Impact Project

Impact Research Centre, The University of Melbourne,
Baldwin Spencer Building (2nd floor) PARKVILLE, VIC. 3052 AUSTRALIA

Telephone: (03) 344 7417 (from overseas: 61 3 344 7417)
Telex: AA 35185 UNIMEL Telegrams: UNIMELB, PARKVILLE
E-mail: impact@impact.unimelb.EDU.AU
Fax: (03) 347 7539 (from overseas: 61 3 347 7539)

Supply Elasticities in the Presence of Adjustment Costs

by

Peter J. Wilcoxen

Impact Project Research Centre
University of Melbourne

Preliminary Working Paper No. IP-46, March 1990

ISSN 1031 9034 *ISBN 0 642 10049 7*

The Impact Project is a cooperative venture between the Australian Federal Government and the University of Melbourne, La Trobe University, and the Australian National University. By researching the structure of the Australian economy the Project is building a policy information system to assist others to carry out independent analysis. The Project is convened by the Industry Commission on behalf of the participating Commonwealth agencies (the Australian Bureau of Agricultural and Resource Economics, the Bureau of Immigration Research, the Bureau of Industry Economics, the Department of Employment, Education and Training, the Department of the Arts, Sport, the Environment, Tourism and Territories and the Industry Commission). The views expressed in this paper do not necessarily reflect those of the participating agencies, nor of the Commonwealth Government.

ABSTRACT

The adjustment-cost model of investment provides a rigorous basis for deriving a firm's price elasticity of output over various lengths of run. Moreover, parameters of the adjustment cost function itself play a prominent role in determining the size of the elasticity over the medium and long run. In this paper, we demonstrate how to derive supply elasticities from the optimization problem of a firm with a Cobb-Douglas production function (the CES case is treated in an appendix). We then compute elasticities for interesting values of the model's parameters, and argue that correct treatment of adjustment costs is essential to obtaining realistic behaviour from exporting sectors in general equilibrium models.

$$\eta_T = \eta_{SR} + \frac{pH^{1/\rho}}{1-\rho}\left[\frac{-\rho}{pH^{1/\rho}-w} + \frac{b}{w(H-b)}\right]\left[\frac{1-e^{-\delta T}}{\phi-1}\right] . \qquad \text{(A40)}$$

Using (A4) to eliminate H from (A40) produces:

$$\eta_T = \eta_{SR} + \sigma p\left(\frac{w}{bp}\right)^{\sigma}\left[\frac{-\rho}{p\left(\frac{w}{bp}\right)^{\sigma}-w} + \frac{b}{w\left(\left(\frac{w}{bp}\right)^{\sigma-1}-b\right)}\right]\left[\frac{1-e^{-\delta T}}{\phi-1}\right] . \qquad \text{(A41)}$$

Simplifying (A41) gives:

$$\eta_T = \eta_{SR} + \left[\frac{1}{1-b\left(\frac{bp}{w}\right)^{\sigma-1}}\right]\left[\frac{1-e^{-\delta T}}{\phi-1}\right] . \qquad \text{(A42)}$$

Thus, the elasticity of output with respect to a permanent change in p for a firm with a CES production function is:

$$\eta_T = \frac{b\sigma}{\left(\frac{w}{bp}\right)^{\sigma-1} - b} + \left[\frac{1}{1-b\left(\frac{bp}{w}\right)^{\sigma-1}}\right]\left[\frac{1-e^{-\delta T}}{\phi-1}\right] . \qquad \text{(A43)}$$

When σ is equal to one, (A43) reduces to the Cobb-Douglas case.[11]

11. It should be noted that (A43) is only valid over a particular domain of the wage/price ratio and the CES parameters b and σ. This domain is the same as that for the CES function in general, and is discussed in Praetz (1968). Thus, (A43) is valid whenever the underlying CES function is valid, but it is not valid for all possible sets of parameters.

$$\eta_T = \eta_{SR} + \left[\frac{1-e^{-\delta T}}{\phi-1}\right]\frac{\partial\Gamma}{\partial p}\cdot\frac{p}{\Gamma}\,. \tag{A35}$$

To obtain the final expression for the long run elasticity, we must differentiate (A20) with respect to p:

$$\frac{\partial\Gamma}{\partial p} = a^{1/\rho}(H-b)^{-1/\rho}\left[H^{1/\rho} + \left[\frac{p}{\rho}H^{\frac{1-\rho}{\rho}} - \frac{1}{\rho}\left[\frac{pH^{1/\rho}-w}{H-b}\right]\right]\frac{\partial H}{\partial p}\right]. \tag{A36}$$

Differentiating (A4) with respect to p produces:

$$\frac{\partial H}{\partial p} = \frac{-\rho H}{p\,(1-\rho)}\,. \tag{A37}$$

Inserting (A37) into (A36) and rearranging gives:

$$\frac{\partial\Gamma}{\partial p} = \frac{a^{1/\rho}(H-b)^{-1/\rho}H^{1/\rho}}{1-\rho}\left[-\rho + \frac{b}{w}\left[\frac{pH^{1/\rho}-w}{H-b}\right]\right]. \tag{A38}$$

Converting (A39) to an elasticity yields:

$$\frac{\partial\Gamma}{\partial p}\cdot\frac{p}{\Gamma} = \frac{pH^{1/\rho}}{1-\rho}\left[\frac{-\rho}{pH^{1/\rho}-w} + \frac{b}{w\,(H-b)}\right]. \tag{A39}$$

Inserting (A39) into (A35) gives:

Table of Contents

List of Tables

$$\frac{\partial \bar{q}(T)}{\partial K(T)} = \frac{\bar{q}(T)}{K(T)} . \qquad (A30)$$

Since the capital accumulation equation, (A28), is the same as it was in the Cobb-Douglas case, the change in $K(T)$ for a given permanent change in λ is given by equation (42), repeated below:

$$\frac{dK(T)}{d\lambda} = \frac{1}{\delta}\left[\frac{1}{\phi-1}\right]\left[\frac{1}{\phi p_k}\right]^{1/(\phi-1)} \lambda^{1/(\phi-1)-1}(1-e^{-\delta T}) . \qquad (A31)$$

Assuming, for convenience, that the firm was initially at the steady state, in the absence of the change in p the capital stock at time T would have been given by:

$$K(T) = \frac{I}{\delta} = \frac{1}{\delta}\left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} . \qquad (A32)$$

Substituting (A32) into (A31) allows the latter to be written:

$$\frac{dK(T)}{d\lambda} = \frac{K(T)}{\lambda}\left[\frac{1-e^{-\delta T}}{\phi-1}\right] . \qquad (A33)$$

Differentiating (A27) gives the change in λ for a permanent change in p:

$$\frac{\Delta\lambda}{dp} = \frac{1}{r+\delta}\cdot\frac{\partial\Gamma}{\partial p} . \qquad (A34)$$

Inserting (A30), (A33) and (A34) into (A29) and simplifying produces the following:

$$\dot{\lambda} - (r+\delta)\lambda = -\Gamma(p,w)\ , \tag{A25}$$

$$\dot{K} = \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} - \delta K\ . \tag{A26}$$

When p, w and r are expected to be constant in the future, (A25) can be integrated to obtain:

$$\lambda = \frac{\Gamma(p,w)}{r+\delta}\ . \tag{A27}$$

Similarly, equation (A26) can be integrated to give an expression for the capital stock at future points in time:

$$K(t) = K(0)e^{-\delta t} + \int_0^t \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} e^{-\delta(t-s)}ds\ . \tag{A28}$$

The elasticity of output at time T in response to a permanent change in p occurring at time zero is given by:

$$\eta_T = \frac{\partial \bar{q}(T)}{\partial p}\cdot\frac{p}{\bar{q}} + \frac{\partial \bar{q}(T)}{\partial K(T)}\cdot\frac{dK(T)}{\Delta\lambda}\cdot\frac{\Delta\lambda}{dp}\cdot\frac{p}{\bar{q}}\ . \tag{A29}$$

The first term on the right-hand side of (A29) is the short-run elasticity, η_{SR}. To obtain the second term, we will first construct each of its components. The derivative of $\bar{q}$ with respect to the capital stock can be obtained from (A9):

Supply Elasticities in the Presence of Adjustment Costs

by

Peter J. Wilcoxen[1]

1. Introduction

An industry's price elasticity of supply is a useful summary of how the sector responds to changes in the price of its output. In the short run, when capital stocks are fixed, it is straightforward to calculate the elasticity from the industry's optimization problem. Over longer periods of time, however, the capital stock in the industry can change. When it does, the elasticity of supply will increase, so in the medium or long run the elasticity may be considerably larger than its short-run value. Just how large the longer-run elasticities are depends on the speed with which capital can enter or leave the industry.

How fast an industry's capital stock changes depends on the nature of its investment decision. If it buys fully-functional capital goods on the open market for a price it takes to be exogenous, then its capital stock can change very quickly and its medium and long-run supply elasticities will be quite large. On the other hand, if it cannot change its capital stock at all, the longer-run elasticities will be identical to the short-run value. A convenient formalization of the investment problem that encompasses both of these cases and the entire range between them is the adjustment-cost model. In the adjustment-cost model a firm creates its own capital stock by buying raw capital goods and installing them in its production process. When the installation step involves diminishing returns, the firm will face convex costs of creating new capital

1. The author is grateful to Alan Powell for many helpful comments on an earlier draft of this paper.

goods. This, in turn, gives the firm a well-defined investment function which can be used to determine the rate at which the firm's capital stock changes.

In the remainder of this paper we will show how output supply elasticities for any desired length of run can be derived from the firm's optimization problem under the adjustment-cost model. Although our analysis will be fairly general, it is motivated by a particular practical problem: how should exporting industries be represented in general equilibrium models? In many GE models, industries are assumed to have constant returns to scale, and to use a single, malleable capital stock.[2] For exporting sectors, this can lead to unrealistically large changes in output for fairly small changes in world prices. Blampied, Horridge and Powell (1986) addressed this problem for mining industries in Australia by arguing that models which fail to take into account that ore deposits diminish in quality as mining proceeds will overstate the supply elasticities of extractive industries. In the present paper we will show that adjustment costs can reduce medium and long-run partial equilibrium supply elasticities, even for industries which do not use a resource whose supply is fixed or which degrades with use.

2. The Short Run

A firm's short-run supply elasticity can be derived fairly easily from its short-run optimization problem. Consider a price-taking firm choosing variable inputs to maximize its profits, given a fixed capital stock K and a production function $q(K,L)$. Profits will be given by:

2. Some examples are Hudson and Jorgenson (1974), Wilcoxen (1988), and McKibbin and Sachs (forthcoming).

text we know that the first-order conditions for intertemporal optimization involve the derivative of E with respect to the capital:

$$\frac{\partial E}{\partial K} = a^{1/\rho}(pH^{1/\rho} - w)(H-b)^{-1/\rho} . \qquad \text{(A19)}$$

For convenience, we can represent this by a new function, $\Gamma(p,w)$:

$$\Gamma(p,w) = a^{1/\rho}(pH^{1/\rho} - w)(H-b)^{-1/\rho} . \qquad \text{(A20)}$$

The firm's first-order conditions for an intertemporal optimum can be obtained by inserting the appropriate partial derivatives into equations (18) through (20). Using (A19), (A20), and the investment cost function given in equation (22), the first-order conditions can be shown to be:

$$\lambda = \phi p_k I^{\phi-1} , \qquad \text{(A21)}$$

$$\dot{\lambda} - (r+\delta)\lambda = -\Gamma(p,w) , \qquad \text{(A22)}$$

$$\dot{K} = I - \delta K . \qquad \text{(A23)}$$

As before, we can solve (A21) for investment as a function of the other variables:

$$I = \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} . \qquad \text{(A24)}$$

Substituting (A24) into (A22) and (A23) produces the model's equations of motion:

$$\eta_{SR} = \frac{b\sigma}{(\frac{w}{bp})^{\sigma-1} - b} , \tag{A15}$$

where σ is the elasticity of substitution between capital and labour, and is related to ρ as shown:

$$\sigma = \frac{1}{1-\rho} . \tag{A16}$$

Consulting equation (A5) shows that η_{SR} will be positive whenever the firm uses a positive amount of labour. In addition, notice that when σ is one, (A15) collapses to the Cobb-Douglas case given in equation (8), where $\alpha=1-b$.

To obtain the medium and long run supply elasticities, we follow the adjustment cost approach used in the text. The first step is to derive the firm's earnings function. Earnings are the difference between revenue and variable costs:

$$E = pq - wL . \tag{A17}$$

Inserting $\tilde{q}$ from (A9) and L from (A6) and rearranging slightly produces the earnings function below:

$$E = a^{1/\rho} K (pH^{1/\rho} - w)(H-b)^{-1/\rho} . \tag{A18}$$

Equation (A18) gives the maximum earnings that can be obtained on a given capital stock after the input of labour has been chosen optimally. From the analysis in the

$$\pi = pq(K,L) - wL , \tag{1}$$

where p is the price of the firm's output, w is the wage rate, and L is labour input. Maximizing (1) over choices of L subject to the constraint on K produces a first-order condition which can be written:

$$\frac{\partial q}{\partial L} = \frac{w}{p} . \tag{2}$$

Now suppose the firm has a Cobb-Douglas production function with constant returns to scale, so $q(K,L)$ has the form:[3]

$$q(K,L) = AK^{\alpha}L^{1-\alpha} , \tag{3}$$

where A and α are parameters. Differentiating (3) with respect to L produces:

$$\frac{\partial q}{\partial L} = (1-\alpha)AK^{\alpha}L^{-\alpha} . \tag{4}$$

Inserting (4) into (2) gives the explicit first-order condition for the firm:

$$(1-\alpha)AK^{\alpha}L^{-\alpha} = \frac{w}{p} . \tag{5}$$

Rearranging (5) gives an expression for the optimal labour input, given prices and the

3. The CES case is discussed in an Appendix.

capital stock:

$$L = K\left[\frac{(1-\alpha)pA}{w}\right]^{1/\alpha} . \tag{6}$$

Using (6) to eliminate L from (3) produces a new function, $\tilde{q}$, which gives the firm's optimal output as a function of prices, wages and its capital stock:

$$\tilde{q}(K,p,w) = AK\left[\frac{(1-\alpha)pA}{w}\right]^{(1-\alpha)/\alpha} . \tag{7}$$

Finally, the firm's short-run price elasticity of supply, η, can be obtained from (7) by logarithmic differentiation:

$$\eta = \frac{\partial \ln\tilde{q}}{\partial \ln p} = \left[\frac{1-\alpha}{\alpha}\right] . \tag{8}$$

Several inferences can be drawn from (8). First, as production becomes more capital intensive (α increases), the elasticity decreases. Conversely, when capital is relatively unimportant in production (α is small), the elasticity is very large. Finally, when the input cost shares are equal (α=0.5), the elasticity is one.

We can obtain the short-run supply elasticity from $\tilde{q}$ using the following formula:

$$\eta = \frac{d\tilde{q}}{dp}.\frac{p}{\tilde{q}} . \tag{A10}$$

Differentiating (A9) with respect to p produces:

$$\frac{d\tilde{q}}{dp} = \frac{a^{1/\rho}KH^{1/\rho}(H-b)^{-1/\rho}}{\rho}\left[\frac{1}{H} - \frac{1}{H-b}\right]\frac{dH}{dp} . \tag{A11}$$

Simplifying gives:

$$\frac{d\tilde{q}}{dp} = \frac{\tilde{q}}{\rho}\left[\frac{1}{H} - \frac{1}{H-b}\right]\frac{dH}{dp} . \tag{A12}$$

Differentiating (A4) with respect to p produces:

$$\frac{dH}{dp} = \frac{-\rho H}{p(1-\rho)} . \tag{A13}$$

Inserting (A13) into (A12) and simplifying yields:

$$\frac{d\tilde{q}}{dp} = \frac{b\tilde{q}}{p(1-\rho)(H-b)} . \tag{A14}$$

Finally, inserting (A14) in (A10) and rearranging gives the firm's short-run elasticity of output with respect to a change in p:

$$H(p, w) = \left[\frac{w}{bp} \right]^{\frac{\rho}{1-\rho}} . \tag{A4}$$

Using H, (A3) can be rearranged and written:

$$L^{\rho} = aK^{\rho}(H-b)^{-1} . \tag{A5}$$

Finally, raising each side of (A5) to the power $1/\rho$ gives an expression for optimal labour input:

$$L = a^{1/\rho}K(H-b)^{-1/\rho} . \tag{A6}$$

Inserting (A6) into (A1) and rearranging produces:

$$\tilde{q}^{\rho} = aK^{\rho} + baK^{\rho}(H-b)^{-1} , \tag{A7}$$

where we have used $\tilde{q}$ to denote the firm's optimal output for a given capital stock. Equation (A7) can be simplified to give:

$$\tilde{q}^{\rho} = aK^{\rho}H(H-b)^{-1} . \tag{A8}$$

Raising each side to the power $1/\rho$ gives:

$$\tilde{q} = a^{1/\rho}KH^{1/\rho}(H-b)^{-1/\rho} . \tag{A9}$$

3. Longer Periods of Time

Over longer periods of time the firm will be able to adjust its capital stock in response to a change in p. This, in turn, means that the firm's price elasticity of output will be higher than in the short run. Precisely how much higher depends on the length of run and the ease with which capital can enter or leave the industry. To formalize the notion of how easy it is for the industry to change its capital stock, we will now develop a model of investment subject to adjustment costs.

Consider a firm whose short and long-run optimizations are separable, so its short-run decision on variable inputs can be determined independently of its level of investment.[4] Let the function $E(K, p, w)$ represent the firm's short-run profits–which we will usually refer to as its "earnings"–on a given capital stock after choosing its variable inputs optimally. The firm in section (2), for example, would have an earnings function given by:

$$E(K, p, w) = \alpha K(pA)^{1/\alpha} \left[\frac{(1-\alpha)}{w} \right]^{(1-\alpha)/\alpha} . \tag{9}$$

Equation (9) was obtained by inserting (6) and (7) into (1). For the moment, however, we will not assume that $E(K, p, w)$ takes any particular form. Thus, a firm investing at rate I will have an instantaneous rate of profit given by:

$$\pi = E(K, p, w) - C(I) , \tag{10}$$

4. This form of separability holds when investment does not appear in the production function. When separability does not hold, the model becomes somewhat more complicated because the first-order conditions for variable inputs have to be handled simultaneously with those for investment. However, the interesting features of the solution do not change substantially, so we have chosen to assume separability holds.

where C is a function giving the cost of I units of new investment.

Now suppose the firm chooses investment to maximize the present value of its future stream of instantaneous profits.[5] For convenience, assume that the interest rate is expected to be constant at r for the foreseeable future.[6] Finally, suppose the firm's initial capital stock is K_0, and that its capital depreciates exponentially at rate δ. Under these assumptions, the investment problem can be written as:

$$\max_{I()} \int_0^\infty (E(K,p,w) - C(I)) e^{-rs} ds \ , \tag{11}$$

$$\text{subject to } \dot{K} = I - \delta K \ , \tag{12}$$

where $\dot{K}$ is the derivative of K with respect to time. For convenience, we will assume that I is non-negative at the solution, so the usual sign constraint can be ignored.

The problem expressed in (11) and (12) can be solved using the method of optimal control.[7] The first step is to form the Hamiltonian, H:

$$H = (E(K,p,w) - C(I)) e^{-rs} + \Lambda(I - \delta K) \ . \tag{13}$$

Taking first-order conditions produces the following three equations:

5. For a derivation of this assumption from basic principles of arbitrage, refer to Wilcoxen (1989).
6. When the interest rate can vary over time, the exposition of the model becomes more complicated without changing the results significantly. In particular, discount factors then involve integrals.
7. See Kamien and Schwartz (1981) for a detailed description of optimal control.

6. Appendix: The CES Case

This appendix derives the short and long run supply elasticities for a firm with a CES production function. The approach used is identical to that described in the text for the Cobb-Douglas case, except that the algebra involved is considerably less tidy. Because the algebra is so formidable, we have included below many intermediate steps that would ordinarily be left out.

We begin by assuming that the firm's production function has the form:

$$q = (aK^\rho + bL^\rho)^{1/\rho} \ , \tag{A1}$$

where q, K and L have their usual meanings, and a, b and ρ are parameters. From this, the first-order condition for optimal labour input can be shown to be:

$$bL^{\rho-1} \left[aK^\rho + bL^\rho \right]^{\frac{1-\rho}{\rho}} = \frac{w}{p} \ , \tag{A2}$$

where w is the wage rate and p is the price of the firm's output. Rearranging (A2) produces:

$$aK^\rho + bL^\rho = L^\rho \left[\frac{w}{bp} \right]^{\frac{\rho}{1-\rho}} \ . \tag{A3}$$

For convenience, we can define a new function, $H(p,w)$, as follows:

Wilcoxen, Peter J. (1989) "Intertemporal Optimization in General Equilibrium: A Practical Introduction", IMPACT Project *Preliminary Working Paper* No. IP-45, University of Melbourne.

$$\frac{\partial H}{\partial I} = -\frac{\partial C}{\partial I}e^{-rs} + \Lambda = 0\,, \tag{14}$$

$$\frac{\partial H}{\partial K} = \frac{\partial E}{\partial K}e^{-rs} - \delta\Lambda = -\dot{\Lambda}\,, \tag{15}$$

$$\frac{\partial H}{\partial \Lambda} = I - \delta K = \dot{K}\,, \tag{16}$$

where $\dot{\Lambda}$ is the derivative of Λ with respect to time. Expressions (14) and (15) can be simplified by defining a new variable, λ, as follows:

$$\Lambda = \lambda e^{-rs}\,. \tag{17}$$

Thus, Λ is the present value of the new variable λ. Using (17) to simplify (14) and (15), the first-order conditions can be shown to be:

$$\lambda = \frac{\partial C}{\partial I}\,, \tag{18}$$

$$\dot{\lambda} - (r+\delta)\lambda = -\frac{\partial E}{\partial K}\,, \tag{19}$$

$$\dot{K} = I - \delta K\,. \tag{20}$$

Equations (18) through (20) are very general and apply to any problem of the form set out in expressions (11) and (12). To obtain explicit first-order conditions for the particular problem at hand we must specify functional forms for C and E. Assuming as we did in the short-run problem that the production process is a constant-returns Cobb-Douglas function of labour and capital, equation (9) gives the firm's earnings function. Thus, differentiating (9) produces one of the necessary partial derivatives:

$$\frac{\partial E}{\partial K} = \alpha(pA)^{1/\alpha}\left[\frac{(1-\alpha)}{w}\right]^{(1-\alpha)/\alpha} . \qquad (21)$$

We also need to choose a functional form for investment costs, $C(I)$. For convenience, we will assume that $C(I)$ is given by the expression below:

$$C = P_k I^{\phi} , \qquad (22)$$

where P_k is the price of raw capital goods, I is the quantity of investment, and ϕ is a parameter. Usually we will assume that $\phi \geq 1$, so the cost of investment will be convex in I. Differentiating (22) produces:

$$\frac{\partial C}{\partial I} = \phi p_k I^{\phi-1} . \qquad (23)$$

Using (21) and (23), the first-order conditions in (18) to (20) can be written as shown:

$$\lambda = \phi p_k I^{\phi-1} , \qquad (24)$$

$$\dot{\lambda} - (r+\delta)\lambda = -\alpha(pA)^{1/\alpha}\left[\frac{1-\alpha}{w}\right]^{(1-\alpha)/\alpha} , \qquad (25)$$

$$\dot{K} = I - \delta K . \qquad (26)$$

Next, equation (24) can be solved for I as a function the other variables:

5. References

Blampied, Chris W., Mark Horridge and Alan A. Powell (1986) "The Behaviour of the Major Extractive Industries in Long-Run Closures of ORANI: A Proposal", IMPACT Project *Preliminary Working Paper* No. OP-55, University of Melbourne.

Hudson, Edward A. and Dale W. Jorgenson (1974) "U.S. Energy Policy and Economic Growth, 1975-2000", *Bell Journal of Economics and Management Science*, 5(2), pp. 461-514.

Kamien, Morton I. and Nancy L. Schwartz (1981) *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, Amsterdam: North-Holland.

McKibbin, Warwick J. and Jeffrey Sachs (forthcoming), *Macroeconomic Interdependence and Cooperation in the World Economy.*

Praetz, Peter D. (1968) "The Permissible Range of the CES Production Function", *Review of Economics and Statistics*, 50(2), pp. 287-288.

Summers, Lawrence H. (1981) "Taxation and Corporate Investment: A q-Theory Approach", *Brookings Papers on Economic Activity*, No. 1, pp. 67-127.

Wilcoxen, Peter J. (1988) "The Effects of Environmental Regulation and Energy Prices on U.S. Economic Performance", Ph.D. Dissertation, Harvard University.

4. Conclusion

The analysis above shows that the adjustment cost formulation provides a rigorous and intuitive framework for deriving a firm's price elasticity of supply over various lengths of run. Moreover, the approach is general enough to capture many appealing hypotheses as special cases. In addition, the results derived in the last section show that medium and long-run supply elasticities are very sensitive to adjustment costs. Thus, using the adjustment cost model to represent exporting industries in general equilibrium models could reduce the output elasticities for those sectors to realistic levels. In fact, since the elasticity is sensitive to even small departures of the adjustment cost parameter ϕ from unity, the usual assumption that there are no adjustment costs at all is very strong.

$$I = \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} . \qquad (27)$$

Finally, using (27) to eliminate I from (26) produces the model's equations of motion:

$$\dot{\lambda} - (r+\delta)\lambda = -\,\alpha(pA)^{1/\alpha}\left[\frac{1-\alpha}{w}\right]^{(1-\alpha)/\alpha} , \qquad (28)$$

$$\dot{K} = \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} - \delta K . \qquad (29)$$

To derive the supply elasticity, the next task is to integrate (28) and (29). Multiplying both sides of (28) by $e^{-(r+\delta)s}$ and integrating over the interval $[t,\infty)$ gives:

$$\lim_{s\to\infty} \lambda(s)e^{-(r+\delta)s} - \lambda(t)e^{-(r+\delta)t} = -\int_t^\infty \alpha(pA)^{1/\alpha}\left[\frac{1-\alpha}{w}\right]^{(1-\alpha)/\alpha} e^{-(r+\delta)s}\,ds . \qquad (30)$$

We now assume that as $s\to\infty$, $\lambda(s)$ grows more slowly than $r+\delta$, so the following transversality condition holds:[8]

$$\lim_{s\to\infty} \lambda(s)e^{-(r+\delta)s} = 0 . \qquad (31)$$

Using (31), equation (30) can be simplified to give:

8. Refer to Wilcoxen (1989) for a detailed interpretation of this condition.

$$\lambda(t) = \int_t^{\infty} \alpha(pA)^{1/\alpha} \left[\frac{1-\alpha}{w}\right]^{(1-\alpha)/\alpha} e^{-(r+\delta)(s-t)} ds \ . \qquad (32)$$

Finally, when the future values of w, p and r are expected to be constant forever, (32) can be integrated easily, giving the following expression for λ:

$$\lambda = \frac{\alpha(pA)^{1/\alpha}}{r+\delta} \left[\frac{1-\alpha}{w}\right]^{(1-\alpha)/\alpha} . \qquad (33)$$

A similar approach can be applied to solve (29) for the path of the capital stock. First, rearrange (29) as shown:

$$\dot{K} + \delta K = \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} . \qquad (34)$$

Next, multiply both sides by $e^{\delta s}$ and integrate over the interval $[0, t]$:

$$K(t)e^{\delta t} - K(0) = \int_0^t \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} e^{\delta s} ds \ . \qquad (35)$$

Finally, rearrange (35) to obtain the following:

$$K(t) = K(0)e^{-\delta t} + \int_0^t \left[\frac{\lambda}{\phi p_k}\right]^{1/(\phi-1)} e^{-\delta(t-s)} ds \ . \qquad (36)$$

Table 1: Ten-year Supply Elasticities for Various Parameter Values[a]

Capital Share (α)	η_0	Adjustment Cost Parameter (ϕ)						
		1.01	1.10	1.25	1.50	2.00	3.00	5.00
0.10	9.00	641.12	72.21	34.28	21.64	15.32	12.16	10.58
0.20	4.00	320.06	35.61	16.64	10.32	7.16	5.58	4.79
0.30	2.33	213.04	23.40	10.76	6.55	4.44	3.39	2.86
0.40	1.50	159.53	17.30	7.82	4.66	3.08	2.29	1.90
0.50	1.00	127.42	13.64	6.06	3.53	2.26	1.63	1.32
0.60	0.67	106.02	11.20	4.88	2.77	1.72	1.19	0.93
0.70	0.43	90.73	9.46	4.04	2.23	1.33	0.88	0.65
0.80	0.25	79.27	8.15	3.41	1.83	1.04	0.65	0.45
0.90	0.11	70.35	7.13	2.92	1.52	0.81	0.46	0.29
1.00	0.00	63.21	6.32	2.53	1.26	0.63	0.32	0.16

[a] Shown are values of the ten-year supply elasticity, η_{10}. The column labelled η_0 gives the short-run (zero-year) elasticity for comparison. In all cases, the depreciation rate is 10 per cent.

Finally, η_T also depends on α in an intuitive way. As α goes toward zero and capital becomes unimportant, η_T goes to infinity. In contrast, as production becomes very capital intensive ($\alpha \to 1$), changes in output come entirely from capital accumulation so η_T becomes dominated by the length of run and the magnitude of adjustment costs, $\phi-1$:

$$\lim_{\alpha \to 1} \eta_T = \left[\frac{1}{\phi-1}\right](1-e^{-\delta T}) . \qquad (46)$$

Equation (44) can also be evaluated for specific numerical values of the parameters. Table 1 gives the value of η_T when T is ten years and the depreciation rate, δ, is ten per cent. For reference, the short run elasticity is shown in the column labeled η_0. To interpret the numbers in the table, remember that in the absence of adjustment costs, ϕ=1 and η_T is infinity. Thus, even a small degree of adjustment costs, such as ϕ=1.01, reduces the elasticity considerably, especially for firms that are capital intensive. As ϕ rises higher, η_T quickly becomes quite small, especially considering that the shock is permanent and ten years are allowed for adjustment. The precise value of ϕ is an empirical question, but the assumption that ϕ=2 is not unusual in the adjustment cost literature.[10] At that point, the elasticity has dropped from infinity down to unity.

10. See Summers (1981), for example.

It is now possible to compute the price elasticity of supply at any point in time. In particular, we would like to know how a permanent change in p occurring at time 0 affects $\tilde{q}$ at time T. That is, we are interested in the following elasticity:

$$\eta_T = \frac{d\tilde{q}(T)}{dp} \cdot \frac{p}{\tilde{q}(T)} , \qquad (37)$$

where the subscript T is used to indicate that the elasticity measures the response of $\tilde{q}$ at time T. Since the change in p is permanent, we have not subscripted dp by time. Referring back to (7) shows that the change in $\tilde{q}(T)$ for an infinitesimal change in p is given by:

$$\frac{d\tilde{q}(T)}{dp} = \frac{\partial \tilde{q}(T)}{\partial p} + \frac{\partial \tilde{q}(T)}{\partial K(T)} \cdot \frac{dK(T)}{dp} . \qquad (38)$$

The first term on the right is easy to evaluate: it can be obtained by differentiating equation (7). The second term is more formidable. The change in $\tilde{q}$ caused by a change in $K(T)$ is straightforward, but the change in $K(T)$ produced by a change in p will depend on the entire history of investment between the time the shock occurs and T. That is, if the shock occurs at 0, $K(T)$ will depend on the path of investment over the interval $[0,T]$. From (27), however, we know that investment depends on λ, and from (33) that λ depends on p. Thus, using the chain rule allows the rightmost term in (38) to be rewritten as:

$$\frac{\partial \tilde{q}(T)}{\partial K(T)} \cdot \frac{dK(T)}{dp} = \frac{\partial \tilde{q}(T)}{\partial K(T)} \cdot \frac{dK(T)}{\Delta\lambda} \cdot \frac{\Delta\lambda}{dp} , \qquad (39)$$

where $\Delta\lambda$ represents the change (or variation) in the path of λ over the interval $[0,T]$. The term $dK(T)/\Delta\lambda$, therefore, gives the change in the capital stock at T produced by the entire history of changes in λ over $[0,T]$. The final term, $\Delta\lambda/dp$, is the change in the path of λ produced by a permanent (and constant) change in p.

Fortunately, evaluating (39) is straightforward when the exogenous variables are expected to be constant in the future. The first term can be obtained by differentiating equation (7):

$$\frac{\partial \bar{q}}{\partial K(T)} = A\left[\frac{(1-\alpha)pA}{w}\right]^{(1-\alpha)/\alpha} . \qquad (40)$$

Next, we observe that the change in p occurs at time 0 and is permanent, so evaluating $\Delta\lambda/dp$ can be accomplished by differentiating (33) with respect to p, giving:

$$\frac{\Delta\lambda}{dp} = \frac{\lambda}{\alpha p} . \qquad (41)$$

From (41) it is clear that the change in λ is constant over time, so the remaining term, $dK(T)/\Delta\lambda$, can be obtained by differentiating (36) with respect to λ:

$$\frac{dK(T)}{\Delta\lambda} = \frac{dK(T)}{d\lambda} = \int_0^t \left[\frac{1}{\phi-1}\right]\left[\frac{1}{\phi p_k}\right]^{1/(\phi-1)} \lambda^{1/(\phi-1)-1} e^{-\delta(T-s)} ds . \qquad (42)$$

Evaluating (42) when all variables are constant gives:

$$\frac{dK(T)}{d\lambda} = \frac{1}{\delta}\left[\frac{1}{\phi-1}\right]\left[\frac{1}{\phi p_k}\right]^{1/(\phi-1)} \lambda^{1/(\phi-1)-1}(1-e^{-\delta T}) . \qquad (43)$$

Finally, by inserting all of the relevant derivatives into (37), and assuming for convenience that the firm is initially at a steady state,[9] the long-run elasticity of output can be shown to be:

$$\eta_T = \left[\frac{1-\alpha}{\alpha}\right] + \frac{1}{\alpha}\left[\frac{1}{\phi-1}\right](1-e^{-\delta T}) . \qquad (44)$$

Equation (44) has a number of interesting and intuitive properties. First, as investment costs become linear ($\phi \rightarrow 1$), the elasticity becomes infinite. This is exactly what we would expect: in a constant returns world with no costs of adjustment, a small change in p elicits an enormous change in output. In contrast, as $\phi \rightarrow \infty$, η_T approaches its short-run value. Again, this makes intuitive sense: if it is very costly to change the capital stock, K will remain constant and changes in output will only come from changes in the use of variable factors.

Equation (44) also illustrates the importance of the length of run in computing the elasticity. For example, as $T \rightarrow 0$, η_T gradually drops to its short-run value. In contrast, as T goes to infinity, the elasticity approaches a constant:

$$\lim_{T\rightarrow\infty} \eta_T = \frac{1}{\alpha}\left[\frac{\phi}{\phi-1} - \alpha\right] . \qquad (45)$$

9. In particular, we assume that before the shock the following relationship holds: $I_0=\delta K_0$. Eliminating this assumption makes the elasticity depend on the initial capital stock, but does not change the character of the results.