14. Weiss, Randall, D., 'Elasticities of Substitution among Capital and Occupations in U.S. Manufacturing', Journal of the American Statistical Association, Vol. 72, December 1977, pp. 764-771.

15. Wymer, C.R., Computer Programs : RESIMUL Manual (Washington, D.C. : International Monetary Fund, 1977).

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ESTIMATION OF OCCUPATIONAL DEMANDS IN AUSTRALIAN INDUSTRY

by

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and
D. J. Parham

Industries Assistance Commission

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REFERENCES

1. Allen, R.G.D., Mathematical Analysis for Economists, Macmillan, London, (1938).

2. Barten, Anton, P., 'The Systems of Consumer Demand Functions Approach : A Review', Econometrica, Vol. 45, January 1977, p23-51.

3. Berndt, E.R. and Christensen, L.R., 'Testing for the Existence of a Consistent Aggregate Index of Labour Inputs', American Economic Review, Vol. 64, June 1974, p391-404.

4. Bowles, Samuel, 'Aggregation of Labour Inputs in the Economies of Growth and Planning : Experiments with a two level CES Function', Journal of Political Economy, Vol. 78, 1970, p68-81.

5. Carter R.A. and Nagar A., 'Coefficients of Correlation for Simultaneous Equation Systems', Journal of Econometrics, Vol. 6, June 1977, p39-50.

6. Hanoch, G., 'CRESH Production Functions', Econometrica, Vol. 39, September 1971, p695-712.

7. ________, 'Production and Demand Models with Direct and Indirect Implicit Additivity', Econometrica, Vol. 3, May 1975, p395-419.

8. Hawkins, R.G., 'Factor Demands and the Production Function in Selected Australian Manufacturing Industries, Australian Economic Papers, Vol.16, June 1977, p97-111.

9. Parham, D.J. and Ryland, G.J., 'Models of Skill Substitution and Transformation in an Occupationally Disaggregated Labour Market', IMPACT of Demographic Change on Industry Structure in Australia, Preliminary Working Paper, OP-18, Industries Assistance Commission, Melbourne, March 1978(a) (mimeo) p37.

10. ________________________, 'ABS Labour Force Survey and Income Distribution Survey Data : Preliminary Analysis, IMPACT of Demographic Change on Industry Structure in Australia, Preliminary Working Paper, IP-05, Industries Assistance Commission, Melbourne, May 1978(b).

11. Ryland G.J. and Vincent D.P., 'Empirical Estimation of the CRESH Production Function', IMPACT of Demographic Change on Industry Structure in Australia, Preliminary Working Paper, No. OP-12, Industries Assistance Commission, Melbourne, June 1977.

12. Sato, Ryuzo, 'Homothetic and Non-homothetic CES Production Functions', American Economic Review, Vol. 67, September 1977, p559-569.

13. Tinbergen, Jan, 'Substitution of Academically Trained by Other Manpower', Weltwirtschaftliches Archive, Band 111, p.466-475, 1975.

APPENDIX - INDUSTRY CLASSIFICATIONS

	IMPACT Industry	CLI	ASIC
1.	Agriculture	02-06	01-02
	Forestry & Fishing	00-01, 07	03-04
2.	Mining - Metallic & Non-metallic Minerals	08-09	11, 14, 15, 16
	Mining - Coal & Crude Petroleum	Nil - included in 3.	12-13
3.	Food, Beverages & Tobacco	26-34	21-22
4.	Textiles, Clothing & Footwear	21-25	23-24
5.	Wood & Wood Products; Paper & Paper Products	35-38	25-26
6.	Basic Chemicals; Other Chemicals & Related Products	16, 39, 40	271-272
	Petroleum Refining : Petroleum & Coal Products	11	273-274
7.	Glass, Clay & Other Non-metallic Mineral Products	10	28
8.	Basic & Fabricated Metal Products	12-15	29, 31
9.	Transport Equipment	20	32
10.	Industrial Machinery & Household Appliances	17-19, 44	33
11.	Rubber & Plastic Products Manufacturing	43, 45-46	342, 343
12.	Other Manufacturing Industries	41, 42, 47	341, 344
13.	Electricity, Gas, Water, Sewerage & Drainage	48-49	36-37
14.	Building & Construction	50-51	41-42
15.	Wholesale & Retail Trade	61-64	46-48
16.	Transport & Storage	52-56	51-55
17.	Communication	57	56
18.	Finance, Insurance, Real Estate & Business Services	58-60, 74	61-63
19.	Public Administration & Defence	65-67	71-72
20.	Community Services & Health	68-73	81-84
21.	Entertainment & Recreational Services	75-79	91-94
22.	Other - Non Classifiable Establishments	80-99	99

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ABSTRACT

The focus of this paper is on the estimation of elasticities of demand by Australian industry for labour of different occupational groups with respect to relative wages and total employment demand. We develop a conceptual framework in terms of both homothetic and non-homothetic production technology but restrict the presentation of results to the homothetic case. The framework incorporates implicit restrictions on the occupational demand equations which carry over from the familiar neo-classical optimizing model of production. In our empirical study we use a variant of the Rotterdam demand system in which the restrictions hold locally at the central values of occupational cost shares. Parameter estimates obtained with this model are contrasted with estimates obtained with Hanoch's CRESH and CONDESH models, both of which are generalizations of the CES function. Our tentative conclusion is that wage substitution effects, although small among many pairs of occupations, are significant in some cases. We hope to report at a later date estimates of substitution elasticities and expansion elasticities derived from Hanoch's CRES and CONDES models, both of which are generalizations of Sato's 'almost homothetic' CES function.

usual alternative has been to assume infinite substitution possibilities, which has serious consequences for the projection of occupational demands. Our results, therefore, are suitable for the solution of general equilibrium models in which relative wages are potentially important determinants of occupational employment demands.

The second area in which our results have important implications falls in a wider perspective of manpower policy. Projections of occupational demand based on the usual requirements approach are predicated on the assumption of zero wage substitution effects and unit expansion effects. The CES results have shown that, in general, wage substitution effects are significantly different from zero and are in excess of 0.2. Estimates of substitution elasticities derived from more flexible functional forms, range from zero to 1.74. Thus the requirements approach will be biased by the existence of neglected wage substitution effects, particularly between certain pairs of occupations such as skilled white collar and blue collar groups.

6. CONCLUDING REMARKS

In this paper we have explored the empirical implications of writing down an occupational demand model consistent with the behaviour of a representative producer who acts as if he maximizes his available labour services subject to an overall wage bill constraint. The restrictions on the occupational demand functions derived for this model also carry over to the familiar labour input demand specifications consistent with producers who act as if they minimize wage expenditures subject to a labour aggregation function (possibly modified to incorporate flexible scale effects).

The above conceptual framework for our model is too general to permit empirical application. We have augmented this general approach with a variant of the Rotterdam model assuming the restrictions on the demand functions hold locally at the central values of the cost shares of each occupation. This device allows us to study both homothetic and non-homothetic production functions, including Sato's special family of non-homothetic functions, known as the 'almost homothetic' class. Hanoch's CRES and CONDES models and Sato's N-H CES display this property and each may be specialized to its homothetic form.

Our results have some important implications for projecting the occupational composition of the labour market. In two areas they can be used to provide improvements on the most common practice.

First, we provide estimates of labour-labour substitution parameters which enable the solution under conditions of changing wage relativities of a general equilibrium model (such as ORANI) that incorporates labour of different types. In contrast, the solution of complete models with zero labour-labour substitution possibilities is indeterminate. The

ESTIMATION OF OCCUPATIONAL DEMANDS IN AUSTRALIAN INDUSTRY *

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1. INTRODUCTION

In recent years a number of empirical studies have appeared whose main concern is the extent to which relative wage movements influence quantitative movements in demands for the services of diverse occupational groups.[1] The empirical results, which are derived from a variety of methods and data sources, have been mixed. The demand elasticities of substitution between different types of labour in these studies have estimated values ranging from infinity (Bowles (1970)) to values between zero and one (Weiss (1977)).[2] Tinbergen (1975) surveyed the literature and concluded that the evidence, properly interpreted, is consistent with a unitary elasticity of substitution between different occupations.

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* The authors thank Alan Powell and Peter Dixon for valuable advice and encouragement throughout the period of this investigation. The empirical study would never have been possible without access to the Wymer (1977) RESIMUL FIML software which makes possible the estimation of the new generation systems models reported in this paper. All remaining errors and omissions are ours.

1. Tinbergen (1975) provides a comprehensive survey of the literature on occupational demand.

2. Weiss found zero labour-labour substitution in two out of the twelve industries in his investigation. In some other industries, he was forced to accept zero substitution between certain occupations, since unconstrained estimation violated the quasi-concavity postulate.

The extent to which relative wages influence substitution among occupational groups is important for several reasons. First, manpower projections based on the customary Leontief analysis which ignores relative wage effects will be biased to the extent that occupational demands are responsive to changes in relative wages. Second, infinite substitutability among occupations would imply that employers do not discriminate among occupations in their labour demands. This implicit homogeneity of labour would suggest that retraining schemes and a social policy of reducing income inequality, through increased expenditure on technical and professional education, would be ineffective. Third, from the viewpoint of production function analysis, the summation of imperfectly substitutable labour inputs to form a single composite index of labour services will bias the estimate of the elasticity of the substitution between capital and labour as a whole (Berndt and Christensen (1973)).

Estimates of substitution elasticities of demand by Australian industry for five different types of labour are presented in this paper. The data base consists of cross sectional information across industries on the characteristics of the Australian workforce at two points in time. This study differs from other econometric studies of occupational demand in several respects. First, we obtain estimates of occupational demand for five occupational groups differentiated on the basis of type of work performed. This contrasts with the vast majority of previous studies[1] which distinguish only two occupational groups differentiated on the basis of type of work - - production versus non-production workers (Bowles (1970)) - - and level of

1. Weiss (1977), whose study distinguishes ten occupational groups, is a recent exception.

Apart from these specific occupation pairs, the evidence contained in Table 4 suggests that changes in relative wages have had relatively little influence on the change in the occupational composition of the workforce experienced over the period 1968/69 to 1973/74.

The compensated wage elasticities of occupational demand for the H-Rotterdam system are given in Table 5 together with R^2 for each individual equation computed from the asymptotic variance : covariance matrix of the reduced form residuals. Of 25 estimated elasticities all but 8 have the correct sign. None of these 8 differs significantly from zero. However, among the 17 correctly signed values only 5 would differ significantly from zero at conventional levels. The highest (in absolute value) and the most precisely estimated compensated own wage elasticity is for skilled white collar workers (- 0.54). This group has the highest estimated compensated cross elasticities, namely those showing its response to the wage rates for skilled and unskilled blue collar groups.

TABLE 5 : COMPENSATED CROSS AND OWN WAGE ELASTICITIES FOR H-ROTTERDAM SYSTEM

	P	SWC	USWC	SBC	USBC	R^2
Professional (P)	- 0.109 (0.80)	- 0.028 (0.25)	0.111 (0.71)	0.072 (0.45)	- 0.046 (0.23)	0.381
Skilled White Collar (SWC)	- 0.020 (0.25)	- 0.540 (3.90)	- 0.118 (0.89)	0.319 (2.18)	0.359 (2.07)	0.578
Unskilled White Collar (USWC)	0.061 (0.71)	- 0.089 (0.89)	- 0.118 (0.57)	- 0.022 (0.15)	0.167 (0.94)	0.600
Skilled Blue Collar (SBC)	0.046 (0.45)	0.276 (2.18)	- 0.025 (0.15)	- 0.367 (1.63)	0.069 (0.31)	0.685
Unskilled Blue Collar (USBC)	- 0.017 (0.23)	0.179 (2.07)	0.110 (0.94)	0.040 (0.31)	- 0.312 (1.53)	0.556

TABLE 4 : ESTIMATES OF SUBSTITUTION ELASTICITIES IN OCCUPATIONAL DEMAND MODELS

(Ratio of estimate to asymptotic standard error in parentheses)

Occupation[(a)]	Substitution Elasticities (AES)			
	H-Rotterdam	CRESH	CONDESH	H-CES
P/SWC	- 0.175 (0.25)	0.146 (0.24)	0.210 (0.74)	0.213 (2.06)
P/USWC	0.520 (0.71)	0.007 (0.15)	- 0.474 (1.33)	"
P/SBC	0.388 (0.45)	- 0.016 (0.16)	0.029 (0.08)	"
P/USBC	- 0.144 (0.23)	- 0.011 (0.16)	0.393 (1.20)	"
SWC/USWC	- 0.551 (0.89)	- 0.753 (1.28)	0.017 (0.05)	"
SWC/SBC	1.720 (2.18)	1.736 (2.27)	0.520 (1.41)	"
SWC/USBC	1.112 (2.07)	1.125 (2.21)	0.884 (2.72)	"
USWC/SBC	- 0.116 (0.15)	0.085 (0.23)	- 0.165 (0.45)	"
USWC/USBC	0.517 (0.94)	0.055 (0.23)	0.199 (0.51)	"
SBC/USBC	0.215 (0.31)	- 0.127 (0.24)	0.702 (1.79)	"

(a) Key to occupational groups is as follows : (P) Professional ; (SWC) Skilled White Collar ; (USWC) Unskilled White Collar ; (SBC) Skilled Blue Collar ; and (USBC) Unskilled Blue Collar .

schooling - - graduate versus other manpower (Tinbergen (1975)). Second, we assume that the parameters of the occupational demand specification may be determined in the absence of a comprehensive analysis of the labour market, i.e., one which also includes a labour supply function.[1] Tinbergen (1975) has put forward the criticism that studies which ignore a labour supply function tend to produce over-estimates of the demand elasticities of substitution among occupations, and hence of wage demand elasticities for particular occupations. This criticism would apply with greater force to studies which use the economy as the unit of observation than it would to those which use a lower level of aggregation such as an industry or a firm. This is because wage rates are more accurately portrayed as being predetermined in the latter case. Third, we base our empirical application on a tightly specified (but nevertheless quite general) conceptual framework for analysing occupational demand from the viewpoint of a representative producer who behaves as if he minimises his labour costs subject to a labour aggregation constraint which embodies the technology of substitution among occupations. We include several aggregation functions, all of which are generalisations of the familiar homothetic CES functions.

Without pre-empting the detailed conclusions reported below, our most general result is that, with few exceptions, the influence of wage substitution effects on occupational demands in Australian industry is likely to be small in the intermediate run (5 years).

1. The authors (Parham and Ryland (1978)) have considered several theoretical models of skill substitution (demand) and transformation (supply) in the context of a labour market assuming equilibrium or disequilibrium closing conditions. The present paper analyses the demand side only of the labour market with the implied assumption that only the supply schedule of labour has shifted such that all realizations lie on the demand schedule.

2. CONCEPTUAL FRAMEWORK

(i) *'Cost' Approach to Occupational Demand*

The empirical implications of the class of occupational demand models we wish to analyse may best be illustrated by considering the actions of the representative firm which acts as if it maximizes a quasi-concave labour aggregation or production function[1] of the form

(1) $$L = f(L_1, L_2, \ldots, L_n) \quad ,$$

subject to a budget constraint ,

(2) $$\Sigma W_i L_i = C \quad .$$

In (1), L_i is the demand for occupation of type i , L is an index of aggregate labour services utilised by the firm in each period, W_i is the exogenously specified wage rate for occupation i , and C is the firm's budgeted expenditure on wages or total wage bill. The labour aggregation function specified in (1) may take several different forms. The simplest one, which holds no interest in our paradigm, is an additive index $L = \Sigma_i L_i$. Below, we consider alternative forms that the

1. In a wider frame of reference we can specify the firm's production function to produce a given level of output as $Y = h(K, L, M)$ where Y is output , K is level of capital stock , L is aggregate labour services, and M is a composite index of other inputs to the firm. We specify that aggregate labour L is strongly separable with respect to the partitioning of factor inputs which is tantamount to assuming that marginal rates of substitution between each occupation and other non-labour inputs are equal. We have nothing further to say about the extent to which demand for capital services influence the demand for each occupation (the so called capital/skill complementarity hypothesis). The latter remains, nevertheless, an empirically relevant hypothesis so far as Australian industry is concerned. In a recent empirical study of U.S. manufacturing industry, Weiss (1977) rejected the complementarity hypothesis for all but a few isolated industries.

TABLE 3 : LIKELIHOOD RATIO TESTS OF EACH MODEL SPECIFICATION
(Degrees of freedom in parentheses)

Null Hypothesis	Alternative Hypothesis			
	H-Rotterdam	CRESH	CONDESH	H-CES
H-Rotterdam	-	3.43 (5)	6.15 (5)	10.52 (9)
CRESH		-	n.a.*	7.08 (4)
CONDESH			-	4.36 (4)
H-CES				-

* not applicable

The main results of this study, however, are contained in Table 4 where we provide estimates of AES for each occupational pair.[1] In general, the AES estimates range between zero and one, but few differ significantly from zero at conventional levels. This observation is confirmed by the H-CES estimates where the AES is approximately 0.2 which in this case is significantly different from zero. The highest detected AES are between blue collar groups and skilled white collar groups (rows 5 and 6 of Table 4).

1. We also tried an alternative index of aggregate labour services obtained by simply adding up man-hours across occupation groups in each industry. This form of aggregation, although inconsistent with our imperfect substitution hypothesis, produced similar estimates of elasticities of substitution between occupation pairs.

TABLE 2 : SUMMARY OF STATISTICS OF ALTERNATIVE MODELS OF OCCUPATIONAL DEMAND

Model	Likelihood Value	R^2 (Carter-Nagar)
H-Rotterdam	171.053	0.409
CRESH	169.336	0.386
CONDESH	167.977	0.396
H-CES	165.795	0.341

systems correlation coefficient R^2 (Carter-Nagar (1977)). It is not surprising that the flexible Rotterdam specification with 14 parameters has the highest log likelihood value and systems R^2. Using Table 2, however, we can perform tests of significance of the differences between each model specification based on the conventional likelihood ratio (LR) test statistic $\lambda = -2(L_1' - L_0')$ which is distributed as Chi-Square with degrees of freedom equal to the reduction in the number of parameters estimated to obtain L_1', the log likelihood value of the alternative hypothesis, relative to L_0', the log likelihood value of the null hypothesis.

The LR test statistics are summarized in Table 3 where, for each model specification, the null hypothesis given by the row is tested against the alternative hypotheses across the columns. This demonstrates no significant difference among alternative models based on conventional levels of significance.

aggregation function may take depending on the labour technologies inherent in the representative firm.

Maximization of (1) subject to (2) implies the familiar first order conditions :

(3) $$\lambda W_i = \partial f / \partial L_i \qquad (i = 1, \ldots, n)$$

where λ is the marginal cost of an additional labour unit.

Assuming, as we have, that wages are exogenously determined and that the wage bill C remains fixed, the endogenous variables L_i may be solved in terms of the exogenous variables (wages and total labour costs) to give the occupational demand system :

(4) $$L_i = L_i(W_1, W_2, \ldots, W_n ; C) \qquad (i = 1, 2, \ldots, n) .$$

The primary purpose of this formulation is to obtain the restrictions on the partial derivatives of the demand functions which carry over from the maximization problem (1), (2). These restrictions can best be illustrated by totally differentiating (4) to give

(5) $$dL_i = \sum_j \partial L_i / \partial W_j \, dW_j + \partial L_i / \partial C \, dC .$$

On substituting $Z \, d \, \ell n \, Z$ for each derivative $d \, Z$ in (5) and dividing by L_i, equation (5) may be expressed in elasticity form as :

(6) $$\dot{L}_i = \sum_j \eta_{ij} \dot{W}_j + \mu_i \dot{C} \qquad (i = 1, 2, \ldots, n)$$

where

η_{ij} = own or cross uncompensated elasticity of demand for occupation i with respect to the j^{th} wage $(i, j = 1, 2, \ldots, n)$, and

μ_i = elasticity of demand for occupation i with respect to the total wage bill.

Above, a dot over a variable signifies the differential of its logarithm.

The satisfaction of the budget constraint (2) implies

$$\sum_i W_i \frac{\partial L_i}{\partial C} = 1 = \sum S_i \mu_i \quad , \qquad (7)$$

where

$S_i = W_i L_i / \sum_k W_k L_k$ (share of occupation i in the firm's total labour costs) .

Equation (7) is the aggregation condition of ordinary demand systems. Other restrictions imposed on the demand system (6) involve the familiar Slutsky decomposition of a price change which, in this case, involves a wage substitution effect and a cost or wage expenditure effect. The total cross wage elasticities of (4) may be partitioned into substitution and expansion effects as follows :

$$\frac{\partial L_i}{\partial W_j} = E_{ij} - (\partial L_i/\partial C) \cdot L_j \quad ,$$

which, in elasticity terms, may be expressed as

The data from these surveys include observations which are null or subject to high sampling errors of estimate. Consequently we had to aggregate occupations and industries while attempting to preserve on the one hand homogeneity of each O:I group while retaining variation between occupations and industries on the other. Our procedures provided us with a comprehensive but disaggregated coverage of Australian industry classified into five occupational groups among 21 industries.[1] Observations on weekly hours worked and number of persons employed from the LFS were converted to a financial year basis corresponding to the IDS sample years 1968/69 and 1973/74.

From these basic data sets we obtained estimates of hours worked, hourly wages (calculated as the ratio of labour income per person to hours per person) for each O:I classification and the index of aggregate labour services. This meant that the survey data yielded five occupational demand equations each with 21 cross section industry observations when the data were transformed to proportionate change form. Our data sources and transformation and aggregation procedures including an analysis of the reliability of the original data are documented in Parham and Ryland (1978(b)). This paper is available on request from the authors.

5. EMPIRICAL RESULTS

In Table 2, we summarize the results for our homothetic variant of the Rotterdam system, CRESH, CONDESH and CES.[1] Alternative model specifications are compared using, as our criteria, the log likelihood value and the

1. The occupational groups were Professional (P), Skilled White Collar (SWC), Unskilled White Collar (USWC), Skilled Blue Collar (SBC), and Unskilled Blue Collar (USBC). Our industry classification is given in the Appendix.
2. Results for the non-homothetic forms of these models will be presented at a later date.

on the assumption that the demands for capital, aggregate labour services, and for materials are determined at a higher level in the hierarchy of decision-making. Fourth, the variable L is unobservable. In log differential form, however, in any industry

$$\dot{L} = \sum_i S_i \dot{L}_i$$

for all models which exhibit homogeneity of degree one. Thus data on $\dot{L}$ can be constructed from a knowledge of occupational wage rates and occupational hours worked.

4. DATA SOURCES

The data used in the empirical application consist of occupation-specific observations on hours worked, annual pre-tax earnings net of unearned income and number of employed persons. The data were collected from cross section surveys for two time periods, 1968/69 and 1973/74. These periods correspond to the only collections of earned income information in the Australian Bureau of Statistics (ABS) Income Distribution Surveys (IDS). Data on hours worked, disaggregated by industry and occupation, were obtained from the Labour Force Surveys (LFS) conducted quarterly by the ABS. Estimates of numbers of employed persons are available for both surveys. Although these data are not primarily designed for assessing hours of work and earned income by occupation and industry (O:I) at a highly disaggregated level, we nevertheless believe that they contain useful information at the level of disaggregation at which we work.

(8) $$\eta_{ij} = S_j\left(\frac{E_{ij}}{S_j} W_j/L_i - \mu_i\right)$$

$$= S_j\left(\sigma_{ij} - \mu_i\right) .$$

In (8) E_{ij} is the cost compensated derivative of a wage change -- i.e., the derivative with respect to W_j of the function (4) -- and the second term is the 'cost' effect. The Allen-Uzawa elasticities of substitution (AES) are related to E_{ij} by $\sigma_{ij} = E_{ij} W_j/(L_i S_j)$ (Allen (1938)). Using (8) it is now straightforward to obtain the remaining restrictions on the demand system (6) in terms of the AES, σ_{ij} .

The own wage substitution effect is always negative implying

(9) $$\sigma_{ii} < 0 ,$$

and the homogeneity requirement is

(10) $$\sum_j S_j \sigma_{ij} = 0 ,$$

so that each demand equation is homogeneous of degree zero in wages and total labour cost. Finally, the symmetry requirement is

(11) $$\sigma_{ij} = \sigma_{ji} .$$

Substitution of (8) in (6) gives the system :

(12) $$\dot{L}_i = \sum_j S_j \sigma_{ij} \dot{W}_j + \mu_i\left(\dot{C} - \sum_j S_j \dot{W}_j\right) \qquad (i = 1, \ldots, n) ,$$

on which the constraints (7), (9), (10), (11) may be imposed as linear restrictions on the coefficients of the system σ_{ij} , μ_i . Employing these restrictions, the system may be parameterized in terms of $(n - 1)$ independent cost elasticities and $\frac{1}{2}n(n - 1)$ independent cross AES. The adding up restriction (7) determines the value of the n^{th} cost elasticity, the symmetry requirement (11) yields the missing $\frac{1}{2}n(n - 1)$ cross AES, while the homogeneity constraint (10) provides the n own AES, which exhausts the coefficient set for (12).

Equation (12) constitutes a Rotterdam constant elasticity demand system in which the restrictions are required, in our empirical work, to hold locally at mean values of the cost shares. These systems have played a very useful role in contemporary consumer demand analysis (Barten (1977)), and there seems no reason whatsoever why they should not be applied to analyse labour demand relationships.

(ii) *'Production' Oriented Appraoch to Occupational Demand*

In a more conventional framework of analysis the representative firm's occupational demand relationships are derived from the first order conditions by minimizing the wage bill ,

(13) $$\Sigma W_i L_i \quad ,$$

subject to the production function (1) to give a set of labour demand relationships, analogous to (6), of the form :

(14) $$\dot{L}_i = \sum_j \eta_{ij} \dot{W}_j + \varepsilon_i \dot{L} \quad ,$$

where

ε_i = elasticity of demand for i^{th} occupation with respect to total labour demand - - the expansion elasticity of i , for short.

In this system specification the matrix $\underline{C}$ contains the over-identifying restrictions (7), (10), (11) for the Rotterdam model or is specialized via equation (8) for each model summarized in Table 1. We assume the residuals are well behaved and are generated according to a multivariate normal distribution with $E(V') = 0$ and $E(V'V) = \Omega$ (the contemporaneous variance : covariance matrix). With these functional parameter restrictions embodied in the problem together with the assumed behaviour of the residuals, the model (6) can be estimated easily and efficiently using the outstanding Wymer (1977) RESIMUL package which utilizes non-linear FIML estimation procedures.

Prior to estimation, however, there are several preliminary issues which must be resolved. First, Hanoch's implicitly additive models CRES(H) and CONDES(H) impose additional parameter restrictions on the substitution and expansion parameters. Generally, we have ignored these restrictions. In those cases where the parameter restrictions have been violated the parameter estimates have usually been so imprecise that they would not have differed significantly from zero at conventional levels. Second, we have augmented the model (18) by including a constant term in each equation (a) in order to obtain a proper evaluation of the likelihood function for the unrestricted reduced form (which is needed for later use in likelihood ratio tests), and (b) to guard against a possible mis-specification problem with respect to the assumed zero-mean error property of the structural form residuals. Although inclusion of a constant term improved the fit of each equation, it did not change the parameter values greatly. Below we have reported all estimates of parameters without the constant term. Third, aggregate labour services, L (which appears in the Z matrix), is treated as exogenous to be estimated are shown. In addition we give the parameter restriction consistent with the regularity conditions imposed on the functional forms chosen for the models. As an initial approximation we have estimated the models assuming CRTS.

3. ECONOMETRIC SPECIFICATION OF OCCUPATIONAL DEMAND

It is convenient to re-write the Rotterdam system (12) which is the parent specification of the 'almost homothetic' and homothetic models considered briefly in 2 (iii) above using matrix notation as

(18) $$L' = CZ' + V' ,$$

where

L' = N:K matrix of observations on the proportionate change in the number of man-hours worked in each occupation i ($i = 1, 2, \ldots, N$) in industry j ($j = 1, \ldots, K$) ;

Z' = P:K matrix of observations on the proportionate change in each exogenous variable, Z_ℓ, ($\ell = 1, \ldots, P$) in each industry j ;

V' = N:K matrix of residuals ; and

C = N:P matrix of coefficients of the elasticity of hours worked with respect to unit changes in each of the exogenous variables .

To obtain the equivalence between the 'cost' approach (6) and the familiar 'production' approach (14) we define the elasticity of costs with respect to aggregate labour services parameter , $\phi = \partial \log C / \partial \log L$ with wages held constant so that $1/\phi$ is the 'scale' elasticity. Then, holding wages constant, (14) and (6) imply :

(15) $$\varepsilon_i = \dot{L}_i / \dot{L} = \frac{C}{\dot{L}} \frac{\dot{L}_i}{\dot{C}} = \phi \mu_i .$$

Thus by substituting ε_i / ϕ for μ_i in (12), the restrictions imposed on the cost model (7), (9), (10), (11), also carry over to the production model (14). Further, if the returns to scale parameter ϕ is known a priori, then the models (14) and (6) are observationally equivalent.

(iii) *'Almost Homothetic' Occupational Demand Models*

The production function underlying (14) may be specialised to a class of non-homothetic (N-H) functions referred to as 'almost homothetic' in which the expansion paths on a two factor isoquant map are linear when the axes are scaled in terms of the logarithms of the variables.[1] Recently Sato (1977) has discussed some of the properties and has contrasted the

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1. More formally, an 'almost homothetic' production function is any monotonic transformation of the form

$$Y = F\left[f\left(\lambda^{r_1} X_1, \lambda^{r_2} X_2\right)\right] = F\left[\lambda^{\rho} f\left(X_1, X_2\right)\right] , \qquad F' > 0 ,$$

which implies that if inputs X_1, X_2 are increased by r_1, r_2 per cent, respectively, then output, Y, increases by ρ per cent. A unique property of this class of function is that the ratio of cost shares S_1/S_2 remains unchanged at different output levels generated by increasing X_1 at a geometric rate r_1, and X_2 at a geometric rate r_2 - - see Sato (1977), p. 564.

empirical performance of this class of N-H model with the familiar homothetic (H) model in the context of the CES factor demand system.[1]

Using the Rotterdam type model (14) as a point of reference, a number of N-H models, all of which exhibit the 'almost homothetic' property, may be analysed. Hanoch (1975) has analysed a class of N-H models which impose implicit strong separability in which the isoquants are additive with respect to factor quantities while the explicit parent production function need not necessarily be additive. Implicit functional forms analysed by Hanoch are a direct aggregation function (CRES) analogous to (1) using the 'production' approach and a reciprocal indirect production function (CONDES) optimized within the framework of the 'cost' approach. Throughout we assume that these functional forms hold locally at the central values of factor shares so that they provide convenient testable alternative hypotheses, each of which is nested within the general approach of the Rotterdam demand system.

A. Constant Ratio of Elasticity of Substitution Models (CRES)

The specialised CRES model analysed by Hanoch is of a form similar to (1) :

$$\sum_{j s.t. q_j \neq 0} Q_j \left(L_j/L^{\ell_j}\right)^{q_j} + \sum_{j s.t. q_j = 0} B_j \log\left(L_j/L^{\ell_j}\right) \equiv 1 \quad , \tag{16}$$

1. Some of the properties of the N-H CES model discussed by Sato are : (i) N-H allows variable marginal substitution rates even when factor ratios remain constant; (ii) Factor income shares in the case of N-H do not exhaust total output; (iii) N-H permits the identification of bias in the so-called factor augmenting technical change hypothesis by explicitly considering the influence of expansion as well as substitution effects.

Assuming CRTS ($\phi = 1$), Hanoch (1975) demonstrates the implied restrictions on the behaviour of the expansion path in the context of cost minimization in competitive factor markets. A U-shaped average cost (AC) curve, one which is compatible with profit maximization in a competitive market, requires that $\phi < 1$ (increasing returns to scale), at low outputs. This implies that $\partial\phi/\partial L$ must always be positive, $\partial\phi/\partial L > 0$ (marginal cost must always be increasing). The latter requirement implies that the substitution parameter be $a < 1$ and the maximum expansion parameter be of the form $\max_i \ell_i > 1$.

The models outlined above are summarized in Table 1 where for each model specification the number of substitution and expansion parameters

TABLE 1 : SUMMARY OF OCCUPATIONAL DEMAND MODELS

Model	Production Technology	Number of Substitution Parameters*	Number of Expansion Parameters*
Rotterdam	Non-Additive Non-Homothetic	$\frac{1}{2}n(n-1)$	$n - 1$
H-Rotterdam	Non-Additive Homothetic	$\frac{1}{2}n(n-1)$	-
CRES	Implicitly Additive Non-Homothetic	n $[q_i \leq 0$ or $0 < q_i < 1]$	$n - 1$ (CRTS) $[\ell_i > 0]$
CRESH	Implicitly Additive Homothetic	n [same as CRES]	-
CONDES	Implicitly Additive Non-Homothetic	n $[d_i \leq 0$ or $0 < d_i < 1]$	$n - 1$ (CRTS) $[\ell_i > 0]$
CONDESH	Implicitly Additive Homothetic	n [same as CRES]	-
N-H CES	Explicitly Additive Non-Homothetic	1 $[0 < a < 1]$	$n - 1$ $[\mathrm{Max}_i\ \ell_i > 1]$
CES	Explicitly Additive Homothetic	1	-

* Intervals within which parameters must lie shown in square parentheses.

The CONDES model imposes the requirement that the differences in elasticities of substitution remain constant implying

$$\sigma_{ij} - \sigma_{\ell m} = \alpha_i + \alpha_j - \alpha_\ell - \alpha_m ,$$

which, in common with the CRES model, reduces the number of substitution parameters to be estimated to <u>n</u> . However, the CONDES model is more flexible than CRES permitting global complementarity in cases where α_i, α_j are small relative to the weighted sum $\sum_k S_k \alpha_k$. The CONDES model may be specialized even further by setting $\ell_i = \ell$ and for CRTS, $\mu_i = 1$ to give the CONDESH model. The latter has been analysed by Hawkins (1976), Weiss (1977) and Parham and Ryland (1978).

C. N-H CES

The 'almost homothetic' CES model analysed by Sato (1977) is weakly self-dual (production and cost functions have similar functional forms but different parameters), and may be derived from either the CRES or CONDES models. The N-H CES model is obtained by setting $a_j = a$ (CRES) or $\alpha_i = a$ (CONDES) so that all cost compensated cross wage elasticities are equal to $S_j a$ everywhere in the inputs-outputs space :

$$S_j \sigma_{ij} = S_j a - \delta_{ij} a ,$$

while the cost elasticities are

$$\mu_i = a + \frac{1}{\phi} \ell_i (1 - a) .$$

where

$\{q_j\}$ = n substitution parameters either $0 < q_j < 1, \forall j$ or $q_j \leqslant 0 , \forall j$;

$\{\ell_j\}$ = n expansion parameters , $\ell_j > 0$, and

$\{Q_j, B_j\}$ = n distribution parameters such that $Q_j q_j > 0$.

From the first order conditions obtained from minimizing (13) subject to (16), Hanoch (1975, pp. 403-7) obtains the cost compensated wage elasticity :

$$S_j \sigma_{ij} = S_j a_i a_j / \sum_k S_k a_k - \delta_{ij} a_i ,$$

and cost elasticity ,

$$\mu_i = \frac{1}{\phi}\left[\ell_i\left(1 - a_i\right) + \frac{\sum_k S_k a_k \ell_k}{\sum S_k a_k} a_i\right] ,$$

where

$a_i = 1/(1 - q_i)$;

$\delta_{ij} = 1(i = j)$ and $\delta_{ij} = 0$ otherwise , and

$1/\phi$ = returns to scale elasticity .[1]

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1. The 'almost homothetic' property of CRES can be demonstrated by showing that $\left.\frac{\partial \log L_j}{\partial \log L_i}\right|_L$ is the same for all L provided S_i/S_j is constant.

This follows immediately since

$$\left.\frac{\partial \log L_j}{\partial \log L_i}\right|_L = - \frac{\ell_i q_i Q_i L_i^{q_i \ell_i} L^{q_j \ell_j - q_i \ell_i}}{\ell_j q_j Q_j L_j^{q_j \ell_j}}$$

while S_i/S_j is equal to the negative of the same expression.

The returns to scale elasticity $1/\phi$ is obtained by Hanoch (1975, p. 404) from setting , $\phi = \frac{\partial \log C}{\partial \log L} = \frac{\partial \log (\Sigma W_i L_i)}{\partial \log L}$ which, at constant prices, is equal to :

$$\phi = \sum_k S_k \frac{\partial \log L_k}{\partial \log L} = \sum_k S_k \ell_k .$$

If constant returns to scale (CRTS) prevail ($\phi = 1$) then $\mu_i = \varepsilon_i$ and $\phi = 1 = \sum_k S_k \ell_k$, implying that in order to satisfy the 'adding up' condition, there are only (n - 1) independent expansion elasticities.

The CRES property imposes the requirement that the ratios of the elasticities of substitution among different pairs of inputs remain globally constant; that is

$$\sigma_{ij}/\sigma_{\ell m} = \frac{a_i a_j}{a_\ell a_m} .$$

When this condition is imposed only n substitution parameters are required to be estimated in order to generate $\frac{1}{2}n(n-1)$ different AES substitution elasticities which would appear to be a usefully parsimonious parameterization.

If $\ell_i = \ell$ (all i) then CRES may be specialized to the homothetic CRESH function discussed previously by Ryland and Vincent (1977) and Weiss (1977). If, in addition, CRTS is assumed ($\ell = 1/\phi = 1$) , only n parameters need be estimated to obtain the entire CRESH system.

B. Constant Difference of Elasticity of Substitution Models (CONDES)

A reciprocal indirect production function analysed by Hanoch is obtained from the optimization problem (1), (2) and takes the form :

(17) $$G\left(W_i/C;\ L\right) = \Sigma B_i \left(W_i/C\right)^{d_i} L^{d_i \ell_i} \equiv 1 ,$$

where

$\{d_i\}$ = n substitution parameters of the CONDES model (either $d_i \leq 0$, $\forall$ i or $0 < d_i < 1$, $\forall$ i ;

$\{B_i\}$ = n distribution parameters with $B_i d_i > 0$; and

$\{\ell_i\}$ = n expansion parameters .

Since (17) represents optimal combinations of occupations we can use Roy's Rule to determine the occupational demand functions from $L_i = G_i' \Big/ \sum_k G_k' \left(W_k/C\right)$ where G_k' is the partial derivative of (17) with respect to normalized wages, W_k/C . After some manipulation - - see Hanoch (1975, p. 412-4) - - (17) may be put in the form of (14) to give cost compensated wage elasticities ,

$$S_j \sigma_{ij} = S_j \left(\alpha_i + \alpha_j - \sum_k S_k \alpha_k\right) - \delta_{ij} \alpha_i ,$$

and cost elasticities ,

$$\mu_i = \frac{1}{\phi}\left[\ell_i (1 - \alpha_i) + \sum_k S_k \alpha_k \ell_k\right] + \left(\alpha_i - \sum_k S_k \alpha_k\right) ,$$

where

$\alpha_i = 1 - d_i$, and

$1/\phi$ = scale elasticity, $\phi = \sum_k S_k \ell_k$ (defined previously) .