Impact Project

Impact Research Centre, Baldwin Spencer Building (2nd floor)
The University of Melbourne, PARKVILLE, VIC. 3052 AUSTRALIA

Telephone: (03) 344 7417 (from overseas: 61 3 344 7417)
Telex: AA 35185 UNIMEL
Telegrams: UNIMELB, PARKVILLE
Fax: (03) 347 7539 (from overseas: 61 3 347 7539)

THE ORANI-MACRO INTERFACE

by

Russel J. Cooper

and

Keith R. McLaren

Department of Econometrics &
Operations Research
Monash University

Preliminary Working Paper No. IP-10 Melbourne May 1980

Appendix 7D Figure Accompanying Section 7.5

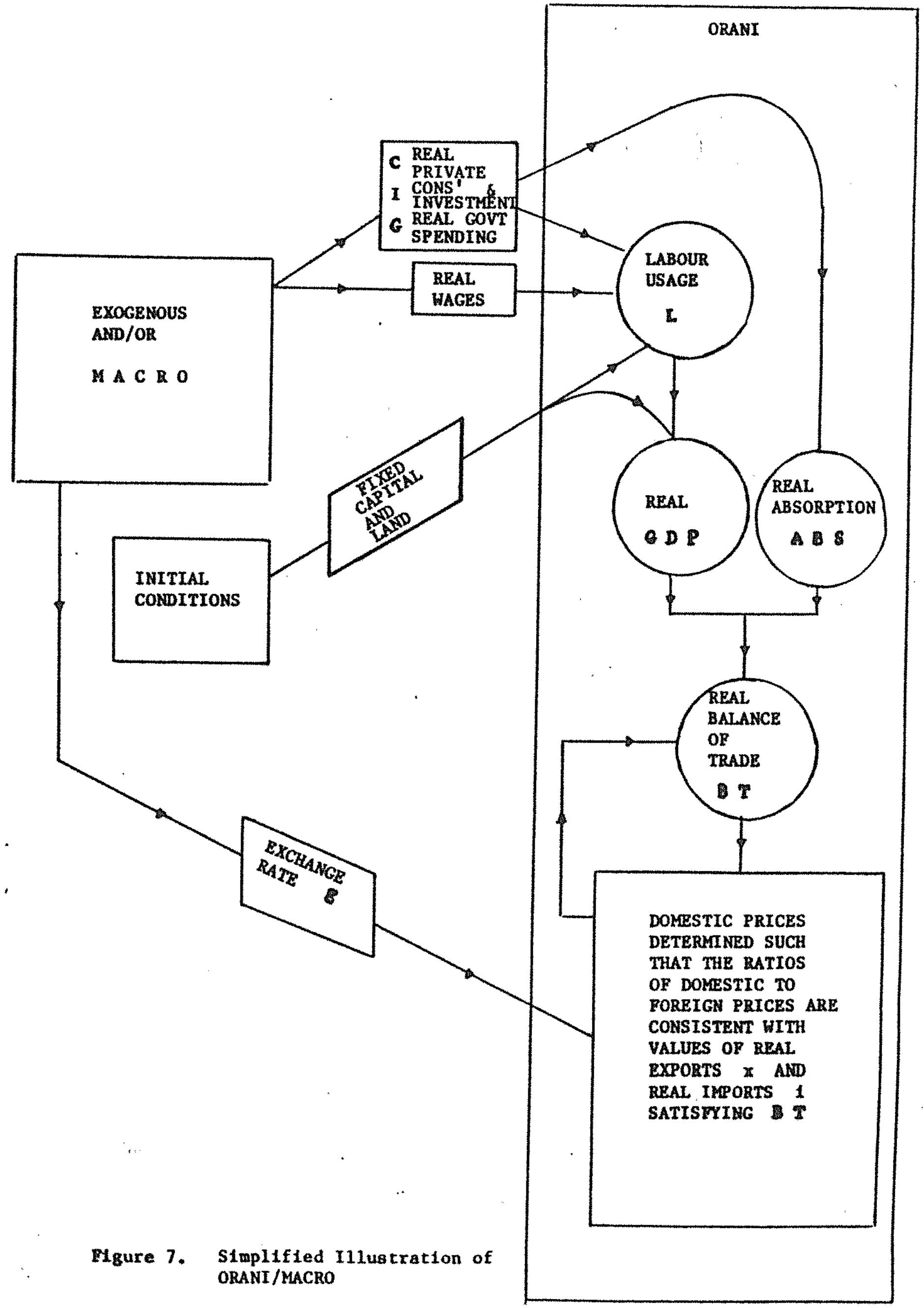


Figure 7. Simplified Illustration of ORANI/MACRO

Printed by C. J. Thompson, Commonwealth Government Printer, Canberra

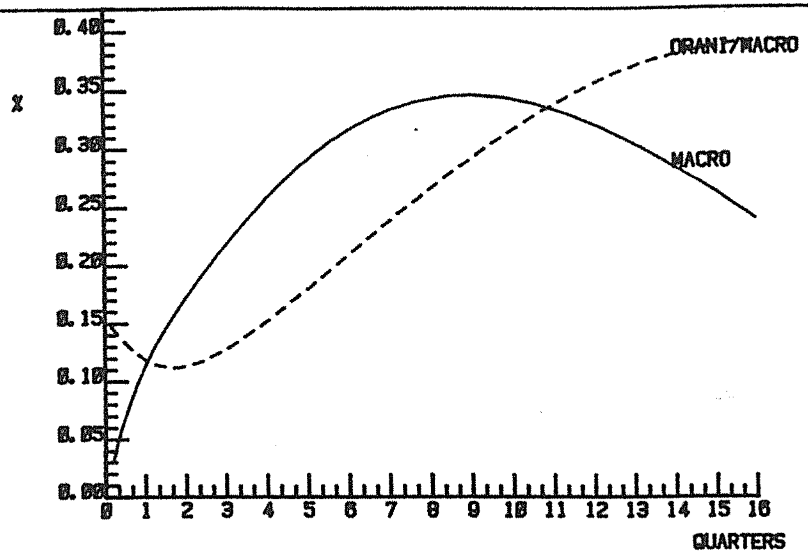


FIGURE 6a. VARIABLE :d log :P: BETA = .918

Contents

Appendices

[First digit of number refers to section in the text to which Appendix relates]

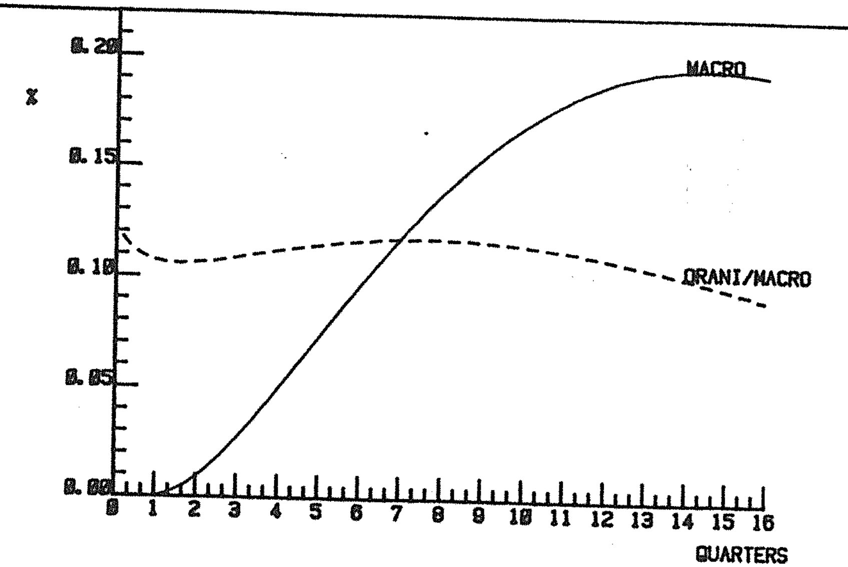


FIGURE 6a. VARIABLE : d log L BETA = .918

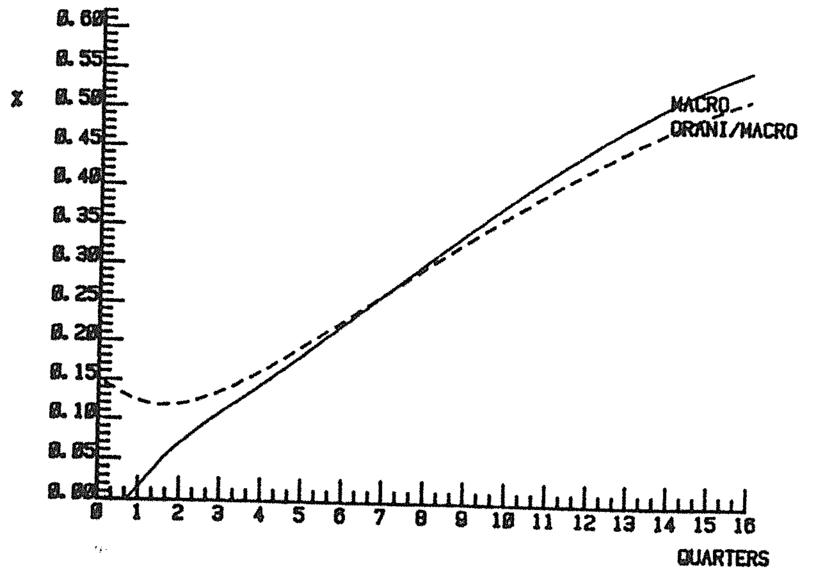


FIGURE 6b. VARIABLE : d log Pyg BETA = .918

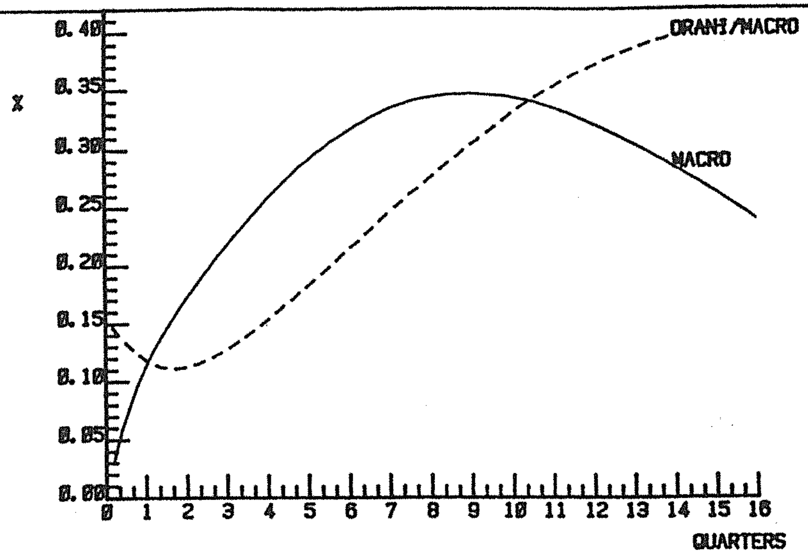


FIGURE 5a. VARIABLE :d log :P: BETA = .883

THE ORANI-MACRO INTERFACE

by

Russel J. Cooper and Keith R. McLaren*

1. Introduction

A convenient device for thinking about the IMPACT Project's medium term model is in terms of three distinct modules :

(i) ORANI, a general equilibrium model specifying the sectoral composition of output, the aggregate volumes and composition of imports and exports, occupation specific demands for labour, and relative commodity prices ;

(ii) MACRO, a macroeconomic model determining aggregate levels of real private consumption and investment, and modelling the financial and monetary markets ;

and

(iii) BACHUROO, a demographic model endogenizing the supply of labour disaggregated by nine occupational groups.[1]

The development of each of these three components has so far proceeded independently. This paper is concerned with the interfacing of the ORANI and MACRO modules.

The idea of constructing ORANI and MACRO as separate modules is appealing from the point of view of division of labour, but may be open to criticism on at least two grounds. The first concerns estimation. Given that ORANI and MACRO do not constitute a fully block

---

* The authors wish to thank Alan Powell for his constructive criticism and guidance, Peter Dixon, Brian Parmenter, Russell Rimmer and Dennis Sams for helpful contributions, and Vivian Lees for computational assistance.

1. For more detail on the original design of IMPACT's medium term model, see Alan A. Powell, The IMPACT Project : An Overview, March 1977, First Progress Report of the IMPACT Project, Volume 1 (Canberra : Australian Government Publishing Service, 1977), pp. xix + 182.

recursive model, their joint estimation would be required to ensure cross-equation consistency and efficiency. However, in the absence (among other things) of

(a) an integrated data base - - ORANI is based largely on input-output data from a given historical year, while MACRO uses quarterly aggregate time series national accounts data ,

(b) a theory of estimation which handles the two classes of data ,

and

(c) computer hardware capable of handling such a massive task ,

reversion to the second-best alternative, namely, separate FIML estimation of each module, is the appropriate course.

The second criticism, made from an economic-theoretic standpoint, is that certain endogenous variables should be determined consistently with the mechanisms posited in both modules, an outcome which cannot be guaranteed when each is developed as a separate model in its own right. This difficulty is the challenge to be faced in linking the two modules, at which stage the possible interdependencies between them must be incorporated.

From this latter perspective, the theory of the interface between the modules becomes a critical methodological issue. This theory must, of course, reflect the theoretical structures of the two modules. Of the various sub-classes of models in the class of general equilibrium models, ORANI is of the Johansen type.[1] A working version

1. The class of applied general equilibrium models pioneered by Leif Johansen in A Multi-Sectoral Study of Economic Growth (Amsterdam : North-Holland, 1960).

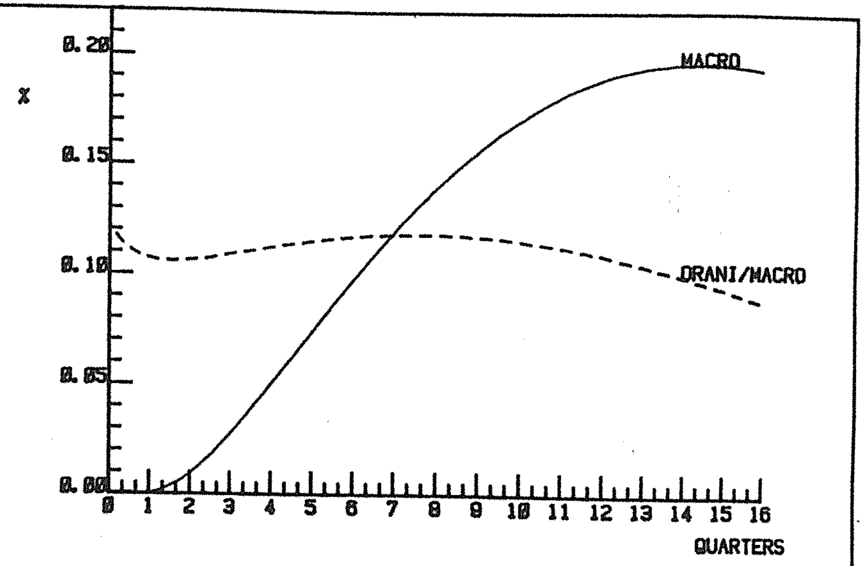


FIGURE 5a. VARIABLE : d log L BETA = .883

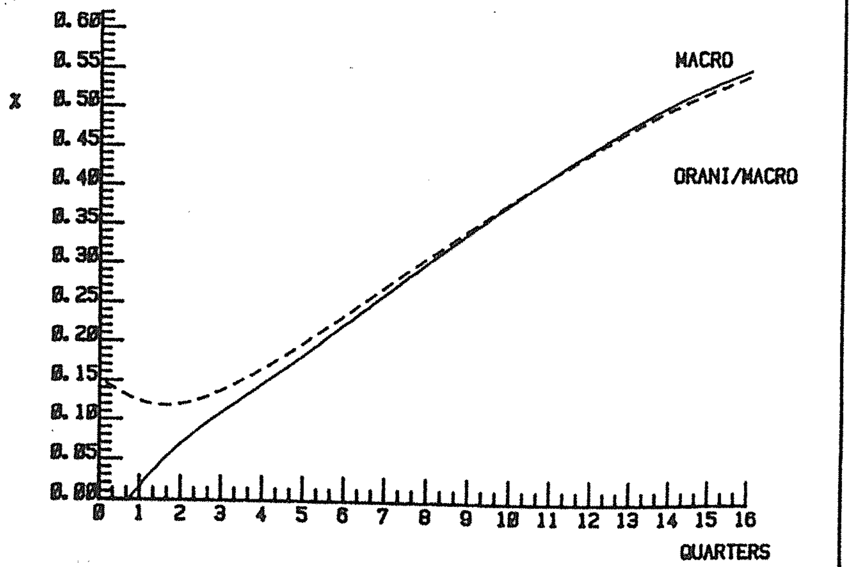


FIGURE 5b. VARIABLE : d log Pyg BETA = .883

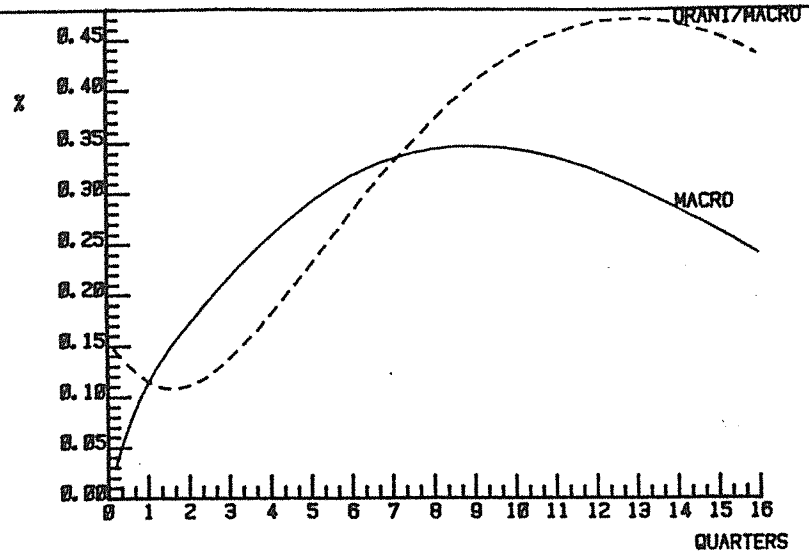


FIGURE 4o. VARIABLE :d log :P1 BETA = .483

of ORANI, ORANI 77, is now available.[1] In the terminology of Challen and Hagger,[2] MACRO might be chosen either from the Keynes-Klein (KK) class, or the Phillips-Bergstrom (PB) class (although there are some strong arguments for choosing the latter).[3] It is proposed that MACRO should be based on the RBA minimal model, a disequilibrium model of the PB class developed by Jonson and co-workers at the Reserve Bank of Australia.[4] The particular features of this model - - its formulation as a disequilibrium system in continuous time, its linearity in differentials of logarithms - - suggest that this model (and others in the same sub-class) be referred to as a Bergstrom model.[5]

The structure of the remainder of this paper is as follows. In Section 2, salient features of the Bergstrom and Johansen sub-classes of models are described. A notation for discussing the interface is introduced in Section 3. In Sections 4 and 5, a general theory for linking a model of the Johansen type with a model of the Bergstrom type is developed. This theory is then specialised to the case of an ORANI-MACRO link in Section 6. Results are presented in the latter part of Section 7. Concluding remarks are offered in Section 8.

---

1. For a comprehensive technical description, see Peter B. Dixon, B. R. Parmenter, G. J. Ryland and John Sutton, ORANI, A General Equilibrium Model of the Australian Economy : Current Specification and Illustrations of Use for Policy Analysis, First Progress Report of the IMPACT Project, Volume 2 (Canberra : Australian Government Publishing Service, 1977), pp. xi + 297.

2. D. W. Challen and A. J. Hagger, "Economy-Wide Modelling with Special Reference to Australia," Economic Society of Australia and New Zealand, 8th Conference of Economists, LaTrobe University, Melbourne, Australia, August 1979 (copies available from Department of Economics, University of Tasmania, Hobart, Tasmania, Australia).

3. For a statement of these arguments, see Alan A. Powell, "The Major Streams of Economy-Wide Modelling : Is Rapprochement Possible?", IMPACT Preliminary Working Paper, I-09, Industries Assistance Commission, Melbourne, April 1980.

4. For a detailed critical assay of the design and performance of the prototype, see Reserve Bank of Australia, Conference in Applied Economic Research, December 1977 (Sydney : Reserve Bank of Australia).

5. For a technical description of continuous time econometric models, see A. R. Bergstrom (ed.), Statistical Inference in Continuous Time Econometric Models (Amsterdam : North-Holland, 1976).

2. The Two Model Types

2.1 The Bergstrom Model

This is a macroeconomic model specified as a first order stochastic differential equation system. It may be represented as:

$$\dot{Y}_B(t) = A_B Y_B(t) + N_B Z_B(t) + u(t) \tag{2.1}$$

where Y_B refers to a vector of the logarithms of endogenous variables, Z_B is a vector of the logarithms of exogenous variables and u is a white noise disturbance vector. The typical equation specification is one of partial adjustment of endogenous variables to their desired or long-run values.

A more rigorous specification of the underlying stochastic differential equation system is:

$$d\, Y_B(t) = A_B Y_B(t)dt + N_B Z_B(t)dt + dv(t) \tag{2.2}$$

where $dv(t)$ is a Gaussian vector process. The solution of this system may be written:

$$Y_B(t) = e^{A_B t} \{Y_B(0) + \int_0^t e^{-A_B \tau} N_B Z_B(\tau) d\tau + \int_0^t e^{-A_B \tau} dv(\tau)\} . \tag{2.3}$$

Let $Z_B(\tau) = Z_B^c(\tau)$ = cet.par. value of a time path of exogenous variables. The control values of Y_B from (2.3) are:

$$Y_B^c(t) = e^{A_B t} \{Y_B(0) + \int_0^t e^{-A_B \tau} N_B Z_B^c(\tau) d\tau + \int_0^t e^{-A_B \tau} dv(\tau)\} . \tag{2.4}$$

Let $Z_B^s(\tau) = Z_B^c(\tau) + z_B$, with z_B a constant vector of exogenous shocks. Then the shocked values of Y_B are:

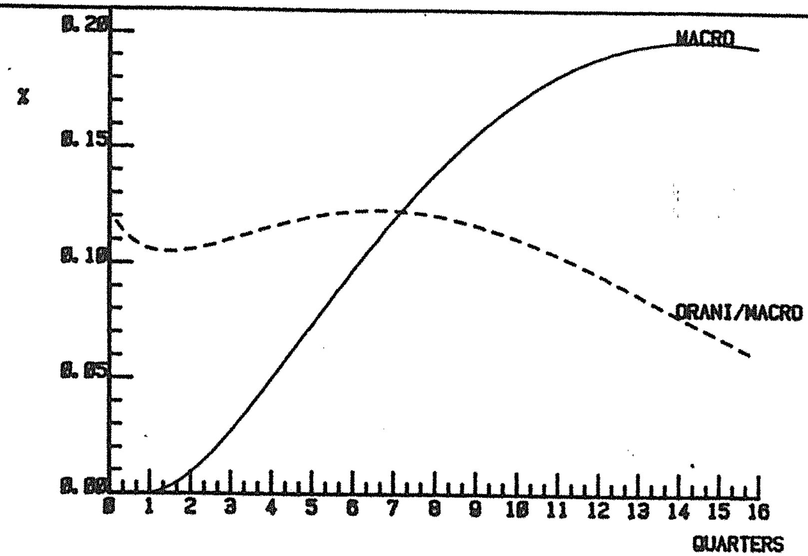


FIGURE 4a. VARIABLE : d log L BETA = .483

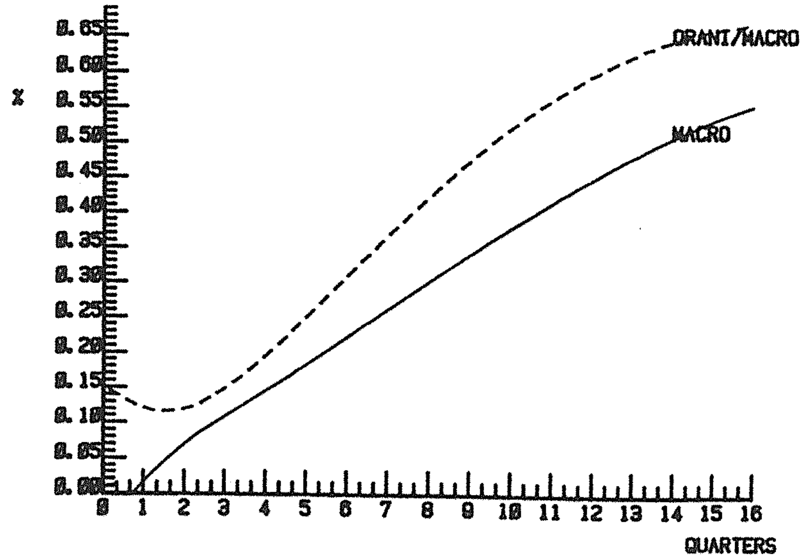


FIGURE 4b. VARIABLE : d log Pyg BETA = .483

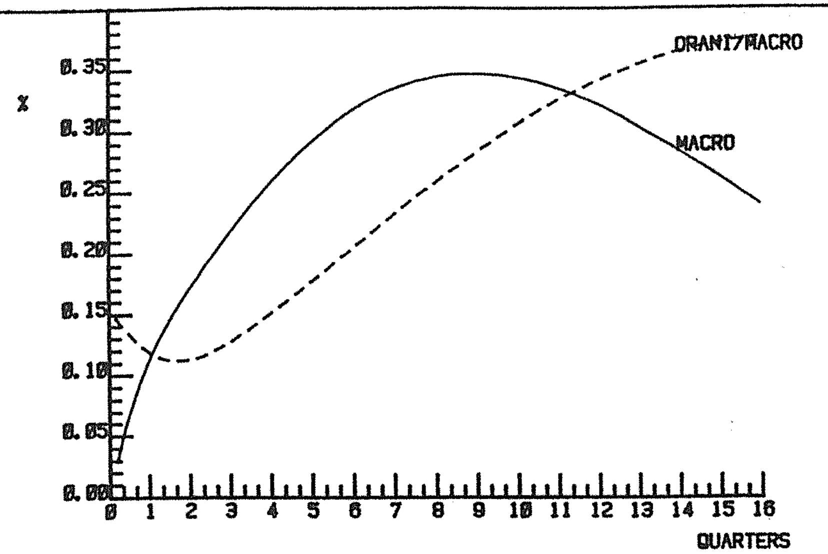


FIGURE 3e. VARIABLE :d log :Pi BETA = .950

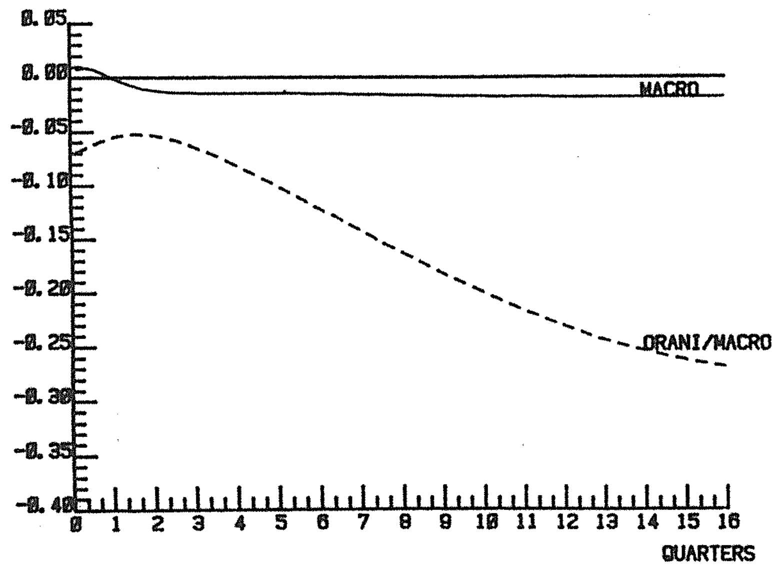


FIGURE 3f. VARIABLE :dlog xPx/E BETA = .950

(2.5) $$Y_B^s(t) = e^{A_B t}\{Y_B(0) + \int_0^t e^{-A_B \tau} N_B Z_B^s(\tau)\, d\tau + \int_0^t e^{-A_B \tau} dv(\tau)\} .$$

Defining the deviation of the shocked value from the control value by lower case letters, (2.5) - (2.4) gives:

(2.6) $$y_B(t) = [\int_0^t e^{A_B(t-\tau)} d\tau]\, N_B\, z_B .$$

Defining:

(2.7) $$C_B(t) = [\int_0^t e^{A_B(t-\tau)} d\tau]\, N_B = A_B^{-1} [e^{A_B t} - I] N_B ,$$

we may write (2.6) as:

(2.8) $$y_B(t) = C_B(t)\, z_B .$$

The interpretation of (2.8) is as follows. Since Y_B and Z_B are logarithms of variables, y_B and z_B are proportional changes. The latter, z_B, represent proportional changes which occur in the levels of the exogenous variables Z_B at time 0 and which are maintained throughout the interval [0,t]. The former, y_B (t), represent the accumulated proportional changes in the Bergstrom model's endogenous ariables up to the point in time t .

2.2 The Johansen Model

In its reduced form, a general equilibrium model may be characterised by the set of equilibrium conditions relating the levels of the endogenous variables Y to the levels of the exogenous variables Z

(2.9) $$Y = f(Z) .$$

Movement from one equilibrium position to another, generated by a small change in the exogenous variables Z , is represented as:

(2.10) $$dY = f_{Z'}(Z)dZ$$

where the ij^{th} element of the Jacobian is $\partial f_i(Z)/\partial Z_j$, f_i is the i^{th} element of the vector function f , and Z_j is the j^{th} element of Z. A Johansen model is characterised by the assumption that the reduced form elasticities are constant for practical purposes. Thus (2.10) is written in proportional change form as:

(2.11) $$y_J = C_J z_J$$

where C_J is treated as a constant matrix determined by the behavioural parameters of f and the conditions prevailing in the initial equilibrium.

Although there is no explicit analysis of the path by which y_J is achieved in a Johansen model, some assumption about the length of time needed for y_J to eventuate is implicit in the estimation of the elasticities C_J and may also be related to the choice of the endogenous/exogenous split.

2.3 Compatibility of the Models

A comparison of (2.8) and (2.11) reveals the potential for compatibility of Bergstrom and Johansen models. The final form of each model is linear in percentage changes. This raises the possibility of endowing the Johansen model with explicit dynamics which are compatible with those of the Bergstrom model. This approach is developed in later sections, and allows the interface of the two types of models.

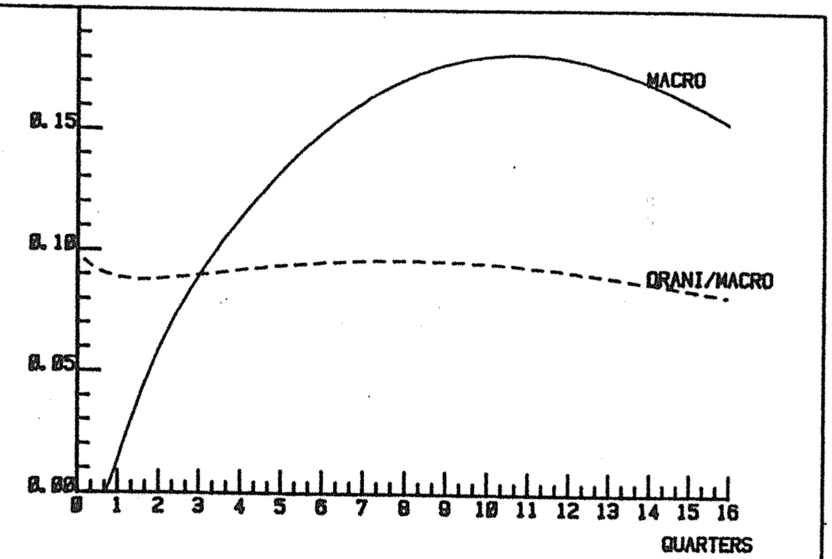


FIGURE 3a. VARIABLE : d log yg BETA = .950

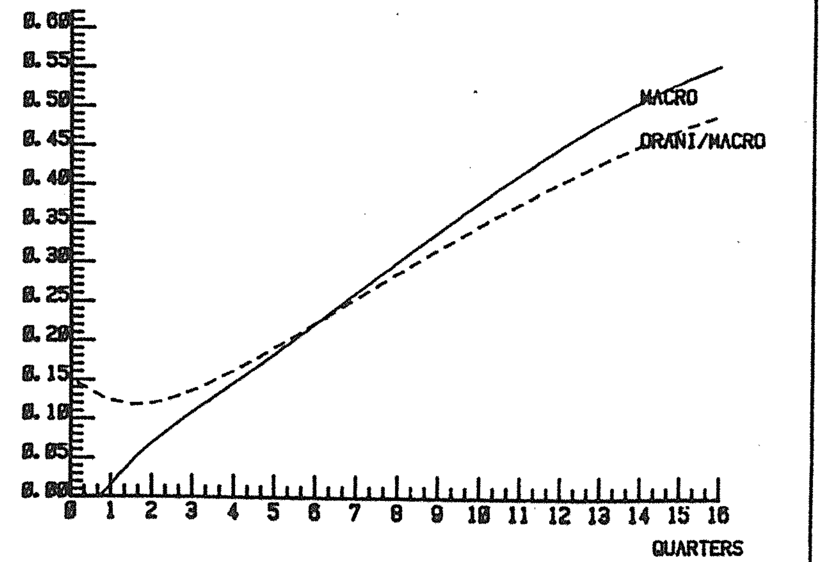


FIGURE 3d. VARIABLE : d log Pyg BETA = .950

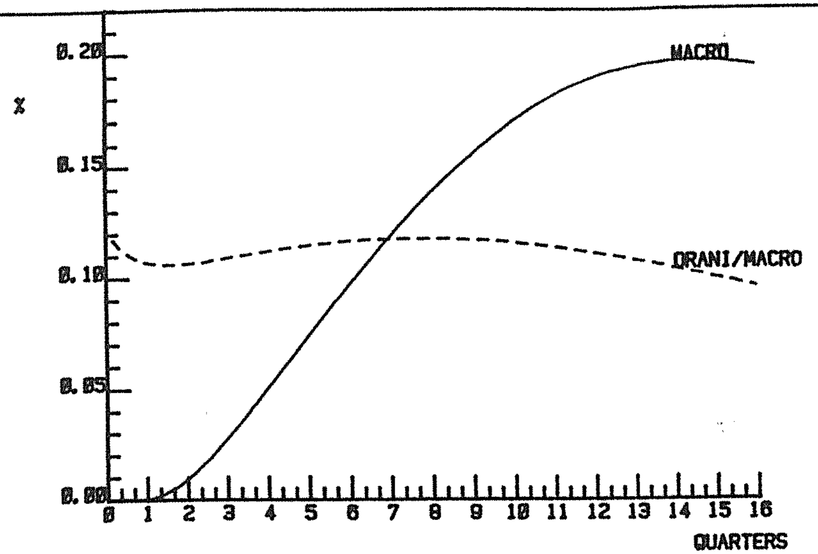


FIGURE 3a. VARIABLE : d log L BETA = .950

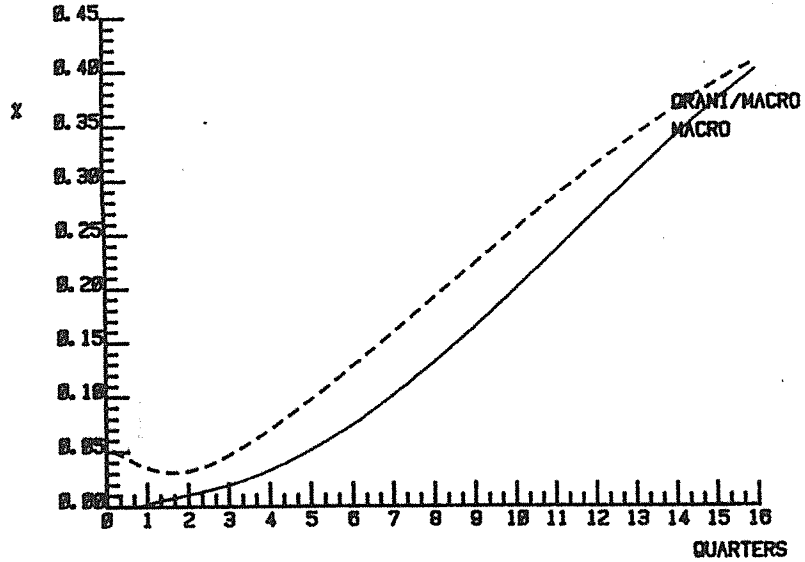
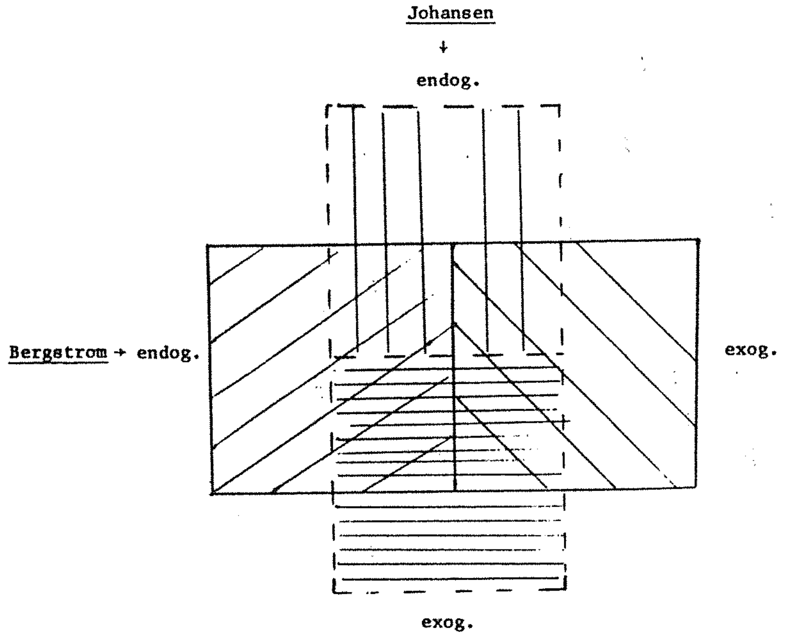


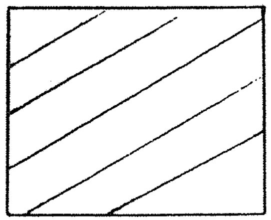
FIGURE 3b. VARIABLE : d log P BETA = .950

3. Notation for the Interface

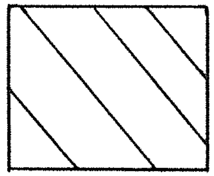
To develop a notation which is consistent in its treatment of the endogenous/exogenous variables in Johansen/Bergstrom, consider the following Venn diagram:



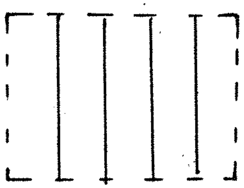
Let y_B = vector of all variables endogenous to the full Bergstrom system. In the above diagram,



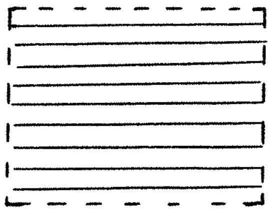
denotes the set containing the elements of y_B . Let z_B = vector of all variables exogenous to the Bergstrom model. In the diagram, z_B is denoted by:



Let y_J = vector of all variables endogenous to the Johansen model. y_J is denoted by:



Let z_J = vector of all variables exogenous to the Johansen model. z_J is denoted by:



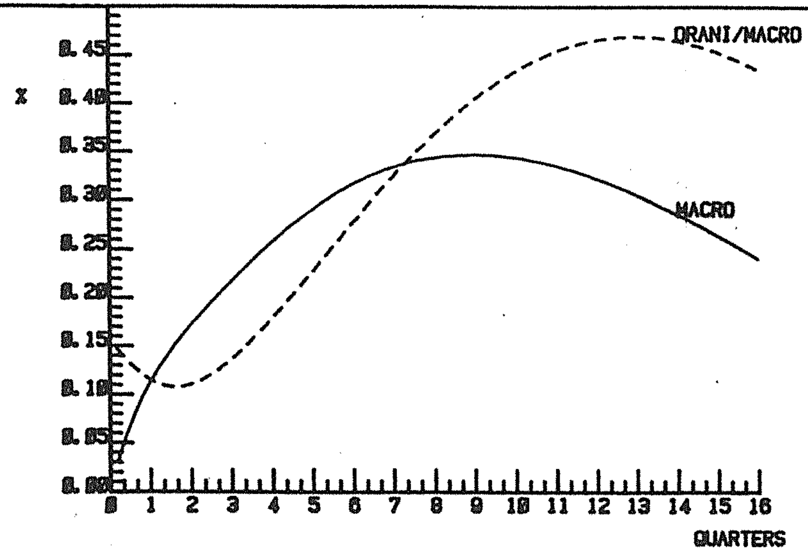


FIGURE 2e. VARIABLE : d log iPi BETA = .500

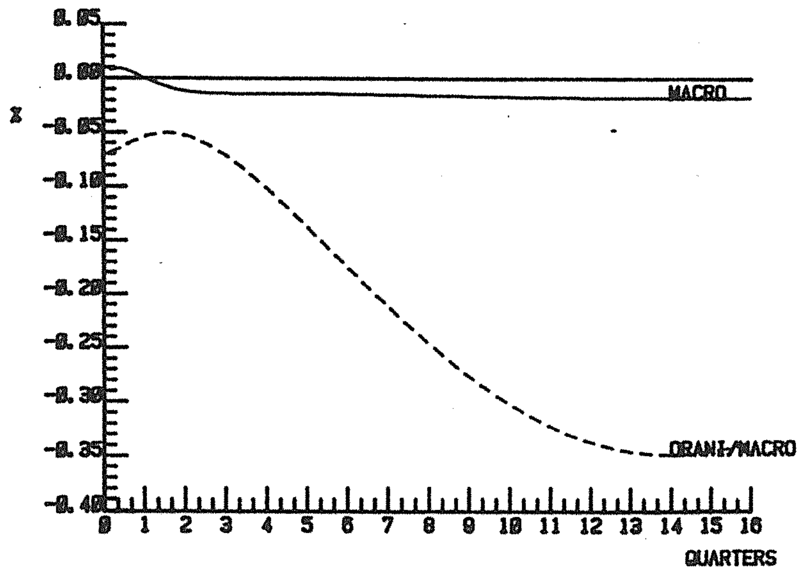


FIGURE 2f. VARIABLE : dlog xPx/E BETA = .500

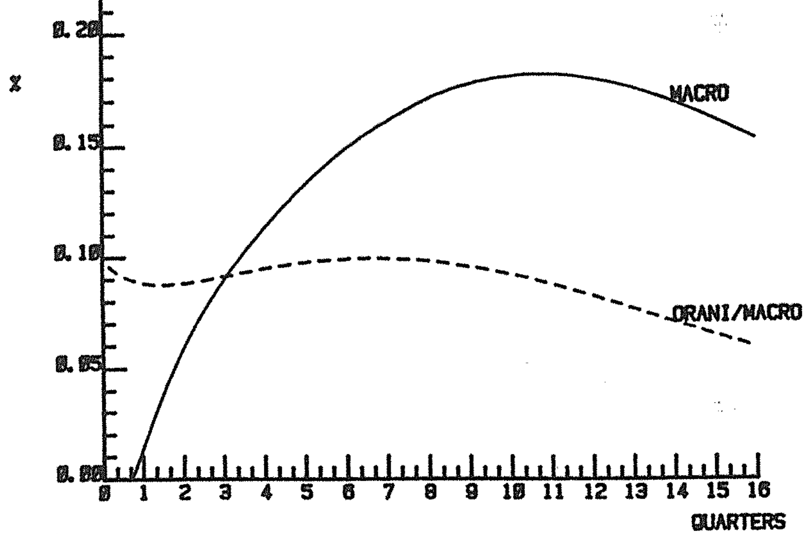


FIGURE 2c. VARIABLE : d log yg BETA = .500

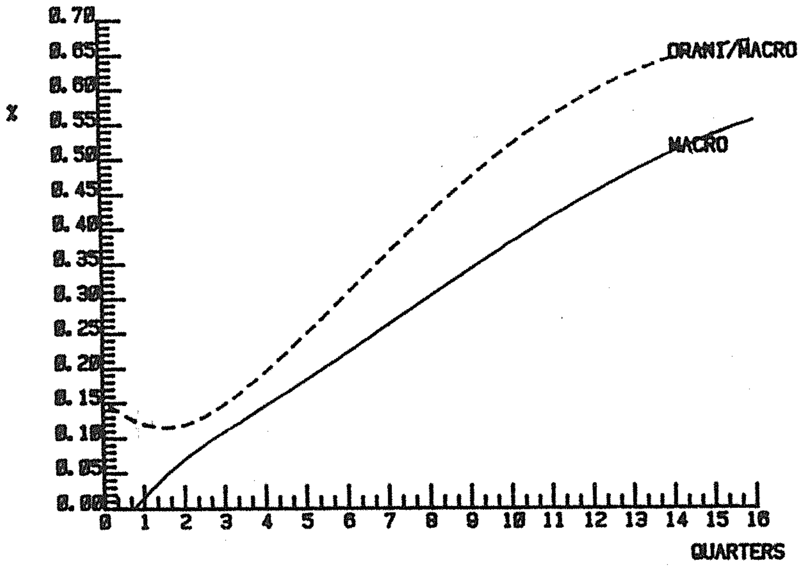
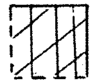


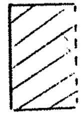
FIGURE 2d. VARIABLE : d log Pyg BETA = .500

Variables in the cross-hatched areas of the diagram may now be defined in terms of y_B, y_J, z_B and z_J as follows:

$$y_B = \begin{pmatrix} y_B y_J \\ y_B z_J \\ y_B - y_B x_J \end{pmatrix}$$

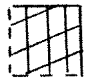
$y_B y_J$ ⟵ the set  in vector form

$y_B z_J$ ⟵ the set  in vector form

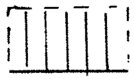
$y_B - y_B x_J$ ⟵ the set  in vector form

where $y_B y_J$ is a vector of variables in the intersection of the sets y_B and y_J, etc., and $x_J = \begin{pmatrix} y_J \\ z_J \end{pmatrix}$, $x_B = \begin{pmatrix} y_B \\ z_B \end{pmatrix}$.

$$y_J = \begin{pmatrix} y_J y_B \\ y_J z_B \\ y_J - y_J x_B \end{pmatrix}$$

$y_J y_B$ ⟵ the set 

$y_J z_B$ ⟵ the set 

$y_J - y_J x_B$ ⟵ the set 

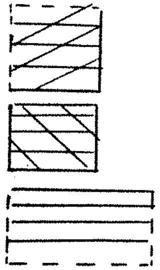
$$z_B = \begin{pmatrix} z_B y_J \\ z_B z_J \\ z_B - z_B x_J \end{pmatrix}$$

$z_B y_J$ ⟵ the set 

$z_B z_J$ ⟵ the set 

$z_B - z_B x_J$ ⟵ the set 

$$z_J = \begin{pmatrix} z_J y_B \\ z_J z_B \\ z_J - z_J x_B \end{pmatrix} \begin{array}{l} \longleftarrow \text{ the set} \\ \longleftarrow \text{ the set} \\ \longleftarrow \text{ the set} \end{array}$$



In this notation, the vector $y_J y_J$ contains the same set of elements as $y_J y_B$, etc. Thus the first block of elements of y_B is the same as the first block in y_J, the second in y_J the same as the first in z_B, the second in z_B the same as the second in z_J, and the second in y_B the same as the first in z_J. The vectors y_B and y_J respectively contain all the variables endogenous to the Bergstrom model and to the Johansen model. The three sub-categories of endogenous variable represented by the partitioning are: endogenous to both models, endogenous to own model but exogenous to other model, and endogenous to own model but not appearing in other model. The three subcategories of the exogenous variable sets z_B and z_J are: exogenous to own model, but endogenous to other model, exogenous in both models, and exogenous to own model but not appearing in other model.

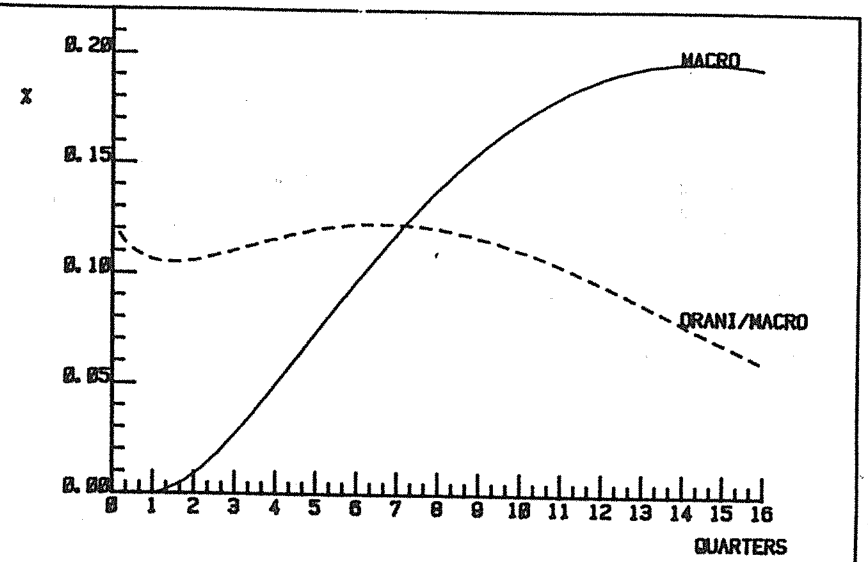


FIGURE 2a. VARIABLE : d log L BETA = .500

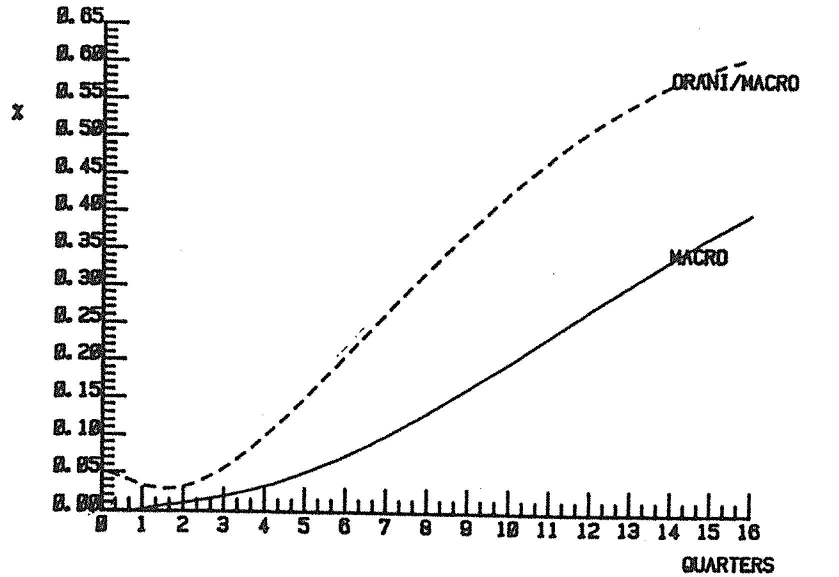


FIGURE 2b. VARIABLE : d log P BETA = .500

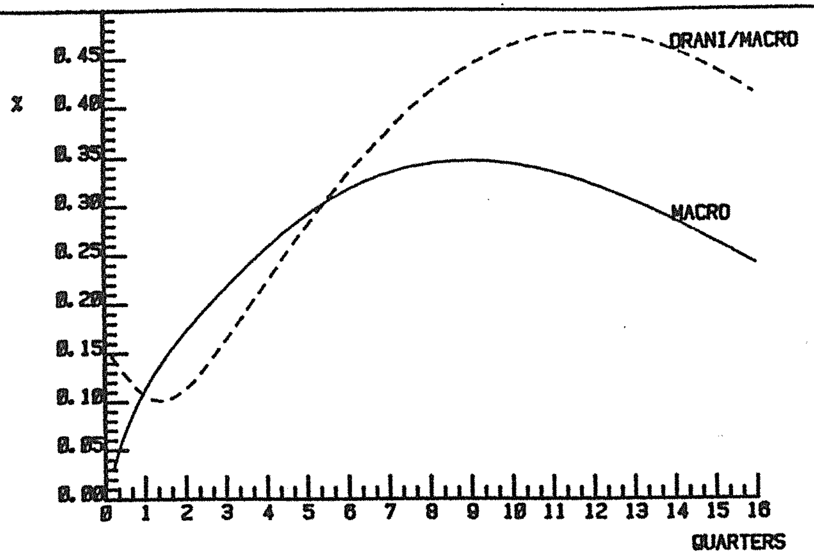


FIGURE 1e. VARIABLE : d log iPi BETA = .050

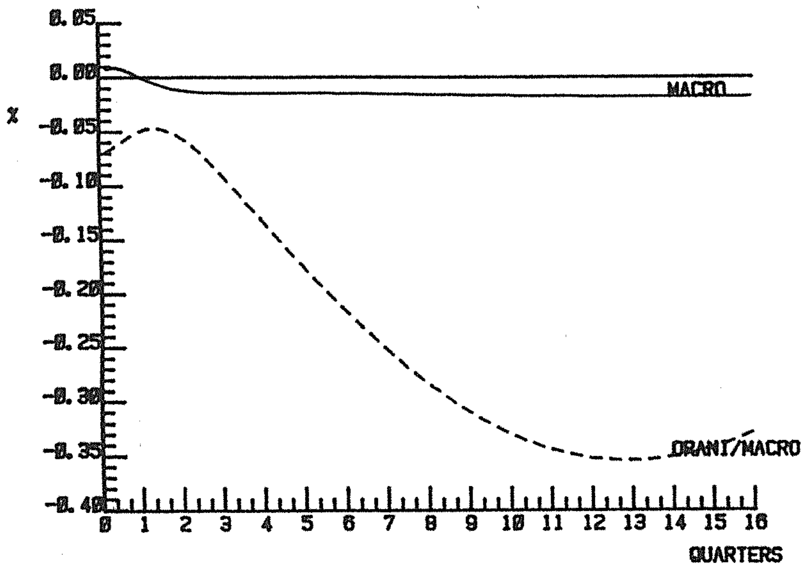


FIGURE 1f. VARIABLE : dlog xPx/E BETA = .050

4. The Models Combined

Rewriting (2.11) and (2.8) in the new notation gives

$$(4.1)\quad \begin{matrix}(a)\\(b)\\(c)\end{matrix}\begin{pmatrix} y_J y_B \\ y_J z_B \\ y_J - y_J x_B \end{pmatrix} = \begin{bmatrix} C_J^{11} & C_J^{12} & C_J^{13} \\ C_J^{21} & C_J^{22} & C_J^{23} \\ C_J^{31} & C_J^{32} & C_J^{33} \end{bmatrix} \begin{pmatrix} z_J y_B \\ z_J z_B \\ z_J - z_J x_B \end{pmatrix} \quad [\textit{Johansen}]$$

$$(4.2)\quad \begin{matrix}(a)\\(b)\\(c)\end{matrix}\begin{pmatrix} y_B y_J \\ y_B z_J \\ y_B - y_B x_J \end{pmatrix} = \begin{bmatrix} C_B^{11} & C_B^{12} & C_B^{13} \\ C_B^{21} & C_B^{22} & C_B^{23} \\ C_B^{31} & C_B^{32} & C_B^{33} \end{bmatrix} \begin{pmatrix} z_B y_J \\ z_B z_J \\ z_B - z_B x_J \end{pmatrix} \quad [\textit{Bergstrom}]\ .$$

Consideration of (4.1) - (4.2) combined raises a number of points. Firstly, equations (4.1) and (4.2) separately represent closed linear models, each having as many linearly independent equations as endogenous variables. But the set of variables $y_J y_B$ is endogenous to both models and the equations separately describing these variables in each model will, in general, have een estimated without consideration of this implied constraint. Therefore there is no justification in principle for deleting (4.1a) while maintaining (4.2a) nor for adopting the converse approach. The only sensible reduced form that is implied by the joint system is that which includes both $y_J y_B$ and $y_B y_J$.

Secondly, an attempt to put (4.1) and (4.2) directly in reduced form would ignore a timing problem. (4.2) is the final form of a dynamic model. Thus, for example, in the set of equations (4.2) (b), $y_B z_J$ is interpreted as an accumulated response after t periods to a shock to z_B which is sustained over $[0,t]$. On the other hand, where $y_B z_J$ occurs as $z_J y_B$, a sub-component of the exogenous variables driving the system (4.1), it has the interpretation as a sustained shock over $[0,t]$ and cannot be directly identified with a continuously accumulating response.

Thirdly, three special cases of the combined system (4.1) - (4.2) may be noted. If $z_B y_J$ is null then a recursive system results in which the Bergstrom model drives the Johansen model. If the Bergstrom model is a small aggregative macroeconomic model and the Johansen model is a large disaggregated microeconomic model then this case may be the typical one. Conversely, if it is $z_J y_B$ which is null, the Johansen model would drive the Bergstrom model. In each of these cases there remains a one-way timing problem such as that illustrated in the second point above, as well as the problem of double endogeneities. On the other hand, in the case where both $z_B y_J$ and $z_J y_B$ are null only the problem of double endogeneities remains.

Fourthly, the set of equations (4.1) provides the accumulated response of the endogenous variables y_J after an elapsed time, t^*, where t^* is implicit in the structure of the Johansen model. On the other hand, (2.7) demonstrates that the derivation of C_B from the estimates of A_B and N_B requires an explicit value of t. Consistency requires that $t=t^*$ and hence an explicit estimate of t^* is required.

Section 5 provides a methodology for simultaneously solving the above-mentioned problems and for deriving an appropriately interfaced model.
In considering the case of a one-way linkage it will be assumed for illustrative purposes that the Bergstrom model drives the Johansen model. In this case the combined models (4.1) - (4.2) simplify to:

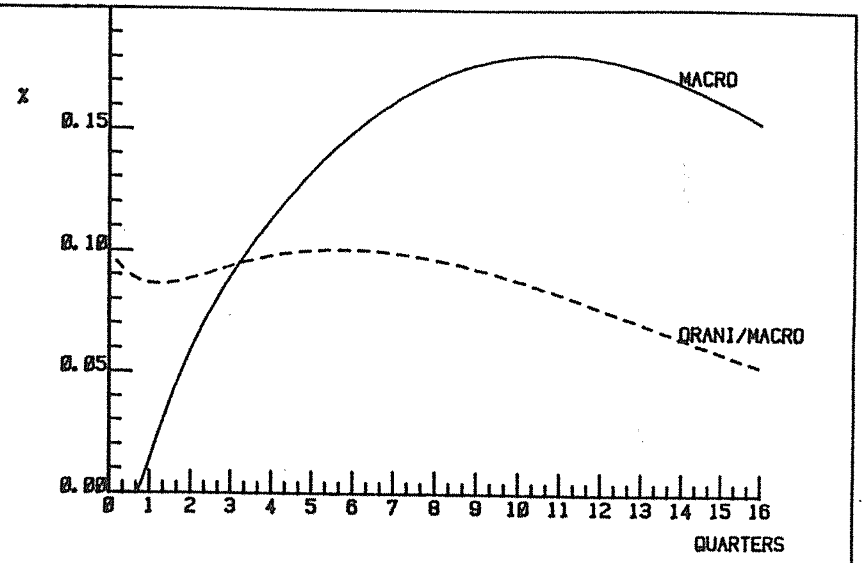


FIGURE 1a. VARIABLE : d log yg BETA = .050

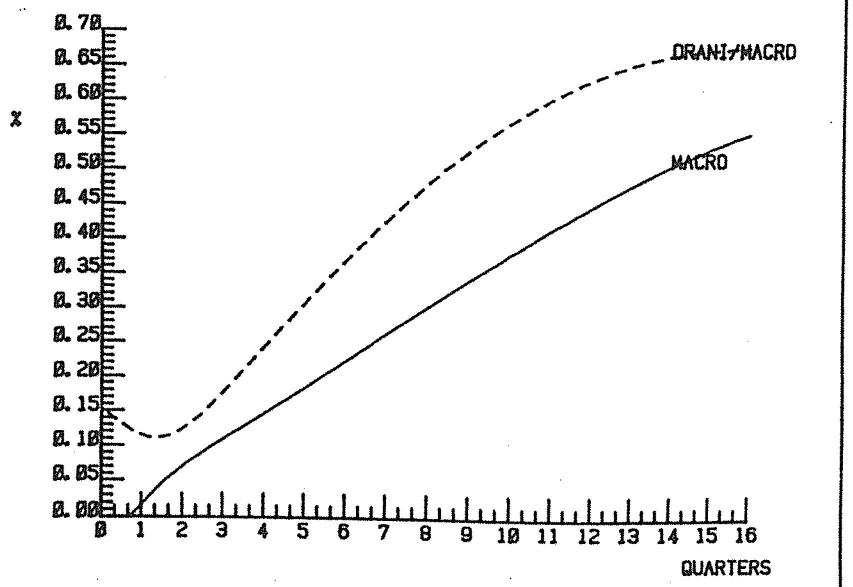


FIGURE 1d. VARIABLE : d log Pyg BETA = .050

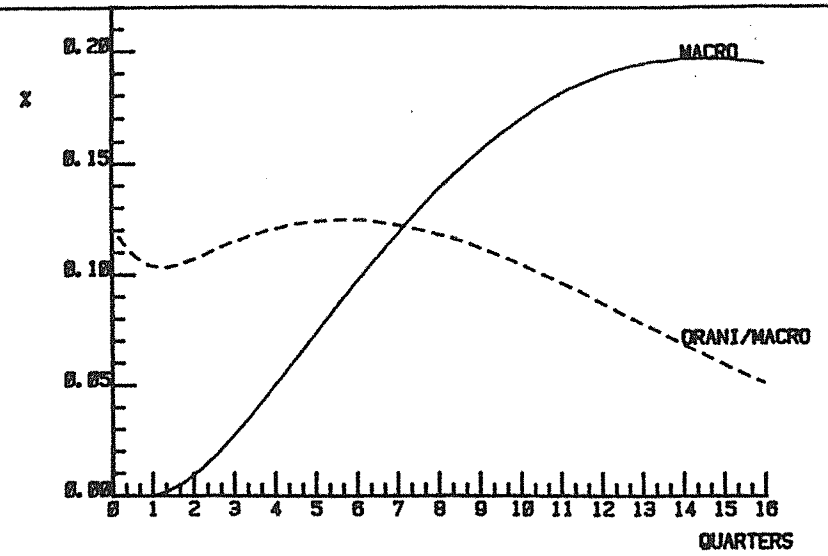


FIGURE 1a. VARIABLE : d log L BETA = .050

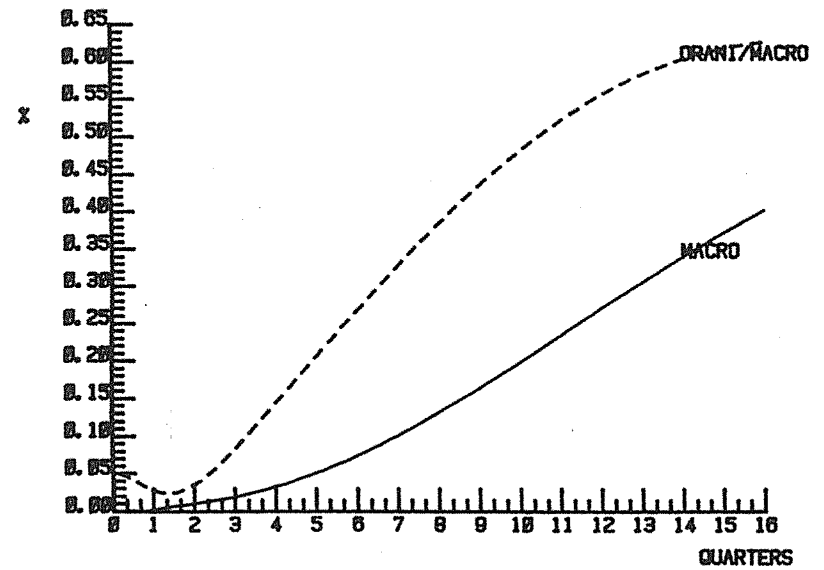


FIGURE 1b. VARIABLE : d log P BETA = .050

$$(4.1^*)\qquad \begin{matrix}(a)\\(c)\end{matrix}\begin{pmatrix} y_J y_B \\ y_J - y_J x_B \end{pmatrix} = \begin{bmatrix} C_J^{11} & C^{12} & C^{13} \\ C_J^{31} & C_J^{32} & C_J^{33} \end{bmatrix} \begin{pmatrix} z_J y_B \\ z_J z_B \\ z_J - z_J x_B \end{pmatrix}$$

$$(4.2^*)\qquad \begin{matrix}(a)\\(b)\\(c)\end{matrix}\begin{pmatrix} y_B y_J \\ y_B z_J \\ y_B - y_B x_J \end{pmatrix} = \begin{bmatrix} C_B^{12} & C_B^{13} \\ C_B^{22} & C_B^{23} \\ C_B^{32} & C_B^{33} \end{bmatrix} \begin{pmatrix} z_B z_J \\ z_B - z_B x_J \end{pmatrix}$$

The model (4.1*) - (4.2*) is dealt with in more detail than the general model (4.1) - (4.2), since the recursive structure is the prototype for the ORANI/MACRO link. It is also convenient in Section 5 to consider firstly a simplified version of the recursive structure in which $z_J = y_B$. In this case the models (4.1*) - (4.2*) simplify further to:

$$(4.1^{**})\qquad \begin{matrix}(a)\\(c)\end{matrix}\begin{pmatrix} y_J y_B \\ y_J - y_J y_B \end{pmatrix} = \begin{bmatrix} C_J^{11} \\ C_J^{31} \end{bmatrix} \begin{pmatrix} z_J y_B \end{pmatrix},$$

$$(4.2^{**})\qquad (b)\quad \begin{pmatrix} y_B z_J \end{pmatrix} = \begin{bmatrix} C_B^{22} & C_B^{23} \end{bmatrix} \begin{pmatrix} z_B z_J \\ z_B - z_B x_J \end{pmatrix},$$

in which the fully recursive structure is revealed by the identity of the left-hand variable of (4.2**) with the right-hand variable of (4.1**).

5. Timing Aspects of the Interface

5.1 The Dynamics Behind the Johansen Model

Recognising the fundamental similarity of (2.8) to (2.11), it is convenient to define matrices A_J and N_J such that

$$C_J = A_J^{-1} [e^{A_J t^*} - I] N_J \tag{5.1}$$

where t^* is the Johansen response interval. Clearly, without further restrictions A_J and N_J are not unique.

Given (5.1), a natural assumption concerning the dynamics of the Johansen response is:

$$C_J(s) = A_J^{-1} [e^{A_J s} - I] N_J , \quad 0 \leq s \leq t^* . \tag{5.2}$$

Thus

$$y_J(s) = [e^{A_J s} - I][e^{A_J t^*} - I]^{-1} y_J(t^*), \ 0 \leq s \leq t^* , \tag{5.3}$$

illustrates the fact that, provided A_J is stable, equilibrium is approached along a geometrically decaying path.

Once the Johansen model has an assumed dynamic structure, linking of the two models is feasible. The rest of this Section explains the way in which timing differences between the two models may be corrected, by decomposing a time path of shocks into a sequence of constant sustained shocks. A more rigorous but less intuitive approach is provided in Appendix 5.

Appendix 7C Graphical Analysis of ORANI Short Run

The following graphs present the responses for MACRO in stand alone form and for ORANI linked with MACRO which result from a one percent shock to government spending. Figures 1, 2 and 3 represent values of β of .05, .5 and .95 respectively while the sub-cases a through f correspond to the double endogeneities: (a) employment: d log L; (b) prices: d log P; (c) real output: d log yg; (d) nominal output: d log Pyg; (e) foreign currency value of imports: d log iPi; (f) foreign currency value of exports d log xPx/E . These graphs are to be interpreted in conjunction with Table 1 of Section 7.

Figures 4, 5 and 6 illustrate the final choice of the (t^*,β) combinations, and are to be interpreted in conjunction with Tables 2 and 3 of Section 7.

The interpretation of Figures 1 to 6 is as follows. The MACRO path is the actual dynamic response path of the relevant MACRO endogenous variable. The ORANI/MACRO path is a conditional response path i.e., any point on that path is to be interpreted as the interfaced ORANI short run response provided that the corresponding point in time can be interpreted as the ORANI short run t^*. Thus in each graph the point of intersection of the paths gives a potential (t^*,β) combination in which the relevant double endogeneity is predicted consistently by both MACRO and the interfaced version of ORANI.

Figures 1 to 3 summarise a broad search for (t^*,β) combinations. Figures 4 to 6 represent the final choice, where a unique (t^*,β) combination is defined for two double endogeneities simultaneously.

(b) ORANI Elasticities File

The format of each card is (A1, I4, AI, I4, F10,0). The two permissible types are

(a) Y---I Y---J ELAS

if YJ is exogenous to ORANI but endogenous to MACRO

(b) Y---I Z---J ELAS

if ZJ is exogenous to ORANI and MACRO

where ELAS = elasticity value

(c) MACRO Variable Names File

Two files of names may be provided, one for MACRO exogenous variables and one for MACRO endogenous variables. These files associate mnemonic names with variable numbers. Format is one name per line in format A10.

5.2 The Recursive Linkage

5.2.1 A Simplified Case

We consider firstly the case $z_J = y_B$. Thus the combined model, uncorrected for timing, is (4.1**) - (4.2**). In this sub-section the shorthand notations y_J, y_B, z_J, z_B, C_J, C_B are understood to represent the vectors and matrices of the system (4.1**) - (4.2**).

Consider now a shock to the Bergstrom model which occurs at time 0 and is sustained over $[0,t]$. Represent this by $z_B[0,t]$. This produces the Bergstrom response:

$$(5.4) \qquad y_B(s) = C_B(s)z_B[0,t] \ , \ 0 \leqslant s \leqslant t \ ,$$

where
$$C_B(s) = A_B^{-1}[e^{A_B s} - I]N_B \ .$$

Identifying y_B with z_J leads to a time varying shock $z_J(s)$ to the Johansen model. Representing $z_J(s)$ as the integral of its derivatives:

$$(5.5) \qquad z_J(s) = \int_0^s \left(\frac{dz_J(\tau)}{d\tau}\right) d\tau$$

allows each of the derivatives to be interpreted as a shock sustained over the period $[\tau,t]$:

$$(5.6) \qquad \frac{dz_J(\tau)}{d\tau} = z_J[\tau,t] \ , \quad 0 \leqslant \tau \leqslant t \ .$$

The effect of this latter shock on the Johansen model can be derived by use of (5.2), with $s = t - \tau$. That is:

(5.7) $$A_J^{-1}\,[e^{A_J(t-\tau)}-I]\,N_J z_J[\tau,t]\ .$$

By the linearity of the Johansen model, the total response to the time varying response $z_J(s)$, $0 \leq s \leq t$, is therefore the integral of expression (5.7) over $[0,t]$:

(5.8) $$y_J(t) \;=\; \int_0^t A_J^{-1}[e^{A_J(t-\tau)}-I]N_J z_J[\tau,t]d\tau$$

Now,

$$z_J[\tau,t] \;=\; \frac{dz_J(\tau)}{d\tau} \;=\; \frac{dy_B(\tau)}{d\tau} \;=\; e^{A_B\tau}\,N_B z_B[0,t]$$

and hence,

(5.9) $$y_J(t) \;=\; \int_0^t A_J^{-1}\,[e^{A_J(t-\tau)}-I]N_J e^{A_B\tau}d\tau N_B z_B[0,t]\ .$$

Thus the accumulated Johansen response at time t can be expressed as a matrix:

$$\left(\int_0^t A_J^{-1}[e^{A_J(t-\tau)}-I]N_J e^{A_B\tau}d\tau\right)N_B$$

times a sustained shock $z_B[0,t]$ to the Bergstrom model. Equation (5.9) is the reduced form (adjusted for timing) of the recursive system (4.1**) - (4.2**) . It is convenient to express (5.9) in structural form as:

(5.10)
$$y_J(t) \;=\; C_{J/B}\,y_B(t)$$
$$y_B(t) \;=\; C_B z_B[0,t]$$

where: $$C_{J/B} \;=\; \int_0^t A_J^{-1}\,[e^{A_J(t-\tau)}-I]\,N_J e^{A_B\tau}\,d\tau\,[e^{A_B t}-I]^{-1}A_B\ .$$

IFDYN	= 1	to generate dynamic ORANI/MACRO path for t=T1 to TSTAR
	= 0	to generate conditional ORANI/MACRO response
TSTAR	=	length of ORANI start-run (if IFDYN=1)

12. NEXOG (I5)

NEXOG = no. of exogenous variables to be perturbed

13. I, VALUE (I5, F10.0)

exogenous variable I perturbed by VALUE

repeat 13 NEXOG times

sets 12 - 13 may be repeated

Program terminates when NEXOG = -1 is encountered.

7. (IFINT ≠ 0) BK, BDK, BICE, BI, BY, BYG (6F10.0)

BK	=	base value of K
BDK	=	base value of δK
BICE	=	base value of investment in construction and equipment
BI	=	base value of total investment (ICE + ID)
BY	=	base value of output net of depreciation
BYG	=	base value of output including depreciation

8. (IFINT > 0) NOE, TITLED, IFPO (I5,AL,2X,I5)

NOE	=	no of non-zero ORANI elasticities
TITLED	=	title of ORANI elasticities file
IFPO	= 1	if elasticities to be printed
	= 0	otherwise

9. (IFINT > 0) NINTER, IFGRAPH (2I5)

NINTER	=	no of double endogeneities to be calculated
IFGRAPH	= 1	if plots of MACRO and MACRO/ORANI responses for double endogeneities required
	= 2	if binary disc file for plotter required
	= 0	otherwise

10. (IFINT > 0) INDY (I), I=1, NINTER (16I5)

INDY(I) = index of I'th double endogeneity

11. T1, T2, T3, IFDYN, TSTAR (3F5.0, I5, F5.0)

T1	initial value of t
T2	final value of t
T3	increment

5.2.2 The General Recursive Linkage

In the case where $z_J \neq y_B$ the combined model, uncorrected for timing, is (4.1*) - (4.2*). In this sub-section the shorthand notations y_J, y_B, z_J, z_B, C_J, C_B are understood to represent the vectors and matrices of the system (4.1*) - (4.2*).

Again consider a sustained [0,t] shock to variables exogenous to the Bergstrom model. As in 5.2.1 the Bergstrom response is given by (5.4). Now, however, it is a subset of y_B, namely $y_B z_J$, which leads to a time varying shock $_{z_J}y_B(\tau)$ to the Johansen model. Again this time path is represented as an integral of its derivatives. The typical derivative represents a sustained shock from τ to t, and by linearity the timing-corrected Johansen response can be represented as the integral of the responses to this overlapping sequence of sustained shocks. In addition, there is an avenue for a direct Johansen response to shocks other than those generated by the Bergstrom response. That is:

(5.11) $$y_J(t) = \int_0^t A_J^{-1}\,[e^{A_J(t-\tau)} - I]N_J \begin{pmatrix} _{z_J}y_B \\ 0 \\ 0 \end{pmatrix} [\tau,t]d\tau + C_J \begin{pmatrix} 0 \\ _{z_J}z_B \\ z_J - {_{z_J}x_B} \end{pmatrix} [0,t] .$$

Now, by definition, $_{z_J}y_B\,[\tau,t] = \frac{d}{d\tau}\; {_{z_J}y_B}(\tau)$.

But:

$$\frac{d}{d\tau}\begin{pmatrix} y_B y_J \\ y_B z_J \\ y_B - y_B x_J \end{pmatrix}(\tau) = e^{A_B\tau} N_B z_B[0,t] .$$

Thus the complete timing - corrected Johansen response may be written:

$$y_J(t) = \int_0^t A_J^{-1}[e^{A_J(t-\tau)} - I]N_J \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{A_B\tau} d\tau \, N_B z_B[0,t] + C_J \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} z_J[0,t] \quad (5.12)$$

and thus the structural form generalisation of (5.10) for the general one-way link is

$$\begin{aligned} y_J(t) &= C_J K z_J[0,t] + C_{J/B}\, y_B(t) \\ y_B(t) &= C_B z_B[0,t] \end{aligned} \quad (5.13)$$

where:

$$C_{J/B} = \int_0^t A_J^{-1}[e^{A_J(t-\tau)} - I]\, N_J V e^{A_B\tau} d\tau \, [e^{A_B t} - I]^{-1} A_B ,$$

$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \text{ and } V = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .$$

2. IFIN, TITLE1, IFNAMES, TITLEY, TITLEZ (I5, A13, 2X, I5, A13, 2X, A13)

IFIN	= 1	if $M, L, \lambda_1, P, \lambda_1^{-1}, P^{-1}$, NM on binary disc file
	= 2	if M, L, A_M, N_M on binary disc file
	= 3	if M, L, A_M, N_M on formatted disc file
	= 4	if M, L, λ, P on formatted disc file
TITLE1	=	title of disc file
IFNAMES	= 1	if variable names file present
	= 0	otherwise
TITLEY	=	title of names file containing endogenous variables
TITLEZ	=	title of names file containing exogenous variables.

3. (IFIN = 3 or 4) FORMAT1 (A20)

FORMAT1 = format of input file

4. IFOUT, TITLE2 (I5,A13)

IFOUT	= 1,2,3,4	to produce output files for future use as in 2.
	= 0	if no output file required.
TITLE2	=	title of output file

5. (IFOUT = 3 or 4) FORMAT2 (A20)

FORMAT2 = format of output file

6. BETA1, BETA2, BETA3 (3F10.0)

BETA1	=	initial value of BETA (0 is admissable)
BETA2	=	final value of BETA
BETA3	=	increment

note: a blank card is sufficient to produce MACRO stand-alone simulations.

Appendix 7B Format of Program Input Files

(a) Steering Cards for Program INTER

1. M, L, IFZ, IFINT, IFPT, IFPTINV, ITESTI, ITEST2, LORD (16I5)

M	=	No. of MACRO endogenous variables (excluding appended identities)
L	=	No. of MACRO exogenous variables
IFZ	= 1	if constant expanded to M exogenous variables (N = L+M-1)
	= 0	otherwise (N=L)
IFINT	= -1	to generate MACRO "as if" s
	= 0	to generate MACRO stand-alone simulations
	= 1	to interface MACRO and ORANI
	= 2	to interface with MACRO in short run (K fixed)
IFPT	= 1	to print eigenvalues (λ_i) of MACRO model
	= 2	to print eigenvalues and eigenvectors (P)
	= 0	otherwise
IFPTINV	= 1	to print λ_i^{-1}
	= 2	to print P^{-1}
	= 0	otherwise
ITEST1	= 1	to print (computed) imaginary part of Y
	= 0	don't print unless $\Sigma\|Imag(Y_i)\| \geq .00001$
ITEST2	= 1	compute and print $A_M = P \Lambda P^{-1}$
	= 0	otherwise
LORD	=	order of Taylor series approximation (program MACROSIM only)

6. Specification of the MACRO and ORANI Models

6.1 Introduction

In this section we specify the versions of the models to be used and detail modifications to be carried out preliminary to the interface. These modifications are designed to achieve consistency of variable definition and consistency of dynamics between the two models. A remaining form of inconsistency may lie in the basic philosophy of construction of the two models. A maintained hypothesis underlying the use of MACRO to determine the ORANI short run is that the economic forces producing the time path of the MACRO endogenous variables which are also endogenous in ORANI are modelled in a similar way to those forces producing the corresponding ORANI short run results. But this hypothesis is unlikely to be reasonable for the whole set of doubly endogenous variables. For example, the import sectors in the two models are specified in a quite dissimilar manner. As another example, the innovative role of monetary disequilibrium effects in the real components of the MACRO model suggests that, even if the models are consistent in the projected responses of nominal values to a given shock, the decomposition of the response into price and quantity effects may well be different in the two models. Thus one would expect to adopt a pragmatic approach to the selection of those double endogeneities to be used in the determination of the ORANI short run.

6.2 The MACRO Model[1]

Version: The version of the model to be used (as defined by the equation specification) will be referred to as MACRO79. This is based on the mid 79 version of the minimal model of the economy, supplied by the Reserve Bank in July 1979. The equations defining MACRO79 are provided in Appendix 6A.

Variant: The variant of MACRO79 to be used is that which results when the parameters take values obtained from FIML estimation using Wymer's program RESIMUL over the sample period 1959 (3) - 1977 (3). The resulting

1 A glossary of the IMPACT nomenclature used in this sub-section and the next (e.g., 'version', 'variant'), is given in Appendix 6E .

variant is stable, with 12 real roots ranging in value from -2.499 to -0.021 and 7 pairs of complex conjugate roots with real parts ranging from -1.653 to -.032. The parameter file is provided in Appendix 6B.

The MACRO model at this stage consists of the 26 equations M1 to M26. In section 6.4 further identities are appended to construct variables which are consistent with variable definitions in ORANI.

6.3 The ORANI Model

Version: The version is ORANI77 as presented in Volume Two of the First Progress Report of the Impact Project [1].

Variant: The variant is also that used in generating the results of Volume Two.

Mode: The variant is run in short run neoclassical mode. The precise endogenous/exogenous classification is chosen to be as closely consistent with MACRO as possible. This classification is listed in Appendix 6C.

Basic Solution: That component of the ORANI basic solution which is relevant to the computations involved in the determination of the short run is provided by the table of elasticities presented in Appendix 6D.

Of the ORANI exogenous variables, the set (exchange rate, wage shift variable, aggregate real household expenditure, aggregate real private investment) are subsequently endogenised by MACRO at the interface. That is, these variables constitute the set $z_J y_B$ in the notation introduced above.

1 Dixon et al., op.cit.

Appendix 7A INTERFACE Computer Programs

The calculations reported in this paper were performed in a number of computer programs, some of which were developed especially for the project. With minor modifications, these programs are capable of handling the interface of an arbitrary model of the Bergstrom class with an arbitrary model of the Johansen class.

Calculations are carried out by the use of:

(a) RESIMUL: Wymer's program RESIMUL was modified to produce a binary disc file of the MACRO structural form matrices on convergence.

(b) MAKEMAC: takes as input the output file from RESIMUL amd constructs a disc file of the MACRO reduced form matrices.

(c) INTER: takes as input the reduced form of MACRO from MAKEMAC, a file of ORANI elasticities, and has two files of variable names. INTER allows iteration over t^* and β and provides graphical and disc output of the MACRO and ORANI/MACRO time response paths to determine the length of t^*. Intermediate calculations for a particular variant of MACRO may be stored on disc and used as future input. Steering cards for INTER and the format of the ORANI elasticities file are provided in Appendix 7B.

(d) HPPLOT: disc files of time response paths produced by INTER are graphed by the HP graph plotter.

The executable version of INTER consists of the main program INTER linked with the INTER object library and the IMSL object library. The INTER object library contains specialist routines for manipulating complex matrices, while the IMSL routines are used to compute the eigenvalues and eigenvectors of a complex matrix.

Appendix 6E Glossary of IMPACT Terminology Used in Sections 6.2 and 6.3

1. In this guide definitions are given for the following terms:
Version (of a model)
Variant (of a version of a model)
Mode (of use of ORANI for simulations);
Basic solution (of ORANI).

2. Version A version of a model is a completely algebraically specified structural form of a model.

3. Variant The same version may be presented with different sets of initial conditions and/or with different parameter files. This generates different variants of the same version.

4. Mode Any variant of any version of ORANI may be run for simulation purposes in a large number of different modes. A mode of use of ORANI for simulations is defined by the partitioning of the ORANI variables into endogenous and exogenous sets.

5. Basic solution A basic solution of ORANI is a matrix of the elasticities of the endogenous variables with respect to exogenous variables. For a given variant of a given version of ORANI this matrix depends only on the mode of use.

6.4 Consistency of Variable Definition

In this section we describe the construction of additional MACRO variables which provide consistency with ORANI definitions. Of the set of ORANI exogenous variables which are endogenised by MACRO, two require adjustment for consistency of definition: (a) investment, (b) real wages. Of the set of doubly endogenous variables, three require adjustment: (c) real output, (d) imports, (e) exports. In addition it will be useful to modify both ORANI and MACRO to provide a further double endogeneity (f) nominal output. Details of these constructions are:

(a) Investment

Let ID be real investment in dwellings and let ICE be gross private fixed capital expenditure on construction and equipment. Let $I = ID + ICE$. In ORANI, $i_R = dI/I$. Now

$$\frac{dI}{I} = \frac{dID + dICE}{I}$$

$$= \frac{dID}{ID}\ \frac{ID}{I} + \frac{dICE}{ICE}\ \frac{ICE}{I} \ .$$

However $\frac{dID}{ID}$ can be identified as d log d in MACRO. Consider now the second term. Using K to denote the capital stock, $\dot{K}$ for its rate of growth (real \$ per unit time) and δ for the depreciation rate,

$$\frac{dICE}{ICE} = \frac{d(\dot{K} + \delta K)}{ICE}$$

$$= \frac{d\dot{K}}{\dot{K}}\ \frac{\dot{K}}{ICE} + \delta \frac{dK}{K}\ \frac{K}{ICE} \ .$$

Of the terms on the right, MACRO endogenizes $\frac{dK}{K}$ = d log K and dk where $k = \dot{K}/K$.

Now,

$$dk = d(\dot{K}/K) = \frac{\dot{K}}{K}\left(\frac{dK}{\dot{K}} - \frac{dK}{K}\right) .$$

Therefore,

$$\frac{d\dot{K}}{\dot{K}} = \frac{dk}{k} + d \log K .$$

Collecting terms:

$$\frac{dI}{I} = \frac{ID}{I} \; d \log d$$

$$+ \frac{ICE}{I}\left[\frac{\delta K}{ICE} \; d \log K + \frac{\dot{K}}{ICE}\left(\frac{dk}{k} + d \log K\right)\right] .$$

To evaluate this expression, d log d , d log K and dk are provided by the appropriate MACRO simulation. The shares should be set at appropriate base period values. For the purposes of the experiments considered below, sample means of ID, I, ICE, δK, K and k are used.

To MACRO in stand alone mode we append the following identities:

(i) $d \log NICE = dk/\bar{k} + d \log K$

as equation M27 to explain the proportional change in net investment in construction and equipment;

(ii) $d \log ICE = (\overline{\delta K} / \overline{ICE}) \; d \log K$
$+ (1 - \overline{\delta K} / \overline{ICE}) \; d \log NICE$

as equation M28 to explain the proportional change in gross investment in construction and equipment; and finally

(iii) $d \log I = (\overline{ICE} / \bar{I}) \; d \log ICE$
$+ (1 - \overline{ICE} / \bar{I}) \; d \log d$

as equation M29.

The last of these endogenizes the ORANI exogenous variable i_R , the

Appendix 6D Table of ORANI 77 Elasticities used in Interfacing Experiments

Variable	ORANI responses resulting from a 1 per cent increase in					
	Exchange rate	Wage shift variables	Aggregate real household expenditure	Aggregate real private investment	Shift term for other usage (domestic and imported components)	Power of Tariff
Aggregate employment	.061747	-.644833	.131555	.225434	.123211	-.008064
Foreign currency value of exports	.111526	-1.191931	-1.220817	-.213164	-.070367	-.407991
Foreign currency value of imports	-.112635	1.116679	1.568266	.539563	.156044	-.413544
Balance of trade	.008766	-.090111	-.109727	-.030071	-.009020	.001833
ORANI consumer price index	.795594	1.323715	1.535071	.174832	.052583	.484073
ORANI capital goods price index	.852196	1.400794	1.167231	.264367	.052881	.583263
Price deflator for aggregate production	.849796	1.670113	1.637739	.248117	.052540	.419029
Real GDP	.039834	-.416362	.088420	.149725	.099672	-.002771

The Endogenous Export Industries in ORANI 77

Industry: ORANI No.	Industry: IO Code	Description	Exports as a Percentage of Total Sales
1	01.01	Sheep	68.5
2	01.02	Cereal grains	56.5
9	04.00	Fishing, trapping, hunting	50.2
10	11.01	Iron	63.0
11	11.02	Other metallic minerals	33.4
12	12.00	Coal and crude petroleum	26.3
15	21.01	Meat products	30.2
22	21.08	Food products n.e.c.	30.3
27	23.01	Prepared fibres	58.1
60	29.01	Basic iron and steel	16.5
61	29.02	Other basic metal products	37.9

The Exogenous Investment Industries in ORANI 77

Industry: ORANI No.	Industry: IO Code	Description
14	16.00	Services to mining
81	36.01	Electricity
82	36.02	Gas
83	37.01	Water, sewerage and drainage
100	61.06	Ownership of dwellings
101	71.01	Public administration
102	72.01	Defence
103	81.01	Health
104	82.01	Education, libraries, etc.
105	83.01	Welfare services
109	99.01	Business Expenses

proportional change in real investment.

(b) Real Wages

MACRO provides (nominal) average weekly earnings, W (equation M9), and price of output, P (equation M6). The identity:

$$d \log (W/P) = d \log W - d \log P$$

is appended as M30 and W/P is used to endogenise the real wage shift term in ORANI.

(c) Output

MACRO provides real output net of depreciation, y (equation M5), and capital stock, K (equation M24). Then real output before depreciation, y_g, is equal to $y + \delta K$, and

$$\frac{dy_g}{y_g} = \frac{dy}{y}\frac{y}{y_g} + \delta \frac{dK}{K}\frac{K}{y_g},$$

and the identity

$$d \log y_g = (\bar{y} / \bar{y}_g) d \log y + (1 - \bar{y} / \bar{y}_g) d \log K$$

is appended as M31, providing a variable which is consistent with the ORANI definition of the proportional change in real GDP.

(d) Imports

MACRO provides real imports of goods and services, i (equation M4), and the price of imports, P_i, is exogenous to both MACRO and ORANI. The identity:

$$d \log (iP_i) = d \log i + d \log P_i$$

is appended as M32 and iP_i is consistent with the ORANI definition of foreign currency value of imports. We note that the d log P_i will be zero except for simulations involving a shock to Australian import prices.

(e) Exports

MACRO provides real exports, x (M3), the price of exports, P_x, in Australian dollars (M8), and the exchange rate, E, measured as \$A/\$US (M18). The identity:

$$d \log (xP_x/E) = d \log x + d \log P_x - d \log E$$

is appended as M33 and xP_x/E is identified with the ORANI definition of foreign currency value of exports.

(f) Nominal Output

The identity:

$$d \log (Py_g) = d \log P + d \log y_g$$

is appended to MACRO as M34 to define the proportional change in nominal GDP. A similar variable is defined in ORANI by summing the elasticities for the price deflator for aggregate production and real GDP.

6.5 Direction of the MACRO/ORANI Interaction

The choice of the MACRO and ORANI models is such that MACRO drives ORANI. Hence the set y_0z_M is null and the appropriate linkage is that outlined in Section 5.2.2. Thus the interfaced model is derived from (5.13) as:

Appendix 6C Mode of Use of ORANI 77 in Interfacing Experiments

List of Exogenous Variables[1]

Variable	Subscript Range	Number	Description
p^m_{j2}	$j = 1,\ldots g$.	g	C.i.f. foreign currency import prices.
p^m_{g+1}		1	
t_j	$j = 1,\ldots,g+1$.	g+1	One plus the ad valorem tariffs.
ϕ		1	The exchange rate, \$A per \$US, say.
s_j	$j \in G$.[2]	g	One plus the ad valorem export subsidies.
$x^{(4)}_j$	$j \notin G$.		Export demands.
$k_j(0)$	$j = 1,\ldots,g$.	g	Current capital stocks.
c_R		1	Real aggregate household expenditure.
i_R		1	Aggregate real private investment.
n		1	Supply of agricultural land.
$f_{(g+2)1m}$	$m = 1,\ldots,M$.	M	Wage shift variables.
$f^{(5)}_{is}$	$i = 1,\ldots,g$, $s = 1, 2$.	2g	"Other" demand shift terms.
$f^{(5)}_{g+1}$		1	
$f^{(2)}_j$	$j \notin J$.[3]	$g-J^*$	Exogenous investment.
f^e_j	$j = 1,\ldots,g$.	g	Shifts in foreign export demands.
$f_{(g+3)j}$	$j = 1,\ldots,g$.	g	Shifts in the real price of "other" cost tickets.
q		1	Number of households.

1. The notation in this Appendix (but not generally elsewhere in the paper) follows Dixon et al. - - see footnote 1, p. 3).

2. G is a subset of $\{1,\ldots,g\}$. In ORANI 77, g = 109. The industries belonging to G -- the endogenous export industries -- are shown on p.58.

3. J is the subset of $\{1,\ldots,g\}$ in which investment is endogenous. The industries not belonging to J are shown on p.58.

$\alpha_{16,1}$	55	0.242	0.062	3.89
$\alpha_{16,2}$	56	0.102	0.023	4.34
α_{17}	57	0.030	0.003	9.22
β_{38}	58	0.006	0.000	9.10
α_{18}	59	0.041	0.009	4.22
β_{17}	60	2.103	0.639	3.29
β_{20}	61	-0.080	0.015	5.20
$\alpha_{13,1}$	62	0.100	0.033	3.01
β_{32}	63	13.170	4.197	3.14
β_{33}	64	1.045	0.392	2.66
$\alpha_{15,1}$	65	0.756	0.091	8.31
β_{41}	66	-0.003	0.002	1.71
α_{14}	67	0.200	0.021	9.20
$\alpha_{12,1}$	68	0.160	0.033	4.83
β_{15}	69	-0.183	0.051	3.58
β_{40}	70	-0.003	0.000	4.15
β_{4}	71	-1.995	0.694	2.87
β_{5}	72	-0.238	0.277	0.86
$\alpha_{12,2}$	73	6.293	1.141	5.51
$\alpha_{19,1}$	74	0.103	0.061	1.67

(6.1)
$$y_O(t^*) = C_O K z_O[0,t^*] + C_{O/M}(t^*) y_M(t^*)$$
$$y_M(t^*) = C_M(t^*)\, z_M[0,t^*]$$

where: C_O = the matrix of ORANI short run elasticities;

$$C_{O/M}(t^*) = \int_0^{t^*} A_O^{-1}[e^{A_O(t^*-\tau)} - I]\, N_O V e^{A_M \tau}\, d\tau\, [e^{A_M t^*} - I]^{-1} A_M ;$$

$$V = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} ;$$

and $C_M(t^*) = A_M^{-1}\, [e^{A_M t^*} - I]\, N_M$.

6.6 Consistency of Dynamics

Previously it has been pointed out that a Bergstrom model typically allows the estimation of the dynamics of the model expressed in A_B and N_B and hence allows the derivation of C_B by (2.7). On the other hand a Johansen model typically allows only the direct estimation of C_J, a matrix which does not uniquely identify the pair A_J, N_J. Further parameterisation is required to uniquely identify the pair A_J, N_J. As pointed out in Appendix 5A, a convenient parameterisation is to choose A_J a scalar matrix. Adopting this parameterisation for ORANI, we have:

(6.2)
$$A_O = (\log \beta)\, I , \qquad 0 < \beta < 1 .$$

The implications of this parameterisation can be seen by looking at the response to a sustained shock $z_O[0,t^*]$ of either (a) the rates of change of the proportional deviations of ORANI endogenous variables, which follow a declining geometric progression:

$$\dot{y}_0(\tau) = \beta^{\tau}\, \dot{y}_0(0) \quad , \quad 0 \leq \tau \leq t^* ,$$

$$= 0 \quad , \quad \tau > t^* ;$$

or (b) the accumulated proportional deviations, which follows:

$$y_0(\tau) = \frac{\beta^{\tau}-1}{\beta^{t^*}-1}\, y_0(t^*) \quad , \quad 0 \leq \tau \leq t^* ,$$

(6.3)

$$= y_0(t^*) \quad , \quad \tau \geq t^* .$$

In Section 7 we document a set of experiments which allow the use of double endogeneities to determine values of t^* and β which make the dynamics of MACRO and ORANI as consistent as possible. Empirically this amounts to choosing β to be some appropriate weighted average of the eigenvalues of the MACRO model.

Specialising the interfaced model (6.1) by use of assumption (6.2) allows the integral component of $C_{O/M}$ to be evaluated explicitly. This gives:

$$(6.4) \quad C_{O/M}(t^*) = \frac{C_0 V}{\beta^{t^*}-1} \{[A_M-(\ln\beta)I]^{-1}[e^{A_M t^*}-e^{(\ln\beta)t^*}I][e^{A_M t^*}-I]^{-1}A_M - I\} .$$

$\alpha_{2,2}$	27	0.157	0.023	6.59
α_{10}	28	0.486	0.110	4.40
β_{25}	29	0.665	0.064	10.39
β_{26}	30	0.150	0.222	6.66
β_{44}	31	0.030	0.100	0.31
$\alpha_{15,3}$	32	0.297	0.102	2.90
β_{27}	33	0.062	0.023	2.64
γ_3	34	-0.205	0.110	1.85
β_{28}	35	0.021	0.100	0.21
$\alpha_{3,1}$	36	2.731	0.435	6.27
$\alpha_{3,2}$	37	0.135	0.032	4.19
β_{41}	38	0.009	0.013	0.67
β_{29}	39	8.655	2.993	2.89
β_{43}	40	0.073	0.005	14.53
β_{30}	41	1.351	0.183	7.38
β_{24}	42	0.115	0.013	8.74
β_{21}	43	0.139	0.050	2.77
β_{10}	44	0.572	0.106	5.38
β_{31}	45	-0.012	0.002	4.51
β_{45}	46	-0.009	0.004	2.19
β_{34}	47	-0.027	0.006	4.05
β_{48}	48	0.037	0.002	13.32
γ_4	49	0.173	0.039	4.41
β_{36}	50	-0.807	0.076	10.59
β_{35}	51	-6.228	0.959	6.49
β_{11}	52	-0.011	0.003	3.53
$\alpha_{15,2}$	53	0.131	0.037	3.51
β_6	54	0.038	0.065	0.59

Appendix 6B   MACRO 79 Parameter Estimates

		PARAMETERS	ASYMPTOTIC STANDARD ERRORS	"t-VALUES"
α_1	1	0.547	0.079	6.87
γ_1	2	-0.083	0.052	1.58
α_4	3	2.201	0.298	7.37
$\alpha_{13,2}$	4	-0.058	0.256	0.23
β_{16}	5	0.435	0.198	2.20
β_{13}	6	-1.066	0.344	3.19
β_{42}	7	0.122	0.010	11.74
$\alpha_{2,1}$	8	1.502	0.251	5.98
β_9	9	-0.641	0.086	7.41
β_{23}	10	0.530	0.152	3.48
β_{47}	11	-0.068	0.048	1.41
$\alpha_{19,2}$	12	0.137	0.038	3.55
β_{14}	13	0.368	0.077	4.73
β_{49}	14	-0.128	0.037	3.44
α_5	15	0.609	0.101	6.01
β_2	16	-0.202	0.073	2.74
β_{39}	17	0.020	0.013	1.54
β_{19}	18	0.738	0.161	4.58
β_{18}	19	-0.352	0.108	3.25
β_3	20	-1.380	0.636	2.17
$\alpha_{7,1}$	21	0.718	0.110	6.48
γ_2	22	-0.391	0.065	5.96
α_8	23	0.808	0.163	4.95
β_{22}	24	0.935	0.041	22.32
α_9	25	0.999	0.180	5.55
α_6	26	2.173	0.407	5.33

7. The Length of the ORANI Short Run

7.1 The Set of Double Endogeneities

As stand alone models, ORANI and MACRO allow the following two variables to be identified as double endogeneities:

(1) aggregate employment from ORANI and employment (M25) from MACRO,

(2) price deflator for aggregate production from ORANI and price of output (M6) from MACRO.

In addition, the previous section has described the way in which further MACRO and ORANI variables have been defined to achieve consistency. This has also allowed the identification of four further double endogeneities:

(3) real GDP from ORANI and real output (including depreciation), y_g (M31) appended to MACRO,

(4) nominal GDP appended to ORANI and to MACRO (M34),

(5) foreign currency value of imports from ORANI and iP_i (M32) appended to MACRO,

(6) foreign currency value of exports from ORANI and xP_x/E (M33) appended to MACRO.

7.2 Framework for the Experiments

The basic experiment to determine the ORANI short run consists of a shock to an exogenous variable. The time response of MACRO endogenous variables is mapped out. The ORANI response consists of two parts, which are discussed in general terms below:

(1) The response of ORANI as a stand alone model to the given shock.

(2) The response of ORANI to those ORANI exogenous variables which have been endogenised by MACRO.

For the purposes of comparison with the MACRO time response, the first of these ORANI responses could be represented as a horizontal line. The height of the line would represent the size of the response. In the horizontal direction the line would not represent a correct time profile, however. For t earlier than the short run the line would overestimate the ORANI response. Nevertheless, if there were no further interaction between ORANI and MACRO, the ORANI short run could be determined as the time t when the horizontal ORANI line crossed the MACRO time-response line for the same endogenous variable.

We now turn to (2), the response in ORANI induced via MACRO. Since MACRO generates a time profile of responses and since some of these responses create additional shocks in ORANI, there is in fact a time profile of shocks to ORANI which generate ORANI responses. An ORANI response at time t^* to a particular shock induced by MACRO at time s is given by (6.3) as $y_0(\tau)$ where $\tau = t^* - s$. Cumulating these responses for s varying over the interval $[0, t^*]$ gives the total ORANI response to the time profile of MACRO-induced shocks.

Thus the complete ORANI response, made up of factors (1) and (2) above, may be mapped as a time profile in a different sense to the MACRO response. The ORANI response is "short run time" dependent. Therefore, a graph on which both the ORANI and MACRO responses are mapped for a particular doubly endogenous variable has the following interpretation. The MACRO path gives the "actual time" profile. The ORANI path represents a series of conditional short run responses only one of which is correct, namely that which corresponds in timing to the actual short run. If MACRO and ORANI are consistent with respect to the modelling of the underlying economic processes which describe the behaviour of the particular variable in question then the

W_A^*	real award wages
W	average weekly earnings
x	real exports of goods and services
x_W^*	real world exports of goods and services
y	real output (net of depreciation)
Z^*	population of working age
λ_1^*	target rate of growth of real output (.012)
λ_2^*	target rate of monetary growth (.021)
λ_3^*	trend rate of technical progress (.0045)
λ_4^*	trend rate of growth of labour productivity (.0065)
λ_5^*	trend rate of growth of real award wages (.0029)
λ_6^*	differential growth rate between sales and stock of inventories of goods (-.0011)

P_{wl}*	price of wool ($US)
P_x	price of exports
QA*	dummy variables for requests to limit advances, 1961
QDS*	synthetic variable for US dock strike, 1969
QER*	synthetic variable for timing of exchange rate changes, 1972, 1973, 1974, 1976
Q_1*	dummy variable for shake out effect in labour market 1974-76
Qr*	dummy variable for increases in commercial bill rate, 1974
QS*	synthetic variable for increases in official interest rates, 1961, 1973
Q_t*	time trend starting in 1974(4), zero earlier
QUS*	dummy variable for devaluation of $US, 1973
r	10 year bond rate
r_{bl}	90 day commercial bill rate
r_{eu}*	90 day Eurodollar rate
r_w*	10 year US bond rate
R	gold and foreign exchange reserves
S	sales
S^e	expected sales
SD*	real statistical discrepancy
t*	time
t_c*	company tax rate
t_1	average personal income tax rate
t_{10}*	index of income tax rate schedule
t_2*	average rate of tax on expenditure
t_3*	average rate of tariffs
t_4*	average rate of payroll tax
T_1	receipts of direct taxes
T_2	receipts of indirect taxes
v	real stock of inventories of goods

ORANI short run will be the time at which ORANI and MACRO give consistent results, that is when the mapped out response paths cross.

The experimental framework is as follows. An exogenous variable is shocked. For each element of the set of doubly endogenous variables the MACRO time response path and the ORANI conditional response path are mapped. Attention is then concentrated upon those members of the doubly endogenous set for which it appears MACRO and ORANI can be consistently compared. The precise formulae for the response paths are now set out.

7.3 Formulae for the Response Paths

Consider a shock to a variable exogenous to both MACRO and ORANI, say z_k , $k \varepsilon \{z_O z_M\}$. The MACRO stand alone response after the elapse of any time interval, t , may be represented by:

$$(7.1) \qquad d \log y_i = \varepsilon^M_{ik} \, d \log z_k$$

for any $i \varepsilon \{y_M\}$. We are interested in $i \varepsilon \{y_M y_O\} \subset \{y_M\}$. The two part ORANI response is:

$$(7.2) \qquad d \log y_i = [\varepsilon^O_{ik} + \sum_j \varepsilon^{O/M}_{ij} \varepsilon^M_{jk}] \, d \log z_k$$

(where the summation ranges over the set $j \varepsilon \{y_M z_O\}$) for any $i \varepsilon \{y_O\}$.

In the above equations, ε^M_{ik} is the MACRO t-period stand alone elasticity, the ik^{th} element of $C_M(t)$; ε^O_{ik} is the ORANI stand alone elasticity, the ik^{th} element of C_O; $\varepsilon^{O/M}_{ij}$ is the interfaced ORANI/MACRO t-period elasticity, the ij^{th} element of $C_{O/M}(t)$.

The latter elasticity is designed such that $\varepsilon^{O/M}_{ij}$ captures the response of ORANI to an ORANI exogenous variable, z_j , which is to be endogenised by MACRO. This response, which was derived in detail in Section 5, is a function of t and β where t is a potential value of the ORANI short run, t* , and β represents the speed of accumulation of the overall ORANI response.

Concentrating upon that subset of $\{y_M y_0\}$ for which MACRO and ORANI are structurally consistent, then, provided β is correctly chosen, solution of (7.1) and (7.2) for d log y_i will coincide when t is the ORANI short run.

7.4 Results of the Experiments

The major experiment reported in this section is that in which an important variable exogenous to both models, real government spending, is subjected to a one percent shock. For the six doubly endogenous variables described in Section 7.1, the time response paths generated by MACRO as solutions of (7.1) are compared with the conditional response paths generated by MACRO/ORANI as solutions of (7.2). As indicated by (7.2) these latter paths are dependent upon β . We iterate over β on the interval [0,1] . These experiments are carried out using the set of computer programs INTERFACE developed by Keith R. McLaren and documented in Appendices 7A and 7B .

The results of an initial search for β over the range [0,1] are discussed below. These results are illustrated by Figures 1a - 3f in Appendix 7C and are summarised in Table 1 below:

Variables used in the model are:[1]

A	bank advances to private sector
B	bonds held by private non-bank groups
b	B/P
c*	real cash benefits to persons
C	domestic credit
CTB*	effective company tax base
d	household expenditure
E	exchange rate ($A/$US)
F	net Australian capital owned by overseas residents
f	F/P
g_1*	real government current expenditure
g_2*	real government capital expenditure
g_3*	real public authorities capital expenditure
h*	required liquidity ratio of the banking sector
i	real imports of goods and services
I*	interest payments on government debt
k	proportionate change in business fixed capital
K	real stock in business fixed capital
l	proportionate change in employment
L	employment
M	stock of money (M3)
m	M/P
MISC*	miscellaneous items in the money supply identity
N	labour supply
P	price of output
P_g	price of government consumption expenditure
P_i*	Australian import prices ($US)
P_w*	world prices ($US)

1 The items marked with an asterisk are exogenous for estimation.

M25. Labour demand

$$\frac{DL}{L} = 1$$

M26. Expected sales

$$D\log(S^e/S_0\, e^{\lambda_1 t}) = .5\,(\log(S/S_0\, e^{\lambda_1 t}) - \log(S^e/S_0\, e^{\lambda_1 t}))$$

$$S = d + x + DK + g_1 + g_2 + g_3$$

Table 1

IMPLIED LENGTH t* OF ORANI SHORT RUN

(quarters)

Double Endogeneity	Speed of Accumulation of Overall ORANI Response, β						
	.05	.2	.35	.5	.65	.8	.95
Employment	7.2	7.2	7.2	7.2	7.1	7.1	7.0
Price of Output	-	-	-	-	-	-	16.9
Real GDP	3.2	3.1	3.1	3.1	3.0	3.0	3.0
Nominal GDP	-	-	-	-	-	-	6.0
Imports	5.5	6.0	6.6	7.3	8.2	9.5	11.4
Exports	-	-	-	-	-	-	-

Within the framework of analysis there appears to be no way of reconciling the ORANI and MACRO export responses. This is probably a function of fundamentally different approaches to the modelling of that sector. Thus the export sector is not considered to be useful in further experimentation on the determination of the ORANI short run.[1]

It is also not possible to reconcile the price of output. On the other hand real GDP gives an implied value for the short run which is unacceptably small. Further analysis of the price response suggests a short run value which is substantially above twenty quarters. This suggests that the two models may be inconsistent in the way in which they split an exogenous shock into price and quantity effects while the overall response of nominal output may be consistent. Results for nominal GDP support this proposition, giving an implied short run which is a decreasing function of β and which takes plausible values for β in the region of .9 . See Table 3 below.

1 The exports equation in MACRO follows conventional macroeconometric practice in placing heavy emphasis on demand factors. By contrast ORANI treats the supply side in considerable detail.

The remaining two double endogeneities, employment and imports, generate plausible short run values over a wide range of β . In fact, the employment response is to all intents and purposes independent of β and gives an extremely plausible value for the short run of seven quarters. In contrast to the result for nominal GDP, the short run implied by the imports responses is increasing with β .

The choice of those doubly endogenous variables which appear to suggest a reasonable degree of consistency between the underlying modelling of MACRO and ORANI is thus narrowed to the three variables : employment, nominal GDP and imports. The behaviour of these variables is such that any two of them may be used to determine a unique (t^*, β) pair. While one approach may be to use a weighted criterion involving all three variables, we feel that it may be more useful from the point of view of particular user applications to present the three possible unique (t^*, β) pairs.

Table 2

LENGTHS OF THE ORANI SHORT RUN WHICH MAKE THE MACRO AND ORANI RESPONSES EQUIVALENT FOR CERTAIN PAIRS OF DOUBLY ENDOGENOUS VARIABLES

(quarters)

Pair of Doubly Endogenous Variables	ORANI Short Run t^*	Speed of Accumulation Parameter β
Employment, Nominal GDP	7.0	.918
Employment, Imports	7.2	.483
Nominal GDP, Imports	10.4	.883

Table 3 below is an enlargement of Table 1 which is presented to illustrate the behaviour of the results in the neighbourhood of the preferred (t^*, β) combinations.

Table 3

EXPANSION OF TABLE 1 FOR SELECTED VARIABLES *

Double Endogeneity	Value of β								
	.463	.483	.503	.873	.883	.893	.916	.918	.920
Employment	7.2	7.2	7.2	7.0	7.0	7.0	7.0	7.0	7.0
Nominal GDP	-	-	-	15.6	10.4	8.8	7.1	7.0	6.9
Imports	7.1	7.2	7.3	10.3	10.4	10.6	10.9	10.9	10.9

* The number in the body of the table is the estimated value of the length of the ORANI short run, t^* .

M17. Bond rate

$$Dr = \alpha_{17}(\hat{r} - r) + \beta_{38}QS + \beta_{39}\log {}^{L}/\phi_0 N + \beta_{40}\log {}^{R}/\theta_0 M + \beta_{41}\ \mathrm{Dlog}\ R$$

$$\hat{r} = 4.0\ \mathrm{Dlog}\ M$$

M18. Exchange rate

$$\mathrm{Dlog}\ E = \alpha_{18}\log {}^{P/P}w/_{E} + \beta_{42}QUS + \beta_{43}QER + \beta_{44}\log {}^{L}/\phi_0 N + \beta_{45}\log {}^{R}/\theta_0 M + \beta_{41}\ \mathrm{Dlog}\ R$$

M19. Bill rate

$$Dr_{b1} = \alpha_{19,1}\ \hat{Dr}_{b1} + \alpha_{19,2}\ (\hat{r}_{b1} - r_{b1}) + \beta_{47}\ \mathrm{Dlog}\ M/A + \beta_{48}Qr$$

$$\hat{r}_{b1} = r_{eu} + \beta_{49}\ \log {}^{EP}w/_{P}$$

M20. Foreign Reserves

$$DR = P_x x - EP_i i + DF$$

M21. Volume of money

$$DM = DR + DC$$

M22. Change in inventories

$$Dv = y + i - d - DK - x - g_1 - g_2 - g_3 - SD$$

M23. Domestic credit expansion

$$DC = P_g g_1 + Pg_2 + Pc + I - T_1 - T_2 - DB + DA + DMISC$$

M24. Business fixed capital stock

$$\frac{DK}{K} = k$$

M12. Non-bank demand for government securities

$$\text{Dlog } B = \alpha_{12,1}(\log \hat{Pb} - \log B) + \alpha_{12,2}(Dr - Dr_w)$$

$$+ \gamma_3(\log \hat{Pm} - \log M)$$

$$\log \hat{b} = b_0 + \log Y + \beta_{29}\, r - \beta_{29}\, r_w + \beta_{30}\log {}^{EP_w}/_P$$

M13. Net capital inflow

$$\text{Dlog } F = \alpha_{13,1}\,(\log \hat{Pf} - \log F) + \alpha_{13,2}(Dr_{b1} - Dr_{eu})$$

$$+ \beta_{31}Q_t$$

$$\log \hat{f} = f_0 + \log y + \beta_{32}\, r_{b1} - \beta_{32}\, r_{eu} + \beta_{33}\log {}^{EP_w}/_P$$

M14. Bank Advances

$$\text{Dlog } A = \alpha_{14}(\log \hat{A} - \log A) + \gamma_4(\log \hat{Pm} - \log M) + \beta_{34}QA$$

$$\log \hat{A} = A_0 + \log(1-h)M + \beta_{35}\, r - \beta_{35}\, r_w + \beta_{36}\log {}^{EP_w}/_P$$

M15. Direct taxes

$$\text{Dlog } T_1 = \alpha_{15,1}\, [\alpha_{15,2}(\log \hat{T}_{11} - \log T_1) + \alpha_{15,3}\text{Dlog } \hat{T}_{11}]$$

$$+ (1 - \alpha_{15,1})\ \text{Dlog } t_c CTB$$

$$\log \hat{T}_{11} = T_{01} + \log t_1 + \log WL$$

$$t_1 = t_{10}W^{.71}$$

M16. Indirect taxes

$$\text{Dlog } T_2 = \alpha_{16,1}(\log \hat{T}_{21} - \log T_2) + \alpha_{16,2}(\log \hat{T}_{22} - \log T_2)$$

$$\log \hat{T}_{21} = T_{02} + \log t_2 + \log P.d$$

$$\log \hat{T}_{22} = T_{03} + \log t_4 + \log W.L$$

7.5 Assessment of Model Compatibility

In this section we address the question of the general compatibility of the MACRO and ORANI models. The interface considered in this paper takes as given the theoretical structures of the two models. Since these are fundamentally different there is no way to demonstrate any functional equivalence between the two models. The best that one could hope for would be to demonstrate that the models would generate numerical results with respect to double endogeneities which are reasonably consistent. In the previous section we found that ORANI can be interfaced with MACRO in such a way as to produce reasonably consistent results for employment, imports and nominal output. On the other hand, inconsistent results are obtained for real output, prices and exports. We turn now to a discussion of these inconsistencies.

The key to the inconsistencies seems to be in the export sector. Equation (M8) of MACRO explains P_x . The major endogenous explanatory variable is E which enters (M8) in such a way as to induce a virtually instantaneous equiproportionate response in P_x . Thus P_x/E is virtually exogenous. Furthermore equation (M3) of MACRO demonstrates that if P_x/E is effectively exogenous then so is x. On the other hand x is a true endogenous variable in ORANI, as will be illustrated below. Since real gross domestic product = absorption + x - i , since absorption is endogenised by MACRO and since the results for imports are reasonably consistent in the two models it follows that an inconsistency in x will lead to an inconsistency in real GDP. A consideration of the structure of ORANI will show that the inconsistency in x will also lead to an inconsistency in prices.

---

1. This section draws heavily upon comments made by Alan A. Powell and Brian R. Parmenter on an earlier draft.

In Appendix 7D is illustrated the relevant aspects of ORANI's behaviour. With fixed capital and land and with a slack labour market, real GDP is determined in ORANI via the real wage, because it determines labour usage L. On the other hand, real absorption ABS is exogenous to ORANI, being endogenised by MACRO. Thus the real balance of trade BT is determined as the difference between GDP and ABS. The domestic to foreign price ratio must then adjust to be consistent with the volume of exports and imports which will result in the required trade balance. Thus in ORANI exports and the domestic price level are closely linked. This is in contrast to the essentially exogenous role of exports in MACRO.

The implications of the inconsistency in x can be seen from the identity

$$GDP = ABS + x - i .$$

Following an increase in absorption balance is achieved in the two models as follows

$$0 = \Delta x_M > \Delta x_0 ,$$

$$\Delta i_M = \Delta i_0 > 0 ,$$

$$\Delta GDP_M > \Delta GDP_0 > 0 .$$

Now suppose for the moment that ORANI were to be modified by constraining exports to be exogenous. Refer to this hypothetical model by 0* . Clearly

$$\Delta i_{0*} > \Delta i_0 ,$$

$$\Delta GDP_{0*} > \Delta GDP_0 .$$

The extent of the difference between GDP_{0*} and GDP_0 would be bounded by $|\Delta x_0|\ x / GDP$. Reference to Figures 3a to 3f indicate that consistency

M6. Price of Output

$$D\log P = \alpha_{7,1}(\log \hat{P} - \log P) + .13\ D\log P_x + \gamma_2(\log \hat{Pm} - \log M) + \beta_{21}(\log \hat{v} - \log v)$$

$$\log \hat{P} = P_0 + \log\ .7\ (^{WL}/_{y})$$

M7. Price of government current expenditure

$$D\log P_g = \alpha_8(\log \hat{P}_g - \log P_g)$$

$$\log \hat{P}_g = P_{g_0} + \beta_{22} \log W + (1 - \beta_{22}) \log P$$

M8. Price of exports

$$D\log P_x = \alpha_9(\log \hat{P}_x - \log P_x) + \beta_{23}(\log \hat{v} - \log v)$$

$$\log \hat{P}_x = P_{x_0} + \beta_{24} \log (EP_{w1}) + (1 - \beta_{24}) \log (EP_1)$$

M9. Average weekly earnings

$$D\log W = \alpha_{10}[\log .7 \bar{y}/_L - \log {}^W/_P] + \beta_{25} \log L/\phi_0\hat{N} + \beta_{26} \log ({}^{W_A}/_{w_0 e^{\lambda_5 t}})$$

M10. Labour supply

$$D\log N = .13\ (\log \hat{N} - \log N) + \beta_{27} \log {}^L/_{\phi_0 N}$$

$$\log \hat{N} = N_0 + \beta_{28} \log \frac{W(1 - t_1)}{P' w_0 e^{\lambda 4 t}} + \log Z$$

M11. Rate of Growth in Labour Demand, normal output

$$Dl = \alpha_{3,1}[\hat{l} - l] + \beta_{10}(\log y - \log \bar{y}) + \beta_{11} Q_1$$

$$\hat{l} = \lambda_1 - \lambda_3 + \alpha_{3,2}(.7\ {}^{(\hat{y})}/_L - .805\ \frac{W(1 + t_4)}{P})$$

$$\bar{y} = y_0 e^{\lambda_3 t} L^{.7} K^{.3}$$

Appendix 6A Equations of MACRO 79

Notation: $D = \frac{d}{dt}$. A zero subscript indicates an unidentified constant.

M1. Household expenditure, demand for money

$$D\log P'd = \alpha_1 (\log \hat{Pd} - \log P'd) + \gamma_1 (\log \hat{Pm} - \log M)$$

$$\log \hat{Pd} = d_0 + \log (Py - T_1 + Pc) + \beta_2(r - 4.0\, D\log P)$$

$$\log \hat{m} = m_0 + \log y + \beta_3 r + \beta_4 r_w + \beta_5 r_{b1} + \beta_6 \log EP_{w/P}$$

$$\log P' = P'_0 + .2\log EP_i + .8 \log P$$

M2. Rate of Growth in Business Fixed Capital Stock

$$Dk = \alpha_{2,1} [\hat{k} - k]$$

$$\hat{k} = \lambda_1 + \alpha_{2,2}(1.2\, (\hat{y} - g_1)/_K - r + 4.0\, D\log P) + \beta_9\, D\log P$$

M3. Exports, desired inventories

$$D\log x = \alpha_4(\log \hat{x}^d - \log x) + \beta_{13}(\log \hat{v} - \log v) + \beta_{14}\, QDS$$

$$\log \hat{x}^d = x_0 + \log x_w + \beta_{15} \log(^{Px}/EP_w)$$

$$\hat{v} = v_0[S^e - g_1]\, e^{\lambda_6 t}$$

M4. Imports

$$D\log i = \alpha_5(\log \hat{i} - \log i) + \beta_{16}(\log \hat{v} - \log v) + \beta_{17}(\log S - \log S^e)$$

$$\hat{i} = [i_0(\frac{EP_i(1 + t_3)}{P})^{\beta_{18}}]S^e$$

M5. Output

$$D\log y = \alpha_6(\log \hat{y} - \log y) + \beta_{19}(\log \hat{v} - \log v) + \beta_{20}(\log S - \log S^e)$$

$$\hat{y} = [1 - i_0\, (\frac{EP_i(1 + t_3)}{P})^{\beta_{18}}]S^e$$

could be achieved between this modified ORANI and MACRO for the variables L, y_g (GDP), iP_i and xP_x/E at a short run in the region of 6 to 8 quarters. On the other hand, in this hypothetical example the inconsistency in prices would be increased, since the additional adjustment in imports in 0* would require a further increase in domestic prices. However consistency in price level effects could not be expected in any event because of the role of the financial sector in determining prices in MACRO. The price determination mechanism in ORANI is such as to raise domestic prices in order to increase imports and decrease exports to restore balance. However, in MACRO the consequent decrease in reserves would be expected to lead to a partially offsetting fall in money supply and hence prices via (M21) and(M6). While the endogenisation of the exchange rate by MACRO provides a channel for this influence in ORANI/MACRO, an examination of the exchange rate equation (M18) and the relevant parameter estimates suggests a negligible feedback. We note of course that the above discussion throws no light on the reasons why the inconsistencies in P and y_g should be offsetting.

The discussion of the hypothetical model 0* has been useful to fix ideas on the nature of the inconsistencies between MACRO and ORANI/MACRO. However the model 0* is not suggested as the preferred procedure for the resolution of the inconsistencies. A preferable approach may be to allow ORANI to endogenise exports for the system. One may also consider the possibility of removing the inconsistency in prices by allowing MACRO to determine the price level. This probably would necessitate further work on exchange rate determination in the combined system.

8. Concluding Remarks

In this paper we have considered the general problem of the interfacing of a previously estimated model of the Bergstrom type with one of the Johansen type. After some discussion of the general problem of the two-way linkage between the models we turned to the one-way case in which the Bergstrom model drives the Johansen model. The theory was then applied to the case in which the Bergstrom model was a slightly modified version of the Reserve Bank minimal model of the macro-economy and the Johansen model was the ORANI module of the IMPACT Project. The basic result was that the models are best interfaced at an ORANI short run of between six and eight quarters.

The paper suggests a number of areas for future work. With respect to the specific MACRO-ORANI application a natural direction for research would be the modification of the modules in order to achieve a cleaner interface. The results of Section 7 suggest an approach in which the general price level is exogenous to ORANI, while exports and the exchange rate are determined by equations designed explicitly to take into account features of both models. These modifications may require further development of the mathematics of the two-way linkage.

Solution of (5A.15) amounts to choosing F and P_J such that P_J is the matrix of characteristic vectors corresponding to the roots of $[0 \;\; C_J^1 \;\; 0]\, F$. At the same time, solution of (5A.16) amounts to choosing F, F^*, P_J and Λ_J such that P_B is the matrix of characteristic vectors corresponding to the roots of $FP_J \Lambda_J P_J^{-1} F^*$. The simplest solution is achieved by setting $\Lambda_J = \gamma I$, and this is the approach adopted in Sections 6 and 7 in the ORANI/MACRO interface. A more general approach which remains to be explored would be the adoption of some optimization criterion, say involving the values of the double endogeneities at time t^* , subject to constraints (5A.15) and (5A.16) .

Denoting A_J by $P_J \Lambda_J P_J^{-1}$, condition (iii) is met by choice of diagonal Ψ with $|\Psi| = 0$ to define:

$$LF = P_J \Psi P_J^{-1} . \tag{5A.12}$$

Condition (iv) is met by ensuring

$$\Omega_i \neq \Lambda_{B_i} \ \forall \ i . \tag{5A.13}$$

Condition (i) is met by choice of F of full row rank and choice of E as F^* where F^* is a right generalized inverse of F.

Now $L = [0 \ N_J^1 \ 0]$, and given the matrix of Johansen elasticities C_J, it follows from (5.1) that L must satisfy:

$$L = [e^{\Lambda_J t^*} - I]^{-1} \ A_J \ [0 \ C_J^1 \ 0] . \tag{5A.14}$$

Thus, to ensure satisfaction of (5A.12) and (5A.14) together it is necessary to choose F, P_J, Λ_J to define diagonal Ψ where :

$$\Psi = [e^{\Lambda_J t^*} - I]^{-1} \Lambda_J P_J^{-1} [0 \ C_J^1 \ 0] \ F \ P_J . \tag{5A.15}$$

On the other hand, to ensure satisfaction of (5A.11) it is necessary to choose F, P_J, Λ_J to define diagonal Ω where:

$$\Omega = P_B^{-1} \ F P_J \ \Lambda_J \ P_J^{-1} \ F^* \ P_B . \tag{5A.16}$$

Thus the conditions of Theorem 1 and of consistency of L and A_J given data on C_J are fully satisfied by (5A.15) and (5A.16) where diagonal Ψ has rank less than or equal to the number of Johansen exogenous variables which are endogenised by Bergstrom, and provided $\Omega_i \neq \Lambda_{B_i}$.

APPENDICES

Appendix 5A Dynamics of the Recursive Linkage

A mechanistic procedure which avoids the introduction of timing problems is to integrate the two models into a dynamic structural model and then to solve for the final form of this joint system. Consider for convenience a deterministic formulation:

$$\dot{Y}_B(t) = A_B Y_B(t) + N_B Z_B(t)$$

$$\dot{Y}_J(t) = A_J Y_J(t) + N_J Z_J(t)$$

and suppose firstly that $Y_B(t) = Z_J(t)$. Then the appropriate joint structural system is

$$(5A.1) \quad \begin{pmatrix} \dot{Y}_B \\ \dot{Y}_J \end{pmatrix} = \begin{bmatrix} A_B & 0 \\ N_J & A_J \end{bmatrix} \begin{pmatrix} Y_B \\ Y_J \end{pmatrix} + \begin{bmatrix} N_B \\ 0 \end{bmatrix} Z_B$$

which may be solved as:

$$(5A.2) \quad \begin{pmatrix} y_B(t) \\ y_J(t) \end{pmatrix} = \begin{bmatrix} A_B & 0 \\ N_J & A_J \end{bmatrix}^{-1} \left\{ e^{\begin{bmatrix} A_B & 0 \\ N_J & A_J \end{bmatrix} t} - I \right\} \begin{bmatrix} N_B \\ 0 \end{bmatrix} z_B[0,t]$$

where $y_B(t) = Y_B^C(t) - Y_B^S(t)$, and so on, using the notation of Section 2. Let

$$(5A.3) \quad \begin{bmatrix} P_B & 0 \\ R & P_J \end{bmatrix}^{-1} \begin{bmatrix} A_B & 0 \\ N_J & A_J \end{bmatrix} \begin{bmatrix} P_B & 0 \\ R & P_J \end{bmatrix} = \begin{bmatrix} \Lambda_B & 0 \\ 0 & \Lambda_J \end{bmatrix}$$

where Λ_B and Λ_J are diagonal. Then (5A.2) may be written:

$$+ \frac{t^2}{2!} [LA_B^2 + A_J LA_B - LA_B FA_J E - A_J LFA_J E]$$

$$+ \;.....$$

$$= t[LA_B - A_J L]$$

$$+ \frac{t^2}{2!} [LA_B^2 - A_J^2 L] + \;.....$$

$$= Le^{A_B t} - e^{A_J t} L .$$

In the case where the number of endogenous variables in the Bergstrom model is greater than the number of endogenous variables in the Johansen model, condition (i) cannot be met. In this case the corresponding theorem is:

<u>Theorem 1'</u>:

If matrices E, F, A_J and L can be constructed such that (v) EF = I, (vi) A_J commutes with $EA_B F$, (vii) FL commutes with A_B , and (viii) $|EA_B F - A_J| \neq 0$, then:

$$(5A.10) \quad \{Re^{\Lambda_B t} - e^{A_J t} R\} P_B^{-1}$$

$$= (EA_B F - A_J)^{-1} (Le^{A_B t} - e^{A_J t} L) .$$

Returning to the case where the number of endogenous variables is greater in the Johansen model, we may choose E, F, A_J and L to satisfy conditions (i) - (iv) as follows: Given $A_B = P_B \Lambda_B P_B^{-1}$, condition (ii) is met by choice of diagonal Ω with $|\Omega| \neq 0$ to define:

$$(5A.11) \quad FA_J E = P_B \Omega P_B^{-1} .$$

$$R\Lambda_B^2 - A_J^2 R = (L\Lambda_B + A_J L)P_B \, .$$

In the cube case:

$$R\Lambda_B^3 - A_J^3 R = [L\Lambda_B^2 + A_J(L\Lambda_B + A_J L)]P_B \, ,$$

and similarly:

$$R\Lambda_B^4 - A_J^4 R = \{L\Lambda_B^3 + A_J[L\Lambda_B^2 + A_J(L\Lambda_B + A_J L)]\}P_B \, ,$$

$$\vdots$$

We require a closed form expression for $\{Re^{\Lambda_B t} - e^{A_J t} R\}P_B^{-1}$. Under certain assumptions on A_J and L, this is provided by the following result:

__Theorem 1:__

If matrices E, F, A_J and L can be constructed such that (i) $FE=I$, (ii) $FA_J E$ commutes with Λ_B, (iii) LF commutes with A_J, and (iv) $|\Lambda_B - FA_J E| \neq 0$, then:

(5A.9) $$\{Re^{\Lambda_B t} - e^{A_J t} R\} P_B^{-1}$$

$$= (Le^{\Lambda_B t} - e^{A_J t} L)(\Lambda_B - FA_J E)^{-1} \, .$$

__Proof__

$$\{Re^{\Lambda_B t} - e^{A_J t} R\} P_B^{-1} (\Lambda_B - FA_J E)$$

$$= \{t[R\Lambda_B - A_J R] + \frac{t^2}{2!}[R\Lambda_B^2 - A_J^2 R]$$

$$+ \frac{t^3}{3!}[R\Lambda_B^3 - A_J^3 R] + \ldots \} P_B^{-1} (\Lambda_B - FA_J E)$$

$$= t[L\Lambda_B - LFA_J E]$$

(5A.4) $$\begin{pmatrix} y_B(t) \\ y_J(t) \end{pmatrix} = \begin{bmatrix} C_{BB} & 0 \\ C_{JB} & C_{JJ} \end{bmatrix} \begin{pmatrix} N_B z_B[0,t] \\ 0 \end{pmatrix}$$

where

(5A.5) $$\begin{bmatrix} C_{BB} & 0 \\ C_{JB} & C_{JJ} \end{bmatrix}$$

$$= \begin{bmatrix} \Lambda_B^{-1}\{a^{\Lambda_B t} - I\} & 0 \\ -A_J^{-1} N_J \Lambda_B^{-1}\{e^{\Lambda_B t} - I\} + A_J^{-1}\{Re^{\Lambda_B t} - e^{A_J t} R\}P_B^{-1} & A_J^{-1}\{e^{A_J t} - I\} \end{bmatrix}$$

$=$ C, say; and where R is the solution of:

(5A.6) $$R\Lambda_B - A_J R = N_J P_B \, ,$$

a relationship which follows from (5A.3).

If A_J and N_J are known then the evaluation of (5A.4) is a numerical exercise analogous to the solution of the Bergstrom stand-alone model. In this case, however, since the dynamics of both models would be separately pre-specified, they would be consistent in their determination of double endogeneities only with probability zero. On the other hand, the purpose of this paper has been to consider the case in which only C_J is pre-specified. This raises the possibility of using the consistency of double endogeneities as a criterion for choice of A_J and N_J. This approach requires a closed form solution to (5A.4). This issue is explored further below.

The system (5A.4) provides an alternative expression for $y_J(t)$ to the integral representation (5.9). The equivalence of the two representations can be demonstrated for those cases where (5A.6) can be solved explicitly for R (and equivalently for those cases in which the integral in (5.9) can be explicitly solved). Before specialising for this purpose, consider the more relevant recursive representation of the models in which $Y_B(t) \neq Z_J(t)$. More precisely, let:

$$Y_B = \begin{pmatrix} Y_B Y_J \\ Y_B Z_J \\ Y_B - Y_B X_J \end{pmatrix} , \quad Z_J = \begin{pmatrix} Z_J Y_B \\ Z_J Z_B \\ Z_J - Z_J X_B \end{pmatrix}$$

The system (5A.1) is now written as:

$$\text{(5A.7)} \quad \begin{pmatrix} \dot{Y}_B \\ \dot{Y}_J \end{pmatrix} = \begin{bmatrix} A_B & 0 \\ L & A_J \end{bmatrix} \begin{pmatrix} Y_B \\ Y_J \end{pmatrix} + \begin{bmatrix} N_B & 0 \\ 0 & Q \end{bmatrix} \begin{pmatrix} Z_B \\ Z_J \end{pmatrix}$$

where $L = \begin{bmatrix} 0 & N_J^1 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} 0 & N_J^2 & N_J^3 \end{bmatrix}$,

and where N_J^i, $i = 1,2,3$ are sub matrices of $N_J = \begin{bmatrix} N_J^1 & N_J^2 & N_J^3 \end{bmatrix}$ corresponding to the partitioning of Z_J above.

By the essential similarity of (5A.7) and (5A.1) it follows that:

$$\text{(5A.8)} \quad \begin{pmatrix} y_B(t) \\ y_J(t) \end{pmatrix} = \begin{bmatrix} C_{BB} & 0 \\ C_{JB} & C_{JJ} \end{bmatrix} \begin{pmatrix} N_B \, z_B \, [0,t] \\ Q \, z_J \, [0,t] \end{pmatrix}$$

where (5A.5) once again defines C provided that L replaces N_J in (5A.5) and in (5A.6).

It is useful to consider under what conditions a closed form expression for the sub-matrix C_{JB} can be obtained. A sufficient condition

is that $R\Lambda_B - A_J R = LP_B$ be analytically solvable for R. Since R is not in general square we cannot strictly consider as a sufficient condition the commutativity of A_J and R. A simple sufficient condition would, however, be that $A_J = \gamma I$, γ a scalar. Then $R = LP_B (\Lambda_B - \gamma I)^{-1}$ and C_{JB} reduces to:

$$C_{JB} = -A_J^{-1} L A_B^{-1} \{e^{A_B t} - I\}$$
$$+ A_J^{-1} L [A_B - \gamma I]^{-1} \{e^{A_B t} - e^{\gamma t} I\} .$$

In this case, $y_J(t)$ implied by (5A.8) is identical to that implied by (5.12) if the integral in (5.12) is calculated explicitly for $A_J = \gamma I$. In the empirical application to the ORANI/MACRO linkage reported in this paper, the above simplification is used. In the remainder of this appendix, however, a more general approach to the determination of C_{JB} is explored.

It has previously been noted that the lower left sub-set of equations in the definition:

$$\begin{bmatrix} P_B & 0 \\ R & P_J \end{bmatrix}^{-1} \begin{bmatrix} A_B & 0 \\ L & A_J \end{bmatrix} \begin{bmatrix} P_B & 0 \\ R & P_J \end{bmatrix} = \begin{bmatrix} \Lambda_B & 0 \\ 0 & \Lambda_J \end{bmatrix}$$

defines a relationship in R:

$$R\Lambda_B - A_J R = LP_B .$$

But by considering in a similar fashion the definition of the roots of the square of $\begin{bmatrix} A_B & 0 \\ L & A_J \end{bmatrix}$, we note that: