IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

ESTIMATION OF THE CONSUMPTION FUNCTION : A SYSTEMS APPROACH TO EMPLOYMENT EFFECTS ON THE PURCHASE OF DURABLES

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ABSTRACT

In this paper we analyse a systems approach to employment effects by formulating CONDELES, the aggregate consumption function associated with the Extended Linear Expenditure System incorporating Durable Goods, and emphasize the effects of employment-related variables. The complete system, with disposable income, prices, the real interest rate and lagged stocks as explanatory variables, in which five commodity aggregates are distinguished, is fitted to quarterly Australian data using Full Information Maximum Likelihood methods. Highly stable and significant estimates of subsistence parameters, marginal budget shares and the marginal propensity to consume are obtained. Unfortunately, however, implausibly low marginal budget shares were found for durables and high positive serial correlation in the residuals was widespread. On the credit side, the pure time rate of discount was estimated to have a plausible value (about 8 per cent per annum). The estimates also suggest strongly that, over the sample period, Australian consumers acted as if they believed that the rate of growth of their disposable permanent money incomes would be only about 80 per cent of the inflation rate. The estimation algorithm appears to provide "unique" estimates.

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1. INTRODUCTION

1.1 The Role of this Paper in the IMPACT Research Program

The IMPACT framework[1] as initially conceived proposes to model the Australian economy in some detail by means of three interacting modules, each having a specialized role. The modules are MACRO,[2] a minimal macroeconomic model based on the RBA76 model of the Reserve Bank; ORANI, a large general equilibrium model of Australian industry structure and international trade; and BACHUROO, an economic-demographic model designed to endogenize the structure of the workforce and the supplies of various types of labour. In the initial specification, besides taking care of national income accounting, the MACRO module was assigned the tasks of

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* Without implicating them in remaining errors, the authors would like to thank Ken Clements and Gordon Fisher for their comments on an earlier draft. Help with data was kindly provided by Robert Walters.

1. See Powell and Lawson [30]. Figures in square brackets relate to the references listed in the bibliography.

2. See Bacon and Johnston [1].

(i) determining the aggregate levels of consumption and private investment (and in so doing, transmitting business cycle impulses into the detailed disaggregated demands for, and supplies of, products and intermediate products, as modelled in ORANI) ;

(ii) encapsulating the effects of monetary and fiscal variables for the entire system ; and

(iii) modelling all disequilibrium behaviour other than in the labour market, which was to be handled at the interface of the ORANI and BACHUROO modules.

The above modular structure has many advantages, not least of which is in the realm of management. Moreover, initial indications are that, as a basis for a model with medium term focus, the existing modular boundaries are likely to be satisfactory. One very ambitious ultimate goal of the IMPACT research program, however, is to obtain a model which will track the economy 'adequately' according to the usual criteria[1] applied in simulation experiments with much smaller (viz., macro-) models. It seems that for this purpose the presentation by the MACRO module of a scalar aggregate consumption function to the ORANI module for sectoral decomposition along neo-classical lines, is too simple. We expect macro and monetary variables to have differential impacts on durable and consumable goods. To model such effects it will probably be ultimately necessary to redraft the modular boundaries between MACRO and ORANI by allowing more complicated signals to pass across the interface. As a

1. See Sowey [34].

first step in this direction, in the present paper we explore the disaggregation of the consumption function into five commodities, including two categories of durables.[1] Employment-related variables are explicitly incorporated into the equations describing the purchase of new durable goods. The five-element consumption vector, whose behavioural basis is derived in this paper, after empirical assessment will be considered as a possible replacement for the existing scalar consumption function in MACRO. Complementary developments in ORANI to enable it to receive this vector signal will be explored in later work.

1.2 General Approach

It is proposed to use the demand systems specification of Dixon and Lluch [5] to incorporate two durable items, Motor Vehicles and Household Durables, into a system where their parameter values are simultaneously determined with those of the three consumable items, Food, Rent and Other Non-Durables. Industries Assistance Commission (IAC) work on automobiles by Filmer, Talbot and others[2] has led to the construction of unique data series for this category, covering an index of the real stock of cars (along the lines of "new 19XY Holden equivalents"), vehicle and service survival functions (covering depreciation, life expectancy and obsolescence), and much more.[3]

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1. The number of disaggregated commodities need not, of course, be restricted to five.

2. See, for example, Filmer and Talbot [6], page 57.

3. See IAC mimeograph [12].

In the case of Household Durables, no sufficiently accurate data on stocks is available (with the exception of some sketchy data on a few components such as refrigerators and colour televisions). Fortunately the existence of stock data is not a precondition for the use of the Dixon/Lluch model,[1] a feature shared by the "dynamic demand equation" of Houthakker and Taylor [11] and many other models of stock accumulation.[2] It is our intention, therefore, to make good use of the existing IAC work on automobiles by specifying the demand equation for new vehicle purchases in terms of adjustment from the observed incoming stock level to a new desired level, whilst simultaneously eliminating stocks from the final form of the demand equation for Household Durables. The running expenditures of Motor Vehicles can be estimated from the IAC model, in the light of the size of the stock of vehicles and the relevant prices of goods and services making up current outlays on maintenance, fuel, registration and so forth.

Although the derivation of the Dixon/Lluch model has been discussed in [5], with a constant interest rate, it is introduced in the following section since we wish to allow for variable interest rates and also as it will be useful for future reference.[3]

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1. The flexibility of the model provides an option of specifying the demand equation for new purchases of durables by either adjusting the incoming stock instantaneously to the desired level or by eliminating stocks altogether from the final form of the demand equation. In implementing the model for purchases of new Motor Vehicles, both options were considered.

2. See, for example, Stone and Rowe [36].

3. Since the market interest rate is a variable in this paper, we note that a central theme of the Dixon/Lluch model, the "shift of plan" term, is also a variable and not a constant as presented in [5]. As a result, the significance or otherwise of the shift term parameter cannot be empirically tested in our formulation.

2. THE EXTENDED LINEAR EXPENDITURE SYSTEM WITH DURABLE GOODS

The consumer's planning problem under certainty is to maximize the present value of a utility functional (1) subject to a life-time budget constraint (2) and a stock-flow identity (3), which assumes that durables decay exponentially. That is, choose $C_{it}(\tau)$ and $R_{jt}(\tau)$ (all i, j and $t \leqslant \tau < \infty$) such that

$$\int_t^{\infty} e^{-\delta(\tau-t)}[\sum_i \beta_i \, \ell n(C_{it}(\tau) - \gamma_i) + \sum_j \eta_j \, \ell n(K_{jt}(\tau) - \Theta_j)]d\tau \, , \qquad (1)$$

is maximized, subject to

$$Z_t/\rho_t = \int_t^{\infty} e^{-\rho_t(\tau-t)} [\sum_i P_{it}C_{it}(\tau) + \sum_j \pi_{jt}R_{jt}(\tau)]d\tau \, , \qquad (2)$$

and

$$\dot{K}_{jt}(\tau) = R_{jt}(\tau) - \sigma_j K_{jt}(\tau) \, , \quad \text{(all j)} \, , \qquad (3)$$

given $(\rho_t, \delta, Z_t, \{P_{it}\}, \{\pi_{jt}\})$. The following notation is used :

the γ_i, β_i, Θ_j and η_j (all i, j) are positive parameters of the instantaneous Klein-Rubin[1] utility function with $\sum_i \beta_i + \sum_j \eta_j = 1$;

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1. See Klein and Rubin [13]. Although γ_i and Θ_j are usually <u>assumed</u> to be positive, it is only necessary that $\gamma_i < C_{it}(\tau)$ and $\Theta_j < K_{jt}(\tau)$, all i, j, t and τ .

δ is the consumer's subjective rate of time preference; ρ_t is the market interest rate the consumer at time t expects to hold throughout the planning period;[1] P_{it} and π_{jt} are the stationary prices the consumer expects consumable i and durable j, at time t, to be sold throughout the planning period;[1] $C_{it}(\tau)$ and $R_{jt}(\tau)$ are the time paths of planned purchases of consumable i and durable j; $K_{jt}(\tau)$ is the planned holding of stock of durable j; σ_j is the decay rate for durable j; Z_t/ρ_t is the present value of the consumer's expected life-time income, at time t, from personal financial assets and sales of labour; τ is planning time.

It should be noted that :

(i) The utility function is assumed to be intertemporally separable[2] ;

(ii) The assumption of a constant subjective discount rate, δ, implies that the consumer's planning behaviour is Strotz-consistent[3] ;

(iii) The consumer expects to be able to borrow and lend at the market interest rate ρ_t ;

(iv) We could attach t subscript and τ argument to γ_i, β_i, Θ_j, η_j, δ and σ_j, but for simplicity these parameters are treated as time invariant.

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1. Since there is no inflation, no distinction is made between nominal and real interest rates. It is a simple matter to formulate prices as functions of planning time τ by making explicit assumptions concerning growth laws governing these time paths. This is considered in section 7 below.

2. See Parks [23] and Motley [20].

3. See Strotz [37], Pollak [26], Blackorby et al. [2] and Hammond ([9] and [10]).

The solution techniques for the paths $C_{it}(\tau)$ and $R_{jt}(\tau)$ to be optimal are well known. Using Euler/Lagrange maximization methods, the planned stock holding of durable j at time t is given by [1]

$$\pi_{jt}K_{jt} = \pi_{jt}\Theta_j + \mu_t\eta^0_{jt} Z^*_t \quad , \tag{4}$$

where

$$\mu_t = \delta/\rho_t \quad , \tag{4.1}$$

$$\eta^0_{jt} = \frac{\eta_j}{(\sigma_j+\rho_t)(1-\delta\alpha_t)} \quad , \tag{4.2}$$

$$\alpha_t = \sum_j \frac{\eta_j}{(\sigma_j+\rho_t)} \quad , \tag{4.3}$$

and

$$Z^*_t = Z_t - \sum_i P_{it}\gamma_i - \sum_j \sigma_j\Theta_j\pi_{jt} \quad . \tag{4.4}$$

If we wish to derive the actual as distinct from the planned, expenditure equations, we need to introduce the notion of continuous replanning so that the expenditure equations may be thought of as the first-period solutions of continually revised optimal plans. It is assumed that, at each point of time t, the consumer adjusts his stocks to their currently optimal levels and that his expectations about prices, income and interest rate are changing smoothly (i.e., P_{it} , π_{jt} , Z_t

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1. If we assume that the present value of expected changes in labour income is zero, permanent income, as defined by Z_t , will be equal to measured income.

and ρ_t are continuous and differentiable functions of real time t).
Then the actual expenditure equations for consumable i and durable j at t are given by[1]

$$p_{it}C_{it} = p_{it}\gamma_i + \mu_t\beta^*_{it} Z^*_t \; , \tag{5}$$

where

$$\beta^*_{it} = \frac{\beta_i}{(1-\delta\alpha_t)} \; , \tag{5.1}$$

and

$$\begin{aligned}\pi_{jt}R_{jt} = {} & \sigma_j\Theta_j\pi_{jt} + \mu_t\eta^*_{jt} Z^*_t \\ & - \mu_t\eta^0_{jt}\left[\frac{\dot{\pi}_{jt}}{\pi_{jt}} + \frac{\dot{\rho}_t}{\rho_t} + \frac{\dot{\rho}_t(1-\delta\alpha_t+(\sigma_j+\rho_t)\delta\phi_t)}{(\sigma_j+\rho_t)(1-\delta\alpha_t)}\right. \\ & \left. + \frac{\sum_i \gamma_i\dot{p}_{it}}{Z^*_t} + \frac{\sum_j \sigma_j\Theta_j\dot{\pi}_{jt}}{Z^*_t} - \frac{\dot{Z}_t}{Z^*_t}\right] Z^*_t \; ,\end{aligned} \tag{6}$$

where

$$\eta^*_{jt} = \eta^0_{jt}(\sigma_j + \rho_t - \delta) \; , \tag{6.1}$$

and

$$\phi_t = \sum_j \frac{\eta_j}{(\sigma_j+\rho_t)^2} \; . \tag{6.2}$$

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1. A critical step in the derivation of equation (6) is the partitioning of the change in the consumer's holdings of durables into two parts - the movement along the plan and the movement of the plan. A diagrammatic account of the partitioning is given in Appendix A.

When the durability of durable goods is implicitly excluded from the analysis, the utility function parameters η_j and θ_j and the stock-flow identity (3) disappear from the planning problem and there is no distinction made between consumables and durables. This results in the following actual expenditure equation for the i^{th} good at time t :

$$p_{it}C_{it} = p_{it}\gamma_i + \mu_t\beta_i(Z_t - \sum_i p_{it}\gamma_i) , \tag{7}$$

with $\sum_i \beta_i = 1$ and $\gamma_i < C_{it}$ for all i and t . Equation (7) is Lluch's extended linear expenditure system[1] (ELES), which is a special case of Dixon and Lluch's ELES with Durable Goods (DELES) when all goods are treated as consumables.

The above equations have been derived on the assumption that variables are in continuous time. For empirical purposes, it will be necessary to adopt a discrete approximation for an observed interval t . One method of discretizing the above continuous time formulation is to use the operators A, B and C where, for an operand x involving a single time-subscripted variable,[2,3]

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1. See Lluch [16].

2. This approach is similar to that adopted by the "Rotterdam School" of demand theorists. See Theil [38].

3. Where the operand x contains several time-subscripted variables, say $x_t = x(a_t , b_t , c_t , \dots)$, the above operators are to be applied term by term to the time-subscripted arguments of x . Thus, for example, Ax_t is to be interpreted as $x(Aa_t , Ab_t , Ac_t , \dots)$. It follows that A, B and C are linear operators only in the case of simple operands $x_t = x(a_t)$.

$$Ax_t = (x_t - x_{t-1})/([\tfrac{1}{2}(x_t + x_{t-1})] \text{ replaces } \frac{\dot{x}_t}{x_t} \; ; \qquad (8.1)$$

$$Bx_t = (x_t - x_{t-1}) \text{ replaces } \dot{x}_t \; ; \qquad (8.2)$$

$$Cx_t = \tfrac{1}{2}(x_t + x_{t-1}) \doteqdot x_{t-\frac{1}{2}} \text{ replaces } x_t \; . \qquad (8.3)$$

If x_t is the value of x during period t (say the mid-point), the first-differences involving the operator A are finite time analogues of differentials of logarithms; the difference operator B gives the change in x from the mid-point of period (t-1) to the mid-point of period t ; the operator C gives the mean value of x at the mid-point of period t and period (t-1) , and effectively measures x at the beginning of period t . While it is the use of these operators that transforms the variables from the value <u>at</u> t to the value <u>during</u> t , the operators will not be used in the definition of variables for purposes of simplicity and will be reserved solely for behavioural equations.

The system given by (5) and (6) is a set of seemingly unrelated regression equations[1] and an updated version of Wymer's RESIMUL package[2] will be used to estimate the system by Full Information Maximum Likelihood (FIML).

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1. See Zellner [48].

2. See Wymer [46]. There are several versions of the RESIMUL program and the one actually used, MACRES , is a "stretched" version necessary to handle the large number of functions of parameters and variables embodied in equations (5) and (6). John Sutton of the IAC ably modified the RESIMUL program to obtain MACRES.

3. EMPLOYMENT EFFECTS - DEMAND RELATIONS OF THE UNEMPLOYED

Let N_t be the population of Australia during period t (or at the mid-point of t[1]). Partition N_t into two components, M_t corresponding to the members of households whose heads are employed during t , and L_t corresponding to the members of households whose heads are not employed during t .[2,3] It is assumed that unemployment is exogenous to this model.

Partition C_{it} , the real consumption of the ith consumable of the economy during period t , into two components, C_{it}^L and C_{it}^M , the real consumption of the ith consumable by the unemployed and employed respectively, during t . We assume that the unemployed consume the three consumable items Food, Rent and Other Non-Durables (for which i = 1, 2 and 3 respectively). As the unemployed are assumed to be unable

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1. We could just as easily specify N_t to be the average population for period t .

2. The fiction is that a household head is either unemployed in all of period t , or none of t . As a first step, we could use L as the number of idle man-year equivalents occurring during t through unemployment.

3. In what follows, we use the phrase "a member of L (M) during t " interchangeably with "a member of a household whose head is unemployed (employed) during t ".

to purchase new durables,[1] then the "representative unemployed consumer" is supposed to have a Klein-Rubin utility function defined on the above three commodities which leads to an ELES system of expenditure equations similar to equation (7):

$$C(P_{it}c_{it}^{L}) = CP_{it}\gamma_{iL} + \beta_{iL}\, C(\mu_{tL}Z_{tL}^{*}) , \qquad (9)$$

with $\sum_{i=1}^{3} \beta_{iL} = 1$,

in which

$$c_{it}^{L} \equiv C_{it}^{L}/L_t \qquad (10)$$

= the per capita consumption of the i^{th} consumable by a member of L during t[2] ;

P_{it} = the "price" of the i^{th} consumable during t (viz., implicit deflator);

$$Z_{tL}^{*} = Z_{tL} - \sum_{i=1}^{3} P_{it}\,\gamma_{iL} \qquad (10.1)$$

= supernumerary, or discretionary, permanent per capita income of a member of L during t[3] ;

1. Unemployment probably modifies the consumption behaviour of households, at least in the case of purchases of major durable goods such as new motor vehicles. It therefore seems interesting to take account of the linkages between employment status and the demand for durable goods.
2. As the analysis presented here is in terms of the "representative consumer", either a member of L or M , the operator "C" is used to descretize the per capita consumption of good i during t . This means, in effect, that total consumption and total population during t are not treated as separate arguments of per capita consumption during t .
3. See the previous footnote concerning per capita measures and the "representative consumer".

Z_{tL} = permanent per capita income of those belonging to households whose head is unemployed during t, net of the per capita cost of running that fraction of the existing stock of motor vehicles owned by households whose head is unemployed;

μ_{tL} = δ_L/ρ_{tL}

= marginal propensity to consume out of permanent income during t by a member of L ;

δ_L = subjective time preference discount rate for a member of L ;

ρ_{tL} = market interest rate at which the unemployed consumer is supposed to be able to lend or borrow during t ;

β_{iL} = marginal budget share of the ith consumable in the budget of a member of L ;

γ_{iL} = per capita "basic needs" requirement of consumable i for the unemployed consumer;

and C is the operator defined by (8.3).

4. DEMAND FOR CONSUMER DURABLES BY THE EMPLOYED

The utility index for the "representative member of a household whose head is employed" is supposed to be a Klein-Rubin function defined on five aggregate commodities, namely, the three consumables as for the unemployed consumer, plus two additional items, the flows of services derived from the stocks of Motor Vehicles and Household Durables. The services flowing from durables are assumed to be proportional to the stocks held which, consequently, should be measured in units which recognize both the quality and age aspect of the vintage composition of the stocks held. For instance, the appropriate unit for the stock of Motor Vehicles might be "brand-new 1962 standard Holden equivalents."

Let K_{1t} be the total stock of Motor Vehicles held in the economy during period t. The total stocks of Motor Vehicles held by the employed and unemployed consumer, respectively, during t are denoted

$$K_{1t}^{M} = \psi_{1tM}K_{1t} \qquad (11.1\ a)$$

and

$$K_{1t}^{L} = \psi_{1tL}K_{1t} \ , \qquad (11.1\ b)$$

where the factors of proportionality for the employed and unemployed during t, ψ_{1tM} and ψ_{1tL}, satisfy

$$\psi_{1tM} + \psi_{1tL} = 1 \quad \text{(all t)} \ . \qquad (11.1\ c)$$

Naturally,

$$k_{1t}^{M} \equiv K_{1t}^{M}/M_t \tag{12.1 a}$$

and

$$k_{1t}^{L} \equiv K_{1t}^{L}/L_t \quad , \tag{12.1 b}$$

where k_{1t}^{M} (k_{1t}^{L}) is the per capita stock of Motor Vehicles held during t by the representative employed (unemployed) consumer.

We can also let K_{2t} be the total stock of Household Durables held in the economy during period t . By analogy with the above, we have

$$K_{2t}^{M} = \psi_{2tM}K_{2t} \tag{11.2 a}$$

and

$$K_{2t}^{L} = \psi_{2tL}K_{2t} \tag{11.2 b}$$

where

$$\psi_{2tM} + \psi_{2tL} = 1 \quad \text{(all t)} \quad . \tag{11.2 c}$$

Furthermore, the per capita stocks of Household Durables held during t by the representative employed and unemployed consumer are given respectively by

$$k_{2t}^{M} \equiv K_{2t}^{M}/M_t \tag{12.2 a}$$

$$k_{2t}^{L} \equiv K_{2t}^{L}/L_t \quad . \tag{12.2 b}$$

It is assumed that $\psi_{jt\ell}$ $(j = 1, 2 \; ; \; \ell = M, N)$ can be estimated within the model or obtained extraneously.

4.1 Planned Stock of Motor Vehicles

The stocks held during t by the representative employed consumer will be denoted

$$k_{1t}^{M} \equiv K_{1t}^{M}/M_t \ , \qquad (12.1)$$

$$k_{2t}^{M} \equiv K_{2t}^{M}/M_t \ , \qquad (12.2)$$

for Motor Vehicles and Household Durables respectively. Under the assumptions of DELES, the per capita stock of Motor Vehicles planned to be held by the employed during period t is given by[1]

$$C(\pi_{1t}k_{1t}^{M}) = C\pi_{1t}\theta_1 + C(\mu_{tM}\eta_{1tM}^{0}Z_{tM}^{*}) \ , \qquad (13)$$

in which

π_{1t} = an appropriate price indicator for Motor Vehicles during t, inclusive of quality corrections ;

θ_1 = per capita "basic needs" of automotive services - measured notionally as the number of "19xy brand-new Holden equivalent per period service flows" required to establish a "basic" standard of living ;

θ_2 = per capita "basic needs" of Household Durable goods ;

$$Z_{tM}^{*} = Z_{tM} - \sum_{i=1}^{3} P_{it}\gamma_{iM} - \sum_{j=1}^{2} \sigma_j\theta_j\pi_{jt} \qquad (13.1)$$

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1. While the superscript M could be attached to η_1, η_2, θ_1, θ_2, σ_1 and σ_2, omission will not lead to confusion as there are no equivalent parameters with the superscript L.

= supernumerary, or discretionary, permanent per capita income of those belonging to households whose head is employed during t

= the permanent per capita income of a member of M , minus the per capita cost of purchasing, at current prices of consumables, the "basic bundle" of three consumables and net of the per capita cost of meeting "minimum" depreciation of the two durables at the current prices of new durables (including the per capita cost of running that fraction of the existing stock of Motor Vehicles owned by the employed) ;

σ_1 = per cent per period depreciation on Motor Vehicles[1] ;

σ_2 = per cent per period depreciation on Household Durables ;

η^0_{1tM} = marginal share of Motor Vehicles in the budget of a member of M during t

$$= \frac{\eta_1}{(\sigma_1+\rho_{tM})(1-\delta_M\alpha_{tM})} ,$$

where

η_1 = a parameter of the utility function[2]

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1. The depreciation rates σ_1 and σ_2 might be treated as parameters or may be extraneously estimated. Previous empirical work would lend support to the latter approach.

2. See equation (1) above.

$$\alpha_{tM} = \sum_{j=1}^{2} \frac{n_j}{(\sigma_j + \rho_{tM})}$$

$$\mu_{tM} = \delta_M / \rho_{tM}$$

= marginal propensity to consume out of both per capita permanent income and per capita supernumerary permanent income during t by the employed ;

δ_M = subjective time preference discount rate for a member of M^1 ;

ρ_{tM} = market interest rate during t at which the employed consumer is supposed to be able to lend or borrow.[1]

4.2 Demand for New Motor Vehicles

As accurate stock data are available for Motor Vehicles, a discrete time version of the stock flow identity (3), together with the planned stock equation (13), will lead to an estimable demand equation for new Motor Vehicles. Firstly, we note that R_{1t} is the real consumption (in quantity terms) of new Motor Vehicles during period t throughout the economy. Since, by assumption, the unemployed do not purchase new vehicles, the level of purchases, per capita, by the employed is given by

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1. We strongly recommend not attempting to estimate δ_M and ρ_{tM} individually (or, for that matter, δ_L and ρ_{tL}). At best one could hope to strike an average, whilst the limited evidence available suggests that their individual estimates are likely to be erratic, though not so the estimate of their ratios, μ_{tM} and μ_{tL} . See Powell [28]. It is hoped that if plausible *a priori* values of ρ_{tM} and ρ_{tL} are used, then the conditional estimates of δ_M and δ_L will lead to plausible values for μ_{tM} and μ_{tL} respectively.

$$r^M_{1t} \equiv R_{1t}/M_t \quad . \tag{14}$$

We use the difference operator B defined in (8.2) and discretize (3) in per capita terms at time t .[1] After substituting for k^M_{1t} from (13), this leads to the per capita purchases equation of the employed during time t ,

$$Cr^M_{1t} = \left[\theta_1 + \frac{C(\mu_{tM} n^0_{1tM} Z^*_{tM})}{C\pi_{1t}}\right] - (1-\sigma_1)Ck^M_{1,t-1} \quad , \tag{15}$$

which is equivalent to expressing purchases of new vehicles during period t as the difference between the planned stock of new vehicles during t and the incoming stock less the depreciation during t .[2] In (15), the $\{k^M_{1,t-1}\}$ series would come from the IAC study.[3] The equation for total purchases, in current dollars, throughout the economy, would be

$$\pi_{1t} R_{1t} = \pi_{1t} r^M_{1t} M_t \quad . \tag{16}$$

---

1. The reader will notice that the discretized version of equation (3) in per capita terms during t is

$$k_{1t} - k_{1,t-1} = r_{1t} - \sigma_1 k_{1,t-1} \quad . \tag{A}$$

The second term on the right-hand side in (A) differs slightly from its corresponding term in (3) as, in the discrete time formulation, depreciation of stock during t takes place on the incoming stock , $k_{1,t-1}$, the per capita stock of motor vehicles held during the previous period.

2. It is implicit that the coefficient of adjustment from the current to the planned stock is unity, since it reflects the instantaneous adjustment in continuous time.

3. See IAC mimeograph [12].

4.3 Demand for New Household Durables

For this category it is not possible to base our modelling for estimation purposes on the stock equation (13), as sufficiently accurate data on stocks of Household Durables are unavailable. With stocks eliminated, a DELES expenditure equation similar to (6) is obtained. Using the operators given by (8.1), (8.2) and (8.3), the expenditure equation for Household Durables during t is given by

$$C(\pi_{2t} r_{2t}^{M}) = \sigma_2 \Theta_2 C\pi_{2t} + C(\mu_{tM} \eta^*_{2tM} Z^*_{tM})$$

$$- C(\mu_{tM} \eta^0_{2tM}) \left[A\pi_{2t} + A\rho_{tM} + \frac{(1-\delta_M C\alpha_{tM} + (\sigma_2 + C\rho_{tM})\delta_M C\phi_{tM}) B\rho_{tM}}{(\sigma_2 + C\rho_{tM})(1-\delta_M C\alpha_{tM})} \right.$$

$$\left. + \frac{\sum_{i=1}^{3} \gamma_{iM} BP_{it}}{CZ^*_{tM}} + \frac{\sum_{j=1}^{2} \sigma_j \Theta_j B\pi_{jt}}{CZ^*_{tM}} - \frac{BZ_{tM}}{CZ^*_{tM}} \right] CZ^*_{tM} , \quad (17)$$

in which

π_{2t} = an appropriate price indicator for Household Durables during t ;

η_2 = a parameter of the utility function[1] ;

1. See equation (1) above.

$$\eta^{0}_{2tM} = \frac{\eta_2}{(\sigma_2+\rho_{tM})(1-\delta_M\alpha_{tM})} \quad ;$$

η^{*}_{2tM} = marginal share of Household Durables in the budget of a member of M during t

$$= \eta^{0}_{2tM}(\sigma_2 + \rho_{tM} - \delta_M) \quad ;$$

and

$$\phi_{tM} = \sum_{j=1}^{2} \frac{\eta_j}{(\sigma_j+\rho_{tM})^2} \quad .$$

5. DEMAND FOR CONSUMABLES BY THE EMPLOYED

The employed are assumed to have marginal budget shares for consumables which are in the same ratios among themselves as the corresponding marginal budget shares for the unemployed; viz., as the β_{iM} (i=1, 2, 3) are the marginal budget shares of the employed for consumables, then[1]

$$\frac{\beta_{iM}}{\sum_{i=1}^{3} \beta_{iM}} = \beta_{iL} \qquad (i=1,\ 2,\ 3)\ . \tag{18}$$

As a result of a normalization of the parameters in the representative utility function for the employed, we have

$$\sum_{i=1}^{3} \beta_{iM} + \sum_{j=1}^{2} \eta_j \equiv 1\ , \tag{19.1}$$

and so

$$\sum_{i=1}^{3} \beta_{iM} = 1 - \sum_{j=1}^{2} \eta_j\ . \tag{19.2}$$

Therefore, from (18) we can write

$$\beta_{iM} = \beta_{iL}(1 - \sum_{j=1}^{2} \eta_j) \qquad (i=1,\ 2,\ 3)\ . \tag{20}$$

The DELES per capita expenditure equation for the i^{th} consumable for a member of M during t is

$$C(P_{it}c^M_{it}) = CP_{it}\gamma_{iM} + \frac{C\mu_{tM}\beta_{iL}(1 - \sum_{j=1}^{2} \eta_j)}{(1 - \delta_M C\alpha_{tM})} CZ^*_{tM}\ , \tag{21}$$

---

1. Since $0 < \sum_{i=1}^{3} \beta_{iM} < 1$, $\beta_{iM} < \beta_{iL}$ (i=1, 2, 3).

in which

γ_{iM} = per capita "basic needs" requirement of consumable i for the representative employed consumer.

6. PERMANENT INCOME, ACTUAL INCOME AND MARGINAL PROPENSITIES

We have a system of eight equations involving 20 parameters. The parameter space can be reduced if we assume that

$$\gamma_{iM} = \gamma_{iL} = \gamma_i \qquad (i=1, 2, 3) \qquad (22)$$

so that the system of equations now has the following 17 parameters:

$$\{ \beta_{i\ell}, \gamma_i, \eta_j, \Theta_j, \sigma_j, \delta_\ell \} \qquad (23)$$

for all $i=1, 2, 3$; $j=1, 2$; $\ell=L, M$. The number of "free" parameters can be reduced further by making the following assumptions or by imposing the following restrictions[1]:

(i) Since estimated depreciation rates have typically been "too high" to warrant their interpretation as physical rates of depreciation within the context of dynamic demand models[2], *a priori* values will be imposed for both σ_1 and σ_2[3] ;

(ii) From equation (20) we see that β_{iL} can be written as a function of β_{iM} and $\sum_{j=1}^{2} \eta_j$. Hence the values of β_{iL} may be treated as functions of parameters (all $i=1,2,3$) ;

---

1. If the restrictions are incorrect, all parameter estimates will be inconsistent.
2. See, for example, Stone and Rowe [36] for the single equation model and Phlips [24] for the systems approach.
3. These values are calculated in section 13.2.

(iii) In estimating the marginal budget share for Other Household Non-Durables use will be made of the normalization

$$\beta_{3M} = 1 - \beta_{1M} - \beta_{2M} - \eta_1 - \eta_2 \quad ,$$

so that β_{3M} is no longer a free parameter;

(iv) Many empirical studies have found that estimates of the "subsistence" parameters are unduly high[1], so much so that they exceeded purchases for a number of periods. Apart from resultant difficulties in the utility interpretation of the fitted results, this is also undesirable because it tends to imply implausibly low values of the average substitution elasticity among commodities.[2] In order to gaurd against this possibility, the following restriction is placed in the base year (i.e., when $P_i = \pi_j = 1.00$ for $i = 1, 2, 3$; $j=1, 2$):

$$\sum_{i=1}^{3} \gamma_i + \sum_{j=1}^{2} \sigma_j \Theta_j = \text{constant} ,$$

where the value of the constant is obtained from previous empirical studies after imposing a further restriction[3];

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1. See, for example, Lluch, Powell and Williams [18], Lluch and Williams [19] , Parks [21], Pollak and Wales [27] and Powell ([28] and [29]) .

2. In the Linear Expenditure System, the 'Frisch parameter' increases in absolute value as the 'cost' of subsistence increases relative to total expenditure. Sato [32a] interprets the negative of the reciprocal of the Frisch parameter as an average substitution elasticity.

3. See Lluch, Powell and Williams [18] and section 13.1 below.

(v) Finally, δ_L is dispensed with by making an explicit assumption concerning the ratio δ_L / ρ_{tL}, see below.

The marginal propensity to consume (MPC) for the unemployed, μ_{tL}, should, within a strict ELES framework, be interpreted as the ratio δ_L / ρ_{tL}. Since the concepts MPC and "permanent income" have to be jointly defined, it is best at this stage to have an operational definition of permanent income in focus. Only two studies have attempted to investigate the performance of some simple lag hypotheses as permanent income generators in the context of ELES,[1] the bulk of the empirical work having been done with actual income substituted for permanent income in the final estimating forms. This seems a reasonable starting point for us. If Z_{tL} in (9) is to be interpreted as current income, then, for the unemployed, it seems reasonable to insist that their MPC, μ_{tL}, is unity (for all t).[2]

For the employed, the ratio $\mu_{tM} = \delta_M/\rho_{tM}$ can be interpreted as the MPC out of (discretionary) permanent income in a situation in which changes in interest rates, all prices and expected permanent income just

1. Powell [28] investigates the linear extrapolative expectations hypothesis in the context of a scalar consumption function for the U.S. viewed as an aggregation of a 5-commodity extended linear expenditure system; Williams and Chang [45] investigate various versions of the Koyck lag distribution.

2. The fiction for the unemployed would be that, during unemployment, their credit rating drops and ρ_{tL} rises. Simultaneously, however, their unemployed situation causes them to focus more heavily on the short run, so that δ_L rises more than proportionately, to the point where δ_L / ρ_{tL} equals unity. We also note that an MPC of unity reduces Lluch's ELES (9) to Stone's Linear Expenditure System (see [35]), provided that Z_t is reinterpreted as total expenditure rather than permanent income.

cancel each other out.[1] Sum (17) and (21) over all commodities and observe that μ_{tM} is the MPC out of both Z^*_{tM} and Z_{tM} in the DELES consumption function for a representative member of a household whose head is employed during period t, provided the terms involving operators A and B are zero:

$$Cv_{tM} = \sum_{i=1}^{3} C(P_{it}c^M_{it}) + \sum_{j=1}^{2} C(\pi_{jt}r^M_{jt}) \tag{24}$$

$$= \sum_{i=1}^{3} CP_{it}\gamma_i + \sum_{j=1}^{2} \sigma_j\Theta_j C\pi_{jt} + C(\mu_{tM}Z^*_{tM})$$

$$- \frac{C(\alpha_{tM}\mu_{tM})}{(1-\delta_M C\alpha_{tM})}\left[A\rho_{tM} + \frac{\sum_{i=1}^{3}\gamma_i BP_{it}}{CZ^*_{tM}} + \frac{\sum_{j=1}^{2}\sigma_j\Theta_j B\pi_{jt}}{CZ^*_{tM}} - \frac{BZ_{tM}}{CZ^*_{tM}}\right]CZ^*_{tM}$$

$$- C(\mu_{tM}\sum_{j=1}^{2}\eta^0_{jtM})\left[A\pi_{jt} + \frac{(1-\delta_M C\alpha_{tM} + (\sigma_j + C\rho_{tM})\delta_M C\phi_{tM})B\rho_{tM}}{(\sigma_j + C\rho_{tM})(1-\delta_M C\alpha_{tM})}\right]CZ^*_{tM} \; . \tag{24.1}$$

There does not appear to be any compelling reason for insisting on any particular values for μ_{tM}, although it seems reasonable to suppose that δ_M is a constant over the sample period so that variations in ρ_{tM} will lead to variations in μ_{tM}.

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1. Adopting the expedient (actual income) = (permanent income), this means that actual income, prices and interest rates are all changing in a way that is consistent with the behaviour which would be observed under a long run (i.e., stationary) equilibrium.

7. EXPECTED GROWTH OF PERMANENT INCOME AND PRICES

It was stated in section 2 that inflation could be readily handled by making explicit assumptions concerning the expected time paths of permanent income and prices. Rewrite equation (2) of the consumer's plan as

$$\int_t^{\infty} Z_t(\tau)e^{-\rho_t(\tau-t)}\,d\tau = \int_t^{\infty} e^{-\rho_t(\tau-t)}\left[\sum_i P_{it}(\tau)C_{it}(\tau) + \sum_j \pi_{jt}(\tau)R_{jt}(\tau)\right]d\tau \qquad (2.1)$$

where $Z_t(\tau)$, $P_{it}(\tau)$ and $\pi_{jt}(\tau)$ are the time paths of permanent income, and prices of the i^{th} consumable and j^{th} durable respectively. Since we are interested in solving for the time paths of $C_{it}(\tau)$ and $R_{jt}(\tau)$, we introduce the following growth law assumptions concerning the paths of permanent income and prices:

$$Z_t(\tau) = Z_t e^{\omega_t(\tau-t)} \qquad (2.2.a)$$

$$P_{it}(\tau) = P_{it} e^{g_t(\tau-t)} \quad \text{(all i)} \qquad (2.2.b)$$

$$\pi_{jt}(\tau) = \pi_{jt} e^{g_t(\tau-t)} \quad \text{(all j)} \qquad (2.2.c)$$

where ω_t and g_t are the stationary expectations regarding the rates of growth of permanent income and all prices[1], respectively, over the consumer's plan : t is actual time and τ is planning time. Substitution of conditions (2.2) in (2.1) leads to the new lifetime budget constraint

1. A differentiating characteristic of durables may well be that consumers hold different expectations regarding the expected growth of their prices vis-a-vis those of non-durables. For the purpose of mathematical simplicity we consider only the case where all commodity prices are expected to grow at the same rate.

$$Z_t/\,(\rho_t-\omega_t) \;=\; \int_t^\infty e^{-(\rho_t-\omega_t)(\tau-t)} \left[\sum_i P_{it}\, C_{it}(\tau) + \sum_j \pi_{jt}\, R_{jt}(\tau) \right] d\tau \;. \qquad (2.3)$$

Maximization of (1) subject to (2.3) and (3) leads to a planned stock equation (4) and actual expenditure equations for consumable i and durable j, equations (5) and (6) respectively, subject to the following modifications:

(i) ρ_t is everywhere replaced by (ρ_t-g_t), ρ_t now being the nominal interest rate at time t (with regard to expected increases in prices) and (ρ_t-g_t) is the real interest rate at t;

(ii) $$Z_t^* \;=\; \xi_t\, Z_t - \sum_i P_{it}\, \gamma_i - \sum_j \sigma_j \theta_j \pi_{jt} \;, \qquad (4.4.a)$$

where $$\xi_t = \frac{\rho_t - g_t}{\rho_t - \omega_t} \;,$$

so that the consumer's permanent income is weighted by ξ_t, which is unity when expectations regarding the rate of growth of permanent income and all prices are identical.

The system of equations is essentially unchanged from that of the previous sections, although the following changes should be noted:

$$Z^*_{tL} = \xi_{tL} Z_{tL} - \sum_{i=1}^{3} P_{it} \gamma_i \quad , \tag{10.1.a}$$

$$\text{where} \quad \xi_{tL} = \frac{\rho_{tL} - g_{tL}}{\rho_{tL} - \omega_{tL}} \quad ; \tag{10.1.b}$$

$$\text{and} \quad Z^*_{tM} = \xi_{tM} Z_{tM} - \sum_{i=1}^{3} P_{it} \gamma_i - \sum_{j=1}^{2} \sigma_j \theta_j \pi_{jt} \quad , \tag{13.1.a}$$

$$\text{where} \quad \xi_{tM} = \frac{\rho_{tM} - g_{tM}}{\rho_{tM} - \omega_{tM}} \tag{13.1.b}$$

and the use of the subscripts L and M is understood.

It is assumed that expectations regarding the rate of growth of prices are invariant to employment category, so that

(i) $g_{tL} = g_{tM} = g_t$ (all t) .

Expectations regarding the rate of growth of income will probably not be identical for those in different employment categories (i.e., $\omega_{tL} \neq \omega_{tM}$). We assert that a member of L has identical expectations regarding the rate of increase of permanent income and prices, that is

(ii) $\omega_{tL} = g_{tL} = g_t$ (all t) ,

which implies ξ_{tL} is unity in (10.1.b). As for a member of M , we assume

(iii) $\omega_{tM} = \varepsilon\, g_{tM} = \varepsilon\, g_t$ (all t) , where ε is a constant factor of proportionality.[1] The closer is ε to unity, the more likely will the employed consumer have identical expectations concerning the growth of permanent income and prices.

Substitute assumption (iii) in (13.1.b) to obtain

$$\xi_{tM} = \frac{\rho_{tM} - g_t}{\rho_{tM} - \varepsilon g_t} \qquad (13.1.b.1)$$

so that $\partial \varepsilon_{tM} / \partial \varepsilon \gtrless 0$ as $(\rho_{tM} - g_t) \gtrless 0$. If the real rate of interest is positive (negative), the employed consumer's supernumerary permanent income is higher (lower) as the expected rate of growth of permanent income increases (decreases) relative to the expected rate of growth of prices.

It would be possible to iterate over permissible values of ε , say in the interval [0,2], and the value of ε that maximizes the likelihood function, $\bar{\varepsilon}$ say, could be selected as the maximum likelihood estimate of ε .

We continue to assume, as we did in section 6, that μ_{tL} is unity, where μ_{tL} is now defined to be

$$\mu_{tL} = \delta_L / (\rho_{tL} - g_t) = \delta_L / \rho^*_{tL} \ .$$

---

1. As it is implicit that ε_L is unity, it is not necessary to use an M subscript for ε in order to differentiate between them.

Restricting μ_{tL} to unity results in the omission of ρ_{tL} and ρ^*_{tL} from the ELES system (9). In what follows we will use ρ^*_{tM} and ρ^*_t interchangeably as the real rate of interest for a member of M , that is

$$\rho^*_{tM} = \rho_{tM} - g_{tM} = \rho_t - g_t = \rho^*_t \ .$$

8. OPERATING EXPENSES OF MOTOR VEHICLES

Even though the unemployed are assumed not to purchase new Motor Vehicles, their previously owned stock of vehicles must be taken account of. There is room for a good deal of ambiguity here. By operating in the second-hand market, the unemployed conceivably might keep up, or even increase, their stock of vehicles (quality and vintage adjusted). It is also possible that their stocks might decline to the extent of depreciation. Or finally, by liquidating vehicles in the second-hand market, the stock of vehicles held by the unemployed might diminish faster than would be accounted for by depreciation alone. Clearly, the matter will not be independent of the average, actual and the expected duration of unemployment over the unemployed, nor of the distribution of the unemployed periods (by length) over the workforce. An annual model will not be able to deal with these aspects.

To the relationships introduced in the previous sections we add the following:

$$CS_t = C(P_t Ck_{1t}) = \tfrac{1}{2}C[P_t(k_{1t} + k_{1,\, t-1})] \tag{25}$$

which states that the per capita running expenses during t, S_t, is proportional to the average per capita stock of Motor Vehicles held between t and $(t-1)$, Ck_{1t},[1] with factor of proportionality during t, P_t. The latter is conceptually quite close to the Australian Bureau of Statistics implicit deflator for expenditure on the operation of Motor Vehicles. A component for interest on funds tied up, or external cost of

---

1. The operator C effectively gives the per capita stocks held at the beginning of t.

finance, could be added, as in the IAC study.[1]

Disaggregating the per capita running expenses of currently owned Motor Vehicles, S_t , by employment status, we have

$$S_{tM} = P_{tM} (k^M_{1t} + k^M_{1,t-1}) / 2 \qquad (25\ a)$$

and

$$S_{tL} = P_{tL} (k^L_{1t} + k^L_{1,t-1}) / 2 \qquad (25\ b)$$

where equation (25 a) ((25 b)) states that the per capita running expenses of Motor Vehicles during t for the employed (unemployed) consumer, S_{tM} (S_{tL}), is proportional to the average per capita stock of Motor Vehicles held between t and $t-1$, $(k^M_{1t} + k^M_{1,t-1}) / 2$ $((k^L_{1t} + k^L_{1,t-1}) / 2)$, with factor of proportionality during t of P_{tM} (P_{tL}) .

Supernumerary permanent per capita income of the employed , Z^*_{tM} , is net of S_{tM} which, in turn, is approximated by $\sigma_1\theta_1\pi_{1t}$ (see equation (13)) , the "minimum depreciation" for Motor Vehicles. Since the unemployed consumer is assumed not to purchase durables, Z^*_{tL} is not net of S_{tL} in equation (9). It is therefore necessary to substract S_{tL} from $(Z_{tL} - \sum_{i=1}^{3} P_{it}\gamma_{iL})$ to obtain supernumerary permanent per capita income of the unemployed, Z^*_{tL} . As S_{tL} is data, we can simply define Z_{tL} , the permanent per capita income of the unemployed, to be net of S_{tL} .

Equation (25) endogenizes expenditure on running costs because S_t is endogenous, being determined by a predetermined $k_{1,\,t-1}$ and k_{1t} , which is itself endogenized via (13). Thus, we can write the stock of Motor

1. See IAC mimeograph [12].

Vehicles held in the economy during period t as

$$N_t k_{1t} = M_t r_{1t}^M + N_t(1-\sigma_1)k_{1,t-1} , \tag{26}$$

where r_{1t}^M is determined by (15).

9. SUMMARY OF BEHAVIOURAL EQUATIONS

The model outlined so far consists of nine behavioural relations which are summarized in Table 1.[1] Notice that in the case of the employed, consumption of both durables and consumables is a function of the real rate of interest, ρ^*_{tM} .

1. The likely next step in this development is to disaggregate Motor Vehicles into locally made and imports, using a nested utility function.

TABLE 1

COMPLETE LIST OF 9 BEHAVIOURAL EQUATIONS OF THE PROPOSED CONSUMPTION FUNCTION

Representative member of household whose head is unemployed

Food, Rent, Other Non-Durables (i = 1, 2, 3)

$$C[P_{it}c^{L}_{it}] = CP_{it}\gamma_i + \beta_{iL}\, CZ^{*}_{tL}$$

Representative member of household whose head is employed

Food, Rent, Other Non-Durables (i = 1, 2, 3)

$$C(P_{it}c^{M}_{it}) = CP_{it}\gamma_i + \frac{C\mu_{tM}\beta_{iL}(1-\sum_{j=1}^{2}\eta_j)}{(1-\delta_M C\alpha_{tM})}\, CZ^{*}_{tM}$$

Purchases of New Motor Vehicles

$$C(\pi_{1t}\, r^{M}_{1t}) = C\pi_{1t}\Theta_1 + C(\mu_{tM}\eta^{0}_{1tM}Z^{*}_{tM}) - (1-\sigma_1)C(\pi_{1t}k^{M}_{1,t-1})$$

Purchases of New Household Durables

$$\begin{aligned} C(\pi_{2t}r^{M}_{2t}) = {} & \sigma_2\Theta_2 C\pi_{2t} + C(\mu_{tM}\eta^{*}_{2tM}Z^{*}_{tM}) \\ & - C(\mu_{tM}\eta^{0}_{2tM})\Bigg[A\pi_{2t} + A\rho^{*}_{tM} \\ & + \frac{(1-\delta_M C\alpha_{tM} + (\sigma_2+C\rho^{*}_{tM})\delta_M C\phi_{tM})B\rho^{*}_{tM}}{(\sigma_2+C\rho^{*}_{tM})(1-\delta_M C\alpha_{tM})} \\ & + \frac{\sum_{i=1}^{3}\gamma_i BP_{it}}{CZ^{*}_{tM}} + \frac{\sum_{j=1}^{2}\sigma_j\Theta_j B\pi_{jt}}{CZ^{*}_{tM}} - \frac{BZ_{tM}}{CZ^{*}_{tM}}\Bigg] CZ^{*}_{tM} \end{aligned}$$

Representative member of total population

Per capita running expenses of Motor Vehicles

$$CS_t = C[P_t Ck_{1t}]$$

10. ESTIMATING EQUATIONS (CONDELES) AND ERROR SPECIFICATION

Consumption data cannot be disaggregated by users along the lines followed above. Accordingly, we aggregate the expenditure functions for consumables, (9) and (21), into three aggregate functions, one for each commodity. We obtain for the i^{th} aggregated commodity

$$P_{it}c_{it} \equiv P_{it}C_{it}/N_t \tag{27}$$

$$\equiv P_{it}(C^L_{it} + C^M_{it})/N_t$$

$$\equiv P_{it}c^L_{it}L_t/N_t + P_{it}c^M_{it}M_t/N_t \ . \tag{27.1}$$

Substituting from (9) and (21) into (27.1), and appending an error term,[1] gives

$$C(P_{it}c_{it}) = CP_{it}\gamma_i + \beta_{iL}C[(L_t/N_t)Z^*_{tL} + (M_t/N_t)Z^{**}_{tM}] + e_{it} \ , \tag{28}$$

in which

$$Z^{**}_{tM} = \frac{\mu_{tM}(1 - \sum_{j=1}^{2} \eta_j)}{(1 - \delta_M\alpha_{tM})} Z^*_{tM} \ , \tag{28.1}$$

and e_{it} is an additive, zero mean random disturbance $(i = 1, 2, 3)$. To obtain total consumption of consumables, (28) would be multiplied by

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1. The addition of a stochastic error term to the solution equations of the problem outlined in section 2 is somewhat less ambitious than attempting to derive the estimating equations by the use of stochastic control theory.

the total population during t , N_t . Estimation would be carried out in the form (28), however, in the interests of eliminating at least one possible source of non-stationary variances.

Purchases equations for new Motor Vehicles and new Household Durables would follow (15) and (17) with error terms e_{4t} and e_{5t} respectively added to them, as in Table 2.[1] To convert these equations from per capita to aggregate terms, they would be multiplied through by M_t . The five estimating equations are brought together in Table 2. In the last line of this table the accounting relationship for running expenses is repeated for completeness. The set of consumption equations associated with DELES will hereafter be referred to as CONDELES.

Under the assumption of no serial correlation both within[2] and across equations, the error specification is

$$E(e_t) = 0 \qquad \text{(all } t) \tag{29.1}$$

$$E(e_t \, e_\tau') = \delta_{t\tau}\Omega \quad , \tag{29.2}$$

where $e_t = (e_{t1}, \ldots, e_{t5})'$, $\delta_{t\tau}$ is the Kronecker delta and Ω is a 5×5 compact contemporaneous variance-covariance matrix of the stochastic elements. We further assume that the latter additive shocks are normally distributed.

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1. While the error terms are appended to the estimating equation for convenience, it is worth noting that if they are introduced in an equation similar to (15) but subscripted for the second durable good, they are essentially unchanged in (17). But should they appear in an equation of the form (13), appropriately subscripted for the second durable, substituting out for the unobservable stock of Household Durables leads to a transformed disturbance term of the form $e^*_{5t} = (\sigma_2 - A\pi_{2t})e_{5t}$. This occurs as a result of substituting for k^M_{2t} and $\dot{k}^M_{2t}$ in the stock-flow identity to obtain an equation with r^M_{2t} as the dependent variable.

2. A non-parametric sign test for serial correlation is given in Appendix B.

TABLE 2

COMPLETE LIST OF FIVE STOCHASTIC EQUATIONS TO BE ESTIMATED FOR THE PROPOSED CONSUMPTION FUNCTION

Commodity	Function
Food, Rent, Other Non-Durables (i=1, 2, 3)	$C(P_{it}c_{it}) = CP_{it}\gamma_i + \beta_{iL}C[(L_t/N_t)\ Z^*_{tL} + (M_t/N_t)Z^{**}_{tM}] + e_{it}$.
Purchases of New Motor Vehicles	$C(\pi_{1t}r^M_{1t}) = C\pi_{1t}\theta_1 + C(\mu_{tM}\eta^0_{1tM}\ Z^*_{tM}) - (1 - \sigma_1)\ C(\pi_{1t}k^M_{1,\ t-1}) + e_{4t}$.
Purchases of New Household Durables	$C(\pi_{2t}r^M_{2t}) = \delta_2\theta_2 C\pi_{2t} + C(\mu_{tM}\eta^*_{2tM}\ Z^*_{tM}) - C(\mu_{tM}\eta^0_{2tM}) \left[A\pi_{2t} + A\rho^*_{tM} + \frac{(1 - \delta_M C\alpha_{tM} + (\sigma_2 + C\rho^*_{tM})\ \delta_M C\phi_{tM})\ B\rho^*_{tM}}{(\sigma_2 + C\rho^*_{tM})\ (1 - \delta_M C\alpha_{tM})} + \frac{\sum_{i=1}^{3} \gamma_i BP_{it}}{CZ^*_{tM}} + \frac{\sum_{j=1}^{2} \sigma_j\theta_j B\pi_{jt}}{CZ^*_{tM}} - \frac{BZ_{tM}}{CZ^*_{tM}} \right] CZ^*_{tM} + e_{5t}$.
Per capita running expense of Motor Vehicles	$CS_t = C(P_tCk_{1t})$.

11. METHOD OF ESTIMATION : RESIMUL

RESIMUL is one program in a suite of programs developed by C. R. Wymer for handling linear systems of equations.[1] The program computes Full Information Maximum Likelihood (FIML) estimates of the parameters of virtually any simultaneous equations system which remains linear in the variables.[2] Specifically, non-linear constraints among the elements of the parameter set, both within and across equations, may be readily handled.[3]

Let $\{\Theta , \Omega\}$ be the set of parameters which it is desired to estimate. The elements of Θ determine the coefficients of variables in a simultaneous equations system; Ω is the stationary variance-covariance matrix of the stochastic elements in the system, these elements being normally and independently distributed. The system we wish to estimate can be written as

$$B(\Theta)y_t + C(\Theta)z_t = e_t \quad , \tag{30}$$

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1. See Wymer [46]. We are indebted to him for access to his remarkably flexible programs, without which estimation of CONDELES would not have been possible.

2. Also among Wymer's suite of programs is ASIMUL, designed to handle systems non-linear in the variables as well as in the parameters. A less expensive alternative in terms of computer time, especially for systems which are 'large' in terms of the number of constraints (such as CONDELES), is to approximate the non-linear system with one linear in the variables by a Taylor series expansion and to estimate the linearized system using RESIMUL.

3. The above statement refers to the parameters which determine the coefficients of variables in the equations. The current version of RESIMUL does not allow constraints on the variances and covariances of the stochastic elements.

where y_t is a vector containing the realized values at data point t of 5 jointly dependent variables; z_t is a similar vector containing the realizations on N predetermined variables;[1] e_t is a vector containing realized values of random shocks, and is independently distributed $N(0, \Omega)$; $B(\Theta)$ and $C(\Theta)$ are matrices of structural coefficients with dimensions 5×5 and $5 \times N$ respectively; and Θ is a vector of parameters.

Each element of the matrices $B(\Theta)$ and $C(\Theta)$ can, by assumption, be expressed as a function of the unknown vector of parameters Θ. Constraints among the elements of B and C are accommodated by specifying their functional relationship to Θ; as mentioned above, constraints among the elements of Ω are not allowed.

Given n data points on (30) and the details of the mapping between $\{B, C\}$ and Θ, RESIMUL computes the FIML estimates of the parameters Θ, of the coefficients $\{B, C\}$, and much more,[2] by a Newton-Raphson iterative procedure beginning with arbitrary initial values of the parameters.[3]

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1. Since CONDELES is highly non-linear in the predetermined variables, a Taylor series expansion is required. The order of the expansion used obviously depends on the degree of accuracy required in the approximation. Quasi-linearity (i.e., regarding x and x^2 as separate variables, in which the system remains linear) will suffice for the predetermined variables, z, but will not do in the case of current endogenous variables, y, since the likelihood function will only be calculable, within RESIMUL, if the model remains strictly linear in the current endogenous variables. Since this problem does not arise in CONDELES, it is not necessary to truncate at the first-order term. However, we should bear in mind that the presence of products of predetermined variables appearing non-linearly in CONDELES, together with an upper bound on the number of predetermined variables that may appear in the system, effectively restricts the approximation to one of either the first- or second-order.

2. See Wymer [46].

3. Initial values of parameters for CONDELES are presented in section 13.6.

12. TAYLOR-SERIES LINEARIZATION

Consider an arbitrary non-linear (in the variables) equation system with i^{th} element

$$f_i(y_t \ , \ z_t) \ = \ e_{it} \ , \tag{31}$$

where each element of f_i is twice differentiable. We wish to rewrite (31) as an approximation having the form of (30). Replace (31) by a mixed first- and second-order Taylor-series approximation

$$e_{it} \ \doteq \ v_{it} \ = \ f_i \ (\bar{y} \ , \ \bar{z}) + (y_t - \bar{y})^T \ f'_{iy}(\bar{y} \ , \ \bar{z})$$

$$+ \ (z_t - \bar{z})^T f'_{iz}(\bar{y} \ , \ \bar{z}) + \tfrac{1}{2}(z_t - \bar{z})^T f''_{izz}(\bar{y} \ , \ \bar{z})(z_t - \bar{z}) \ . \tag{32}$$

In (32), $\bar{y}$ and $\bar{z}$ are the sample mean values of y_t and z_t respectively and are the co-ordinates at which the Taylor expansion is to be made, while the superscript T indicates transposition. The vector $f'_{iy}(\bar{y} \ , \ \bar{z})$ has as its j^{th} element the derivative

$$(\partial f_i/\partial y_j)\Big|_{\bar{y},\bar{z}}$$

and similarly the k^{th} element of f'_{iz} is the derivative

$$(\partial f_i/\partial z_k)\Big|_{\bar{y},\bar{z}} \quad ,$$

whilst finally the matrix $f''_{izz}(\bar{y} \ , \ \bar{z})$ has as its $(j, \ k)^{th}$ element the derivative

$$(\partial^2 f_i/\partial z_j \partial z_k)\Big|_{\bar{y},\bar{z}} \quad .$$

In CONDELES each equation is non-linear in the real market interest rate, ρ_t^* ; hence the j^{th} non-linear component of the i^{th} equation (i=1, ..., 5), $f_{ij}(\rho_t^*)$, can be approximated by[1]

$$g_{ij}(\rho_t^*) = a_{ij} + b_{ij}\,\rho_t^* + c_{ij}\,\rho_t^{*2} \quad . \tag{33}$$

We wish to find values of a_{ij} , b_{ij} and c_{ij} (all i, j) such that

$$f_{ij}(\bar{\rho}^*) = g_{ij}(\bar{\rho}^*) = a_{ij} + b_{ij}\,\bar{\rho}^* + c_{ij}\,\bar{\rho}^{*2} \tag{34.1}$$

$$f'_{ij}(\rho^*) = g'_{ij}(\bar{\rho}^*) = b_{ij} + 2c_{ij}\,\bar{\rho}^* \tag{34.2}$$

$$f''_{ij}(\bar{\rho}^*) = g''_{ij}(\bar{\rho}^*) = 2c_{ij} \quad , \tag{34.3}$$

where the superscripts ' and '' denote the first and second derivatives respectively.

The solutions can be obtained by solving the system

$$\begin{bmatrix} f_{ij}(\bar{\rho}^*) \\ f'_{ij}(\bar{\rho}^*) \\ f''_{ij}(\bar{\rho}^*) \end{bmatrix} = \begin{bmatrix} 1 & \bar{\rho}^* & \bar{\rho}^{*2} \\ 0 & 1 & 2\rho^* \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a_{ij} \\ b_{ij} \\ c_{ij} \end{bmatrix} \quad , \tag{34}$$

---

1. The index $j = \begin{cases} 1 & \text{for } i = 1, \ldots, 4 \\ 5 & \text{for } i = 5 \end{cases}$

for version 1 of the model (see section 13). In version 2, when the stock of Motor Vehicles has been eliminated from the estimating equation, j = 5 for i = 4 .

so that

$$\begin{bmatrix} a_{ij} \\ b_{ij} \\ c_{ij} \end{bmatrix} = \begin{bmatrix} 1 & -\bar{\rho}^{*} & \bar{\rho}^{*2}/2 \\ 0 & 1 & -\bar{\rho}^{*} \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} f_{ij}(\bar{\rho}^{*}) \\ f'_{ij}(\bar{\rho}^{*}) \\ f''_{ij}(\bar{\rho}^{*}) \end{bmatrix} . \qquad (35)$$

13. DATA BASE FOR CONDELES

Two versions of CONDELES are estimated : CONDELES 1 is the system of equations given in Table 2 for the five commodity groups Food, Rent, Other Non-Durables, New Motor Vehicles and New Household Durables; CONDELES 2 also has five broad commodity aggregates but New Motor Vehicles and New Household Durables are aggregated into the single category Durable Goods, while Other Non-Durables are split into the two groups {Clothing, Footwear and Textiles} and Other Non-Durables. Note that for model 1 stock data are used for the Motor Vehicles equation but all stocks are eliminated from model 2.

The entire sample period is September quarter 1959 to June quarter 1976 but as the model involves lagged values of some variables, the initial period is March quarter 1960. CONDELES 2 is run over a subset of the data, March quarter 1962 to December quarter 1972. Seasonally adjusted quarterly data are used throughout.

Data requirements are considered in Walters.[1] Working estimates of the real consumption series for non-durables and the real purchases series for durables are provided by the Australian Bureau of Statistics (ABS), as are the working estimates of price indexes for all commodities. The conceptually desirable stock series are obtained from Filmer and Talbot.[2] Adjustments to this series to comply with the requirements of CONDELES are given in section 13.5.

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1. See Walters [42].

2. See Filmer and Talbot [6].

From section 3 we have the relation

$$N_t = M_t + L_t \quad . \qquad (36)$$

The total population also equals the total number of employed and unemployed persons, E_t and U_t respectively, plus those persons defined as not-in-the-work-force, NWF_t, all given during t. If we define the total number of persons in the work force at t as WF_t ($\equiv E_t + U_t$), then

$$N_t = M_t + L_t = E_t + U_t + NWF_t = WF_t + NWF_t \quad . \qquad (36.1)$$

Since CONDELES distinguishes only between those employed and unemployed, it is not intended to explain the consumption patterns of those persons not in the work force. As a result, NWF_t is allocated to M_t and L_t in the proportions E_t/WF_t and U_t/WF_t respectively, so that the total population of Australia at any given time t is partitioned into the two components

$$M_t = \left\{ 1 + \frac{NWF_t}{WF_t} \right\} E_t$$

and

$$L_t = \left\{ 1 + \frac{NWF_t}{WF_t} \right\} U_t \quad .$$

Data on total population and on population by employment status are obtained by the ABS from Household Expenditure and Labour Force Surveys.[1]

1. See Walters [42].

No income data by employment status are available from the Labour Force Surveys and the construction of the income series for both the employed and unemployed is discussed in section 13.4. The derivation of subsistence expenditures, a priori depreciation rates, real quarterly interest rates and per capita stocks of Motor Vehicles of the employed are presented below. Since RESIMUL requires initial starting values of the parameters, the calculation of these values is given in the final subsection.

13.1 Subsistence Expenditures of the Employed and Unemployed

Reference was made in section 6 to the estimated values of the "subsistence" parameters which have typically violated the utility specification. To overcome this potential difficulty we impose the restriction that the sum of the subsistence parameters in the base period, when all prices are set to unity, should be strictly less than the quarterly per capita consumption expenditure for the employed.[1] Since the unemployed are deemed not to purchase durables, the subsistence expenditures for the unemployed in the base period will be less than for the employed by the amount of the subsistence depreciation on durables, on the assumption that subsistence expenditure for consumables is invariant to employment status.

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1. While this will not guarantee that each subsistence parameter will be less than its corresponding quarterly per capita expenditure, it enforces compliance with the utility specification on average without restricting the subsistence expenditure for each commodity to be a particular value.

One method of applying this restriction is through the Frisch parameter ω [1], the expenditure elasticity of the marginal utility of expenditure, which enters the relationship between subsistence expenditure and total expenditure. Since subsistence expenditure can be expressed as a function of total expenditure, the Frisch parameter and the variable MPC, in CONDELES it is necessary to obtain information on at least the average MPC, prior to estimation, to impose a restriction on subsistence expenditure. Since it is desired to estimate the MPC for the employed within the model (by estimating the subjective rate of time preference for the employed), it was decided to approximate the relationship in CONDELES by that of the Linear Expenditure System (LES) where no knowledge of the MPC is necessary. The relationship in LES in the base period is[2]

$$\omega = - \left\{ \frac{v}{v - \sum_{i=1}^{3} \gamma_i - \sum_{j=1}^{2} \sigma_j \theta_j} \right\}, \qquad (37)$$

where v is total per capita expenditure in the base period.

In 1966-67 total expenditure in Australia was \$13956 million and the average population was 11,728,250. Hence the annual per capita consumption expenditure on all goods was \$1189.90 so that the quarterly per capita consumption expenditure was \$297.48. On the basis of previous empirical studies[3] an estimate of - 2 for ω is suggested.

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1. See Frisch [8].
2. See Powell ([29], page 41).
3. See Lluch, Powell and Williams [18] and also Frisch ([8], page 189).

The quarterly per capita subsistence expenditure for the employed is then restricted to be

$$\sum_{i=1}^{3} \gamma_i + \sum_{j=1}^{2} \sigma_j\theta_j = - v/2 = 148.74 \qquad (38)$$

in 1966-67 dollars. Quarterly per capita subsistence expenditure for the unemployed is given by $\sum_{i=1}^{3} \hat{\gamma}_i$ in 1966-67 dollars, which will be available <u>after</u> estimation (the "^" denotes estimated value of a parameter). Note that $\sum_{i=1}^{3} \hat{\gamma}_i$ will be less than 148.74 if both θ_1 and θ_2 are estimated to be positive, since both σ_1 and σ_2 are positive.

For CONDELES 2 restriction (38) becomes

$$\sum_{i=1}^{4} \gamma_i + \sigma\theta = 148.74 \quad , \qquad (39)$$

since there are four consumables and only one durable. Quarterly per capita subsistence expenditure of the unemployed is given by $\sum_{i=1}^{4} \hat{\gamma}_i$ in 1966-67 dollars.

13.2 A Priori Depreciation Rates

The depreciation rates σ_1 and σ_2 , for Motor Vehicles and Other Household Durables respectively, are assumed constant in CONDELES. An estimate of σ_1 is derived from an IAC study on Passenger Motor Vehicles.[1] Average depreciation rates for small, medium and large Motor Vehicles are derived for the early and late years of the sample period

1. See IAC mimeograph [12].

(1960-61 and 1972-73 respectively) and the proportions of the automobile market captured by the three types of Motor Vehicles are used as the weights for the three depreciation rates to obtain a weighted depreciation rate for the two years (18.44 per cent and 19.15 per cent respectively). A geometric mean is taken of these two weighted depreciation rates to obtain the a priori depreciation rate of $\sigma_1 = 4.4$ per cent per quarter, equivalent to an annual depreciation rate of 18.8 per cent. We have assumed that the quarterly depreciation rate for Other Household Durables is three quarters of the value for automobiles, namely, $\sigma_2 = 3.3$ per cent, equivalent to an annual depreciation rate of 13.87 per cent.

For CONDELES 2, the single durable good is redefined to include both Motor Vehicles and Other Household Durables. The two rates derived for CONDELES 1 are weighted by the quarterly per capita expenditures on Motor Vehicles and Other Household Durables in 1966-67 ($16 and $23 respectively) to obtain

$$\sigma = \{16(.044) + 23(.033)\}/39 = .0375 ,$$

which is 3.75 per cent per quarter, equivalent to an annual depreciation rate of 15.86 per cent.

13.3 Real Quarterly Interest Rates

A number of steps were involved in calculating ρ^*_t, the real quarterly interest rate. Firstly the annual nominal interest rate series were obtained. Walters[1] supplied data on the overdraft rate for the entire sample period. Annual figures for the hire purchase interest rate

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1. See Walters ([42], Appendix 5).

for 1972-76 were provided by Walters[1] while figures for 1959-68 were obtained from the IAC study.[2] An interpolation was made based on movements in the overdraft rate for the intervening years. The hire purchase and overdraft rates were weighted according to the relative proportions financed by each method, the data for 1959-73 being provided from a survey conducted by the IAC[3] and the remaining three years being extrapolations based on the preceding years.

Nominal annual rates were converted to nominal quarterly rates by using the formula

$$\rho_t = (1 + \rho_{at})^{\frac{1}{4}} - 1 \quad ,$$

where ρ_t (ρ_{at}) is the nominal quarterly (annual) interest rate during t .

Quarterly expected inflation rates were calculated from semi-log ordinary least squares regressions of the Consumer Price Index (CPI) over the preceding eight quarters using the formula

$$g_t = \frac{3}{14}\left\{\frac{1}{9}\sum_{h=1}^{8} h \, \ell n \, CPI_{t+h-9} - \frac{1}{2}\sum_{j=1}^{8} \ell n \, CPI_{t-j}\right\} ,$$

where g_t is the quarterly expected inflation rate for time t and CPI_t is the Consumer Price Index during t . Finally the real quarterly interest rate was determined by subtraction; that is, $\rho_t^* = \rho_t - g_t$.

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1. Ibid.
2. See IAC mimeograph ([12], page 32).
3. Ibid.

13.4 Permanent Per Capita Incomes of the Employed and Unemployed

Both versions of CONDELES incorporate the assumption that the present value of expected changes in labour income for the employed and unemployed is zero so that measured and permanent income are the same.[1] Total income in the economy, $M_t Z_{tM} + L_t Z_{tL}$, is defined as the sume of farm and non-farm incomes, dividends and all transfer payments, net of all taxes levied on income.

Total population is divided by the sum of persons registered for employment, civilian non-farm employment, farm employment and defence employment to obtain the average household size of three persons in CONDELES.[2] Under the assumption that the average household size is invariant to the employment status of the head, the average household size is multiplied by the number of persons registered for employment to obtain L_t . M_t is determined as the residual in equation (36).

According to the Poverty Inquiry[3] the poverty line for three persons in 1975 was $70.40 per week. The quarterly per capita income for an unemployed consumer, Z_{tL} , was obtained by using the formula

$$Z_{tL} = \frac{70.4}{3} \times 13 \times \frac{CPI_t}{CPI_{75}} ,$$

where CPI_{75} (= 1.802) is the average value of the CPI in 1975 relative to the base year 1966-67.

---

1. Williams and Chang [45] have removed this assumption within the context of the ELES for Taiwan by experimenting with four measures of "income." ELES estimates were found to be very similar in all cases. Considerations about permanent income are left for future work. Also see Powell [28].
2. See Walters [42].
3. Ibid.

The quarterly per capita income of the employed consumer, Z_{tM}, is obtained by using

$$Z_{tM} = \frac{\text{Total Income} - L_t Z_{tL}}{M_t} ,$$

and is weighted by the coefficient ξ_{tM} to take account of the expected growth of permanent income and prices.

13.5 Per Capita Stock of Motor Vehicles of the Employed

The stock of services of Motor Vehicles is obtained from the published demand study by the IAC.[1] Since $\{\pi_{1t}\}$ is the quality-adjusted price index series for a new medium-sized Motor Vehicle in 1966-67, it is first necessary to convert the stock series into new 1966-67 medium vehicle equivalents. Given that new small, medium and large Motor Vehicles cost \$1952, \$2516 and \$6009 respectively in 1966-67, the stock figure is divided by 0.24.[2] We then multiply by 2.516 to generate the value series $\{K_{1t}\}$, which is expressed in thousands of dollars.[3]

Letting K_{1t} denote the total number of new 1966-67 medium vehicle equivalents in value terms, it is necessary to allocate the stock of Motor Vehicles to the employed and unemployed. According to the 1971 Population Census,[4] households whose head is employed tend to have a larger number of cars than those whose head is unemployed (an average of 1.332 as against 0.904). Using the notation of this section we may write NMV_t as the total number of Motor Vehicles owned by the work force during t,

---

1. See IAC mimeograph [12].
2. 0.24 = 2516/(1952 + 2516 + 6009).
3. Specifically, a new medium-sized Holden cost \$2516 in 1966-67.
4. See Walters [42].

that is

$$NMV_t = 1.332\ E_t + 0.904\ U_t\ .$$

The factors of proportionality discussed in section 4 are calculated within CONDELES according to the following relationships

$$\psi_{1tM} = 1.332\ E_t/NMV_t$$

and

$$\psi_{1tL} = 1 - \psi_{1tM}\ .$$

Since K_{1t} and ψ_{1tM} are available, K^M_{1t} can be obtained. Dividing K^M_{1t} by M_t leads to k^M_{1t}, the per capita stock of Motor Vehicles of the employed at the end of quarter t.

13.6 Initial Values of Parameters

The parameters for the two versions of CONDELES can be expressed as

$$\{\gamma_1,\ \gamma_2,\ \gamma_3,\ \Theta_1,\ \beta_{1M},\ \beta_{2M},\ \eta_1,\ \eta_2,\ \delta_M\} \tag{40}$$

and

$$\{\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4,\ \beta_{1M},\ \beta_{2M},\ \beta_{3M},\ \eta,\ \delta_M\} \tag{41}$$

respectively, since the other parameters in (23) can be written as functions of parameters using the restrictions given in section 6 and the constraints (38) and (39).[1]

1. Note that the parameter ε, relating the expected growth rates of permanent income and all prices in both versions of CONDELES, does not need a starting value as it will be estimated by iterating over the range [0, 2] at intervals of 0.05.

The estimation algorithm requires starting values to be given for the nine parameters of each model. Initial values for CONDELES 1 are plausible guesses based on previous studies of the ELES,[1] where the "subsistence" parameter estimates are less than the minimum actual consumption values during the sample period. However early computer estimates for CONDELES 1 were also taken into account in selecting initial parameter values for CONDELES 2. Apart from the first two commodities, Food and Rent, the consumption categories are not identical for the two models. Since the third commodity in model 1, Other Non-Durables, has been split into two groups for the second version, the initial value of γ_3 in (40) equals the sum of initial values of γ_3 and γ_4 in (41). Initial values of parameters in CONDELES 1 and 2 respectively are given by[2]

$$\{52, 6, 84, 42, .084, .2275, .05, .05, .01\}$$

and

$$\{52, 3, 61, 23, .084, .2275, .05, .05, .01\} \quad .$$

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1. See Lluch, Powell and Williams [18].

2. There appears to be no evidence to suggest that final parameter estimates are sensitive to the initial values chosen for parameters. Although different starting values may lead to different final estimates, the latter will typically be converging to the previous solution values. See Wymer [46] and section 14 below.

14. EMPIRICAL RESULTS

MACRES, an enlarged version of RESIMUL, is used to estimate CONDELES. In section 14.1 we discuss the results for the original model, CONDELES 1, which includes the use of stock data for Motor Vehicles. Then in section 14.2 the results for CONDELES 2, with a different classification of commodities and a smaller data base, are discussed. Although there are still five commodities, only one durable good is used and it is felt that this is less of a burden on the data in view of the parameterizations involved. We also examine the effects of increasing the real interest rate by five percentage points and discuss the results for CONDELES 2A, a slightly modified version of CONDELES 2, in which only one durable commodity is distinguished.[1] RESIMUL is then tested for sensitivity of final solutions to initial values of parameters and the model is checked for uniqueness of estimates. Finally, CONDELES 2 is transformed to the simple Linear Expenditure System (LES)[2] in section 14.3 to enable comparisons to be made. Tentative conclusions are then presented.

Unless otherwise stated, all parameter estimates are highly significant. The hat above a parameter denotes the corresponding FIML

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1. We did not, in fact, modify the algebraic morphology of the system, because of the massive amount of reprogramming that would have been entailed. Having aggregated Motor Vehicles into the fourth equation of Table 1, we used the third equation of that table for purchases of Clothing (which was disaggregated from Other Non-Durables). Equation 3 was then turned into an equation for consumables by using a depreciation rate of 100 per cent per quarter.

2. The properties and interpretation of the LES are well known. See Powell [29].

estimate and the bar above a variable refers to mean value; for example, $\hat{\bar{\mu}}_M$ is the estimate of the MPC of the employed at the mean quarterly real interest rate (.0213 per cent per quarter, for all cases but one). Both the MPC of the employed and its asymptotic standard error are calculated at the sample mean real interest rate.

14.1 CONDELES with Two Durable Goods : Instantaneous Stock Adjustment for Motor Vehicles

For purposes of comparison the average quarterly per capita expenditure for each commodity and their respective proportions in the base year 1966-67 are listed in Table 3. Dominant features of the results presented are :

(i) Significant serial correlation exists for all equations, particularly for the Motor Vehicle stock equation in which there are only 3 changes of sign in 66 observations.

(ii) All estimates of subsistence parameters are positive and less than the average per capita expenditures. However, the estimate for Rent is relatively low.

(iii) All the marginal budget shares (MBS's) for the unemployed are positive and exceed the respective MBS's for the employed. The only MBS not positive is for Other Household Durables and, although statistically significant, its low absolute value is economically implausible. While the MBS for Motor Vehicles is of a plausible order of magnitude when compared with its proportion of expenditure in 1966-67, it appears to be rather low.

TABLE 3

ESTIMATES OF CONDELES 1, AUSTRALIA 1960 I TO 1976 II

Commodity Group[1]	Mean Quarterly Expenditure in 1966-67 $	Proportion	Subsistence Parameters[2]	Marginal Budget Shares for the Employed	Unemployed	Z^3
Food	64.50	0.217	52.22 (0.302)	0.081 (0.002)	0.088 (0.002)	-6.33
Rent	35.11	0.118	2.16 (0.589)	0.241 (0.003)	0.263 (0.004)	-5.83
Other Non-Durables	161.64	0.543	79.52 (0.705)	0.594 (0.004)	0.649 (0.004)	-6.33
Motor Vehicles	14.47	0.049	6.10 (0.231)	0.084 (0.002)		-7.32
Other Household Durables	21.76	0.073	8.74 (0.596)	-0.00004 (0.000001)		-6.33
TOTAL	297.48	1.000	148.74	1.000	1.000	

Estimates of:

Time Preference Discount Rate of the Employed	δ_M	0.0181 (0.0002)
MPC of the Employed	$\bar{\mu}_M$ $(=\delta_M / \bar{\rho}^*_M)$	0.850 (0.0094)
Ratio of expected rate of growth of permanent income to expected rate of growth of prices	ε	0.800 (n.c.)

Note: Asymptotic standard errors are given in parentheses.

1. The Food category includes Drink and Tobacco; the Other Non-Durables group includes Clothing, Footwear and Textiles.
2. Units are dollars per capita in 1966-67 prices.
3. Z is the test statistic for independence of residuals (see Appendix B).

n.c. Not computed; ML value obtained by iterating with external settings of ε .

(iv) The estimated subjective time preference discount rate of the employed is 1.81 per cent per quarter and the MPC of the employed is 0.85. Although the latter is high, it is not inconsistent with previous results obtained for Australia in the context of the ELES.[1] The MPC of the employed is less than that of the unemployed which is, by construction, set equal to unity.

(v) The ratio of the expected rate of growth of permanent income to the expected rate of growth of prices, ε , is estimated to be 0.8. This estimate differs from the usual assumption of unity in applications of the ELES.[2]

(vi) Convergence is relatively fast, requiring only 8 iterations.

14.2 Aggregation of Motor Vehicles and Other Household Durables : CONDELES with a Single Durable Good

Apart from the existence of serial correlation, the only noticeable problem in CONDELES 1 is the implausibly low negative value of the MBS for Other Household Durables. In an attempt to remedy this, we decided to aggregate the two durables and disaggregate Other Non-Durables into Clothing and Other Non-Durables. We also felt that the sample period may be too long to justify our assuming constancy of parameters.[3] Rather than introduce various hypotheses about the variation of parameters over time[4] we decided to reduce the sample period so that it now began

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1. See Lluch, Powell and Williams ([18], page 45).
2. Ibid., and references to the ELES literature contained therein.
3. See Lluch and Williams [19].
4. For example, see Pollak and Wales [27], Phlips [24] and Lluch [17].

March quarter 1962 and ended December quarter 1972, giving a total of 44 observations. The most striking features of the estimates presented in Table 4 are :

(i) Serial correlation is still highly significant everywhere, particularly for Durable Goods which has only 1 change of sign in 44 observations.

(ii) All estimates of subsistence parameters are positive. The estimate for Durable Goods is marginally higher than its corresponding average quarterly per capita expenditure and is nearly three times as large as the sum of the subsistence estimates for both durables in CONDELES 1 (see Table 3).

(iii) Other than for Durables which has a negative but insignificant MBS, all MBS's are positive. However, the estimate for Clothing is very low. The estimate for Other Non-Durables is a surprise since the MBS for the employed is actually higher than its corresponding estimate in CONDELES 1, even though Clothing and Other Non-Durables are aggregated in CONDELES 1. The corresponding estimate for the unemployed is only marginally higher in CONDELES 2. Since the MBS for Durable Goods is, to all intents and purposes, equal to zero, there is no practical difference between the estimated MBS's for the employed and the unemployed.

(iv) The subjective discount rate of the employed has increased to 2 per cent per quarter while the MPC has increased to 0.939, still a little high for Australia.

TABLE 4

ESTIMATES OF CONDELES 2, AUSTRALIA 1962 I TO 1972 IV

Commodity Group[1]	Mean Quarterly Expenditures in 1966-67 $	Proportion	Subsistence Parameters[2]	Marginal Budget Shares for the Employed	Unemployed	Z[3]
Food	64.50	0.217	48.23 (0.337)	0.103 (0.002)	0.103 (0.002)	-5.34
Rent	35.11	0.118	4.51 (0.854)	0.211 (0.005)	0.211 (0.005)	-5.03
Clothing	28.81	0.097	25.36 (0.301)	0.010 (0.001)	0.010 (0.001)	-5.64
Other Non-Durables	132.83	0.446	31.11 (1.085)	0.676 (0.005)	0.676 (0.005)	-5.64
Durable Goods	36.23	0.122	39.53 (0.792)	-0.0004 (0.001)		-6.25
TOTAL	297.48	1.000	148.74	1.000	1.000	

Estimates of:

Time Preference Discount Rate of the Employed	δ_M	0.0200 (0.0004)
MPC of the Employed	$\bar{\mu}_M$ $(=\delta_M / \bar{\rho}^*_M)$	0.939 (0.0188)
Ratio of expected rate of growth of permanent income to expected rate of growth of prices	ε	0.800 (n.c.)

Note: Asymptotic standard errors are given in parentheses.

1. Clothing includes Footwear and Textiles. Durable Goods include Motor Vehicles and Other Household Durables.
2. Units are dollars per capita in 1966-67 prices.
3. Z is the test statistic for independence of residuals (see Appendix B).

n.c. Not Computed (see footnote to Table 3).

(v) There is no change in the estimate of the ratio of expected growth rates at 0.8.

(vi) Convergence is very slow with 146 iterations. In fact, even after 51 iterations the estimates were still very close to the initial values. The subsistence parameters for Rent and Clothing became negative midway through the iterations but finally ended at 4.51 and 25.36 respectively; the MBS for Durable Goods did go as high, relatively speaking, as 0.01 before ending at - 0.0004.

(vii) To examine whether we had really obtained a unique root of the likelihood equation, another run was made using new starting values.[1] Although the new estimates did not converge to the same solutions immediately, they were heading in the same direction as before. A tight convergence test appears to be all that is necessary to obtain "unique" estimates.

14.2.1 Variations in the Quarterly Real Interest Rate

To gauge the effects of altering the quarterly real rate of interest on expenditures we decided to increase the rate by five percentage points so that $\bar{\rho}^*_M$ = 0.0713. Since the commodity classification is identical to that in the previous section, the mean quarterly expenditures and their respective proportions in 1966-67 are omitted from Table 5 for brevity. The most notable similarities and differences are :

(i) The same problem exists with serial correlation in all equations with only 1 change in sign in the sequence of residuals for Durable Goods.

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1. The new starting values were twice the solution values at iteration 146 minus the solution values at iteration 120.

TABLE 5

ESTIMATES OF CONDELES 2 WITH HIGHER QUARTERLY REAL INTEREST RATES[1], AUSTRALIA 1962 I TO 1972 IV

Commodity Group	Subsistence Parameters[2]	Marginal Budget Shares for the Employed	Marginal Budget Shares for the Unemployed	Z^3
Food	50.46 (0.303)	0.091 (0.002)	0.092 (0.002)	-5.64
Rent	-6.06 (0.655)	0.287 (0.004)	0.290 (0.004)	-5.64
Clothing	21.10 (0.300)	0.018 (0.001)	0.018 (0.001)	-5.03
Other Non-Durables	46.12 (0.765)	0.594 (0.004)	0.600 (0.004)	-5.03
Durable Goods	37.12 (0.789)	0.010 (0.002)		-6.25
TOTAL	148.74	1.000	1.000	

Estimates of:

Time Preference Discount Rate of the Employed	δ_M	0.0120 (0.0004)
MPC of the Employed	$\bar{\mu}_M$ $(=\delta_M / \bar{\rho}^*_M)$	0.168 (0.0056)
Ratio of expected rate of growth of permanent income to expected rate of growth of prices	ε	0.800 (n.c.)

Note: Asymptotic standard errors are given in parentheses.

1. The results presented are for the case where quarterly real interest rates have been increased by five percentage points (ie., $\bar{\rho}^*_M = 0.0713$).

2. Units are dollars per capita in 1966-67 prices.

3. Z is the test statistic for independence of residuals (see Appendix B).

n.c. Not computed (see footnote to Table 3).

(ii) Apart from Rent, where the negative and significant coefficient suggests that demand is elastic with respect to its own price, all subsistence parameter estimates are positive. The estimate for Other Non-Durables has increased by fifty per cent while the estimate for Durable Goods is essentially unchanged, being marginally greater than the corresponding mean quarterly expenditure.

(iii) All estimated MBS's are positive but those for Clothing and Durable Goods are unrealistically low at 0.018 and 0.01 respectively. The MBS's for consumables do not differ significantly between the employed and the unemployed.

(iv) The time preference discount rate of the employed, δ_M , has surprisingly fallen to a low 1.2 per cent per quarter when the real interest rate was increased by five percentage points. This resulted in an extremely low MPC of the employed of 0.168, which is nevertheless significant. The tendency of the subjective discount rate, and hence of the MPC, to vary indirectly with the real rate of interest, was confirmed in other results.

(v) Once again, ε is estimated to be 0.8.

(vi) The estimate converged after 58 iterations. New starting values were used to test for uniqueness. With the exception of the MBS for Durable Goods, η , and δ_M , the solution values for CONDELES 2 were used as new starting values. Initial values for η and δ_M were chosen as 0.6 and 0.067 respectively, the latter being implied by the estimate of the MPC of 0.939 (see Table 4), together with a mean real rate of interest of 0.0713. The estimates did not converge immediately but they apparently were heading back toward the previous solution values after 30 iterations.

14.2.2 Instantaneous Stock Adjustment for Clothing with an Infinite Decay Rate

A revealing experiment was performed with CONDELES. Using the commodity classification for CONDELES 2 (4 consumables, 1 durable) but the model specification for CONDELES 1 (3 consumables, 2 durables, a stock equation for Motor Vehicles), the equation for Motor Vehicles was used for Clothing, together with a quarterly depreciation rate of 100 per cent (equivalent to an infinite instantaneous decay rate in a continuous time model). Imposing such a high rate eliminated lagged stocks as an explanatory variable, so no stock data were required. Since the purpose of this exercise was to examine whether the model specification was sufficiently flexible to allow us to estimate the parameters associated with a consumable from an equation specifically derived for a durable, we ignored the problem of purchases of Clothing by the unemployed.[1] The results are presented in Table 6. Noteworthy points are :

(i) Serial correlation is still severe for all equations.

(ii) Excluding Rent, where the coefficient is negative but insignificant, all subsistence parameters are positive and less than their corresponding mean quarterly expenditures, apart from Durable Goods. The estimate for Rent was slowly approach zero when the system converged.

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1. Alternatively, the model may be conceived of as one in which only the employed are assumed to buy Clothing and Durable Goods.

TABLE 6

ESTIMATES OF CONDELES 2A, AUSTRALIA 1962 I TO 1972 IV

Commodity Group	Subsistence Parameters[1]	Marginal Budget Shares for the Employed	Marginal Budget Shares for the Unemployed	Z^2
Food	48.96 (0.329)	0.097 (0.002)	0.100 (0.002)	-5.34
Rent	-1.00 (0.888)	0.242 (0.005)	0.250 (0.006)	-5.34
Other Non-Durables	36.42 (1.031)	0.629 (0.005)	0.650 (0.005)	-5.34
Clothing	25.29 (0.256)	0.031 (0.001)		-5.64
Durable Goods	39.07 (0.806)	0.0009 (0.0008)		-6.25
TOTAL	148.74	1.000	1.000	

Estimates of:

Time Preference Discount Rate of the Employed	δ_M	0.0202 (0.0004)
MPC of the Employed	$\bar{\mu}_M$ $(\delta_M / \bar{\rho}^*_M)$	0.948 (0.0188)
Ratio of expected rate of growth of permanent income to expected rate of growth of prices	ε	0.800 (n.c.)

Note: Asymptotic standard errors are given in parentheses.

1. Units are dollars per capita in 1966-67 prices.
2. Z is the test statistic for independence of residuals (see Appendix B).

n.c. Not computed (see footnote to Table 3).

(iii) Other than for Durable Goods, where the estimate is positive but insignificant, all MBS's are both positive and significant. The MBS for Clothing is still low at 0.031.

(iv) Since $\bar{\rho}_M^*$ has been reduced to 0.0213, δ_M has increased to 2.02 per cent per quarter. Accordingly, the MPC of the employed has risen to 0.948, the highest estimate yet for any version of CONDELES.

(v) Once again, ε is estimated to be 0.8.

(vi) The estimates converged very slowly, but not monotonically, after 60 iterations.

14.3 Infinite Decay Rates for Durables : the Linear Expenditure System

The overwhelming impression one obtains from the above results is that the MBS for Durables is implausibly low. To assist in understanding the results of CONDELES, a simples LES has been fitted by Lawson[1] using the same data base. These results are reported in Table 7. As is conventional in such allocative systems, the errors are subject to an exact linear constraint so the covariance matrix is singular.[2] Since likelihood methods require inversion of the covariance matrix, the solution involves dropping an arbitrarily selected equation from the system. All estimates in the Aitken family are known to be invariant to the choice of equation deleted.[3] Since Durable Goods are of particular interest, the

1. See Lawson [15].
2. See Parks ([21] and [22]) and Pollak and Wales [27].
3. See Powell [29].

TABLE 7

ESTIMATES OF THE LINEAR EXPENDITURE SYSTEM, AUSTRALIA 1962 I TO 1972 IV

Commodity Group[1]	Sample Mean Expenditures $	Proportion	Subsistence Parameters[2]	Marginal Budget Shares	Asymptotic R^2 values[3]	Z^4
Food	65.95	0.206	69.38 (2.31)	0.102 (0.003)	N.A.	N.A.
Rent	39.41	0.123	42.98 (2.67)	0.140 (0.007)	0.9998	-4.73
Clothing	31.01	0.097	32.95 (1.55)	0.068 (0.003)	0.9998	-3.81
Other Non-Durables	142.97	0.447	158.91 (10.05)	0.484 (0.010)	0.99995	-3.20
Durable Goods	40.64	0.127	48.45 (5.00)	0.206 (0.014)	0.9991	-2.29
TOTAL	319.98	1.000	352.67	1.000	0.99999[5]	

Note: Asymptotic standard errors are in parentheses.

1. The system is estimated without the Food equation.
2. Units are dollars per capita in 1966-67 prices.
3. See Carter and Nagar [3] and Ryland [32].
4. Z is the test statistic for independence of residuals (see Appendix B).
5. Asymptotic R^2 value for the entire system of equations.

N.A. Not Applicable.

expenditure equation for one of the consumables, Food, is not directly estimated.

Since the LES does not distinguish between consumables and durables, we do not proceed with the assumption that only the employed purchase durables. The model is therefore reformulated on a per capita basis and disregards employment status as an identifying household characteristic. Since the burden on the data is felt to be particularly light, no restriction is placed on the subsistence expenditures in the base year. For current purposes, estimates of subsistence parameters in the base year are to be compared with the sample mean per capita expenditures. Key points to note are :

(i) Serial correlation is still strong, but is surprisingly weakest for Durable Goods. It is well known that the LES yields subsistence parameter estimates that are very sensitive to error specification, in particular, the value of the first-order autocorrelation coefficient.[1]

(ii) All subsistence parameters are positive but exceed the corresponding sample mean expenditures for each commodity. Estimates for Rent and Other Non-Durables are substantially higher than their CONDELES counterparts.

(iii) While all MBS's are positive the estimate for Durable Goods is, for the first time in the results reported here, both statistically significant and economically meaningful. This substantial improvement for Durables more closely reflects the

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1. See Lluch and Williams [19].

importance of this commodity in the consumption pattern of the household. The MBS for clothing has more than doubled its previous estimates.

(iv) The overall fit is high, but this is to be expected from time series data.

(v) Compared with the slow convergence of CONDELES, the LES converged after only 4 iterations.

14.4 Tentative Conclusions

The essential features of the empirical results presented in the above tables are summarized below.

(i) The preponderance of serial correlation is particularly disturbing, especially for the Durable Goods equation where there is usually only one change in sign in 44 observations. In some cases these residuals are a very large proportion of the observed values.

(ii) Restricted estimation appears to have worked reasonably well. When the restrictions are valid, estimates will be more efficient than those obtained without restrictions; if they are invalid, the estimates will be inconsistent. The sum of subsistence expenditures in the base year was restricted in an attempt to alleviate the well known erratic behaviour of the individual estimates. This was successful -- in CONDELES, the estimated individual subsistence parameters were very stable. Individual estimates are positive and (serial correlation aside) highly significant, with the exception of two cases for Rent. Moreover, these estimates are less than the average expenditure for each commodity during the sample period, with the exception of Durable Goods, a decided improvement over other results in the literature.[1] (Note the substantial increase in subsistence parameter estimates for the LES when the constraint is not imposed.)

(iii) The MBS's are, with the exception of Durable Goods, all positive and highly significant. They are also of a similar order of magnitude for both the employed and the unemployed. When we insist on real durability, the MBS's for Durables collapse to values in the neighbourhood of zero; if we allow durables to be treated as consumables, as in the case of the LES (where, as an additional difference, employment status is not stressed), we obtain sensible estimates. Clearly, further investigation is warranted.

(iv) Unlike other research related to the ELES, the interest rate was used as a variable rather than as a parameter. This reduced the burden that has been typically foisted upon the ELES, where inferences about both interest rate and utility parameters have been expected. Consequently, the wildly erratic behaviour of the market discount and time preference

1. This feature has led to applied work with shifting subsistence parameters. See Pollak and Wales [27] and Phlips [24].

discount rates was not encountered, unlike previous work by Powell[1] who found that, despite extreme variations in the estimates of the individual parameters, the estimate of their ratio, the MPC, was highly stable. With one exception, estimates of the MPC of the employed were stable and highly significant, although a little high in the context of Australia. However, when we note that the ratio $\hat{\delta}_M/\bar{\rho}^*_M$ $(= \hat{\bar{\mu}}_M)$ is the estimated MPC of the employed, evaluated at the sample mean real interest rate, only in the stationary state, then the estimates may not be larger than expected.[2]

(v) The ratio of the expected rate of growth of permanent income to the expected rate of growth of prices was estimated to be 0.8 in all cases of CONDELES. This was sufficiently different from the implicit assumption of unity in many applications of ELES[3] and is an interesting result.

(vi) Generally speaking, different starting values did not lead to noticeably different solution values. The number of iterations required for convergence differed dramatically, ranging from 146 for CONDELES 2 to 4 for the simplest model tested, the LES.

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1. See Powell [28].

2. See section 6 above.

3. See Lluch, Powell and Williams [18].

15. CONCLUDING REMARKS AND RESEARCH PERSPECTIVES

Emperical results for CONDELES lend some support to the theoretical framework developed in section 2. Nevertheless, there remain unresolved difficulties, some of which are currently under investigation. A partial catalogue of factors which may have contributed to the mixed performance of the model empirically, is given below:

(i) Consumers not in the work force have been dealt rough justice. An additional category for those households whose head is excluded from the work force might be introduced in addition to the two categories already examined in the model, since these consumers may display a markedly different pattern from both the employed and the unemployed.

(ii) The expectations hypothesis needs to be investigated in greater detail. The employed and unemployed may have different price expectations and, even within each employment category, consumers may have different expectations regarding the prices of durables and consumables. Expectations models for permanent income could also be improved.[1] In particular, the employed and unemployed may have different anticipations concerning the present value of expected changes in labour income. The unemployed may also not have a uniform expected inflation rate for both prices and labour income, contrary to our assertion in section 7.

1. See Williams and Chang [45] and Powell [28].

Use of cross-section data may provide additional information on these unsettled issues.

(iii) The Motor Vehicles equation performed poorly, even though what we believe to be relatively accurate stock data were used. This may suggest that the asset data should be restructured. Used Motor Vehicles are converted into new standard (i.e. medium) 1966-67 car equivalents using weights which do not reflect the observed, but as yet unmeasured, time-series variations in new/used car price relativities. Capital gains and losses originating from this source do not appear in our theory.

(iv) Approximating a highly non-linear (in-the-variables) model by a Taylor-series linearization is not ideal. The alternative procedure is to use an estimation package allowing non linearities in the variables (such as Wymer's ASIMUL program). This remains to be investigated.

(v) Further experimentation for depreciation rates may be necessary. If, in fact, the rate of depreciation were to measure simultaneously the physical rate of depreciation and the psychological rate at which the habit of using a durable (such as the number of miles travelled by Motor Vehicles) wears off[1], a higher rate of depreciation than

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1. The literature suggests that the original conception of the depreciation rate in terms of the average service life of a durable good may need to be modified, especially if consumers place a premium on newness. See Houthakker and Taylor [11], Phlips ([24] and [25]) and Sexauer [33].

those used in this study might be in order. Subsistence parameters may vary with the stocks of habits. Under certain assumptions, this can be reduced to the ELES with transformed variables that can be justified in terms of a first-order autocorrelation process.[1]

(vi) The presence of serial correlation has been simply noted, as the consequences are well known. If the continuous time version of the structural from is correct, in the sense that only white noise errors need be added to it in order to obtain a valid representation of consumers' behaviour, then in the discrete time analogue we would expect serially correlated errors. Bergstrom and Wymer [1a, p.36] propose a "COSMA" transformation for pre-whitening the errors of the discrete analogue data. We have not used this transformation. Recent empirical work with an Australian minimal macro model, moreover, suggests that more powerful filters than COSMA would be needed to filter out all autocorrelation induced by movement from continuous to discrete time (Giles [8a]).

(vii) Simultaneity bias may be present owing to the inclusion of measured income as an explanatory variable in each equation of the model. Income is exogenous in the micro model presented in section 2 but the inevitable use of macro

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1. See Lluch [17] and Lluch and Williams [19].

data presages at least some simultaneous feedback of consumption on to income. The magnitude and sign of any bias in the estimates arising from this source are unknown.

(viii) Limitations of the CONDELES model itself should be noted. Planning under certainty, with perfect capital markets, is a strong assumption of the model. Price, income and interest rate expectations are assumed to be continuous and differentiable functions of real time, so that expectations are not permitted to change abruptly. But abrupt changes in expectations may, of course, be a feature of the real world.

(ix) The discrete analogues used here in the empirical implementation of a continuous time model are basically very crude. Whereas they may be adequate for the purposes of most dynamic models, the continuous stock equilibrium assumption of DELES may cause the estimates to be sensitive to the discretization procedures adopted. This aspect remains to be investigated.

(x) Finally, potentially revealing research is progressing in introducing alternative forms of holding wealth so that the theory of consumption and savings may be integrated with the demand for money. This may be accomplished by introducing the stocks of assets of different kinds (including liquid assets) directly into the utility function (Clements [3a]).

APPENDIX A

STOCK HOLDING UNDER CONTINUOUS STOCK EQUILIBRIUM

Given the assumptions in the text, the change in the consumer's holdings of durable j , at time t , is given by totally differentiating (4) with respect to τ and evaluating the result at $\tau = t$. That is,

$$\dot{k}_{jt} = \left[\frac{\partial k_{jt}(\tau)}{\partial \tau}\right]_{\tau=t} + \left[\frac{\partial k_{jt}(\tau)}{\partial \pi_{jt}}\dot{\pi}_{jt} + \frac{\partial k_{jt}(\tau)}{\partial Z_t^*}\dot{Z}_t^* + \frac{\partial k_{jt}(\tau)}{\partial \rho_t}\dot{\rho}_t\right]_{\tau=t} ,$$

where the change in stock holdings has been partitioned into two components, the first term on the right being the movement <u>along</u> the path and the second term being a movement <u>of</u> the path, arising from changes in expectations regarding prices of durables, supernumerary permanent income and the market interest rate. The above result, together with (3) and (4), is used to obtain (6). The following diagrammatic explanation is useful in understanding the mechanics of the partitioning.

In the figure, we have shown five schedules. The three marked with arrow heads are consumer plans. They show how the consumer, at each of the points in real time, T, s, and t^*, plans his future holdings of durable j . For our particular illustration, changes in the consumer's expectations over the period T to t^* have been causing upward revisions - the later planning schedules lie above the earlier ones. The upward revisions are also illustrated by the fourth schedule (shown as joining the □'s). For each point of real time, T, s, t^*, etc., this schedule shows the planned holding of durables for time t^* , i.e., it shows the points $k_{tj}(t^*)$ for all $t \leqslant t^*$. The positive slope implies that changes in the

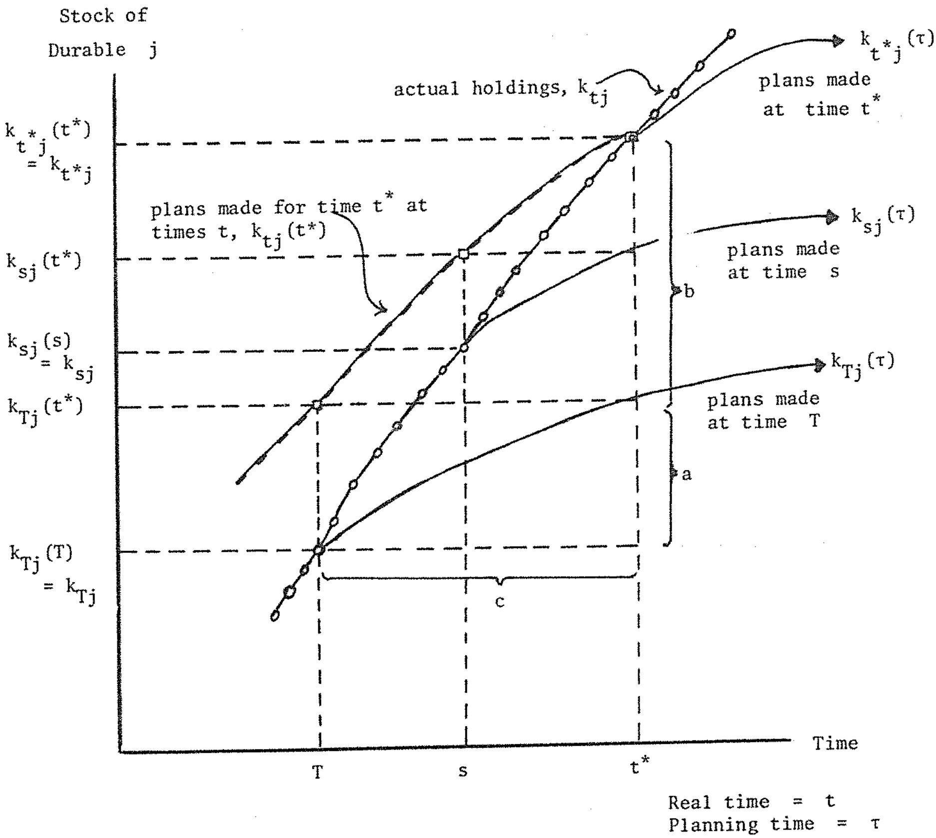

FIGURE 1 : Movements along the path and movements of the path.

consumer's expectations were causing him to increase his holdings of durable j, planned for time t^*, as time approaches t^*. The reader will see that the $k_{tj}(t^*)$ schedule is constructed by taking points at which the planning schedules cross the t^* abscissa. The fifth schedule, shown as joining the dots, is the path of the actual holdings of durable j. Actual holdings are assumed to equal currently planned holdings and therefore the "actual" schedule links the initial points on each of the planned schedules.

The change in actual holdings over the period t to t^* can be written as

$$k_{t^*j} - k_{Tj} = \left[k_{t^*j} - k_{Tj}(t^*)\right] + \left[k_{Tj}(t^*) - k_{Tj}\right] .$$

Then replacing k_{t^*j} and k_{Tj} by $k_{t^*j}(t^*)$ and $k_{Tj}(T)$, and dividing through by $t^* - T$, we obtain

$$\text{(A1)} \quad \frac{k_{t^*j} - k_{Tj}}{t^* - T} = \frac{\left[k_{t^*j}(t^*) - k_{Tj}(t^*)\right]}{t^* - T} + \frac{\left[k_{Tj}(t^*) - k_{Tj}(T)\right]}{t^* - T} .$$

In terms of the diagram, the expression on the left of the equality is $(a + b)/c$, and as T approaches t^*, this becomes the slope of the actual schedule at t^*, i.e., $\dot{k}_{t^*j}$.

The first term on the right of (A1) is represented by b/c. For T close to t^*, this term approximates the slope of the $k_{tj}(t^*)$ schedule for any t in the interval $[T, t^*]$. In the limit as T

approaches t^*, b/c becomes the slope (defined only to the left since $k_{tj}(t^*)$ does not exist for $t > t^*$) of the $k_{tj}(t^*)$ schedule at t^*, i.e.,

$$\frac{b}{c} \to \left.\frac{\partial k_{tj}(t^*)}{\partial t}\right]_{t=t^*}$$

where the derivative is understood to be a left derivative.

The second term on the right side of (A1) is a/c. As T approaches t^*, this term approaches the slope at t^* of the planning schedule made at T, i.e.,

$$\frac{a}{c} \to \left.\frac{\partial k_{Tj}(t^*)}{\partial t}\right]_{t=t^*} ,$$

or equivalently,

$$\frac{a}{c} \to \left.\frac{\partial k_{t^*j}(t)}{\partial t}\right]_{t=t^*} ,$$

where the derivative is understood to be a right derivative. (Note that $k_{t^*j}(t)$ is not defined for $t < t^*$).

We conclude that

$$\dot{k}^*_{tj} = \left.\frac{\partial k_{tj}(t^*)}{\partial t}\right]_{t=t^*} + \left.\frac{\partial k_{t^*j}(t)}{\partial t}\right]_{t=t^*}$$

where the first derivative is defined from the left, while the second is defined from the right.

APPENDIX B

A NON-PARAMETRIC SIGN TEST FOR SERIAL CORRELATION

Let n be the number of observations and let r denote the number of changes of sign of the residuals. If we assume that positive and negative values are equally likely to occur, the probability of a sign change is 0.5 .

Thus r follows the Binomial Distribution with parameters $(n - 1)$ and 0.5.

For n large, we have

$$z = \frac{r - 0.5(n - 1)}{\sqrt{0.25(n - 1)}} \sim N(0, 1) .$$

Under the null hypothesis, the residuals are independently distributed and the test is two-tailed.

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