

Impact Project

Impact Centre
The University of Melbourne
153 Barry Street, Carlton
Vic. 3053 Australia
Phones: (03) 344 7417
Telex: AA 35185 UNIMEL
Telegrams: UNIMELB, Parkville

IMPACT is an economic and demographic research project conducted by Commonwealth Government agencies in association with the Faculty of Economics and Commerce at The University of Melbourne, the Faculty of Economics and Commerce and the Departments of Economics in the Research Schools at the Australian National University and the School of Economics at La Trobe University.

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MACROECONOMIC CLOSURE IN APPLIED GENERAL

EQUILIBRIUM MODELLING :

Experience from ORANI and Agenda for Further Research

by

Alan A. Powell

University of Melbourne

Russel J. Cooper

Macquarie University

Keith R. McLaren

Monash University

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1. Introduction

Perhaps most important among the contentious issues remaining in applied general equilibrium modelling is the role (if any) of conventional macroeconomics. Models in the Walrasian tradition are comparative static in nature; they are not well suited, therefore, to the analysis of transitory phenomena such as the stage of the business cycle. When our policy-analytic models are focussed at one or two years into the hypothetical future, however, it may be impossible to ignore transients and remain convincing. In other words, we may be required to tell a story which is part Walrasian and part macroeconomic. How can such an unlikely hybrid be seeded?

1.1 The Extended Walrasian Paradigm

Several approaches are possible. The cleanest, of course, involves a Walrasian take-over bid for macroeconomics. In such an extended Walrasian paradigm demand and supply functions for money and perhaps other financial assets are added to the real system (Patinkin (1958); Feltenstein

(1981); Vincent (1983))¹. This introduces at least one additional agent (the supplier of money, usually the government), and the opportunity to endogenize such important macroeconomic variables as the price level and the money supply.

In terms of applied work, it is early days yet for the extended Walrasian paradigm. If eventually it turns out that transients with expected lives of (say) less than a year cannot really be modelled successfully using the tools of economic analysis, then the limitations inherent in the comparative static nature of the extended Walrasian method may come to be accepted as the state of the art for the analysis of short-run issues. In the meantime there is a large profession engaged in applied macrodynamic analysis who claim otherwise. Taking their claim (provisionally) at its face value, is there a method of incorporating macrodynamic insights within a Walrasian framework?

1.2 The IMPACT Paradigm

The institutional environment within which the IMPACT Project was set up provided a strong incentive to find an affirmative answer to the preceding question. While necessarily we were required to concentrate on the real economy and relative prices because of our principal backer's (the Industries Assistance Commission, IAC's) interest in international trade and industry protection, and while also we were required to build a model which would produce sectoral policy recommendations in harmony with the thrust of the government's macroeconomic policy, under no circumstances were we to build our own macro models. The latter interdiction was motivated by two considerations: one survival oriented, the second more honourable. The former could be paraphrased as "Keep off the Treasury's

turf, or you know what!", while the latter simply recognized that the community had already made a very large investment in macrodynamic analysis, especially at the Reserve Bank and at the Treasury. Macroeconomic insights, if such were available, were to be harvested from the work of others. In due course these motivations led to the development of the IMPACT paradigm. The latter allows a macrodynamic model to determine all of the major monetary, financial and macroeconomic aggregates simultaneously with the determination by a computable general equilibrium (CGE) model of relative prices and the commodity and factor composition of the economy along strictly Walrasian lines.

1.3 Plan of the Paper

It is a major aim of this paper to summarize and to make accessible an account of our work on the IMPACT paradigm. Necessary initial steps are the definition of macroeconomic closure of a CGE model, and the distinction between such closure in the short and the long run. These preliminaries constitute Section 2. Because of widespread agreement that monetary phenomena are transients lacking a long-run influence, in many respects long-run macroeconomic closure of a CGE model is the simpler of the two cases. IMPACT work on this front is summarized in Section 3. Short-run macroeconomic closure, and the IMPACT paradigm proper, are dealt with in Section 4. In the fifth and final Section we offer concluding remarks and a perspective for future research.

2. Macroeconomic Closure of a CGE:Basic Concepts2.1 Basic Closure

The term 'closure' is used in various ways, and with varying degrees of precision, in our literature. At the most basic level we say that a model is 'closed' if we have sufficient information to compute a solution. It will suffice here to restrict attention to linear models. Let M ,

$$(1) \quad M: \quad Ax = 0 \quad ,$$

be a linear model containing m consistent, linearly independent equations in k variables ($k \geq m$). Then a closure of M , in the basic sense referred to above, consists of:

- (a) a declaration that a certain subset x_2 of x containing $(k-m)$ variables is exogenous;

and

- (b) an assignment of a set of values (say \bar{x}_2) to these exogenous variables.

Sometimes the term 'closure' is used loosely to describe (a) alone. In that case, closure means that we could, if we were to make an assignment (b), solve the model. Since the variables x_2 are exogenous, their assignment can be completely arbitrary, and hence these two primitive notions of closure boil down to the same thing.

2.2 Closure with respect to a Subset of Variables

A second notion of closure is defined relative to some subset of the model's variables. We say that M is closed with respect to the variables x^m ($x^m \subset x$) provided the variables x^m are endogenized by the model in the context of the basic closure selected. Thus M is closed with respect to x^m if the x_2 chosen in (a) is such that x^m and x_2 have no elements in common. 'Macroeconomic closure' is of this second type. In this case the components of x^m will include the real values of major aggregates (consumption, investment) plus a range of monetary and financial variables (the exact list will depend upon the application). Macroeconomic closure is then achieved by ensuring that M has sufficient equations to endogenize these variables in the context of a suitable partition of the variables x into the endogenous and exogenous categories. Here 'suitable' has two implications:

- (i) in partitioning x into $[x_1, x_2]$ (where $x^m \subset x_1$) so that

$$Ax = 0$$

becomes

$$A_1x_1 + A_2x_2 = 0 ,$$

the choice made must result in A_1 having full rank;

and

- (ii) no variable (outside the set x^m) which the model is required to explain may be assigned to the exogenous set x_2 .

Condition (i) is required in order to obtain a unique solution of M ($x_1 = -A_1^{-1} A_2x_2$), while condition (ii) ensures that macroeconomic closure is not obtained at the expense of other purposes for which the model was built.

2.3 Short-run versus Long-run Closure

The first crucial difference between short and long run modelling is in the choice of variables to be included in the model. While transients may play important roles in short-run modelling, by definition they have no place in the long-run. If transient variables are included in a model to be used in a long-run closure, they should appear on the exogenous list where they should be assigned neutral values. Alternatively, it should be a property of the model's structure that such transient variables can be guaranteed endogenously to take on neutral values in a long-run solution. If the focus is on the long run, then in keeping with the long-run neutrality of money, variables included in x^m (and therefore within x_1 in a macroeconomically closed model) should be real stocks or flows, or relative prices.

In the standard version of IMPACT's CGE model, ORANI (Dixon, Parmenter, Sutton and Vincent (1982) -- hereafter DPSV (1982)), provision is made for the user to select his own basic closure (DPSV (1982), Ch.7). The choice of endogenous and exogenous sets x_1 and x_2 (along with some choices about the values of behavioural parameters) crucially affects the interpretation of the comparative static time frame. In ORANI's standard (or neoclassical) short-run closure, the elapsed time between the base-period and the attainment of the new solution is assumed to be sufficiently short to allow one to ignore the impact of the shock under analysis on capital stocks in use in each industry, but sufficiently long for relative prices, consumption, investment demands, and production, of every commodity to have reached a new equilibrium configuration. (As will be seen below, such a short run concept might well correspond in calendar

time to about two years.) With capital stocks fixed, rates of return in the different industries are endogenous. If we reverse the roles of these two sets of variables -- i.e., we take industry rates of return as exogenous and endogenize the vector of capital stocks -- then, clearly, a longer time-frame must be involved. In fact this is what is done in long-run closures of ORANI, where it is assumed that, relative to the world capital market, Australia is a small country. Given that any initial deviations in Australian industries' relative and absolute rates of return from the competitive world-wide values of these variables are eliminated, in the long run, by accumulation or decumulation of capital, what are the sources of such capital?

2.4 ORANI's Lack of Long-run Macroeconomic Closure

It is at this point that ORANI's lack of long-run macroeconomic closure becomes evident. The standard version of ORANI focusses heavily on commodity markets, and on the demand side of the labour market, but does not deal with financial or capital markets. To provide long-run macroeconomic closure it is necessary to add additional equations in order to endogenize the real flow of foreign capital. Inter alia this involves also endogenizing real saving and consumption (not usually endogenous in closures of the standard version of ORANI). The basic approach for such an extension of ORANI has been developed by Dixon, Parmenter and Rimmer (forthcoming 1983).

2.5 ORANI's Lack of Short-run Macroeconomic Closure

In a standard short-run closure of the standard version of ORANI,

the following variables of macroeconomic interest are routinely assigned to the exogenous set x_2 :

- (i) one of the price level or the exchange rate (as numeraire),
- (ii) one of the real wage or the aggregate level of employment,
- (iii) one of real absorption ($C+I+G$) or the balance of trade surplus ($X-M$).

These three choices are forced on users of ORANI (in stand-alone mode) because there are no mechanisms in the model suitable for determining:

- (I) the extent to which induced changes in the real exchange rate will be realized as changes in the domestic inflation rate relative to the foreign rate or as changes in the nominal exchange rate;
- (II) the extent to which induced changes in the buoyancy of the labour market will be realized as changes in real wages or as changes in employment;
- (III) the extent to which induced changes in national income will be realized as changes in aggregate absorption or as changes in the balance of trade.

Thus at the outset three important macro variables must be exogenized in stand-alone applications of ORANI. But there are further macro variables,

such as interest rates, the money stock, the stock of bonds held by the non-bank sector and the level of foreign reserves, which do not appear in ORANI at all. If these are to be included in x^m , then additional equations are needed for short-run macroeconomic closure.

3. Towards a Long-Run Closure of ORANI

As we have seen in sub-section 2.4, there are no behavioural equations in the standard version of ORANI (DPSV (1982)) which would enable an endogenous split of private spending into consumption and investment, nor is there an explicit relationship endogenizing capital flows. Before giving a brief account of what has so far been achieved towards filling this vacuum, some methodological perspectives are first necessary.

3.1 The Nature of an ORANI Solution

The comparative static method can be used in two essentially different ways. In the first, two equilibria which are supposed to occur at different points in time are compared. In the second, the differentials between two equilibria occurring at the same notional period of time are calculated. The difference between the values Y^A and Y^B of a given endogenous variable Y at a given point of time in two equilibrium solutions is then attributed to differing scenarios A and B which vary with respect to the settings of the exogenous variables. Thus the computed value $(Y^A - Y^B)$ answers the question:

By how much would Y differ at time τ as the result of making the assumptions A about the exogenous variables rather than the assumptions B ?

Here the assumptions B define ceteris paribus. In ORANI simulations this second method (contemporaneous differential) comparative statics - cdcs for short) will almost always be the relevant approach. While all pretence of forecasting is eschewed in cdcs, the policy questions addressed come into sharper focus.

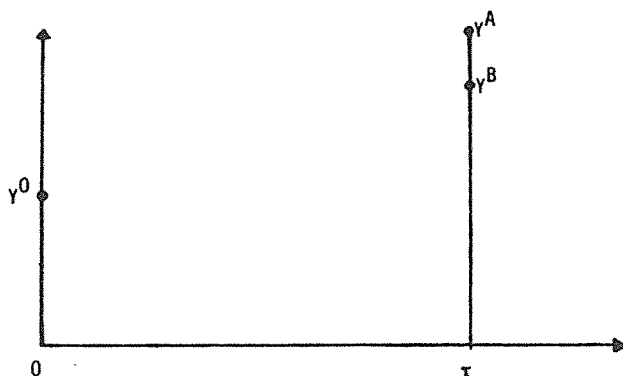


Figure 1. The nature of a contemporaneous differential comparative static solution. Y^0 is the base-period value of some endogenous variable. Y^B is the value projected for $Y(\tau)$ under ceteris paribus assumptions about the future time paths of the exogenous variables. Y^A is the value projected for $Y(\tau)$ when the ceteris paribus time paths of the exogenous variables are disturbed by an exogenously specified (vector) differential.

These ideas are illustrated in Figure 1. We may have in mind some actual historical year $t=0$ in which the economy produces an observation Y^0 on an endogenous variable Y . This is the base period. We then perform the following thought experiment. Suppose the sequence of future realizations, specified as far ahead as τ periods, of the exogenous variables of the system is Z_1, \dots, Z_τ . This forms the ceteris paribus assumption set (set B). Then in τ we would expect Y to have the value Y^B . Now suppose instead

that the sequence of future realizations on the exogenous variables is $Z_1 + \delta_1, \dots, Z_\tau + \delta_\tau$. The resultant projected value of $Y(\tau)$ is Y^A . The proportional difference, $(Y_A - Y_B)/Y_B = y(\tau)$, is attributed to the shock $\delta_1, \dots, \delta_\tau$ in the exogenous variables. Notice that nothing is said about the source of the difference $^2 (Y^B - Y^0)$.

For analytical ease consider a shock in which all but one of the elements of each δ_t is zero ($t=1, \dots, \tau$) and this element is set equal to a constant percentage of the corresponding element of Z_t :

$$(2) \quad \delta_{it} = i^{\text{th}} \text{ element of } \delta_t = k Z_{it} = k (i^{\text{th}} \text{ element of } Z_t) \\ (t = 1, \dots, \tau).$$

This form of shock is a once-off proportional change in the level of the i^{th} exogenous variable relative to its ceteris paribus level. More generally, we could consider shocks of the form $\delta_t = \hat{k} Z_t$, where \hat{k} is a diagonal matrix. This is the most appropriate type of shock with which to present ORANI in both short and long-run closures. Of course, as between the two lengths of run, both the partitioning into endogenous and exogenous sets, and the value of τ will differ. For reasons which are given in Section 4, we favour a value of about 2 years for τ in (neo-classical)³ short-run closures of ORANI. The value of τ in long-run closures is an open question but it must be sufficiently large to make plausible the assumption (see sub-section 2.3 above) that initial short-run disturbances induced in rates of return in different industries are eliminated by capital formation and depreciation. Notice that apart from this plausibility requirement -- effectively that sufficient time be allowed for capital to be mobile between industries -- the long run is an arbitrary number of calendar years.

Typically solutions to ORANI are computed using Johansen linearizations (DPSV(1982), Ch.5). Given a data base (Y^B, Z^B) on endogenous and exogenous variables, and a file θ on behavioural parameters, a solution of the model at period τ can be written:

$$(3) \quad F_{\theta, \tau}(Y^B, Z^B) = 0.$$

The values Y^B of endogenous variables in (3) are assumed to be consistent with the values of the exogenous variables Z^B . Both Y^B and Z^B are assumed to be realizations at period τ . In order for it to be reasonable that the mechanisms in the model (as encapsulated in F) will indeed lead to (3) holding, some regular evolution of Z_t prior to $t = \tau$ may need to be assumed. When the once-off shock $\hat{k}Z_t$ ($t=1, \dots, \tau$) is introduced, so that Z^B is replaced by $(I + \hat{k})Z^B$, the new solution at $t=\tau$ is:

$$(4) \quad F_{\theta, \tau}(Y^A, (I + \hat{k})Z^B) = 0.$$

Log linearization of (3) produces:

$$(5) \quad y = \Pi z,$$

where y and z are vectors of log differentials, while Π is a function of Y^B, Z^B, θ and τ . To compute the Y^A value shown in Figure 1 all we need do is put $z = k$ in (5), where k (the vector corresponding to \hat{k}) is the vector of once-off proportional shocks. This follows since, for small shocks, $(I + \hat{y})Y^B$ (where \hat{y} is the diagonal matrix formed from y) is a good approximation to Y^A .

3.2 The Dixon-Parmenter-Rimmer Approach

We have seen that an essential ingredient for computing an ORANI solution is a data base (Y^B, Z^B) representing the ceteris paribus position of the economy in the year projected. In short-run applications it is often sufficient to ignore the difference between (Y^B, Z^B) and (Y^0, Z^0) -- that is, the judgement is made that the difference between some well documented base-period data base and a reasonable ceteris paribus data base for the year projected would have only minor effects on the results⁴. In long-run applications such an approach is untenable. It thus becomes necessary to erect a scenario generating (Y^B, Z^B) as a separate exercise. While in principle the choice of (Y^B, Z^B) is highly arbitrary, sensible choices are restricted by sets of values of endogenous and exogenous variables which could have been produced by an economy whose mechanisms mirror those incorporated in ORANI. In exploratory work using a miniature version of ORANI, MO81, Dixon Parmenter and Rimmer (DPR) (forthcoming 1983) simply take a photographic enlargement of their base-period data base (Y^0, Z^0) as their data base (Y^B, Z^B) in the projected (or snapshot) year (τ): thus the rate of growth of all real variables between 0 and τ is the same. Further realism could be added by allowing for long-run fixity in the supplies of agricultural land and some other natural resources, and by incorporation of exogenous estimates of technological improvement⁵.

Given the arbitrary nature of the projection-period ceteris paribus data base, is there any essential link between it and an historical base-period data base? To answer this it is necessary first to give a brief description of MO81.

MO81 contains six equation blocks (DPR (forthcoming 1983), Table 1): (I) final demands for commodities (by households, capital goods creators, and by foreigners); (II) demands by industries for inputs and supplies by them of outputs; (III) zero pure profit conditions; (IV) market 'clearing' equations; (V) miscellaneous definitional equations; and (VI) macroeconomic closure. The last of these is related recursively to the earlier blocks in the sense that no variable endogenized within it in standard closures of ORANI appears in any earlier block. It is mainly in block VI that our current interest lies.

Equation (T31) of DPR (forthcoming 1983) gives the log differential form of the consumption function, which may be simplified notationally as follows:

$$(6) \quad c = f_c + \psi_1 a + \psi_2 t - \psi_3 v + \psi_4 (q + d) ,$$

where the lower case Roman letters represent proportional deviations of variables from their ceteris paribus values in year τ . The key to the notation is:

- f_c : average propensity to consume (apc)
- a : labour income
- t : aggregate tariff revenue
- v : aggregate export subsidy
- q : share of the capital stock in Australia owned by
domestic residents
- d : income to capital.

The coefficients $\{\psi_i; i=1,\dots,4\}$ are the shares in domestic income accounted for by wage income, tariff revenue, export subsidies and returns on capital accruing to domestic residents. If the apc is treated as exogenous, with $f_c=0$, then (6) represents a consumption function passing through the origin; the alternative exists to allow f_c to be endogenous, in which case additional equation(s), which define some other sort of consumption function, must be added. However the form of the consumption function is not likely to be critical for results derived under the cdcs methodology; so the simple linear, proportional consumption function involved when f_c is exogenous will be adequate for most purposes. Such a specification has the advantage of preserving, with minimal complication, the first degree homogeneity of M081 in real variables.

Apart from taxes and subsidies on international trade, the consumption effect in the snapshot year, c , is driven by labour income a , income to capital d , and by the change in the share of the capital stock which is domestically owned. The variables a and d respectively are endogenized by employment and the wage rate on the one hand, and by the demand for capital and rental rates, on the other. These variables in turn are endogenized within blocks I - IV. It remains to endogenize q . It is at this point that explicit recognition of intertemporal links connecting the data bases of the base period 0 and the projected (or snapshot) period τ becomes unavoidable. The aim is to model these links in a manner which avoids, as far as possible, commitment to a full dynamic specification (which lies outside our paradigm.) That is, we would hope to generate comparative static solutions which are compatible, to a good degree of approximation, with a wide variety of possible adjustment paths between 0 and τ .

DPR make the following assumptions about capital accumulation by domestic residents:

- (a) Depreciation occurs at an exogenous geometric rate. In the implemented versions of their theory, depreciation rates are uniform across industries.
- (b) In any year t between the base year 0 and the snapshot year τ , the share in the title to newly created capital of domestic residents is just the ratio of domestic saving to total capital formation. This ratio is not industry-specific.
- (c) The shares of the total capital stock in the base period ($Q(0)$) and in the snapshot period ($Q(\tau)$) are 'known' respectively from historical data and the ceteris paribus scenario B.
- (d) The real value of the annual flow of saving grows geometrically between 0 and τ ; that is

$$(7) \quad \frac{S(t)}{\pi(t)} = \frac{S(0)}{\pi(0)} (1+U)^t \quad (t = 1, \dots, \tau),$$

where S and π respectively are the nominal flow of saving by domestic residents and the capital goods price index, and U is the rate of growth in real saving.

The last assumption introduces a primitive dynamics hitherto lacking in ORANI and its miniatures. Our concern is to show that cdcs solutions of M081 are not sensitive to the particular functional form chosen (viz. (7)).

If the depreciation rate is D , domestically owned capital which has been accumulated between 0 and τ and which survives to τ is:

$$\begin{aligned}
 \Delta_0^\tau(KQ) &= \sum_{t=0}^{\tau-1} (S(t)/\pi(t))(1-D)^{\tau-t-1} \\
 (\text{from (7)}) \quad &= \frac{S(0)}{\pi(0)} \sum_{t=0}^{\tau-1} (1+U)^t (1-D)^{\tau-t-1} \\
 &= \frac{S(0)}{\pi(0)} (1-D)^{\tau-1} \sum_{t=0}^{\tau-1} \left\{ \frac{(1+U)}{(1-D)} \right\}^t \\
 (8) \quad &= \frac{S(0)}{\pi(0)} \left\{ \frac{(1+U)^\tau - (1-D)^\tau}{D+U} \right\} ,
 \end{aligned}$$

where K and Q respectively are the aggregate capital stock and the share of this stock which is owned by domestic residents. (Note that the operator Δ_0^τ takes differences between the values of a variable at different points of time.)

Consider what happens to (8) when a shock introduced at $t=0$ induces an endogenous logarithmic differential u in U . Let $\Delta^*(\cdot)$ be the operator which takes the difference between the value of (\cdot) under the shock scenario A and its value at the same point of time under the control

scenario B. (Notice that $uU = \Delta^*U$.) Then the difference made by the shock to the amount of capital owned in τ by domestic residents is $\Delta^*(\Delta_0^\tau(KQ))$. An expression for this difference is obtained by differencing (8):

$$(9) \quad \Delta^*(\Delta_0^\tau(KQ)) = \frac{S(0)}{\pi(0)} \left\{ \frac{(1+U+\Delta^*U)^\tau - (1-D)^\tau}{D+U+\Delta^*U} - \frac{(1+U)^\tau - (1-D)^\tau}{D+U} \right\}.$$

For example, suppose the ceteris paribus growth rate of real saving is 8 per cent per annum ($U=.08$) and the depreciation rate 20 per cent per annum ($D=.20$). Let an exogenous shock at $t=0$ result in a 10 per cent response in $U(u=.1, uU=.008=\Delta^*U)$. Then domestically owned capital stock in year $\tau=10$ would be, according to (9), 0.3706 times base period saving ($S(0)/\pi(0)$) higher than it would have been in the absence of the shock. The growth paths of real saving, with and without the shock are shown in Figure 2a.

Suppose now that instead of (7), the ceteris paribus growth path for real saving $\{S(t)/\pi(t)\}$ is simply taken to be the unspecified sequence $\{S^R(0), S^R(1), \dots, S^R(\tau-1)\}$ which has arbitrary values for its members other than the first. Suppose that under the shock, differences $\{\Delta^*S^R(0), \Delta^*S^R(1), \dots, \Delta^*S^R(\tau-1)\}$ are generated in the annual flows of real saving.

19.

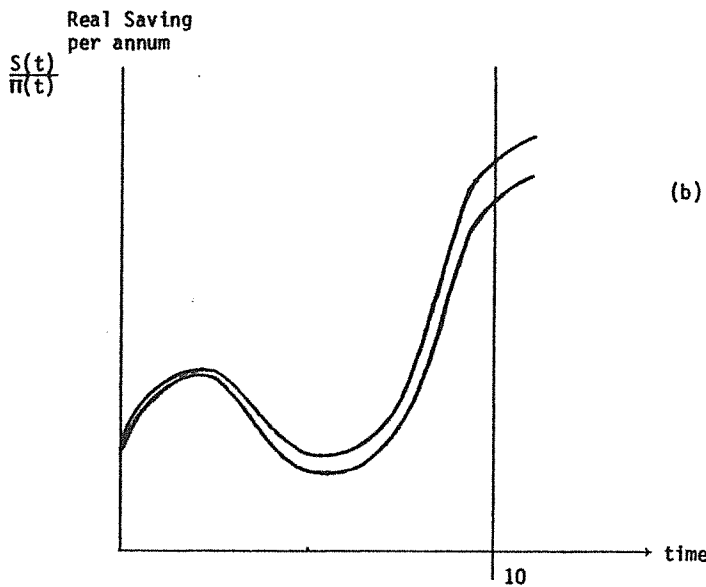
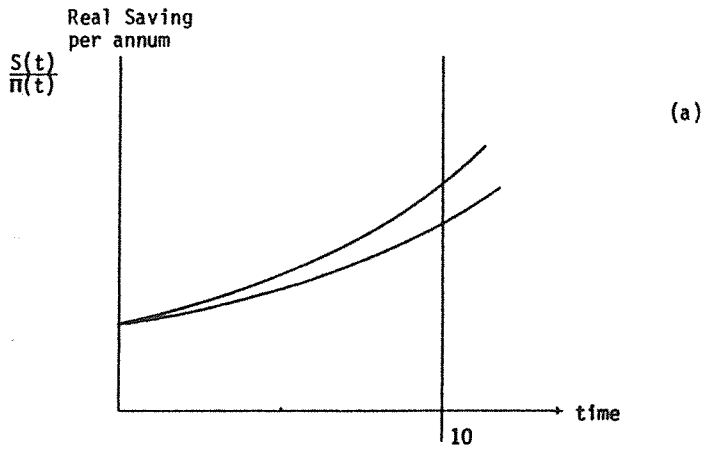


Figure 2. Two possible paths of real saving yielding identical contemporaneous differential comparative static solutions in M081.

Then

$$\begin{aligned}
 \Delta^*(\Delta_0^\tau(KQ)) &= \sum_{\tau=0}^{\tau-1} [S^R(t) + \Delta^*S^R(t)](1-D)^{\tau-t-1} \\
 &\quad - \sum_{\tau=0}^{\tau-1} S^R(t)(1-D)^{\tau-t-1} \\
 (10) \qquad \qquad &= \sum_{\tau=0}^{\tau-1} \Delta^*S^R(t)(1-D)^{\tau-t-1} .
 \end{aligned}$$

Equation (10) demonstrates that the difference made in year τ to the amount of capital owned by domestic residents is a function only of the differential path in real saving, and not of the ceteris paribus path of real saving itself.

The differential path implied by a once-over change in the growth rate U to $(U+\Delta^*U)$ throughout $[0, \tau-1]$ is of course only one possibility (though a plausible one). However, many sequences $\{\Delta^*S^R(0), \dots, \Delta^*S^R(\tau-1)\}$ when substituted into (10) would yield a common value. Because the macroeconomic closure block VI is recursively related to the earlier blocks of M081, it follows that the use of (7) is consistent with a broad range of dynamic stories. Thus we have endowed miniature ORANI with a primitive dynamics which indeed does leave the details concerning the adjustment path of real saving virtually wide open.

The variables (other than base period values) appearing in (9) are $K(\tau)$, $Q(\tau)$ and U : respectively the snapshot year capital stock, the domestically owned fraction of it, and the growth rate between the base year

and the snapshot year of real saving. The log differential $k(=\Delta^*K(\tau)/K(\tau))$ is endogenized within blocks I - V of M081 via the assumption of an open capital market in which Australia is a small country (see sub-section 1.2 above); block VI must endogenize both q and u (where lower case letters are defined similarly to k). (9) provides one equation for this purpose. The additional equation needed is given by taking log differentials of (7) at $t=\tau$, obtaining:

$$(11) \quad s - \pi = \tau \frac{U}{(1+U)} u .$$

There are new variables in (11), namely s and π . The consumption function (6) of course endogenizes s as well as c via an income identity. π (the comparative static log differential at τ in the capital goods price index) is endogenized in earlier blocks. In Johansen solutions $U/(1+U)$ in (11) is evaluated at its ceteris paribus value. Thus macroeconomic closure in (M081) is obtained by adding the consumption function (6), a domestic income identity, a growth path of real saving (7), and an accumulation relation (8) for domestic equity. The log linear versions of these four equations constitute the macroeconomic closure block of M081. Foreign capital flows could be obtained by back solution if wanted.

3.3 Measurement of Prosperity Effects

One feature of the DPR macroclosure of M0-81 may be a problem from the point of view of comparing the effects at $t=\tau$ of different shocks injected at $t=0$. This is the unavailability of a suitable index to measure prosperity effects unambiguously.

Consider a shock in an exogenous variable introduced at $t=0$ (a rise in the overseas price of a commodity for which Australia is a price-taker, say). Suppose that this shock is "good", in the long run, for Australian residents. The benefits could show up as (i) an increase in wealth at τ (viz. as $\Delta^*(\Delta_0^T(KQ))$); or (ii) as a higher average level of consumption over the latent period $[1, \tau]$; or as a mixture of both.

Let D° be the time preference discount rate. Then the present value of the consumption stream over the latent period is:

$$(12) \quad C' = \sum_{t=1}^{\tau} \frac{C(t)}{\Xi(t)} (1-D^\circ)^t,$$

where $\{C(t)\}$ and $\{\Xi(t)\}$ respectively are the sequences of money consumption and of the consumer price index. As in the case of saving, it is convenient to adopt a simple growth path for real consumption :

$$(13) \quad \frac{C(t)}{\Xi(t)} = \frac{C(0)}{\Xi(0)} (1+W)^t \quad (t = 1, \dots, \tau).$$

Then the change in the present value of real consumption over the latent period induced by a shock introduced at $t=0$ is:

$$(14) \quad \Delta^*(C') = \frac{C(0)}{\Xi(0)} \frac{1}{\tau} \left\{ \frac{(1+W+\Delta^*W)(1-D^\circ)}{(W+\Delta^*W-D^\circ-WD^\circ)} \left[(1+W+\Delta^*W)^\tau (1-D^\circ)^\tau - 1 \right] \right. \\ \left. - \frac{(1+W)(1-D^\circ)}{(W-D^\circ-WD^\circ)} \left[(1+W)^\tau (1-D^\circ)^\tau - 1 \right] \right\}.$$

Any given shock which is beneficial to domestic residents produces,

under the DPR closure, a point in the consumption-wealth space. However different shocks produce points along different rays. It is therefore useful to make the trade-off between consumption and wealth explicit. This is done in Figure 3 for the case of a 34 per cent tariff rise injected into M081 (evaluated with the published DPR (forthcoming, 1983) data base).

Percentage Change in Real
Consumption over the
Latent Period

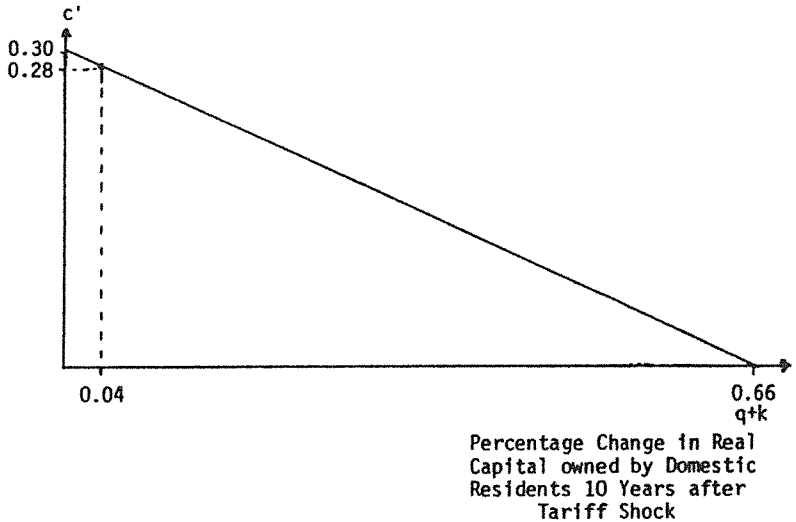


Figure 3. Possibilities frontier associated with a 34 per cent tariff rise in M081. Percentage changes are measured relative to ceteris paribus paths of the variables. The variable on the vertical axis is $100\Delta(C')/C'$ (see equation (14)) evaluated without discounting ($D^0=0$). The shock is an increase of 34 per cent in the tariff on the imported good at $t=0$.

In the DPR tariff solution (point A in Figure 3) the average propensity to consume (apc) is exogenous at every point over the latent period. Allowing the apc to vary generates the frontier shown in Figure 3. (The apparently beneficial effect in MO81 of the tariff increase is probably explained to a substantial extent by terms-of-trade effects associated with the export demand elasticities adopted. Boadway and Treddenick (1978) obtained similar results with a larger model.) The use of frontiers such as that shown in Figure 3 should facilitate comparison of different shocks in the following sense. The selection of a particular point on the frontier corresponds to the adoption of a particular consumption function. The frontier approach automatically provides a sensitivity analysis of a very wide variety of functional forms. It also provides a choice of metric with which to measure the prosperity effects of different shocks, i.e.,

- (i) the change in the amount of capital owned in τ years time with no change in real consumption in the intervening period;

and

- (ii) the change in consumption over the τ years following the shock with no change in the amount of capital owned in τ years time.

4. The IMPACT Paradigm⁶

The IMPACT paradigm is based on the following, strong, maintained hypothesis: in the short run

"... financial and money markets, as well as fiscal actions, are only important for individual industries insofar as they exert a real effect on the big components of national income: namely, private consumption, private capital formation, and government spending" (Powell (1981), p.242).

Under this paradigm the absence of monetary and financial variables in ORANI is no embarrassment since the economy is visualized as being separable between its macro (including financial and monetary) components on the one hand, and its micro (especially sectoral, and relative price) components on the other. Indeed there are some virtues in carrying out simulations with ORANI in an exogenous macroeconomic environment: there are, after all, a set of macro instruments available to sterilize any unwanted macro consequences of sectoral and relative price shocks. Nevertheless we have found that policy advisers are preoccupied with these macro consequences (probably because their potential sizes have been seriously overestimated, especially in the case of tariff-related shocks). Thus the problem has required serious attention.

4.1 Problems of the Interface

Our approach is to use a small macrodynamic model, MACRO, to close ORANI in short-run simulations. MACRO is a slightly revised mid 1979 version of the Reserve Bank of Australia's RBII model (Jonson and Trevor (1981) -- see also Cooper (1983), p.28). This attempt to interface a comparative static model with a macrodynamic model threw up some substantial theoretical challenges. Among the more notable ones are:

- (i) the lack of an explicit dynamics in ORANI,
- (ii) temporal aggregation problems,
- (iii) the presence of variables which are endogenous to both models,
- (iv) differing definitions in the two models of essentially the same variables,
- (v) the presence in MACRO of macrorelations which cannot be derived as explicit aggregates of microrelations in ORANI (e.g., the production function),
- (vi) the difficulty of preserving homogeneity properties in the interfaced system.

The interfacing method developed by Cooper and McLaren (1980, 1982, 1983) seems to resolve the first four of these simultaneously. The difficulties posed by (v) and (vi) seem to be irreducible liabilities of the method. In particular one cannot expect a separately developed macro model to be

formulated in terms of perfect aggregates (where such exist) of the variables appearing in a given CGE model, nor could one expect the interfaced system to display excess supply functions for commodities which are homogeneous of first degree in the monetary stock and in nominal prices (see Weintraub (1974), p.51). On these two points of theoretical consistency the extended Walrasian paradigm offers better prospects.

Some of these problems were minimized by the particular macro model chosen. MACRO is a Bergstrom-Wymer model formulated in continuous time (see Bergstrom and Wymer (1976)). This continuous time formulation made solution of the temporal aggregation problem much easier. The relatively extensive coverage of money and financial markets, both of which are totally lacking in ORANI, was an advantage, as was the small size of MACRO (26 equations) and the consequent relatively infrequent occurrence of double endogeneities. Indeed, as we will see below, the double endogeneities were actually turned to advantage in the development of the interfacing methodology. This was possible because these variables provide a link between ORANI solutions and realizations of observable variables in real time.

4.2 Basic Approach

The cdcs interpretation of ORANI solutions gels nicely with the macrodynamic method: in both cases attention is focussed on deviations from control. In the case of MACRO, such deviations can be computed for all instants of continuous time after the injection of a shock. A short-run solution of ORANI, on the other hand, only projects a single deviation from

control at a single point of time after the shock. The interfacing methodology is based on attempting to identify an instant after an exogenous shock which reconciles these two stories. The elapsed time then provides an estimate of the ORANI short run t^* .

For the moment take it as given that some method of supplying feedbacks to ORANI from MACRO can be found. (With only a minor oversimplification, for the moment we ignore feedbacks from ORANI to MACRO.) Call the resulting system ORANI-MACRO (O-M). The response of this system will be shown below to be a function of the unknown length t^* of the ORANI short run (and of certain other parameters). The response of ORANI within O-M is a cdcs point solution which varies parametrically with t^* . In Figure 4 we concentrate on a doubly endogenous variable, Y . The deviation-from-control of this variable, according to the MACRO equations in O-M, has a path in real time shown by the broken line. The points marked with a cross, on the other hand, do not form a path in real time. Rather, they indicate a point solution for the deviation-from-control of Y as endogenized by ORANI within O-M conditional on the assumption that the length of the ORANI short run is the elapsed time on the horizontal axis. Thus we can select an ORANI short run t^* which makes the MACRO solution for Y at t^* in real time as close as possible to the ORANI solution conditional on a short run of t^* . Such a procedure simultaneously provides an indirect real time validation of ORANI via MACRO and as well furnishes an estimate of the calendar period corresponding to the ORANI short run. The only complications to this story in practice are due to the vector nature of Y , and the need to choose t^* simultaneously with other parameters of the interface.

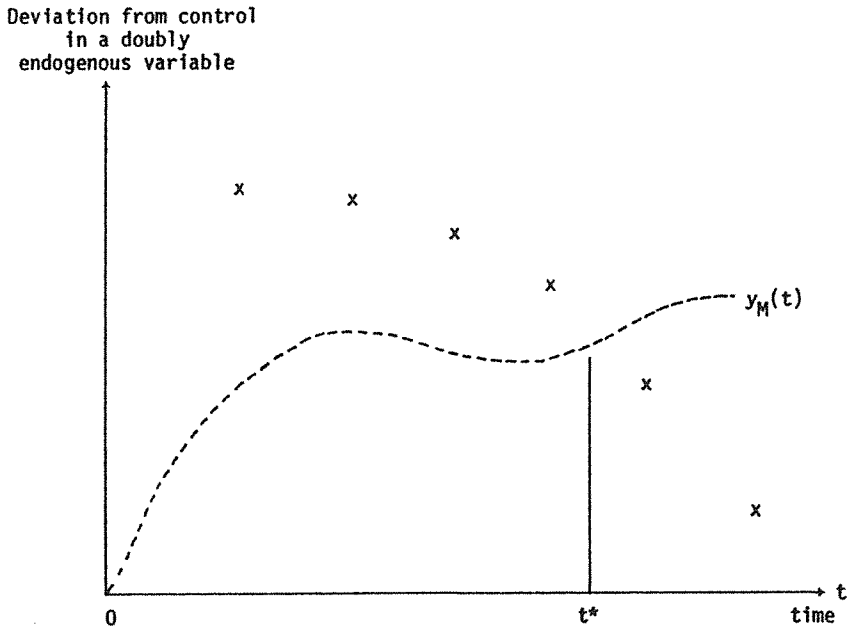


Figure 4. Reconciliation of ORANI and MACRO projections of a doubly endogenous variable. A shock is injected at $t = 0$. The deviation-from-control in a doubly endogenous variable Y , $y_M(t)$, endogenized as a function of real time by MACRO, is shown as the broken line - - -. Within ORANI-MACRO, the ORANI contemporaneous differential comparative static solutions for the same variable at t (where t is the assumed length of the ORANI short run), are shown as $x \ x \ x$. At t^* the MACRO and ORANI stories are reconciled.

4.3 The Two Model Types

- (a) ORANI ORANI (Dixon, Parmenter, Sutton and Vincent (1982)) is a CGE model of the Johansen (1960) class. As we have seen in sub-section 3.1, under a typical allocation of variables to the endogenous and exogenous categories, a cdcs solution to ORANI can be written :

$$(15) \quad y_0 = C_0 z_0 ,$$

where the subscript 0 denotes ORANI variables and parameters; y_0 represents a vector of proportional changes (relative to ceteris paribus) in ORANI endogenous variables; z_0 represents a vector of once-off proportional changes (relative to ceteris paribus) in ORANI exogenous variables, and C_0 is the matrix of ORANI elasticities.⁷ The following quotation illustrates the way in which results generated by (15) may be interpreted: "given a policy change A, then in the macro-economic environment B, variable C will differ in the short-run by x per cent from the value it would have had in the absence of the policy change" (Dixon et al. (1982) p.63). Implicit in the construction and interpretation of the ORANI elasticity matrix C_0 is the notion of the ORANI short run, t^* , which is defined as a period long enough for prices to adjust, for output to be expanded using given plant, for new investment plans to be made and new capital goods to be purchased, but not long enough for the size of capital stock in use to change as a result of the shock.

In the notation of sub-section 3.2, (15) could be written:

$$(16) \quad \Delta^* \ell_n Y_0 = C_0 z_0 ;$$

however we stick to the notation in (15) from this point on. To indicate explicitly the fact that the result (15) compares two solutions of ORANI at t^* , we may write instead

$$(17) \quad y_0(t^*) = C_0(t^*)z_0 .$$

Thus y_0 is the proportional difference between two equilibria at t^* . This difference is conditional on the maintenance of the exogenous macroeconomic environment. Variables embodying this environment are represented in z_0 where they are assigned zero values. However the original shock (i.e., the non-zero elements of z_0) may also cause changes in variables outside ORANI (i.e., in MACRO). Some of the latter variables appear among the elements of z_0 which are set to zero in stand-alone applications of ORANI using (17). Thus when (17) is used alone the implicit assumption is that the induced differentials in the macroeconomic environment are sterilized by off-setting use of fiscal and/or monetary instruments.

(b) MACRO Our MACRO model, MACRO81, is an adaptation (see Cooper (1983)) of the Reserve Bank's RBII model as documented in the appendix to Jonson and Trevor (1981). Underlying it is a first-order stochastic differential equation system which may be written:

$$(18) \quad dY_M(t) = A_M Y_M(t)dt + B_M Z_M(t)dt + dv(t) ,$$

where the subscript M indicates MACRO variables and coefficients, Y and Z respectively are vectors of logarithms of endogenous and exogenous variables respectively, and $dv(t)$ is a Gaussian vector process. The solution of this system may be written:

$$(19) \quad y_M(t) = e^{A_M t} \left\{ y_M(0) + \int_0^t e^{-A_M \tau} B_M z_M(\tau) d\tau \right\} \\ + e^{A_M t} \int_0^t e^{-A_M \tau} dv(\tau) ,$$

where $e^{A_M t}$ is defined by $P e^{\Lambda t} P^{-1}$, in which P is the matrix containing the characteristic vectors of A_M , and Λ is the diagonal matrix containing the characteristic roots of A_M . The matrix $e^{\Lambda \tau}$ has typical element $e^{\lambda_i \tau}$, in which λ_i is the i^{th} characteristic root of A_M . As before, let $\Delta^*(\cdot)$ be the operator taking the differential between (\cdot) when the exogenous variables follow the shocked path $\{Z_M(\tau) + z_M; \tau \in (0, t]\}$ and its value at the same point of time when they follow the control path $\{Z_M(\tau); \tau \in (0, t]\}$. Since Δ^* and the operators on the right of (19) are linear, and since $\Delta^*(Z_M(\tau) + z_M) \equiv z_M$ for all relevant τ , we have:

$$(20.1) \quad \Delta^* y_M(t) = \left[e^{A_M t} \int_0^t e^{-A_M \tau} B_M d\tau \right] z_M ,$$

or

$$(20.2) \quad y_M(t) = C_M(t) z_M \quad (\text{say}),$$

where

$$(20.3) \quad C_M(t) = A_M^{-1} [e^{A_M t} - I] B_M .$$

The notation on the left of (20.1) emphasises that the differentials involved are deviations from control at t . However the notation (20.2) is more convenient and is adopted from here on. The stochastic term on the right of (19) disappears in (20.1) because the introduction of the shock z_M into $\{Z_M(\tau); \tau \in (0, t]\}$ does not affect the realization of $v(t)$. Thus Δ^* differences alternative realizations of the same sample which differ only in their systematic part.

4.4 Temporal Aggregation

Equation (20.2) demonstrates that, for arbitrary t , the MACRO model can be written in a Johansen-like form. It is tempting to combine (17) and (20.2) into one system. Before this can be accomplished successfully, however, it is necessary to take account of the difference between the time profile of shocks z_0 suitable for input into ORANI and the evolving nature of outputs $y_M(t)$ ($t \in [0, t^*]$) from MACRO.

In Figure 5a are shown a once-off proportional shock z in a doubly exogenous variable Z and the response $y_M^0(t)$ in time of a variable y_M^0 which is endogenous to MACRO but exogenous to ORANI. (In typical short-run ORANI configurations, the real wage is such a variable.) Considered as an exogenous input to ORANI, z in Figure 5a is a suitable shock; however $y_M^0(t)$ is not. Notionally $y_M^0(t)$ can be split up into a (large) number of step functions; this is shown in Figure 5b. We are interested in the ORANI response at t^* to the combined stimuli of z maintained throughout $[0, t^*]$ and the series of step shocks in y_M^0 , which are maintained for successively shorter and shorter periods as we move from 0 to t^* . To be able to evaluate the impact of these shocks whose duration is less than t^* it is necessary to endow ORANI with a within-short-period dynamics. The so augmented model will be referred to as ORANI+.

The dynamics in ORANI+ is kept as simple as possible. For each of a small, but exhaustive, number of classes of endogenous variables in ORANI+, it is assumed that the within-short-run response belongs to a one parameter family. Given (20.3) it is natural to assume (by analogy with

Proportional Change

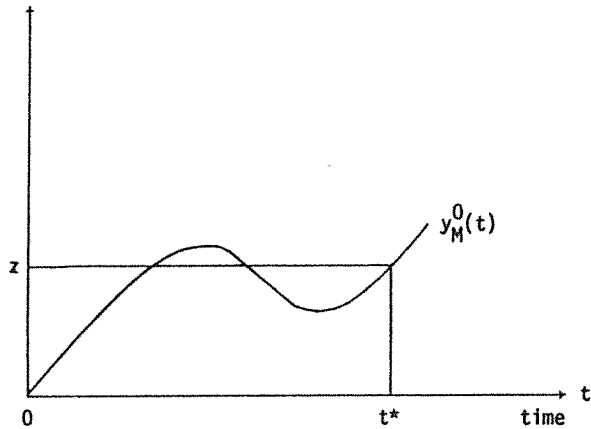


Figure 5a. The shock z in a doubly exogenous variable Z is injected into MACRO at $t=0$. t^* is the length of the ORANI short run. The proportional response in the MACRO endogenous variable y_M^0 is $y_M^0(t)$. ORANI will accept shocks in the step-function form displayed by z ; it cannot accept an arbitrarily evolving shock such as that displayed by $y_M^0(t)$.

Proportional Change

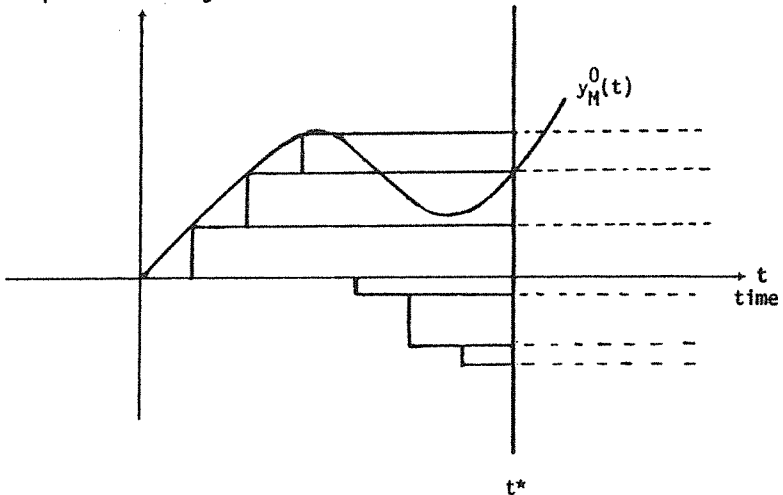


Figure 5b. The log differential $y_M^0(t)$ is endogenous to MACRO but appears also as an exogenous input into ORANI. The continuously varying shock $y_M^0(t)$ can be represented as a sequence of discrete step-function shocks as shown.

Figure 5. Time profiles of a once-off proportional shock suitable for input to ORANI and of a MACRO endogenous variable which is exogenous to ORANI.

MACRO) that the ORANI+ endogenous responses at s , $s \in [0, t^*]$, take the form:

$$(21.1) \quad y_0(s) = C_0(s)z_0 \quad .$$

where

$$(21.2) \quad C_0(s) = A_0^{-1} \left[e^{A_0 s} - I \right] B_0 \quad .$$

From ORANI the right-hand matrix in (21.1) is known only for $s=t^*$. Simplifying assumptions about the structure of A_0 are therefore needed. We assume that A_0 has the form:

$$(22) \quad A_0 = (\mathcal{L}\hat{n}\hat{\beta})I \quad ,$$

where $(\mathcal{L}\hat{n}\hat{\beta})$ is a diagonal matrix which, in the application reported below, has only six distinct elements. Let $(\mathcal{L}n\beta_i)$ be the i^{th} such distinct value, and let $y_{0j}(\tau)$ be the cdcs solution for the j^{th} ORANI endogenous variable, which is assigned the within-short-run adjustment parameter β_i . Then our assumptions imply:

$$(23) \quad y_{0j}(\tau) = \frac{\beta_i^\tau - 1}{\beta_i^{t^*} - 1} y_{0j}(t^*), \quad 0 \leq \tau \leq t^* \quad .$$

Possible within-short-run adjustment paths are shown in Figure 6. Notice that since this dynamics applies only within the short-run, values of $\beta_i > 1$ are admissible, and β_i values in general bear no necessary relation to long-run-stability conditions. Indeed, adjustment paths such as (e) in Figure 6 gibe well with some allegedly realistic descriptions of the adjustment of variables (such as prices of manufactures) which cannot be changed costlessly.

Response of a Variable
Endogenous to ORANI
(proportional change)

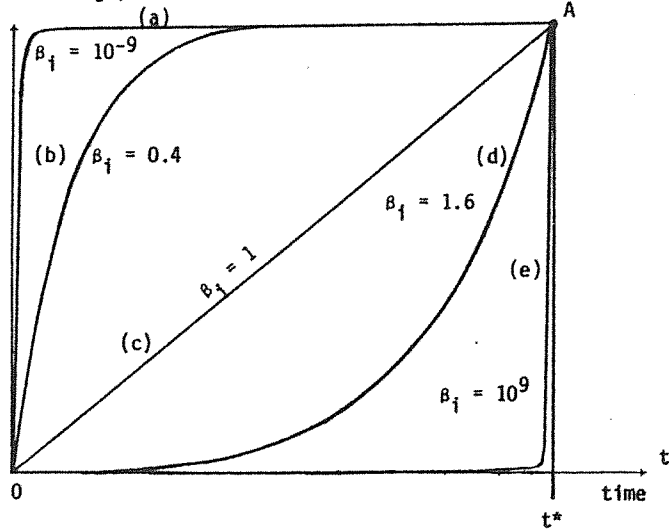


Figure 6. Within short-run dynamics in ORANI. t^* is the ORANI short run. The within-short period dynamics in ORANI+ for an endogenous variable with adjustment coefficient β_i is shown for different values of β_i . The point A is the short run contemporaneous differential comparative static solution for ORANI at t^* . The adjustment paths in $[0, t^*]$ imply nothing about long run dynamics.

4.5 The Interfacing Method

The interface parameters of the ORANI+ MACRO system are the length of the ORANI short run, t^* , and the adjustment parameters β_i . The two models are to be interfaced with these parameters set initially to arbitrary values. A search procedure is then used to optimize their values. In what follows the various coefficient matrices in the system are all known up to an arbitrary specification of t^* and the vector of β values. We start with the assumption that the system is recursive; that is, while we allow ORANI to be driven by variables that are endogenized in MACRO, we assume initially that there are no MACRO exogenous variables endogenized by ORANI.

Consider again the evolution of $y_M^0(t)$ in Figure 5b. Suppose now that this is the path of just one of many such variables; i.e., redefine $y_M^0(t)$ to be vector of exogenous inputs into ORANI which are endogenized in MACRO. In order to take an infinitesimal rather than a discrete step-function decomposition of $y_M^0(t)$, write it as the integral of its own derivatives:

$$(24) \quad y_M^0(t) = \int_0^t \frac{dy_M^0(\tau)}{d\tau} d\tau .$$

Each of these derivatives may be interpreted as an infinitesimal shock to ORANI sustained over the interval $[\tau, t]$. Let L be the linear operator which, when operating on $y_M^0(t)$, produces a vector conformable with z_0 , having as its elements the values of the appropriate elements of $y_M^0(t)$ in positions corresponding to ORANI exogenous variables endogenized by MACRO, and zeros elsewhere. Then $d[Ly_M^0(\tau)] \equiv Ld[y_M^0(\tau)]$ is a shock in a form

suitable for acceptance by ORANI+. Inject these instantaneous shocks for $\tau \in [0, s]$ into ORANI+ via (21.1); given the linearity of the model, the accumulated response at $s, s \in [0, t^*]$, is obtained by summation:

$$(25) \quad y_0(s) = \int_0^s C_0(s-\tau) L d y_M^0(\tau) ,$$

$$= \int_0^s C_0(s-\tau) L Q d y_M(\tau) ,$$

where Q is the linear operator that selects the subset $y_M^0(\tau)$ from $y_M(\tau)$.

Using (20.2), (20.3) and (21.2) in (25), we obtain for $s=t^*$:

$$(26) \quad y_0(t^*) = \int_0^{t^*} A_0^{-1} \left[e^{A_0(t^*-\tau)} - I \right] B_0 L Q e^{A_M \tau} d\tau B_M z_M .$$

This is the ORANI+ response at t^* (i.e., the ORANI response) to the variables exogenous to it which respond endogenously in MACRO to the shock z_M . If z is the initial shock in the double exogeneities, then in (26) z_M can be replaced by:

$$(27) \quad z_M = R z ,$$

where the operator R produces a vector conformable to z_M having as its elements the elements of z where possible, and zeros elsewhere. Given the assumption (22) on A_0 , and given the value of $C_0(\equiv C_0(t^*))$ from ORANI, for any B vector and value of t^* we can compute B_0 from (21.2). Thence the integral term in (26) can be computed (see Cooper and McLaren (1980) p.26

and (1982) p.16 for the details). Let its computed value be:

$$(28) \quad \int_0^{t^*} A^{-1} \left[e^{A_0(t^*-\tau)} - I \right] B_0 L Q e^{A_M \tau} d\tau$$

$$= H \quad (\text{say}) .$$

Then (26) can be written more simply as:

$$(29) \quad y_0(t^*) = H B_M z_M .$$

The right-hand side of (29) can be expressed in terms of $y_M^0(t^*)$ by the following argument:

$$(30) \quad y_0(t^*) = H \left[e^{A_M t^*} - I \right]^{-1} A_M A_M^{-1} \left[e^{A_M t^*} - I \right] B_M z_M .$$

Thus from (20.2) and (20.3) :

$$(31) \quad y_0(t^*) \equiv H \left[e^{A_M t^*} - I \right]^{-1} A_M y_M(t^*)$$

$$= G y_M(t^*) \quad (\text{say}) .$$

The coefficient matrix G in (31) gives the effects at t^* on the ORANI endogenous variables of the shocks endogenized by MACRO as evaluated at t^* . G can be thought of as an operator which corrects the timing of these MACRO variables to make them suitable inputs to ORANI and then performs the required multiplication.

Let S be a selection operator analogous to R in (27) such that putting:

$$(32) \quad z_0 = Sz$$

produces an ORANI shock vector containing zeros except for the doubly exogenous shocks which take on their values as in z . The initial doubly exogenous shock z produces in ORANI a direct effect C_0Sz and an indirect effect $Gy_M(t^*) = GC_M(t^*)Rz$. The total ORANI response is obtained by adding these:

$$(33) \quad y_0(t^*) = C_0Sz + GC_M(t^*)Rz .$$

In the case where the initial shock is not confined to double exogeneities (but in which the shocks z_0 and z_M contain no contradictions), (33) generalizes to:

$$(34) \quad y_0(t^*) = C_0z_0 + GC_M(t^*)z_M .$$

In the recursive case developed above in which there are no reverse linkages from ORANI to MACRO, the complete ORANI-MACRO system consists of (34) and the MACRO solution:

$$(35) \quad y_M(t^*) = C_M(t^*)z_M .$$

In this system certain variables will be endogenized both in y_M and in y_0 . The interfacing method consists of choosing the within-short-run-adjustment

coefficients vector β and the length of the ORANI short run t^* which minimize the inconsistencies between the MACRO and ORANI sides of the model.

In the case where there exist variables driving MACRO which are endogenous to ORANI -- this occurs in fact (see below) -- the same principles are used to interface the models; that is, ORANI is endowed with a within-short-run dynamics conditional on the unknown values of the interface parameters, which are then chosen to minimize inconsistencies in the double endogeneities at t^* . The formalities of this procedure are set out in Appendix 1.

4.6 Calibration of the Interface Parameters

In sub-sections 4.4 and 4.5 we have described an interfacing method that is specific to some injected shock or shocks. Possible approaches to making the procedure operational are:

- (i) the construction of a new interface for each simulation conducted on the O-M system;
- (ii) the selection of interface parameters which optimize a criterion function defined over a nominated set containing a variety of shocks;
- (iii) the selection of a set of interface parameters from a single standard simulation.

We have chosen (iii). In keeping with the literature on the comparative analysis of macrodynamic models (see, e.g., De Bever *et al.* (1979)), we have chosen an exogenous increase in government spending as the standard simulation.

In considering the interplay between ORANI and MACRO variables it is not necessary to carry the complete (and impossibly long) list of ORANI variables. Only macroeconomic variables are involved at the interface. A listing of ORANI variables relevant to the interface is given in Table 1. Table 2 shows a similar partial listing of MACRO variables.

TABLE 1
A PARTIAL LISTING OF VARIABLES APPEARING IN ORANI78

Variables Endogenous to ORANI: The Vector Y_0		
1.	$\log y_g$	Real Gross Domestic Product
2.	$\log P$	Price Deflator for Aggregate Production (\$A)
3.	$\log L_h$	Aggregate Employment Demand (labour-hours)
4.	$\log i$	Real Imports
5.	$\log P_{wl}$	Price of Wool (\$A)
6.	$\log x$	Real Exports
7.	$\log P_x$	Price of Exports (\$US)
8.	$\log (x P_x)$	Foreign Currency Value of Exports (\$US)
Variables Exogenous to ORANI: The Vector Z_0		
1.	$\log E$	Exchange Rate (\$A/\$US)
2.	$\log (W/P)$	Real Wages
3.	$\log C_R$	Real Household Expenditure
4.	$\log I_R$	Real Private Investment
5.	$\log K(0)$	Initial Capital Stocks
6.	$\log g_1$	Government Expenditure
7.	$\log t_3$	Average Level of Tariffs

Source: Cooper (1983).

TABLE 2
A PARTIAL LISTING OF VARIABLES APPEARING IN MACRO81

Variables Endogenous to MACRO81 (A Subset of Y_M)		
1.	$\log y_g$	Real Gross Domestic Product
2.	$\log P$	Price Deflator for Aggregate Production (\$A)
3.	$\log L_h$	Aggregate Employment Demand (labour-hours)
4.	$\log i$	Real Imports
5.	$\log E$	Exchange Rate (\$A/\$US)
6.	$\log W$	Nominal Wages (average weekly earnings, \$A)
7.	$\log d$	Aggregate Real Household Expenditure (including investment in dwellings)
8.	k	Proportional Change in New Investment (=DK/K)
9.	$\log K$	Business Fixed Capital Stock
Variables Exogenous to MACRO81 (A Subset of Z_M)		
1.	$\log P_{w1}$	Price of Wool (\$US)
2.	$\log x$	Real Exports
3.	$\log P_x$	Price of Exports (\$A)
4.	$\log g_1$	Government Expenditure
5.	$\log (1+t_3)$	Power of Tariff
6.	$\log P_i$	Price of Imports (\$US)
7.	$\log P_w$	World Prices (\$US)

Source: Cooper (1983).

From the listings of ORANI and MACRO variables in Tables 1 and 2 respectively it is evident that the following variables are endogenous to both models:

Double Endogeneities

$\log y_g$	Output
$\log P$	Prices
$\log L_h$	Employment
$\log i$	Imports

A set of variables of particular interest in view of the role of MACRO in endogenizing the macroeconomic environment of ORANI is the following group of ORANI exogenous variables:

MACRO to ORANI Feedbacks (ORANI Definitions)

$\log E$	Exchange Rate
$\log (W/P)$	Real Wages
$\log C_R$	Real Household Expenditure
$\log I_R$	Real Gross Private Investment (including investment in dwellings)
$\log K(0)$	Initial Capital Stocks

Variables exogenous to MACRO and endogenized by ORANI are:

ORANI to MACRO Feedbacks

$\log P_{wl}$	Price of Wool
$\log x$	Quantity of Exports
$\log P_x$	Price of Exports

In practice matters are complicated by the need to select various combinations of MACRO endogenous variables to correspond to the definitions of certain ORANI exogenous variables (for details, see Cooper (1983), pp.12-13).

The standard simulation consisted of a 10 per cent increase in real government spending. Other than the average tariff rate, this is the only double exogeneity. The main interface parameters to be estimated are:

- (i) the speeds of adjustment (i.e., the elements of the matrix A_0). For parsimony of parameterisation this matrix is chosen to be diagonal. This has the advantage of assigning at most one adjustment speed parameter to each ORANI equation. Since usually only the aggregate versions of ORANI equations are used explicitly in the interfaced system, this provides a simple identification of an adjustment speed which may subsequently be applied to some subset of the general ORANI disaggregated families of equations. For the model specification given above the

matrix A_0 is of dimension 8 and the respective adjustment speeds relate to the following variables:

<u>ORANI</u> <u>Variable</u>	<u>Adjustment</u> <u>Speed</u>
1. y_g Real Gross Domestic Product	β_1
2. P Price Deflator for Aggregate Production	β_2
3. L_h Aggregate Employment Demand (labour-hours)	β_3
4. i Real Imports	β_4
5. P_{wl} Price of Wool	β_5
6. x Real Exports	β_6
7. P_x Price of Exports (\$US)	β_6
8. $x P_x$ Foreign currency Value of Exports	β_6

Since variable 8 must be assigned an adjustment speed which is consistent with variables 6 and 7, these are constrained to a common adjustment speed (β_6). The remaining five adjustment speeds are freely chosen. The parameterisation is in fact made upon e^{A_0} , so that the field of search is the six-dimensional positive orthant.

- (ii) the ORANI "short run", t^* : the length of time during which the effects calculated from an ORANI stand-alone experiment would be expected to be working themselves out. A priori this interval was expected to be in the range 6 to 8 quarters and an intensive search over 5 to 12 quarters was carried out.

The criterion for choice of the interface parameters was the minimization of the (unweighted) sum of squared deviations of the double

endogeneities when evaluated at t^* . An intensive search over an eight-dimensional space⁸ led to the parameter values shown in column I of Table 3.

TABLE 3
ESTIMATED VALUES OF MAIN INTERFACE PARAMETERS

ORANI Variables		Adjustment Speeds (Diagonal Elements of e^{A_0}) (I)	Solution Values at t^* of the Double Endo- geneities** (II)
y_g	Output	$\beta_1 = 4.39$	1.073
P	Domestic Prices	$\beta_2 = 74.4$	1.358
L_h	Employment	$\beta_3 = 9.8$	1.557
i	Imports	$\beta_4 = 5.9$	1.854
P_{wl}	Price of Wool	$\beta_5 = 10$	n.a.
$x, P_x, (x P_x)$	Export Sector	$\beta_6 = .5$	n.a.

ORANI Response Interval : $t^* = 7.94$ quarters

** Result shown in column II applies to both the ORANI and MACRO projections of the doubly endogenous variable within the interfaced ORANI-MACRO system. At the given parameter settings, the two projections are in fact identical to the fourth decimal place. Further details are provided in Appendix 3, where a comparison is given with solution values for t^* set at 7.93 and 7.95 quarters.

n.a. Not applicable (variable is not doubly endogenous).

Source: Cooper (1983).

Besides the estimates of the main interface parameters, this table also shows the values, at these parameter settings, of the doubly endogenous variables. These figures correspond to a shock of 10.0 to $\log g_1$, government expenditure, and may be interpreted as percentage responses to a 10 per cent shock. The method has been successful in finding a setting for the interface parameters which eliminates inconsistency between ORANI and MACRO under a shock in government spending.

4.7 - Results for a Tariff Shock

Using the above parameter values, in this experiment we shocked t_3 , the average tariff variable in ORANI, by 25 per cent. A search over t_3 to determine the appropriate corresponding shock to $(1 + t_3)$, the power of the tariff variable in MACRO, subject to the criterion of minimizing the squared difference between the ORANI and MACRO projections of the output double endogeneity, led to the following results:

Shock to $(1 + t_3)$:	3.434%
Implied Value of t_3	:	15.923%
Value of real output (y_g) as computed by both the MACRO and ORANI components of the interfaced system	:	-0.089%

This implied value of t_3 (approximately 16 per cent) is of broadly comparable size to more direct estimates by the Industries Assistance Commission (1980, Table 2.1). Using weights based on the unassisted value

of domestic production of commodities, the average tariff levels on manufactured products were estimated by the Commission to be 24 per cent in 1968-69, and 15 per cent in 1977-78.

The principal results of the tariff experiment are given in Table 4. The main feature to note is the strong similarity between columns I and II, and the extent of the divergence between them and column III.

Column II shows the 'macroeconomically compensated' values of the responses of the major macroeconomic aggregates to a 25 per cent increase in tariffs. This is the standard ORANI, stand-alone, story. Column I shows how the ORANI story must be modified to allow for the impact of the tariff shock on the macroeconomic environment. These results in column I are based on the interface parameters shown in Table 3. The overwhelming impression gained by comparison of columns I and II is that the change in the macroeconomic environment induced by the tariff shock has only second order consequences to major macroeconomic aggregates.

Comparison of columns I and III of Table 4 reveals that MACRO and ORANI want to tell somewhat different stories about the impact of the tariff rise. By construction, their output responses were made identical (-0.089%) by selection of a suitable value of t_3 (the average tariff level). It will be noted that the MACRO responses in employment and output are equal, whereas ORANI shows the standard neoclassical short run result in which the employment response is a multiple (namely, 1.9) of the output response. Imports decline according to both sides of the interfaced model; the MACRO

TABLE 4
 PRINCIPAL RESULTS OF A 25 PER CENT TARIFF INCREASE
 IN THE ORANI-MACRO SYSTEM*

Endogenous Variable		ORANI Response within the Interfaced System (I)	Standard ORANI Response (II)	MACRO Response within the Interfaced System (III)
y_g	Output	-0.089	-0.123	-0.089
P	Prices	1.862	1.805	-0.374
L_h	Employment	-0.167	-0.205	-0.082
i	Imports	-1.322	-1.528	-3.625
$(x P_x)$	Value of Exports (\$US)	-2.824	-2.523	n.a.
E	Exchange Rate (\$A/\$US)	n.a.	n.a.	-0.237
W/P	Real Wages	n.a.	n.a.	0.104
k	Rate of Net Investment	n.a.	n.a.	0.003
K	Capital Stock	n.a.	n.a.	0.112
d	Household Expenditure	n.a.	n.a.	-0.053

* Based on the interface parameters shown in Table 3 and $t_3 = 0.15923$.
 For further details, see Cooper (1983). Figures in the table are
 percentage deviations from control at 7.94 quarters after the tariff
 shock.

n.a. not applicable.

Source: Cooper (1983).

response, however, is 2 to 3 times larger in absolute value. The key to this is the behaviour of the price level in MACRO, which is the glaring inconsistency. It seems here difficult to accept the MACRO result that a 25 per cent increase in tariffs could be deflationary at a two year lag. Given this result, however, the consequent increase in the competitiveness of the domestic economy leads in MACRO to a substantial decline in imports. It may be expected that, in the presence of an experiment involving a highly non-neutral shock, the model which has been built more specifically for analysis of the type of shock in question would give the more reasonable results. In the above case it is not surprising that the ORANI results for a tariff shock would be the more reasonable.

5. Conclusion and Prospect for Further Research

The ORANI model is an example of a large CGE model used routinely for policy analysis. It is not macroeconomically closed. For long run applications this lack of closure will be corrected by the addition of equations endogenizing (a) the ownership of the capital stock in a relatively distant 'snapshot' year, and (b) consumption between the introduction of the shock under analysis and the snapshot year. No attempt should be made to endogenize monetary phenomena in long-run closures of ORANI. Current work at IMPACT envisages the full-scale implementation of the scheme developed on a miniature version of ORANI by Dixon, Parmenter and Rimmer (forthcoming 1983). Analysis in Section 3 above indicates that commitment to a particular functional form and parameter estimates for the consumption function is not necessary to obtain useful policy results.

Monetary and other transient phenomena are involved in short-run economic analysis. Essentially two options are available for the short-run macro-closure of ORANI: (i) the extended Walrasian paradigm and (ii) the IMPACT paradigm. (A third option, currently on the back-burner because of the probably insurmountable data problems inherent in the estimation of its additional parameters, endows every ORANI micro-variable with its own adjustment path -- see FitzGerald (1979)). During the last four years work at IMPACT has concentrated on (ii). This work has resulted in an operational method for interfacing a small macrodynamic model in the Bergstrom-Wymer (1976) class with a large CGE model in the Johansen (1960) class (Cooper and McLaren (1980), (1982), (1983); Cooper (1983)). It is hoped that other researchers will extend the range of simulations made with the interfaced system; above we have reported only the (aggregate) results for a shock in government spending (used to calibrate the interface) and for a uniform tariff change. We hope to initiate work on the extended Walrasian paradigm during 1984 with a view to comparative analysis of its performance against that of the IMPACT paradigm.

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FOOTNOTES

1. In his later work, Walras himself took the initial steps in extending his framework to include a monetary asset. For a critique of his attempt, see Patinkin (1965), pp. 541-572. An important initial step in the extension of the Walrasian paradigm in applied work was the 'helicopter drop' assumption made by Clements (1980) who treated domestic credit expansion as an exogenous variable, but one entailing real monetary effects in the short run. Kehoe (1981) also adds government debt (but not money) to the Walrasian real system. Another group of authors (e.g., Piggott (1983) explore the real effects which an exogenous inflation produces via the failure, in varying degrees, of the different elements of the tax system to be fully and instantaneously indexed to the general price level.
2. For an inkling as to why modelling $(Y^B - Y^O)$ may be beyond current possibilities, see Powell (1981), pp. 229-232.
3. Short-run closures of ORANI with a neo-Keynesian flavour are also possible -- see, e.g., Wright and Cowan (1980) or Dixon, Powell and Parmenter (1979), pp. 36-40. In such closures a value of $\tau \ll 2$ years would be appropriate.
4. This approximation should not be confused with that involved in the use of Johansen's method to compute a solution.
5. One set of such estimates is provided by Bureau of Industry Economics (1981).
6. In this Section we draw freely on Cooper and McLaren (1980), (1982), (1983), and on Cooper (1983).
7. The table of elasticities - the derivatives of elements of Y_0 with respect to elements of Z_0 (the matrix C_0) - is given in Appendix 2. The numerical values of these elasticities correspond to the underlying numerical data base and parameter values of ORANI78 used in the tariff experiment reported in Dixon et al. (1982), Chapter 7. The split of variables into endogenous and exogenous sets corresponds to the standard short-run neoclassical mode with a slack labour market. The table of elasticities is therefore a subset of a standard basic solution of ORANI78. It may be noted that the numeraire of the model is the exchange rate and, with respect to this variable, the model is homogeneous of degree zero in reals and one in nominals.
8. MACRO endogenizes the number of persons employed, whereas ORANI endogenizes total labour hours. This led to the addition to the system of the equation

$$d \log L_h = d \log L + n (d \log y - d \log y^*) ,$$

where L_h is labour hours, L is persons employed, y is output and y^* is the output level corresponding to "standard hours". n was treated as an interface parameter; its estimated value, 0.45, was determined jointly with the seven other interface parameter estimates reported in Table 3. For further details, see Cooper (1983), pp. 8-9.

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APPENDIX 1

Interfacing the Models in the Non-Recursive Case

This appendix is self-contained from a notational point of view. Upper case letters here represent logs of variables: Y and Z indicate respectively endogenous and exogenous variables, and subscripts O and M identify ORANI and MACRO. D is the differential operator. A and B are matrices of coefficients appearing in the continuous-time versions of the structural forms of MACRO and ORANI+.

The two models may be written together as:

$$D \begin{pmatrix} Y_M \\ Y_O \end{pmatrix} = \begin{bmatrix} A_M & 0 \\ 0 & A_O \end{bmatrix} \begin{pmatrix} Y_M \\ Y_O \end{pmatrix} + \begin{bmatrix} B_M & 0 \\ 0 & B_O \end{bmatrix} \begin{pmatrix} Z_M \\ Z_O \end{pmatrix},$$

where the speed of adjustment parameters A_O and the matrix B_O must be chosen such that:

$$A_O^{-1} [e^{A_O t^*} - I] B_O = C_O$$

in order for ORANI+ to be consistent at t^* (the ORANI short run) with the original ORANI comparative static specification.

To allow interaction between the models we recognise that certain variables which are exogenous in one model may be endogenized by the other.

We define selection matrices R_M , R_O , Q_M , Q_O such that:

$$R_M Z_M = Q_O Y_O$$

and

$$R_O Z_O = Q_M Y_M .$$

R_M selects from MACRO's exogenous variables those which are endogenous to ORANI; Q_O selects from ORANI's endogenous variables those which are exogenous to MACRO, etc. These equations define the causal links from ORANI to MACRO and from MACRO to ORANI respectively.

Next define P_M , P_O such that:

$$R_M' R_M + P_M' P_M = I$$

and

$$R_O' R_O + P_O' P_O = I .$$

Thus P_M , P_O select from Z_M , Z_O those variables not selected by R_M , R_O . The interfaced system may now be written:

$$D \begin{pmatrix} Y_M \\ Y_O \end{pmatrix} = \begin{bmatrix} A_M & B_M R_O' Q_O \\ B_O R_M' Q_M & A_O \end{bmatrix} \begin{pmatrix} Y_M \\ Y_O \end{pmatrix} + \begin{bmatrix} B_M P_M' P_M & 0 \\ 0 & B_O P_O' P_O \end{bmatrix} \begin{pmatrix} Z_M \\ Z_O \end{pmatrix} ,$$

where

$$B_O = [e^{A_O t^*} - I]^{-1} A_O C_O .$$

This system has the form:

$$DY = AY + BZ$$

and hence has solution in proportional change form:

$$y(t) = C(t)z ,$$

where

$$C(t) = A^{-1} [e^{At} - I]B .$$

Certain variables are explained by both MACRO and ORANI+. These double endogeneities appear twice in the solution vector $y(t)$. Let S_M, S_O select these values as recorded in the MACRO and ORANI subsets respectively. The calibration of the interface parameters - the elements of A_0 together with t^* - may be based experimentally on the minimization of:

$$[S_M y(t) - S_O y(t)]' W [S_M y(t) - S_O y(t)] ,$$

where W is a weighting matrix. In the experiment described in the text, W was set equal to an identity matrix.

APPENDIX 2
The ORANI Elasticities Matrix C_0

Endogenous Variable	Exogenous Variable						
	E	W/P	C_R	I_R	$K(0)$	g_1	t_3
y_g	0.0	-0.4476	0.1092	0.1497	0.5675	0.1060	-0.0049
P	1.0	1.7285	2.0805	0.2299	-1.9715	0.1259	0.0722
L_h	0.0	-0.6968	0.1389	0.2167	0.4221	0.1555	-0.0082
i	0.0	0.6692	1.4576	0.4281	-0.7924	0.1832	-0.0611
P_{wl}	1.0	0.3515	0.2793	0.0156	-0.4885	0.0154	0.0152
x	0.0	-2.2016	-2.5060	-0.3792	2.8457	-0.1514	-0.1123
P_x	0.0	0.2402	0.2546	0.0317	-0.3073	0.0150	0.0115
$x P_x$	0.0	-1.9614	-2.2515	-0.3475	2.5384	-0.1364	-0.1009

APPENDIX 3
Details of Responses to the 10 Per cent Government Spending Shock
Around $t^* = 7.94$ Quarters for Doubly Endogenous Variables with
ORANI Adjustment Speeds Set at Parameter Values Given in Table 3

Variable*	Solution According to	Solution Values for Alternative t^*		
		$t^* = 7.93$	$t^* = 7.94$	$t^* = 7.95$
y_g	MACRO	1.07387	1.07283	1.07174
	ORANI	1.07276	1.07276	1.07276
P	MACRO	1.35629	1.35800	1.35968
	ORANI	1.35800	1.35800	1.35800
L_h	MACRO	1.55702	1.55683	1.55666
	ORANI	1.55680	1.55680	1.55680
i	MACRO	1.86113	1.85417	1.84735
	ORANI	1.85417	1.85417	1.85417
Sum of Squared Differences		.00005	.00000	.00005

* For notation, see Table 3.