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DESIGN OF THE ORANI-MACRO-BACHUROO INTERFACE

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Contents

and where $P^{-1} \Lambda P = A$, with

$$A = \begin{bmatrix} A_M & B_M L'_{OM} K_{MO} & B_M L'_{BM} K_{MB} \\ B_O L'_{MO} K_{OM} & & B_O L'_{BO} K_{OB} \\ B_B L'_{MB} K_{BM} A_M + B_B L'_{OB} K_{BO} B_O L'_{MO} K_{OM} & B_B L'_{MB} K_{BM} B_M L'_{OM} K_{MO} + B_B L'_{OB} K_{BO} A_O & B_B L'_{MB} K_{BM} B_M L'_{BM} K_{MB} + B_B L'_{OB} K_{BO} B_O L'_{BO} K_{OB} \end{bmatrix} ,$$

$$B = \begin{bmatrix} B_M L'_{XM} & 0 & 0 \\ 0 & B_O L'_{XO} & 0 \\ B_B L'_{MB} K_{BM} B_M L'_{XM} & B_B L'_{OB} K_{BO} B_O L'_{XO} & B_B L'_{XB} \end{bmatrix} ,$$

$$H = \begin{bmatrix} 0 \\ 0 \\ A_B \end{bmatrix} .$$

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Russell J. Cooper and Keith R. McLaren

1. INTRODUCTION

The IMPACT Project's short term model is to consist of three modules - ORANI, MACRO and BACHUROO. This paper describes an interface which will provide for interactions among them. The resulting interfaced model will be particularly suited for policy analyses which require a detailed account of interactions among macro, industry, and labour-market variables.

In a previous paper (Cooper and McLaren (1981)), the design features of the ORANI-MACRO interface were developed. The present paper develops the design features of the interface between ORANI-MACRO and BACHUROO.

It is interesting to note that each of the three IMPACT modules is, in its essential structure, representative of one of the three major classes of quantitative economic models: ORANI is a general equilibrium model, MACRO is a continuous time disequilibrium model, and BACHUROO is a discrete time disequilibrium model.

2. THE MODELS

2.1 ORANI

ORANI is a general equilibrium model of the Johansen (1960) class, and is fully documented in Dixon et al. (1982). Under a typical allocation of variables to the endogenous/exogenous categories, the structure of ORANI can be represented by the system:

$$y_0 = C_0 z_0 \quad , \qquad (2.1)$$

where the subscript 0 denotes ORANI variables and parameters; y_0 represents a vector of proportional changes in ORANI endogenous variables; z_0 represents a vector of proportional changes in ORANI exogenous variables, and C_0 is the matrix of ORANI elasticities. The following quotation illustrates the way in which results generated by (2.1) may be interpreted: "given a policy change A , then in the macro-economic environment B , variable C will differ in the short-run by x per cent from the value it would have had in the absence of the policy change". (Dixon et al. (1982) p. 63). Implicit in the construction and interpretation of the ORANI elasticity matrix C_0 is the notion of the ORANI short-run. The short-run is defined as a period long enough for prices to adjust, for output to be expanded using given plant, for new investment plans to be made but not completed, but not long enough for changes in the size of capital stock in use. The value of the short-run will be denoted by t^* (measured in quarters) and to indicate explicitly the fact that the result (2.1) eventuates at the end of t^* quarters, we may write instead:

$$y_0(t^*) = C_0(t^*)\, z_0 \; . \qquad (2.2)$$

Thus y_0 is to be interpreted as the restoration of a conditional equilibrium which takes t^* quarters to eventuate. The equilibrium is

and hence

$$\exp\,(A(t-4))\int_0^{t-4} \exp\,(-A\tau)\; H \exp\,(A\tau)\; d\tau$$

$$= \exp\,(A(t-4))\, \int_0^{t-4} \mathrm{vec}^{-1}\{Q'^{-1} \exp\,(\Phi\tau)\; Q \;\mathrm{vec}\; H\,\}\; d\tau$$

$$= \exp\,(A(t-4))\; \mathrm{vec}^{-1}\;\{Q'^{-1}[\int_0^{t-4} \exp\,(\Phi\tau)\; d\tau]\; Q \;\mathrm{vec}\; H\,\}$$

$$= \exp\,(A(t-4))\; \mathrm{vec}^{-1}\;\{Q'^{-1}\;\Phi^{-1}[\exp\,(\Phi(t-4)) - I]\; Q \;\mathrm{vec}\; H\}.$$

It follows that, for the interface under Method 5, equation (4.3) is replaced by:

$$y(t) \;=\; \tilde{C}(t)\; z \;, \qquad (4.7)$$

where

$$\tilde{C}(t) \;=\; \begin{cases} C(t) & 0 \leq t \leq 4 \\ C(t) + \exp\,(A(t-4))\;\psi(t-4)\; B & 4 \leq t \leq 8 \;, \end{cases}$$

with $\psi(t-4) = \mathrm{vec}^{-1}\;\{Q'^{-1}\;\Phi^{-1}\;[\exp\,(\Phi(t-4)) - I]\; Q \;\mathrm{vec}\; H].$

$$\Phi = \Lambda \otimes -\Lambda \;,$$

$$Q = P^{-1} \otimes P \;,$$

It follows that :

$$y(t) = \begin{cases} C(t)z & 0 \leqslant t \leqslant 4 \\ [C(t) + \int_0^{t-4} \exp\,(A(t-4-\tau))\; H \exp\,(\Lambda\tau)\; d\tau B]z, & 4 \leqslant t \leqslant 8, \end{cases}$$

and calculation of $y(t)$ for $t > 8$ follows a similar pattern. Assuming that $0 \leqslant t^* \leqslant 8$, the above calculations cover the interval of interest. The integral may be evaluated by noting that :

$$\text{vec}\,\{\exp\,(-A\tau)\; H \exp\,(A\tau)\}$$

$$= \text{vec}\,\{P^{-1} \exp\,(-\Lambda\tau)\; PHP^{-1} \exp\,(\Lambda\tau)\; P\}$$

$$= \{[P' \exp\,(\Lambda\tau)\; P'^{-1}] \otimes [P^{-1} \exp\,(-\Lambda\tau)\; P]\}\; \text{vec}\, H$$

$$= \{P' \otimes P^{-1}\}\{\exp\,(\Lambda\tau) \otimes \exp\,(-\Lambda\tau)\}\{P'^{-1} \otimes P\}\; \text{vec}\, H$$

$$= Q'^{-1}\, e^{\Phi\tau}\, Q\; \text{vec}\, H \;,$$

conditional because, although the original shock is represented explicitly by a non-zero element in z_0 , this shock may also cause changes to variables outside ORANI (i.e. in MACRO or BACHUROO). These variables may be identified with other elements of the vector z_0 which would have been set to zero in the "stand-alone" application of (2.2).

2.2 MACRO

The MACRO module is to be viewed as endogenizing the macroeconomic climate within which the other more disaggregated models operate. Thus MACRO is to be a small to medium size macroeconomic model determining such macro-aggregates as real private consumption and investment, and paying particular attention to the financial and monetary markets which are not modelled in the other modules. Adopting for convenience the terminology of Challen and Hagger (1979), existing dynamic macro-econometric models fall into two principal classes:

(i) The Keynes-Klein (KK) class is characterized by a specification in discrete time with the basic adjustment mechanism being in terms of first (or higher order) difference equations. Variables are typically specified in level form, with non-linearities linearized at either the estimation or simulation stages. Although the basic driving force of the dynamics of such models is usually that of adjustment towards equilibrium, this process of adjustment is often merely implicit. Estimation is typically equation by equation, so that there are few cross-equation constraints, and the estimated versions are often quite large, usually with more than 100 equations. After introducing identities to reduce higher order difference equations to first-order, and transforming to reduced form, the typical specification of such models would be :

$$Y_M(t) = A_M Y_M(t-1) + B_M Z_M(t) , \qquad (2.3)$$

where Y_M (Z_M) is a vector of variables endogenous (exogenous) to MACRO. One such model of the Australian economy is NIF-10 (Johnston et al. (1981)).

(ii) The Phillips-Bergstrom (PB) class is characterized by a specification in continuous time with the adjustment mechanism being in terms of first (or higher order) differential equations. Equations are typically linear in logarithms (or if not, linearized in logarithms), and the dynamic process is explicitly one of a process of adjustment towards equilibrium. Such models are usually small (20-30 equations) and estimation is typically by full information maximum likelihood methods, utilizing any available cross-equation constraints. After introducing identities to reduce higher order differential equations to identities, and transforming to "reduced form", the typical specification of such models would be:

$$DY_M(t) = A_M Y_M(t) + B_M Z_M(t) , \qquad (2.4)$$

where variables are now expressed as logarithms and D denotes differentiation with respect to time. Such a model of the Australian economy is provided by the RBII class of models (Jonson and Trevor (1981)).

In terms of the analytical development of the interface, there are probably both advantages and disadvantages with each of the specifications (2.3) and (2.4). Indeed, as will be seen below, BACHUROO is specified in a form much closer in spirit to (2.3) than to (2.4), so that a MACRO-BACHUROO interface would be fairly straightforward if MACRO were in the form (2.3). On the other hand, it was found that the ORANI-

constant shock to DZ_{BX} (alternatively, DZ_{BX} may be thought of as the basic exogenous variable vector rather than Z_{BX}).

The path of responses to shocks of the specified types may be written as :

$$y(t) = \int_{-4}^{t-4} e^{A(t-4-\tau)} HDy_B(\tau)\, d(\tau) + C(t)\, z , \qquad (4.7)$$

where

$$C(t) = A^{-1}[\exp(At) - I]\, B ,$$

$$B = [\tilde{B} \quad J] ,$$

$$z = \begin{pmatrix} z_{MOX} \\ \tilde{z}_{BX} \end{pmatrix} ,$$

and where

$$y_B(\tau) = \begin{cases} 0 & -4 \leqslant \tau \leqslant 0 \\ C(\tau)z & 0 \leqslant \tau \leqslant 4 . \end{cases}$$

A general solution to (4.5) is:

$$Y(t) = e^{At}Y(0) + \int_{-4}^{t-4} e^{A(t-4-\tau)} HDY_B(\tau)d\tau$$

$$+ \int_0^t e^{A(t-\tau)} JDZ_{BX}(\tau)d\tau$$

$$+ \int_0^t e^{A(t-\tau)} \tilde{B}\, Z_{MOX}(\tau)d\tau \,. \qquad (4.6)$$

We wish to compare control paths of (4.6) with shocked paths. The following shocks are admitted:

$$z_{MOX}(t) = \begin{cases} 0 & , \quad t < 0^- \\ Z^s_{MOX}(t) - Z^c_{MOX}(t) = z_{MOX}, & 0^+ < t < t^* \end{cases},$$

$$z_{BX}(t) = \begin{cases} 0 & , \quad t < 0 \\ Z^s_{BX}(t) - Z^c_{BX}(t) = \tilde{z}_{BX}t & , \quad 0 < t < t^* . \end{cases}$$

That is, in interface experiments, variables exogenous to MACRO and ORANI (and not endogenized in the interfaced system) may be subjected to the usual abrupt and sustained shock. However, shocks to variables exclusively exogenous to BACHUROO are to be ramped in (linearly) since the continuous time specification of BACHUROO does not admit of jumps in Y_B and a linearly accumulating shock to Z_{BX} is consistent with a

MACRO interface could be developed quite successfully with a MACRO model in the form of (2.4). The final choice is really one of size and accessibility.

NIF is probably too large and disaggregated for use in the closure of the IMPACT model. In terms of size and coverage of variables, RBII is quite satisfactory in providing short run macroeconomic closure for the overall IMPACT system. Thus from now on, we will identify MACRO with RBII, and take (2.4) as the dynamic form of the MACRO module.

2.3 BACHUROO

BACHUROO is a demographic and labour supply model, based on a demographic core which utilizes conventional demographic accounting to model ageing, sexual composition and marital status of the population, and on an econometric model which utilizes the "new home economics" to model such things as family formation, fertility and labour force participation. (For details see Powell (1980)). The basic model structure is an annual, non-linear difference equation which can be written as ;

$$Y_B(t') = f(Y_B(t'-1), Z_B(t')) \,, \qquad (2.5)$$

where Y_B represents the BACHUROO endogenous variables, Z_B represents BACHUROO exogenous variables, and t' is measured in years. The time variable t' can be related to the earlier use of a time variable, t , by use of the identity $t=4t'$ when the three models are to be combined.

3. GENERAL PROBLEMS OF THE INTERFACE

3.1 Introduction

So far we have identified 3 sets of endogenous variables, Y_M, Y_O and Y_B, and 3 sets of exogenous variables Z_M, Z_O and Z_B. The three models described in the previous section each allow the calculation of the effect of a shock to Z_i on the variable Y_i, (where i is one of M, O, B), ignoring information available from the other two models. But in general $Z_i \cap Y_j \neq \phi$ for $i \neq j$, and the purpose of the interface is to allow for the transmission of information among all three models. Thus while each of (2.1), (2.3) and (2.5) can be considered separately as reduced forms, when treated as a single system we have a structural model in which some of the Z_i are identified with certain Y_j. To transform the complete model to reduced form for purposes of simulation, moreover, requires that each component module be written in a similar form.

3.2 The "ORANI Form" of the Modules

One way to achieve similarity would be to write each model in the same form as ORANI. For example, in Cooper and McLaren (1981) it is shown that (2.4) implies that :

$$y_M(t) = C_M(t) z_M \, , \qquad (3.1)$$

where $C_M(t) = A_M^{-1} [\exp(A_M t) - I] B_M$ (and where lower case letters represent proportional changes), so that if $t = t^*$, MACRO and ORANI are comparable. Similarly, by subjecting each element of Z_B to a shock and computing the corresponding Y_B path, it would be possible to obtain a local constant elasticity approximation for BACHUROO. Following

where

$$Y = \begin{pmatrix} Y_M \\ Y_O \\ Y_B \end{pmatrix}, \quad Z_{BX} = L_{XB} Z_B \, , \quad Z_{MOX} = \begin{pmatrix} L_{XM} Z_M \\ L_{XO} Z_O \end{pmatrix},$$

$$H^* = \begin{bmatrix} 0 \\ 0 \\ A_B \end{bmatrix}, \quad J^* = \begin{bmatrix} 0 \\ 0 \\ B_B L'_{XB} \end{bmatrix}, \quad B^* = \begin{bmatrix} B_M L'_{XM} & 0 \\ 0 & B_O L'_{XO} \\ 0 & 0 \end{bmatrix},$$

$$A^* = \begin{bmatrix} A_M & B_M L'_{OM} K_{MO} & B_M L'_{BM} K_{MB} \\ B_O L'_{MO} K_{OM} & A_O & B_O L'_{BO} K_{OB} \\ 0 & 0 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -B_B L'_{MB} K_{BM} & -B_B L'_{OB} K_{BO} & I \end{bmatrix},$$

which therefore yields the "reduced form" representation:

$$DY(t) = H \, DY_B(t-4) + AY(t)$$

$$+ J \, DZ_{BX}(t) + \tilde{B} \, Z_{MOX}(t) \, , \qquad (4.5)$$

where $H = H^*$, $J = J^*$, $A = M^{-1}A^*$, $\tilde{B} = M^{-1}B^*$.

Appendix B

The Interface for Method 5

The approach to the interface in this case needs to recognise the special nature of the time delay in the BACHUROO model. Equation (3.12) may firstly be expressed in quarterly time units as:

$$DY_B(t) = A_B\, DY_B(t-4) + B_B\, DZ_B(t)$$

Hence, using (4.1) the "structural form" of the interface differs from that outlined in section 4.2 because of the BACHUROO specification:

$$DY_B(t) = A_B\, DY_B(t-4) + B_B\, L'_{MB}\, K_{BM}\, DY_M(t)$$

$$+ B_B\, L'_{OB} K_{BO}\, DY_O(t)$$

$$+ B_B\, L'_{XB} L_{XB}\, DZ_B(t) \;.$$

Appending this specification to the MACRO and ORANI "structural forms" given in section 4.2, one obtains the delay-differential form:

$$MDY(t) = H^* DY_B(t-4) + A^* Y(t)$$

$$+ J^* DZ_{BX}(t) + B^* Z_{MOX}(t) \;,$$

Powell (1981), specify an initial value for $Y_B(0)$ and a control path for Z_B, say $Z^c_B(\tau)$, $\tau = 1, 2, \ldots$. Equation (2.5) can be solved recursively for the control path of Y_B, $Y^c_B(\tau)$, $\tau = 1, 2, \ldots$. Now construct a shocked path of Z_B as $Z^s_B(\tau) = Z^c_B(\tau) + \Delta Z(\tau)$ where $\Delta Z(\tau)$ is defined as $z.I.Z^c_B(\tau)$ and z is a constant, say 0.01, corresponding to a 1 percent stock to Z . Substituting $Z^s_B(\tau)$ in (2.5) allows the construction of $Y^s_B(\tau)$, and we define:

$$y_B(\tau) = \frac{Y^s_B(\tau) - Y^c_B(\tau)}{Y^c_B(\tau)} \;, \quad \tau = 0, 1, \ldots \;.$$

Now carry out this experiment with I replaced by I^i where I^i represents the identity matrix with all but the i^{th} 1 replaced by 0; i.e., the shock is now a 1% shock to the i^{th} variable exogenous to BACHUROO. This allows the calculation of y^i_B , $i=1, \ldots, k$. Now define:

$$C_B(\tau) = [y^1_B \;\ldots\; y^k_B\,] \times 100 \qquad (3.2)$$

and write:

$$y_B(\tau) = C_B(\tau)\, z_B \;, \qquad (3.3)$$

which is formally analogous to the ORANI model. In Powell (1981), $C_B(\tau)$ is given a least squares interpretation, and the possibility of deriving alternative estimators of $C_B(\tau)$ through a weighted least squares method is explored.

3.3 Method 1: Interfacing the "ORANI Forms"

Given the three "ORANI forms", it would then be possible to treat (2.2), (3.1), (3.3) (with $t = \tau = t^*$) as one simultaneous system and obtain the reduced form. While such an approach would overcome the basic problem of passing information across models by identifying variables exogenous to some models with variables endogenous to others, and would be easy to compute, being of the same structure as ORANI for which extensive software exists, it would be at best a poor approximation. This is because where variables appear on the right of (2.2), (3.1), (3.3) as z_O, z_M, z_B they are to be interpreted as constant sustained shocks. However, they do not have the same interpretation where they appear on the left as y_O, y_M or y_B. Equation (3.1), for example, indicates explicitly the continuous time path by which y_M evolves from 0 at t=0 to $y_M(t^*)$ at $t=t^*$. To ignore this time profile when Y_M is supplied as an exogenous input to another model would be to ignore useful information, and to treat the final value $y_M(t^*)$ as a constant sustained shock would clearly be in error. Similarly, some information exists on the way in which y_B evolves. Being a difference equation, this information consists of points along the path. On the other hand, there is no explicit information on the time profile of the evolution of y_O. But again, it is known that y_O eventuates only after the elapse of t^* quarters, so again it would be in error to treat a value y_O which is an exogenous input to BACHUROO or MACRO as a constant sustained shock over the period $[0, t^*]$. Thus to directly obtain the reduced form of the system (2.2), (3.1), (3.3) would involve an unacceptable level of error and the discarding of useful information. This is the problem of temporal aggregation which arises in the transmission of signals across models which are evolving in time. This problem can be overcome by combining models when written in their dynamic form rather than in their final form.

Let $P = [I \quad 0]$ and $Q = [0 \quad I]$

of dimensions such that :

$$\tilde{A}_B P'P = \tilde{A}_B \quad \text{and} \quad \begin{bmatrix} P \\ Q \end{bmatrix} = I \ .$$

Then $P\tilde{A}_B P' = A_{11}$ and hence (3.7) may be replaced by :

$$PY_B(t) = A_{11}\ PY_B(t-1) + P\tilde{B}_B\ Z_B(t) \ , \tag{3.7a}$$

$$QY_B(t) = Q\tilde{B}_B Z_B(t) \ . \tag{3.7b}$$

Equation (3.7a) represents the submodule of BACHUROO to be treated as in the text, with Y_B in the text understood as PY_B, $\tilde{A}_B$ understood as A_{11} and $\tilde{B}_B$ understood as $P\tilde{B}_B$.

Equation (3.7b) may be combined with the representations (4.1) and written in proportional change form as:

$$Qy_B(t) = \left(Q\tilde{B}_B\ L'_{XB}L_{XB}\right) z_B$$

$$+ \left(Q\tilde{B}_B\ L'_{MB}K_{BM}\right) y_M(t)$$

$$+ \left(Q\tilde{B}_B\ L'_{OB}K_{BO}\right) y_O(t)$$

and hence the remaining module $Qy_B(t)$ of BACHUROO may be calculated recursively from the interfaced solution.

Appendix A

The Case of Singular $\tilde{A}_B$

Consider equation (3.7):

$$Y_B(t) = \tilde{A}_B Y_B(t-1) + \tilde{B}_B Z_B(t)$$

and suppose that $\tilde{A}_B$ is singular. We may suppose without loss of generality that :

$$\tilde{A}_B = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix} ,$$

(if this is not the case, replace Y_B by MY_B where M satisfies :

$$M\tilde{A}_B M^{-1} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

and replace $\tilde{B}_B$ by $M\tilde{B}_B$),

where A_{11} is nonsingular.

3.4 Method 2

The models may be put in dynamic form by recognizing that equations (3.3) and (2.2) demonstrate a formal analogy between BACHUROO and ORANI. Hence a possible way to proceed would be to link a combined BACHUROO-ORANI with MACRO in the same manner as was developed in Cooper and McLaren (1981) for the ORANI-MACRO interface. To resolve the temporal aggregation problem in the case of the ORANI-MACRO interface, the approach taken by Cooper and McLaren was to recognize that, since (2.2) and (3.1) are analogous final forms, it must therefore be legitimate to assume that there exists an implicit dynamic structure to ORANI generating (2.2) analogous to the dynamic structure (2.4) generating (3.1). Hence we may write :

$$DY_O(t) = A_O Y_O(t) + B_O Z_O(t) , \qquad (3.4)$$

where A_O, B_O satisfy :

$$C_O(t^*) = A_O^{-1}[\exp (A_O t^*) - I]B_O . \qquad (3.5)$$

Given C_O, A_O and t^* are unknown parameters characterizing the dynamics (3.4) and B_O is defined by (3.5). The approach was then to write the joint system consisting of (2.4) and (3.4), identify the appropriate elements of Z_O with Y_M and of Z_M with Y_O to define a system :

$$DY = AY + BZ$$

and hence derive the system :

$$y = C(t^*)z ,$$

which could be used for policy simulation. The values of the "double endogeneities" in y were used in the choice of t^* and A_O. In this way, all of the information pertaining to the dynamics of MACRO was used, while the ORANI dynamics were assumed to be represented to a first order of approximation by the parameters A_O and t^*. To the extent that ORANI and MACRO "overlapped", this information was used in the determination of the ORANI parameters A_0 and t^* .

While a direct extension of this approach to the ORANI-MACRO-BACHUROO interface would be feasible, such a procedure would ignore the specific information available on the dynamics of BACHUROO.

3.5 The "MACRO Form" of BACHUROO

An alternative approach is to translate the dynamic form of BACHUROO into a form analogous to the MACRO dynamic form. Again, in annual terms, the BACHUROO dynamic form (where t now refers to years) is:

$$Y_B(t) = f(Y_B(t-1), Z_B(t)) \quad (3.6)$$

Under suitable reinterpretation of the function f in (3.6) we may assume that Y_B and Z_0 are measured in terms of logarithms.

For the purposes of the interface we need to write (3.6) in the linear form:

$$Y_B(t) = \tilde{A}_B Y_B(t-1) + \tilde{B}_B Z_B(t) \quad (3.7)$$

One such approximation could be achieved by analytical linearization of (3.6). An alternative approximation could be based on a nonlinear simulation of (3.6) with the generated data subsequently used to provide a numerical fit of (3.7). In the sequel it is assumed that (3.7) refers to that submodule of BACHUROO which is truly dynamic. The case of singular $\tilde{A}_B$ is relegated to Appendix A, where it is shown that a submodule of BACHUROO can be treated in the above manner, and the remainder of BACHUROO (for which no information on

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endogenous variables, while allowing the incorporation of all available information on the time paths of those BACHUROO exogenous variables endogenized elsewhere. To this point the methods presuppose that the one year time delay in the estimated form of the BACHUROO model is not intrinsic to the theory but is merely a reflection of data availability. For completeness, Method 5 considers a possible approach if it were desirable to preserve the one year time delay in the transition from the discrete to the continuous formulation. For the specific version of the BACHUROO model under consideration for the interface, the time delay is not intrinsic to the theoretical specification, and hence Method 4 is the preferred method. The choice between the two possible approaches to the numerical implementation of Method 4 is possibly best determined on an experimental basis.

dynamics exists) can be handled analogously to the back-solution variables in ORANI.

Methods 3 and 4 below are based upon a differential equation specification which approximates (3.7) by :

$$DY_B(t) = A_B Y_B(t) + B_B Z_B(t) \, , \qquad (3.8)$$

which is the analogue of the continuous time MACRO "reduced form" equation (2.4).

3.6 Method 3

The general solution to (3.8) is :

$$Y_B(t) = \exp(A_B t)\, Y_B(0) + \int_0^t \exp(A_B(t-s))\, B_B Z_B(s)ds$$

and thus the discrete time model corresponding to (3.8) is :

$$Y_B(t) = \exp(A_B)\, Y_B(t-1) + \int_{t-1}^{t} \exp(A_B(t-s))\, B_B Z_B(s)\, ds \, . \qquad (3.9)$$

As an approximation, assume $Z_B(s)$ is constant over the interval (t-1, t); then we have :

$$Y_B(t) = \exp(A_B)\, Y_B(t-1) + A_B^{-1}[\exp(A_B)-I]B_B Z_B(t) \, . \qquad (3.10)$$

Comparing (3.10) with (3.7), A_B and B_B can be chosen such that :

$$\exp(A_B) = \tilde{A}_B \, ,$$

$$A_B^{-1}[\exp(A_B)-I]B_B = \tilde{B}_B$$

and this provides one means by which (3.8) can be seen as an approximate continuous time equivalent of (3.7).

While this approach is feasible and clearly preferable to Method 2, the approximation of $Z_B(s)$ by $Z_B(t)$ for $t-1 < s \leq t$ may be a problem in light of the fact that certain elements of Z_B will be endogenized at the interface by Y_M and Y_O, and these are unlikely to be piecewise constant.

3.7 Method 4

Another approximation to (3.8) is obtained by approximating $DY_B(t)$ by $Y_B(t) - Y_B(t-1)$. In the resultant equation:

$$Y_B(t) - Y_B(t-1) = A_B Y_B(t) + B_B Z_B(t) \qquad (3.11)$$

the coefficient matrices A_B and B_B may either be estimated numerically using the data generated by the simulation of (3.6) or be derived as follows :

$$A_B = I - \tilde{A}_B^{-1} \quad ,$$

$$B_B = \tilde{A}_B^{-1}\tilde{B}_B \quad ,$$

where $\tilde{A}_B$ and $\tilde{B}_B$ are estimated numerically from (3.7).

5. USE OF THE INTERFACED MODEL

Consider the interfaced model (4.3) evaluated at the "short-run", $t=t^*$:

$$y(t^*) = C(t^*)z \ . \qquad (5.1)$$

Comparison with the stand-alone ORANI solution,

$$y_O(t^*) = C_O(t^*)z_O \qquad (5.2)$$

suggests that (5.1) may be used in policy analysis in the same manner as the ORANI model is currently used. Indeed we can view (5.1) as a suitable closure of ORANI.

6. CONCLUSION

In this paper five methods for effecting the ORANI-MACRO-BACHUROO interface have been considered. Method 1, the interfacing of the "ORANI forms" of the models, was introduced merely to illustrate the nature of the temporal aggregation problem. Subsequent methods employed the concept of a continuous time dynamic formulation underlying the ORANI and BACHUROO models. Method 2 endowed both ORANI and BACHUROO with assumed dynamics. This method was rejected as it ignored the available information on BACHUROO dynamics. The remaining methods make more explicit use of this information. While Method 3 attempts to pursue the formal analogy between BACHUROO and MACRO, the approximation involved in the treatment of variables exogenous to BACHUROO but endogenized elsewhere was felt to be unacceptable. To avoid this problem, Method 4 concentrated the approximation error on the rates of change of BACHUROO where

$$C(t) = A^{-1}[\exp(At) - I]\,B. \qquad (4.4)$$

In constructing A and B from the individual model ingredients it should be noted that A_M, B_M, A_B, B_B and all L and K matrices are given. With respect to A_O and B_O, solutions of the ORANI-MACRO interface (Cooper and McLaren (1981)) may be used. A preferable approach would be either:

(i) Use double endogeneities between Y_M and Y_O in (4.3) to reassess A_O, t^* and hence B_O (= $[\exp(A_O) - I]^{-1}A_O C_O{}^*$, $C_O{}^*$ given). To the extent that the effecting of the BACHUROO interface will influence considerations of experimental environment, mode of use of ORANI and inter-model feedbacks, this would be a natural extension of the Cooper-McLaren (1981) experiments.

(ii) Estimate (4.2) by econometric methods using time series data, subject to constraints on the relevant components of A and B. For the purpose of the use of this approach the ORANI and BACHUROO models referred to above should be understood as being the respective submodels for which quarterly time series data are available on the endogenous variables. The essence of this approach is also outlined in Cooper and McLaren (1981).

3.8 Method 5

An alternative general approach to the specification of BACHUROO in a MACRO-analogous continuous time form would be :

$$DY_B(t) = \tilde{A}_B DY_B(t-1) + \tilde{B}_B DZ_B(t). \qquad (3.12)$$

Equation (3.12) could be viewed as the appropriate continuous-time representation of (3.7) if one wished to model the one year time delay as inherently important in the behavioural specification of BACHUROO. This represents a methodologically different stance from that involved in Methods 2, 3 and 4 and hence calls for a separate approach to the design of the interface (see Appendix B).

4. DESIGN OF THE ORANI-MACRO-BACHUROO INTERFACE

4.1 Simultaneity Interactions

Interactions between the modules are effected by introducing the linkage equations:

$$L_{OB}Z_B = K_{BO}Y_O \;,\; L_{MB}Z_B = K_{BM}Y_M \;,$$
$$L_{BO}Z_O = K_{OB}Y_B \;,\; L_{MO}Z_O = K_{OM}Y_M \;, \qquad (4.1)$$
$$L_{OM}Z_M = K_{MO}Y_O \;,\; L_{BM}Z_M = K_{MB}Y_B \;.$$

The subscripted symbols K and L are rectangular matrices containing specific patterns of units and zeros. Thus, for example, L_{OB} selects from the BACHUROO exogenous set those variables which are endogenised by ORANI. K_{BO} selects the same variables from the ORANI endogenous set. Let L_{XB} select all those variables from the BACHUROO exogenous set which are not endogenized by either ORANI or MACRO. Obviously the elements of $L_{OB}\,Z_B$, $L_{MB}\,Z_B$ and $L_{XB}\,Z_B$ correspond one-to-one with the elements of Z_B. It follows that $L'_{OB} L_{OB} + L'_{MB} L_{MB} + L'_{XB} L_{XB} = I$. Similarly, $L'_{BO} L_{BO} + L'_{MO} L_{MO} + L'_{XO} L_{XO} = I$ and $L'_{OM} L_{OM} + L'_{BM} L_{BM} + L'_{XM} L_{XM} = I$.

4.2 The Interface for Methods 2, 3 and 4.

The three models may be written in a form analogous to the MACRO continuous time "reduced form" as:

$$DY_M(t) = A_M Y_M(t) + B_M Z_M(t),$$
$$DY_O(t) = A_O Y_O(t) + B_O Z_O(t),$$
$$DY_B(t) = A_B Y_B(t) + B_B Z_B(t).$$

The first two equations are merely (2.4) and (3.4). The BACHUROO equation, however, is (3.8) converted from an annual to a quarterly observation interval by adjusting the time units from annual to quarterly and hence redefining A_B and B_B as one quarter of their previous values.

The aim of the interface is to produce a super-system of the form:

$$DY(t) = AY(t) + BZ(t), \tag{4.2}$$

where

$$Y = \begin{pmatrix} Y_M \\ Y_O \\ Y_B \end{pmatrix},$$

but where Z contains only variables exogenous to all models.

Using (4.1) the "structural form" of the interface is:

$$\begin{aligned} DY_M(t) &= A_M Y_M(t) + B_M L'_{OM} K_{MO} Y_O(t) \\ &\quad + B_M L'_{BM} K_{MB} Y_B(t) + B_M L'_{XM} L_{XM} Z_M(t), \\ DY_O(t) &= A_O Y_O(t) + B_O L'_{BO} K_{OB} Y_B(t) \\ &\quad + B_O L'_{MO} K_{OM} Y_M(t) + B_O L'_{XO} L_{XO} Z_O(t), \\ DY_B(t) &= A_B Y_B(t) + B_B L'_{OB} K_{BO} Y_O(t) \\ &\quad + B_B L'_{MB} K_{BM} Y_M(t) + B_B L'_{XB} L_{XB} Z_B(t), \end{aligned}$$

and hence the interfaced system has the "reduced form" (4.2), where:

$$Z = \begin{pmatrix} L_{XM} Z_M \\ L_{XO} Z_O \\ L_{XB} Z_B \end{pmatrix},$$

$$A = \begin{bmatrix} A_M & B_M L'_{OM} K_{MO} & B_M L'_{BM} K_{MB} \\ B_O L'_{MO} K_{OM} & A_O & B_O L'_{BO} K_{OB} \\ B_B L'_{MB} K_{BM} & B_M L'_{OB} K_{BO} & A_B \end{bmatrix},$$

$$B = \begin{bmatrix} B_M L'_{XM} & & \\ & B_O L'_{XO} & \\ & & B_B L'_{XB} \end{bmatrix}.$$

Following the procedures outlined in Cooper and McLaren (1981) for the ORANI-MACRO interface, the full ORANI-MACRO-BACHUROO interface is operationalised by writing (4.2) in proportional change form:

$$y(t) = C(t)\, z, \tag{4.3}$$