattle and sheep/cattle are lower than might have been expected given the inherent suitability of high rainfall zone pastures to the grazing of sheep and cattle. Traditionally, the high rainfall zone has been dominated by sheep grazing for both wool and sheepmeats. The emergence of a beef cattle industry on a significant scale has taken place only in more recent years. We suspect that our analysis, framed entirely in terms of relative prices, has failed to capture the rate of adoption or adjustment path towards the new cattle/sheep equilibrium. Even though relative prices may clearly suggest a transformation from sheep to cattle, it is likely that, because of the 'newness' of a cattle enterprise in a predominantly sheep grazing region, such a transformation occurs only slowly at first then at an accelerated rate. That is, the lag in acceptance of the relatively new cattle enterprise is progressively overcome as cattle gain greater acceptance among sheep men. One way of allowing for this type of adjustment process is by the incorporation of a suitably shaped trend curve (for example the logistic curve, which is sigmoidal) into the cattle supply equation, and the mirror image of this curve into the wool supply equation. Computer soft- and hardware limitations precluded our following up this approach.

Although they are generally poorly determined, it is of interest to interpret the estimates of the wheat quota parameters for the Wheat/Sheep zone. Our results (see Appendix) indicate that following the imposition of wheat quotas in 1969, planned output of wheat declined sharply. This was offset by very small expansions in the planned outputs of wool and of sheep and much larger expansions in the planned outputs of cattle and of barley. For the period between 1972/73 and 1973/74 when wheat quotas became more or less inoperative, the results indicate a large expansion in planned output of wheat, a large contraction in the planned output of barley and a moderate contraction in the planned output of wool.

5.3 High Rainfall Zone

The six estimated pairwise transformation elasticites are shown in Table 6.

Table 6

Estimated Product Transformation Elasticities[a] : High Rainfall Zone

	Sheep	Cattle	Grains, etc.
Wool	-0.01 (0.20)	-0.04 (0.21)	-0.36 (1.20)
Sheep		-0.07 (0.21)	-0.66 (0.91)
Cattle			-2.13 (1.25)

(a) Ratio of estimated parameter to asymptotic standard error in brackets.

production period (one year) with common starting date. The assumption of constant returns to scale to all inputs implies diminishing returns to scale to the variable inputs. Hence the scale of output is determinate and the model has a solution. At the commencement of each production period, the representative farmer chooses his combination of products and variable inputs given the supply of his fixed inputs (land, capital and his own labour and management).

Let F be the set of subscripts identifying the M^* factors which are fixed among the total of M factors employed. The constrained maximization problem facing the representative farm firm may now be stated as :

Choose the bundle of outputs (Y_i), inputs (X_j), and an overall level of farm activity (Z) to maximize total farm gross margin - - that is,

(6) $$\text{Max} \quad P'Y - \sum_{j \notin F} Q_j X_j$$

subject to

(7.1) $$\sum_{i=1}^{N} \left(\frac{Y_i}{Z}\right)^{k_i} \frac{r_i}{k_i} - \kappa_2 = 0$$ (product transformation constraint) ;

(7.2) $$\sum_{\substack{j \\ h_j \neq 0}} \left(\frac{X_j}{Z}\right)^{h_j} \frac{t_j}{h_j} + \sum_{\substack{j \\ h_j = 0}} \left[\ell n \left(\frac{X_j}{Z}\right)\right] t_j - \kappa_1 = 0$$ (input substitution constraint) ;

(7.3) $$X_j = \bar{X}_j \qquad (j \varepsilon F)$$ (fixed factor constraint) ;

where the $\bar{X}_j$ represent the exogenously given supplies of the fixed factors; P is the vector of net prices for products, i.e., the price after allowance for the costs of product specific inputs, and for $j \notin F$, Q_j is the price of the j^{th} variable input.

In what follows, the Q_j for $j \epsilon F$ are defined to be the shadow prices (Lagrangean multipliers) on the constraints (7.3).[1] Differentiating the Lagrangean [2] corresponding to problem (6), (7.1) - (7.3), with respect to the (M + M* + N + 3) variables X_j, Q_j for $j \epsilon F$, Y_i, Z, Λ and Γ (Λ and Γ are Lagrangean multipliers pertaining to the constraints (7.1) and (7.2) respectively), after eliminating Λ and Γ and taking logarithmic differentials, we obtain the following system of product supply and factor demand equations :

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1. Note that the first-order conditions for problem (6), (7.1) - (7.3) imply that

$$\sum_{i=1}^{N} P_i Y_i = \sum_{j=1}^{M} Q_j X_j = \sum_{j \epsilon F} Q_j X_j + \sum_{j \notin F} Q_j X_j ;$$

that is, the value of output equals the cost of inputs, where fixed input costs are reckoned at their shadow rental value.

2. Our assumptions that $h_j < 1$, $k_i > 1$ for all j , i ensure that there is no danger of corner solutions. The CRESH isoquants either fail to reach the axes or meet them tangentially with slopes of (minus) zero and (minus) infinity at the tangency points. Also, for any given Z , the CRETH transformation frontiers meet the axes at right angles. Hence even when the substitution and transformation elasticities are very high (i.e., h_j , $k_i \rightarrow 1$) , the first order conditions will hold as equalities. It follows that an appropriate choice of parameters enables CRESH/CRETH to approximate to an arbitrarily high degree of accuracy the corner solutions involved in perfect substitutability and/or transformability.

Table 5

Estimated Own and Cross Price Elasticities[a] :

Wheat/Sheep Zone

Per cent response one year later in the planned output of (b) :	Product whose expected price changes :						Sum of Cross Price Responses
	Wool	Sheep	Cattle	Wheat	Barley	'Other'	
Wool	0.256	- 0.012	- 0.020	- 0.152	- 0.014	- 0.044	- 0.242
	(2.38)	(0.52)	(1.27)	(1.52)	(1.12)	(1.82)	
Sheep	- 0.031	0.225	- 0.015	- 0.118	- 0.011	- 0.034	- 0.209
	(0.41)	(0.91)	(0.71)	(0.78)	(0.67)	(0.83)	
Cattle	- 0.077	- 0.023	0.483	- 0.269	- 0.025	- 0.078	- 0.472
	(0.78)	(0.72)	(1.13)	(1.06)	(0.99)	(1.09)	
Wheat	- 0.254	- 0.075	- 0.114	0.766	- 0.080	- 0.245	- 0.768
	(2.05)	(1.96)	(2.42)	(2.59)	(2.37)	(2.51)	
Barley	- 0.078	- 0.023	- 0.036	- 0.270	0.497	- 0.078	- 0.485
	(0.99)	(0.90)	(1.42)	(1.49)	(1.57)	(1.54)	
'Other'	- 0.204	- 0.061	- 0.092	- 0.687	- 0.064	1.111	- 1.108
	(2.06)	(1.93)	(2.63)	(2.71)	(2.56)	(2.87)	

(a) Ratio of estimated coefficient to apparent asymptotic standard error in brackets. (See footnote to Table 4.)

(b) Actual output may differ from planned output due to droughts, etc..

Both own and cross elasticities of supply are high. For example, in the Wheat/Sheep Zone, a 10 per cent rise in the expected price for wheat, with all other product and input prices held constant, leads one year later to an 8 per cent increase in the output of wheat, and to the following percentage contractions in the output of competing products : sheep, 1.2 per cent, barley, 2.7 per cent; wool, 1.5 per cent; cattle 2.7 per cent; and 'other' products, 7 per cent.

the products, the smallest transformation elasticities are between the livestock enterprises (wool-sheep, sheep-cattle and wool-cattle), whilst the largest are among the grains. Transformation elasticities between product pairs involving a crop enterprise and a livestock enterprise are intermediate in magnitude. In the wheat-sheep zone, one can envisage few technical problems in changing the relative acreages of different types of grains in response to changes in relative product prices. On the other hand, there are obvious limits on the extent to which one type of livestock may be replaced throughout a zone with another over a one year period.

In Table 5, we report a matrix of own and cross supply elasticities associated with the reduced form CRETH system. This matrix corresponds to the elements c_{ij} in equation (16). The reduced form elasticities incorporate both transformation and expansion effects. That is, they are derived under the assumption that the scale of output is free to change as transformation from one product to another occurs. For example, an increase in the price of one product is likely to lead to an increase in the overall scale of output while at the same time leading to a reduction in the quantity of a competing product. Hence the size reduction in the competing product is likely to be smaller than would have been the case had the scale of output been held constant.[1]

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1. The relationship between the structural form (SF) and reduced form (RF) cross price elasticities is as follows :

$$\underset{(RF)}{\xi_{ij}} = \underset{(SF)}{\xi_{ij}} + \xi_{iz}\xi_{zpj} ,$$

where ξ_{iz} is the elasticity of output of product i with respect to the overall capacity variable Z (which is unity for all products) and ξ_{zpj} is the elasticity of Z with respect to the price of product j . Estimates of ξ_{zpj} can be deduced from the reduced form analysis via the parameter h (see Appendix for details). They range from 0.001 to 0.006. Hence for all practical purposes, the SF and RF price elasticities can be regarded as equivalent.

(8) $$y_i = z + \frac{1}{k_i - 1}\left[p_i - \sum_{i'=1}^{N} p_{i'} S^*_{i'}\right] ;$$ (CRETH Supply Functions - - Structural Form)

(9) $$x_j = z + \frac{1}{h_j - 1}\left[q_j - \sum_{j'=1} q_{j'} W^*_{j'}\right] ;$$

(10) $$x_j = \bar{x}_j \qquad (j\epsilon F) ;$$

(9)–(10): (Structural Form of CRESH Demand Functions and Fixed Factor Valuation Equations)

(11) $$\sum_{i=1}^{N} p_i S_i = \sum_{j=1}^{M} q_j W_j ;$$ (CRESH/CRETH Input-Output Identity)

where the lower case symbols p_i , q_i , x_j , y_i , z , represent small changes in the logarithms of the corresponding upper case variables P_i , Q_j , X_j , Y_i , Z ; and where

(12.1) $$S_i = P_i Y_i \Big/ \sum_{\ell=1}^{N} P_\ell Y_\ell$$

(the share of product i in total farm revenue) ,

(12.2) $$W_j = Q_j X_j \Big/ \sum_{\ell=1}^{M} Q_\ell X_\ell$$

(the cost share of input j in total costs; for fixed inputs the W_j represent imputed annual rentals as a percentage of total costs) ,

(12.3) $$S^*_i = \frac{\hat{S}_i}{\sum_{\ell=1}^{N} \hat{S}_\ell} = \frac{S_i \big/ (k_i - 1)}{\sum_{\ell=1}^{N} S_\ell \big/ (k_\ell - 1)} ,$$

and

$$W^*_j = \frac{\hat{W}_j}{\sum_{\ell} \hat{W}_\ell} = \frac{W_j \Big/ (h_j - 1)}{\sum_{\ell=1}^{M} W_\ell \Big/ (h_\ell - 1)} \quad . \tag{12.4}$$

Equations (8) and (9) together with (10) and (11) constitute a system of (N + M + M* + 1) equations, linear in the endogenous variables y_i , x_j , q_j for $j \epsilon F$, and z .

2.4 Interpretation of CRETH Supply and CRESH Demand Functions

The interpretation of (8) , the product supply equation, is as follows : the percentage change in output of product i is linear in the percentage change z in overall farm activity, and in the percentage change in the relative price of product i . In determining the latter relative price movement, the percentage change in the absolute price of product i has subtracted from it a weighted average of the percentage changes in the prices of all products, where the weights used depend on the relative shares of the different products in the gross value of production at the farm gate, and on the parameters, $k_1 , \ldots, k_N$.

By setting $p_\ell = 1$, $z = 0$ and $p_{i'} = 0$ for all $i' \neq \ell$, we see that (8) implies that

$$\xi_{i\ell} = - \frac{1}{k_i - 1} S^*_\ell \quad , \qquad i \neq \ell \; , \tag{13}$$

where $\xi_{i\ell}$ is the cross price elasticity of supply of product i with respect to changes in the price of product ℓ at any given level of capacity, Z . Under our restriction that $k_i > 1$ for all i , it is apparent that all product-product cross price elasticities are negative. With the overall level of activity fixed, an increase in the price of product ℓ will lead to a reduction in the output

Planned production of wool in the pastoral zone is considerably less responsive to its expected price than are cattle or 'other' output to their respective prices. This result appears reasonable in view of the traditionally wool oriented activities of the pastoral zone (see Table 1).

5.2 Wheat-Sheep Zone

The fifteen estimated pairwise transformation elasticities, which all have the correct sign, are shown in Table 4.

Table 4

Estimated Product Transformation Elasticities[(a)] :
Wheat/Sheep Zone

	Sheep	Cattle	Wheat	Barley	'Other'
Wool	-0.08 (0.81)	-0.19 (1.09)	-0.58 (2.57)	-0.19 (1.34)	-0.47 (1.82)
Sheep		-0.15 (0.72)	-0.45 (0.94)	-0.15 (0.77)	-0.37 (0.85)
Cattle			-1.00 (1.13)	-0.32 (0.95)	-0.81 (1.12)
Wheat				-1.01 (1.57)	-2.52 (2.81)
Barley					-0.81 (1.39)

(a) Ratio of estimated parameter to apparent asymptotic standard error in brackets. Ratio may be overstated due to iterative treatment of the second term on the RHS of (19).

The estimated transformation elasticities are rather high for a number of product pairs. This is consistent with the view that Wheat-Sheep Zone farmers have considerable possibilities for changing their output mix in response to relative price changes. As one might expect from the nature of

All transformation elasticities have the expected (negative) sign, although the standard errors are disappointingly large. As would be expected, the ease of transformation is highest when the grains, etc. category is one of the product pairs. Nevertheless, for a one-year response, the value obtained for wool : cattle is quite large, implying a cross elasticity of cattle supply with respect to expected wool price of about - 0.3 . The matrix of own and cross product supply elasticities for the pastoral zone is shown in Table 3.[1]

Table 3

Estimated Own and Cross Price Elasticities[(a)] :
Pastoral Zone

Percentage response one year later in the planned output (b) of:	Product whose expected price changes :		
	Wool	Cattle	Grains, etc.
Wool	0.083 (1.25)	- 0.039 (1.05)	- 0.043 (0.96)
Cattle	- 0.332 (1.05)	1.008 (2.38)	- 0.676 (1.46)
Grains, etc.	- 0.929 (0.96)	- 1.724 (1.46)	2.654 (1.75)

(a) Ratio of estimated elasticity to asymptotic standard error in brackets.

(b) Actual output may differ from planned output due to droughts, etc..

1. The own product supply elasticities (ξ_{ii}) can be deduced from the CRETH structural form equation (8) as :

$$\xi_{ii} = (1 - S_i^*)/(k_i - 1) .$$

In applying this formula, S_i^* has been evaluated at the sample mean values of S_i .

of product i, $i \neq \ell$. Under CRETH it also follows that all partial transformation elasticities between products are negative. If $\tau_{i\ell}$ is the pairwise product transformation elasticity between i and ℓ , then as a matter of definition ,

$$\text{(14)} \qquad \tau_{i\ell} \equiv \frac{\xi_{i\ell}}{S_\ell} ,$$

$$= - \frac{1}{k_i - 1} \frac{1}{k_\ell - 1} \frac{1}{\sum_{i'=1}^{N} \hat{S}_{i'}} \qquad (i \neq \ell) .$$

On the input side, we find from (9) that under CRESH,

$$\text{(15.1)} \qquad \eta_{jm} = - \frac{1}{h_j - 1} W_m^* \qquad (j \neq m) ,$$

and that

$$\text{(15.2)} \qquad \sigma_{jm} = - \frac{1}{h_j - 1} \frac{1}{h_m - 1} \frac{1}{\sum_{j'=1}^{M} \hat{W}_{j'}} \qquad (j \neq m) ,$$

where η_{jm} and σ_{jm} are respectively the (output-compensated) cross-price elasticity of demand for factor j with respect to changes in the price of factor m , and σ_{jm} is the (Allen Uzawa) partial elasticity of substitution between j and m . With all $h_j < 1$, the elasticities η_{jm} and σ_{jm} are positive for all pairs of factors. It follows that the possibility of complements is excluded.

3. Econometric Specification and Estimation Method

The behavioural equation set (8) - (11) is, in a certain sense, approximately linear in the variables. An exact linear representation would be possible if the product and factor shares ($\{S_j\}$ and $\{W_i\}$ respectively) were constant. An approximate representation of the system which is linear in the variables, therefore, is obtained by fixing these shares at sample means, say, $\{\bar{S}_j\}$ and $\{\bar{W}_i\}$. Matrix methods then lead immediately to a linear reduced form in which the $(N + M + M^* + 1)$ endogenous variables $\{y_1, \ldots, y_N ; x_1, \ldots, x_M ; q_j, j \varepsilon F ; z\}$ appear on the left, and the $(N + M)$ exogenous variables $\{p_1, \ldots, p_N ; \bar{x}_j, j \varepsilon F; q_j, j \notin F\}$ on the right.[1] For one of the three regions constituting our sample (namely, the wheat-sheep zone), sufficient stability was exhibited in the shares $\{S_j\}$ and $\{W_i\}$, in our opinion, to allow the linearization procedure alluded to above. Furthermore, because of relatively little variation over the time-series sample in the input mix and in input prices, the likelihood surface for the reduced form was extremely flat with respect to the input substitution parameters. This led us to abandon estimation of the CRESH side of the system.[2] Thus, for the wheat-sheep zone, the focus of our attention became the reduced form supply equations in the linearized system, i.e.,

$$(16) \qquad y_i = \sum_{j \notin F} \ell_{ij} q_j + \sum_{j \varepsilon F} m_{ij} \bar{x}_{2j} + \sum_{j=1}^{N} c_{ij} p_j \qquad (i = 1, \ldots, N),$$

where the coefficients $\{\ell_{ij} ; m_{ij} ; c_{ij}\}$ are functions of the CRESH and CRETH parameters, $\{h_j\}$ and $\{k_i\}$, and of the mean shares $\{\bar{S}_j\}$ and $\{\bar{W}_i\}$.

1. For details, see Dixon, Vincent and Powell, op. cit..
2. More detail is given in Vincent, Dixon and Powell, op. cit..

appropriate. In addition, a set of dummy variables to reflect the influences of wheat quotas on planned outputs of wheat and competing products was used for the wheat/sheep zone supply system.[1]

5. The Results[2]

We report the results of our CRETH analysis in the form of the following matrices for each zone :

(i) a symmetric matrix of pairwise product transformation elasticities ;

(ii) an assymetric matrix of own and cross price elasticities of supply.

5.1 Pastoral Zone

Estimation of the three product system yielded the transformation elasticities[3] shown in Table 2.

Table 2

Estimated Product Transformation Elasticities [(a)] :
Pastoral Zone

	Cattle	Grains, etc.
Wool	- 0.39	- 1.08
	(1.05)	(0.96)
Cattle		- 16.91
		(1.46)

(a) Ratio of estimated parameter to asymptotic standard error in brackets.

1. For details of the construction and use of these variables see Vincent, Dixon and Powell, op. cit..
2. A complete listing of parameter estimates of each of the equations systems is contained in the Appendix.
3. Transformation elasticities were calculated from (14) with $\hat{S}_i$ evaluated at sample mean values of S_i.

Table 1

Composition of Output by Zone

Product		Average Share of Output (1952/53 to 1973/74)		
		Pastoral Zone (a)	Wheat/Sheep Zone	High Rainfall Zone
Wool		0.858	0.353	0.545
Sheep		-	0.120	0.158
Cattle		0.102	0.063	0.150
Wheat	Grains, etc.	0.040	0.346	0.147
Barley			0.053	
Other			0.065	

(a) In the pastoral zone, wool and sheep (the product being sheep meat) are modelled as a single product, whereas in the wheat-sheep and high rainfall zones they are treated separately. Sheep are grazed in the pastoral zone entirely for wool production. Income from the sale of old wool sheep for slaughter accrues to the wool enterprise. However, in the other zones, climatic conditions are such that two distinct types of sheep are grazed; sheep bred for wool growing capacities and sheep bred for their meat.

Because we were unable to modify the variables representing outputs of sheep, cattle and 'other products' to abstract from weather influences, a weather variable was constructed and used where

Because of the difficulties encountered on the input demand side, we aggregated land and capital into a single fixed factor, and replaced the CRESH specification by its CES special case. The parametric constraints which bind the coefficients under these assumptions are, first that

(17) $$\ell_{ij} = \bar{W}_j/[(h-1)\bar{W}_M] = \ell_j \qquad (i = 1, \ldots, N, \; j = 1, \ldots, M-1) ,$$

in which the ordering of the inputs places the fixed factor last (i.e., in position M), and in which the $\bar{W}$'s are shares of non-specific inputs in total (non-specific) cost, and h is the common substitution parameter (related to the elasticity of substitution by $\sigma = 1/(1-h)$) ;

(18) $$m_{iM} = 1 , \quad i = 1, \ldots, N ;$$

and finally, that

(19) $$c_{ij} = \frac{\delta_{ij}}{k_i - 1} - \frac{\bar{S}_i}{\sum_{\ell} \bar{S}_{\ell}/(k_{\ell}-1)} + \mu\bar{S}_j ,$$

where

(20) $$\mu = (1 - \bar{W}_M) \Big/ [\bar{W}_M(h-1)] ,$$

and in which δ_{ij} is Kronecker's delta. Our estimation of transformation possibilities in the wheat-sheep zone is of (16), subject to (17) - (20). We assumed that zero-mean white noise errors could be appended to (16).[1] Then estimation was based[2] on full information maximum likelihood using Wymer's

1. Since the mapping of the serial properties of the CRETH system as we move from levels to various systems in differential changes is obviously very complex, alternative assumptions would have enormously complicated estimation. We tested the validity of our assumption ex post using a non-parametric procedure (outlined below).

2. To simplify computing, the second term on the right of (19) was set manually to correspond with an assumed set of values of $\{k_{\ell}\}$, which were then updated in light of the FIML solution obtained. The whole procedure was iterated to convergence. This procedure does not necessarily produce FIML estimates, but we believe that in this case the approximation is adequate.

exceptionally flexible RESIMUL package,[1] which provides estimates both of the parameters $\{h ; k_1, \ldots, k_N\}$ and of the coefficients $\{\ell_j ; c_{ij}\}$, as well as of their asymptotic standard errors.

In the other two agricultural regions we encountered the same difficulties on the CRESH side as in the wheat-sheep zone. In addition the variation over the time series sample in product shares $\{S_i\}$ was too large to contemplate the use of restrictions such as (17) - (20). It seemed better in these cases to accept the risk of some simultaneity bias by working in the structural form of the CRETH supply functions (8), which involves treating z (the proportional change in aggregate inputs) exogenously. From the viewpoint of using RESIMUL it was advantageous to transform the N-equation system (8) into the equivalent N equation system,[2]

$$y_i - \frac{(k_\ell - 1)}{(k_i - 1)} y_\ell - \frac{k_i - k_\ell}{k_i - 1} z - \frac{p_i - p_\ell}{(k_i - 1)} = 0 \qquad \text{(for all } i \neq \ell\text{)}, \tag{21.1}$$

$$z - \sum_{i=1}^{N} y_i S_i = 0 . \tag{21.2}$$

This is because the non-linearities in variables in (21.1) - (21.2) are confined to the last equation in which the shares $S_i \; [\equiv P_i Y_i / \sum_{i'} P_{i'} Y_{i'}]$ are, strictly speaking, endogenous. In estimation we treated them as exogenous constants. This amounts to an index-number error in the measurement of changes (z) in generalized capacity to produce, occasioned by the use of fixed (rather than variable) shares as weights. (Clearly the larger is N, the better are the chances that compensating errors will occur in (21.2)).

1. C. R. Wymer, "Computer Programs : RESIMUL Manual," International Monetary Fund, Washington, D.C., 1977 (mimeo). This work could not even have been attempted without the access generously made available by Dr. Wymer to his extremely advanced programs.
2. The choice of a particular value for ℓ in (21.1) is arbitrary. It does not affect the estimates for the k_i's .

In the cases of the high rainfall and pastoral zones, we assume that normally distributed zero-mean white noise errors could be appended to (21.1). These equations together with the constant-shares version of (21.2), treated as an identity, were estimated as a system by FIML, again using Wymer's RESIMUL package.

4. The Data

We have been provided with a time series of the average values across the sample of key input, output revenue and cost variables from the Bureau of Agricultural Economics (BAE) Australian Sheep Industry Survey (ASIS) spanning the period 1952/53 to 1973/74. The data were analysed on a zonal basis with the zones classified broadly according to rainfall. This disaggregation is particularly important in the Australian environment where because of climatic and biological influences, the same product may be produced using quite different technologies in different zones.

We have distinguished different product categories in each zone according to zonal production patterns. The product categories distinguished in our analysis and their relative importance in total revenue for each zone are shown in Table 1. All variables are conceptually differentials of logarithms. Operationally we have used the simplest discrete analogue; i.e., $d \log (a_t)$ is replaced by $[(a_t - a_{t-1})/a_t]$. Where possible, we modified the raw data on outputs so that the series represent the planned outputs of producers. The price series were modified to reflect producers' expectations using the Koyck/Nerlove adaptive expectations model. All lags in observed response to prices, whether due to adjustment frictions or the updating of expectations, under out treatment are located on the expectational side.