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ESTIMATES OF THE GENERALIZED WORKING MODEL *

Murray D. SMITH

Monash University, Clayton, Victoria 3168, Australia

Kenneth W. CLEMENTS

The University of Western Australia, Nedlands, Western Australia 6009, Australia

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This paper presents estimates of the one-parameter generalization of Working's (1943) model proposed by Laitinen et al. (1983).

Let p_i, q_i be the price and quantity consumed of good i, $M = \sum_{i=1}^{n} p_i q_i$ total expenditure or ('income') and $w_i = p_i q_i / M$ a budget share. Working's (1943) model expresses the budget share as a linear function of the logarithm of income,

$$w_i = \alpha_i + \beta_i \log\left(\frac{M}{M_0}\right), \tag{1}$$

where M_0 is the income unit. Laitinen et al. (1983) proposed a one-parameter extension of (1),

$$w_i = \alpha_i \left(\frac{M}{M_0}\right)^{\gamma-1} + \frac{\beta_i}{\gamma-1}\left[\left(\frac{M}{M_0}\right)^{\gamma-1} - 1\right], \tag{2}$$

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with $\Sigma_i \alpha_i = 1$, $\Sigma_i \beta_i = 1 - \gamma$. Eq. (2) reduces to (1) for $\gamma \to 1$ and implies linear Engel curves for $\gamma = 0$. The income elasticity for good i implied by (2) is

$$\frac{\partial(\log q_i)}{\partial(\log M)} = \gamma + \frac{\beta_i}{w_i}, \tag{3}$$

which provides a broader range of behavior of the income elasticities when income changes than do the elasticities derived from (1). These

Table 1
Budget shares of alcoholic beverages and total expenditure: Australia, 1975–1976. [a]

Houshold group h	Sample size N_h	Budget shares $w_{ih} \times 100$: Beer	Wine	Spirits	Total expenditure M_h^*
Capital cities					
1	313	1.60	0.38	0.35	1.00
2	333	2.27	0.60	0.52	1.73
3	510	2.73	0.45	0.50	2.24
4	463	2.13	0.74	0.59	2.87
5	535	2.58	0.58	0.58	3.37
6	659	2.25	0.73	0.71	4.45
Other urban					
7	427	1.99	0.31	0.47	1.06
8	330	2.69	0.39	0.70	1.77
9	480	3.07	0.45	0.27	2.26
10	356	3.36	0.25	0.41	2.62
11	338	3.29	0.29	0.47	3.21
12	294	3.68	0.43	0.61	3.99
Rural					
13	187	1.32	0.31	0.46	1.21
14	155	1.95	0.15	0.23	1.59
15	158	2.29	0.22	0.43	1.99
16	123	2.94	0.36	0.58	2.37
17	93	2.20	0.23	0.23	2.66
18	115	2.36	0.58	0.19	3.54
Mean	-	2.48	0.41	0.46	-

[a] The entries in the last four columns are group means, with total expenditure normalized so that the poorest household group = 1. Source: Australian Bureau of Statistics (1975–1976, tables 4.4, 4.5 and 4.6).

Table 2
Estimates of generalized Working model: Australia, 1975–1976. [a,b]

$w_{ih} = (\alpha_i + \gamma_i Z_{1h} + \delta_i Z_{2h})(M_h^*)^{\gamma - 1} + \frac{\beta_i}{\gamma - 1}[(M_h^*)^{\gamma - 1} - 1]$.

Beverage	$\alpha_i \times 100$	$\gamma_i \times 100$	$\delta_i \times 100$	$\beta_i \times 100$	Income elasticity $\gamma + \beta_i / w_i$
Beer	1.648	0.710	0.024	0.327	1.30
	(0.460)	(0.466)	(0.293)	(1.745)	
Wine	0.466	−0.188	−0.223	0.069	1.34
	(0.115)	(0.128)	(0.151)	(0.297)	
Spirits	0.462	−0.070	−0.154	0.021	1.21
	(0.165)	(0.184)	(0.235)	(0.374)	
			$\gamma = 1.169$		
			(0.642)		

[a] As the data are in grouped form, observations are weighted by the square roots of the sample sizes given in table 1. The variable $Z_{1h} = 1$ if $h \in$ other urban, 0 otherwise; $Z_{2h} = 1$ if $h \in$ rural, 0 otherwise. The income elasticities are evaluated at the means of the budget shares.

[b] Asymptotic standard errors in parentheses.

latter income elasticities take the form $1 + \beta_i / w_i$. For some further properties of (2), see Tran Van Hoa (1983).

The objective of this note is to estimate (2) with the data given in table 1 relating to expenditure on beer, wine and spirits in Australia.[1] Here $M^* = M/M_0$ with the poorest household group being the income unit. The ML estimates of (2), with regional intercept dummies, are given in table 2. As can be seen, γ is not significantly different from 1, implying that the data are compatible with the Working model (1). Although the estimates of the three β_i's are individually insignificant, on the basis of a likelihood ratio test we can firmly reject the joint hypothesis $\beta_1 = \beta_2 = \beta_3 = 0$. For comparison, in table 3 we give the weighted least-squares estimates of (1). As expected, the two sets of income elasticities are very similar.

To compare the fit of the two models we use the information inaccuracy. Let $w_{1h}, \ldots, w_{4h}$ be the observed budget shares for household h and $\hat{w}_{1h}, \ldots, \hat{w}_{4h}$ the predicted shares, where $w_{4h} = 1 - \Sigma_{i=1}^{3} w_{ih}$ and similarly

[1] See Tran Van Hoa et al. (1983) for an application of (2) to energy consumption.

Table 3
Estimates of Working model: Australia, 1975–1976. [a,b]
$w_{ih} = \alpha_i + \gamma_i Z_{1h} + \delta_i Z_{2h} + \beta_i \log M_h^*$.

Beverage	α_i $\times 100$	γ_i $\times 100$	δ_i $\times 100$	β_i $\times 100$	Income elasticity $1+\beta_i/w_i$
Beer	1.583 (0.223)	0.794 (0.186)	0.017 (0.260)	0.752 (0.194)	1.30
Wine	0.458 (0.066)	−0.215 (0.055)	−0.257 (0.077)	0.146 (0.057)	1.35
Spirits	0.463 (0.082)	−0.076 (0.068)	−0.168 (0.095)	0.105 (0.071)	1.23

[a] See note a to table 2.
[b] Standard errors in parentheses.

for $\hat{w}_{4h}$. The information inaccuracy of these predictions is

$$I_h = \sum_{i=1}^{4} w_{ih} \log \frac{w_{ih}}{\hat{w}_{ih}}. \tag{4}$$

When the predictions are prefect, $I_h = 0$; otherwise $I_h > 0$. We also have the inaccuracy for good i and its complement,

$$I_{ih} = w_{ih} \log \frac{w_{ih}}{\hat{w}_{ih}} + (1 - w_{ih}) \log \frac{1 - w_{ih}}{1 - \hat{w}_{ih}}. \tag{5}$$

Using the two models to provide the predictions, we take weighted means of (4) and (5) over the 18 observations, where the weights are proportional to the sample size N_h. This gives ($\times 10^6$):

Good	Generalized Working model (2)	Working model (1)
Beer	207.28	211.69
Wine	98.57	104.87
Spirits	146.79	152.21
All other	117.70	110.88
Total	450.59	466.32

This shows that (2) beats (1) for the three alcoholic beverages and for the total, but not for all other goods.

We can adjust the total information inaccuracy for the fact that (2) contains one additional parameter by means of a simple degrees of freedom correction [Theil (1980, pp. 174–175)]. This amounts to multiplying the inaccuracy by the ratio of the number of observations (18) times the number of independent equations (3) to this product less the number of unconstrained coefficients. With the regional dummies included, the number of such coefficients for (2) is 13, while for (1) it is 12. Thus the correction ratio for (2) is $18 \times 3/(18 \times 3 - 13) = 54/41$, and $54/42$ for (1). Application of this correction yields:

	Generalized Working model (2)	Working model (1)
Total	$450.59 \times \frac{54}{41} = 593.46$	$466.32 \times \frac{54}{42} = 599.55$

Although the difference is now smaller, the generalized model is still superior.

We note in conclusion that, as is the case with most surveys on alcohol consumption, the data in table 1 systematically understate consumption. Accordingly, not too much weight should be placed on the precise numerical values of the income elasticities.

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