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REGIONAL DISAGGREGATION OF RESULTS FROM ORANI 78 : THE UNDERLYING THEORY

by

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and

$$x_{(L1)}^{(0)} = B_{LN}^{(1)*} z_N + B_{LM}^{(1)*} z_M + B_{LN}^{(2)*} y_N + B_{LM}^{(2)*} y_M + \sum_{h=3,5} B_{L(US)}^{(h)*} x_{(US)}^{(h)} + B_{L(U1)}^{(4)*} x_{(U1)}^{(4)} . \quad (42.35)$$

Equation (42.34) follows from (12.83) and (12.81) where we recall that we have assumed zero technical change and that local industries produce only one product and that local products are produced by only one industry. Equation (42.35) is implied by the market clearing condition (20.6). Given the definitions of $\tilde{x}_{(L1)}^{(0)}$, $\tilde{z}_M$, etc., (42.33) establishes the validity of the aggregation conditions (42.1) - (42.10).

In summary, we have shown in this appendix that, under the assumptions marked with a (*), the regional results from our modified LMPST method are compatible with the economy-wide ORANI results. That is, the regional results generated by ORES aggregate according to (42.1) - (42.10). A check through section 40 reveals that the (*) assumptions are compatible with or even explicitly made in our estimation of the various "shares" parameters used in ORES.

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With the addition of equations (42.29) - (42.32), the number of equations relating to tilde variables has increased by $2M^* + 3U^*$ to equal the number of variables. We assume that our expanded equation system has a unique solution for the tilde variables. It is easy to check that this solution must be

$$\left.\begin{aligned}\tilde{x}^{(0)}_{(L1)} &= x^{(0)}_{(L1)}\\ \tilde{z}_M &= z_M\\ \tilde{z}_N &= z_N\\ \tilde{y}_M &= y_M\\ \tilde{y}_N &= y_N\\ \tilde{x}^{(3)}_{(US)} &= x^{(3)}_{(US)}\\ \tilde{x}^{(5)}_{(US)} &= x^{(5)}_{(US)}\\ \tilde{x}^{(4)}_{(U1)} &= x^{(4)}_{(U1)}\\ \tilde{v} &= v\\ \text{and}\quad \tilde{x}_{(g+1,1)K} &= x_{(g+1,1)K}\end{aligned}\right\} \quad (42.33)$$

(42.33) satisfies (42.22), (42.23), (42.26), (42.28) and (42.29) - (42.32) trivially. It also satisfies (42.21) and (42.20) because ORANI computations are constrained so that

$$z_m = x^{(0)}_{(m'1)}, \quad m\epsilon M,\ m'\epsilon L \qquad (42.34)$$

where m' is the local product produced by local industry m

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Finally we require an expression for the LHS of condition (42.10). Equation (39.21) implies that

$$\tilde{x}^{(1)}_{(g+1,1)k} = x^{(1)}_{(g+1,1,)k} + \sum_k z^r_k S^r_{10,k} - z_k , \quad k\epsilon K . \quad (42.27)$$

If we assume that

$$S^r_{10,k} = S^r_{1k} , \quad k\epsilon N \quad (*)$$

and

$$S^r_{10,k} = S^r_{4k} , \quad k\epsilon M , \quad (*)$$

i.e., that the share of region r in total employment in industry k is equal to the region's share in the industry's total output, then (42.27) reduces to

$$\tilde{x}^{(1)}_{(g+1,1)k} = x_{(g+1,1)k} + \tilde{z}_k - z_k , \quad k\epsilon K . \quad (42.28)$$

Equations (42.20), (42.21), (42.22), (42.23), (42.26) and (42.28) form a system of $(L^* + M^* + M^* + 2U^* + 1 + K^*)$ linear equations in the $(L^* + M^* + N^* + M^* + N^* + 2U^* + 2U^* + U^* + 1 + K^*)$ variables $\tilde{x}^{(0)}_{(L1)}$, $\tilde{z}_M$, $\tilde{z}_N$, $\tilde{y}_M$, $\tilde{y}_N$, $\tilde{x}^{(3)}_{(US)}$, $\tilde{x}^{(5)}_{(US)}$, $\tilde{x}^{(4)}_{(U1)}$, $\tilde{v}$ and $\tilde{x}_{(g+1,1)K}$, where L^*, M^*, N^*, U^* and K^* are, in turn, the numbers of elements of the sets L, M, N, U and K. The validity of (42.1), (42.2), (42.7) and (42.8) has already been established. Consequently we can write

$$\tilde{z}_n = z , \quad n\epsilon N , \quad (42.29)$$

$$\tilde{y}_n = y , \quad n\epsilon N , \quad (42.30)$$

$$\tilde{x}^{(5)}_{(us)} = x^{(5)}_{(us)} , \quad u\epsilon U, \; s=1,2 , \quad (42.31)$$

and

$$\tilde{x}^{(4)}_{u1} = x^{(4)}_{u1} , \quad u\epsilon U . \quad (42.32)$$

where we assume that

$$S^r_{6(us)} = S^r_9 , \quad u\ U, \quad s=1,2, \quad r=1,\dots,R ,$$

i.e., we assume that region r's share in aggregate household consumption of each commodity is equal to the region's share in the economy-wide wage bill.

Equation (39.18) yields the following expression for the LHS of condition (42.9)

$$\tilde{v} = \sum_r \sum_{k\epsilon K} p^{(1)}_{(g+1,1,)k} W^r_k S^r_9 + \sum_r \sum_{k\epsilon K} x^{(1)r}_{(g+1,1)k} W^r_k S^r_9 . \qquad (42.24)$$

Note that $W^r_k S^r_9$ is the share of the economy's wage bill which is accounted for by industry k in region r. An alternative expression for this is $W_k \bar{S}^r_k$ where $\bar{S}^r_k$ is defined as the share of region r in the aggregate wage bill of industry k. Thus, by using (39.21), we can write (42.24) as

$$\tilde{v} = \sum_{k\epsilon K} W_k \left(p^{(1)}_{(g+1,1,)k} + x^{(1)}_{(g+1,1,)k}\right) + \sum_{k\epsilon K} W_k \left(\sum_r z^r_k \bar{S}^r_k - z_k\right) . \qquad (42.25)$$

If we assume that

$$\bar{S}^r_k = S^r_{1k} , \qquad k\epsilon N \qquad (*)$$

and

$$\bar{S}^r_k = S^r_{4k} , \qquad k\epsilon M , \qquad (*)$$

i.e., that regional shares in industry wage bills equal regional shares in industry outputs, then (42.25) becomes

$$\tilde{v} = v + \sum_{k\epsilon K} W_k \left(\tilde{z}_k - z_k\right) . \qquad (42.26)$$

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37. Introduction

The model described in the previous chapters is capable of producing results only at the national level. For many policy purposes some regional disaggregation may be required. Users of ORANI might, for example, be interested in the likely implications of tariff reform (say) for employment at the regional as well as the national level. In order to meet such needs we have developed a regional disaggregation programme which can be run sequentially with the main ORANI programmes. The regional disaggregation programmes take ORANI results at the Australia-wide level as an input and produce results for each of the six Australian states as an output.

The method we have used is an adaptation of that proposed by Leontief, Morgan, Polenske, Simpson and Tower (1965) (hereafter

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* This is a draft chapter for a book on the ORANI model being prepared by P.B. Dixon, B.R. Parmenter, J.M. Sutton and D.P. Vincent. Consequently there are occasional references to other chapters. The first two sections of the paper (sections 37 and 38) can be read independently of any other material. For a detailed understanding of the rest of the paper, readers would need to be familiar with the theory of ORANI 78 (see Dixon, P.B., "The Theoretical Structure of ORANI 78", IMPACT Preliminary Working Paper No. OP-27, IMPACT Research Centre, Melbourne University, June 1980).

LMPST). The main advantage of the LMPST method is that it is very economical in its data demands. In particular, it avoids the necessity for detailed information about inter-regional trade flows which is simply not available for the Australian economy. Whilst our modified LMPST method could, in principle, be used to disaggregate ORANI results to regional bases other than the states, we have to date worked only at the state level, for two reasons. Firstly, such regional data as are required are most readily available at the state level. This is due primarily to the collection and publication of integrated economic census data by state offices of the Australian Bureau of Statistics (ABS). Secondly, there are good geographical reasons, peculiar to Australia, for expecting the simple LMPST methodology to be successful at the state level.

In this chapter we first review the original LMPST model in the context of a multi-regional input-output framework. Then in sections 39 and 40 we describe the theoretical structure and empirical implementation of our modified LMPST approach. These sections are supplemented by an appendix, section 42, which establishes the basic aggregation proposition that the results coming out of our regional computations are consistent with the economy-wide ORANI results going into them. A summary of the chapter and some concluding remarks are given in section 41. Results from regional computations are presented with those from ORANI itself in Chapter 7.

In deriving (42.21) we use the fact that $S^r_{3m} = S^r_{4m}$. Because we assume that each local product is produced by a unique single-product industry, it follows that regional shares in the outputs of local products can be equated with regional shares in the outputs of local industries.

Next, we obtain an expression for the LHS of (42.5) by noting that (39.12) implies that

$$\tilde{y}_m = y_m + \tilde{z}_m - z_m , \quad m \in M , \qquad (42.22)$$

where we assume that

$$S^r_{5m} = S^r_{4m} \quad \text{for all} \quad r=1,\ldots,R, \quad m \in M , \qquad (*)$$

i.e., we assume that the share of region r in the total investment of industry m is equal to region r's share in the total output of industry m .

An expression for the LHS of (42.6) can be derived from equations (39.14) to (39.16).[1] These give

$$\tilde{x}^{(3)}_{(us)} = x^{(3)}_{(us)} + \gamma\, \varepsilon_{(us)} (\tilde{v} - v) ,$$

$$u \in U, \quad s=1,2, \quad r=1,\ldots,R , \qquad (42.23)$$

---

1. To derive (42.23) we use the equation

$$x^{(3)r}_{(us)} = x^{(3)}_{(us)} + \gamma\, \varepsilon_{(us)} (v^r - v) ,$$

$$u \in U, \quad s=1,2, \quad r=1,\ldots,R .$$

This is not part of ORES. It is, however, implied by ORES.

giving us (42.12). For readers who wish to check the other relationships, it might be helpful to recall that in deriving our regional sales share matrices, we not only assumed that industry technology is the same for all regions, but also that household consumption patterns are identical across regions. Thus, in deriving (42.16), it is valid to equate $S^r_{6(us)}$ for all u and s with region r's share in total consumer spending. By using (42.12) - (42.18) we simplify (42.11) to

$$\tilde{x}^{(0)}_{(i1)} = \sum_{n\epsilon N} \left(B^{(1)*}_{LN}\right)_{in} \tilde{z}_n + \sum_{m\epsilon M} \left(B^{(1)*}_{LM}\right)_{im} \tilde{z}_m$$

$$+ \sum_{n\epsilon N} \left(B^{(2)*}_{LN}\right)_{in} \tilde{y}_n + \sum_{m\epsilon M} \left(B^{(2)*}_{LM}\right)_{im} \tilde{y}_m$$

$$\sum_{h=3,5} \sum_{u\epsilon U} \sum_{s=1}^{2} \left(B^{(h)*}_{L(US)}\right)_{i(us)} \tilde{x}^{(h)}_{(us)} + \sum_{u\epsilon U} \left(B^{(4)*}_{L(U1)}\right)_{iu} \tilde{x}^{(4)}_{(u1)} ,$$

$$\text{for all } i\epsilon L , \qquad (42.20)$$

where, as mentioned before, $\tilde{z}_n$, $\tilde{y}_n$, $\tilde{x}^{(0)}_{(i1)}$, etc. are the LHS's of (42.1), (42.2), (42.3), etc..

To derive an expression of the LHS of (42.4), we use (39.28). This gives

$$\tilde{z}_m = \tilde{x}^{(0)}_{(m'1)} , \quad m\epsilon M, \ m'\epsilon L \qquad (42.21)$$

where m' is the local product produced by local industry m.

38. The LMPST Model in the Multi-Regional Input-Output Framework

The regional model devised by LMPST can be represented as a multi-regional input-output (hereafter MRIO) model. MRIO models express outputs by commodity, industry and region as linear functions of regional final demands. A general form of MRIO models for an economy of U commodities, K industries and R regions can be written

$$X = BZ + Y , \qquad (38.1)$$

where X is a $(UR \times 1)$ vector of outputs by commodity and region, i.e., a vector composed of R regional subvectors each of U commodity output levels; Z is a $(KR \times 1)$ vector of outputs by industry and region; Y is a $(UR \times 1)$ vector of final demand for commodities distinguished by region of production; and B is a matrix of coefficients whose typical component, b_{urks}, is the input of commodity u from region r required per unit output of industry k in region s.[1] We have assumed, consistent with ORANI, that technology is industry, rather than commodity, specific.

The industry output vector (Z) can be expressed as a function of the commodity output vector by assuming that, within regions, industries maintain constant shares in the total outputs of commodities,[2] i.e., by defining

$$Z = DX , \qquad (38.2)$$

---

1. Alternatively X, Z and Y can be interpreted as vectors of changes in commodity outputs, industry outputs and final demand. Corresponding reinterpretations of the coefficient matrices (B, D and C) are, of course, necessary for the incremental form.

2. In the special case where all industries produce only one commodity and each commodity is produced by only one industry, $K = U$ and D is an identity matrix.

where D is a (KR × UR) block diagonal matrix composed of R , (K × U) matrices. The typical element, d^r_{ku} shows, for the r^{th} region, the share of commodity u which is produced in industry k .[1] On combining equations (38.1) and (38.2) we can solve for X as

$$X = (I - BD)^{-1} Y \quad . \qquad (38.3)$$

If solutions for Z are required, they may be obtained by substituting from (38.3) into (38.2).

In the representation of the MRIO model given in equations (38.1) - (38.3), the matrix B combines both technological and sourcing assumptions. In its most general form this matrix requires details of flows of commodities from each region to industries in each region. Such data are seldom available.[2] The implementation of the MRIO model typically proceeds by the imposition of simplifying assumptions about industries' regional sourcing of their inputs. The technological and sourcing assumptions are, for example, often separated by writing

$$B = CA \quad , \qquad (38.4)$$

where A is a (UR × KR) block diagonal technology matrix. Along its diagonal are R , (U × K) regional technology matrices. The

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1. In interpreting sums of commodity outputs as industry outputs, we assume that quantity units are defined so that commodity prices are unity.

2. Pioneering work on the construction of such data sets for 80 industries and 50 states in the United States has been completed by Polenske (1972-78). No similar results are available for Australia.

$$\left(B^{(2)r*}_{LM}\right)_{im} S^r_{3i} = \left(B^{(2)*}_{LM}\right)_{im} S^r_{5m} \ , \qquad i\epsilon L, \quad m\epsilon M \ , \qquad (42.15)$$

$$\left(B^{(3)r*}_{L(Us)}\right)_{iu} S^r_{3i} = \left(B^{(3)*}_{L(Us)}\right)_{iu} S^r_{6(us)} \ , \quad i\epsilon L, \quad u\epsilon U, \quad s=1,2 \ , \qquad (42.16)$$

$$\left(B^{(5)r*}_{L(Us)}\right)_{iu} S_{3i} = \left(B^{(5)*}_{L(Us)}\right)_{iu} S^r_{8(us)} \ , \quad i\ L, \quad u\ U, \quad s=1,2 \ , \qquad (42.17)$$

and

$$\left(B^{(4)r*}_{L(U1)}\right)_{in} S_{3i} = \left(B^{(4)*}_{L(U1)}\right)_{in} S^r_{7u} \ , \qquad i\epsilon 1, \quad u\epsilon U, \qquad (42.18)$$

where the B* matrices without the r superscripts are the economy-wide analogs of the regional B* matrices. For example, the i,n^{th} elements of $B^{(1)*}_{LN}$ is the share of the economy-wide sales of good (i1) which is used as an intermediate input to industry n and as a margin service on the delivery of intermediate inputs to industry n . Rather than providing derivations of all the relationships (42.12) - (42.18), we will illustrate general approach by considering only (42.12). This is derived as follows : from (40.1) we have

$$\left(B^{(1)r*}_{LN}\right)_{in} S^r_{3i} = \frac{\left(A^{(1)}_{(i1)n} + \sum_{u\epsilon U}\sum_{s=1}^{2} A^{(us)n1}_{(i1)}\right)\bar{Z}^r_n}{\bar{X}^{(0)r}_{(i1)}} \ \frac{\bar{X}^{(0)r}_{(i1)}}{\bar{X}^{(0)}_{(i1)}} \ ,$$

that is,

$$\left(B^{(1)r*}_{(LN)}\right)_{in} S^r_{3i} = \frac{\left(A^{(1)}_{(i1)n} + \sum_{u\epsilon U}\sum_{s=1}^{2} A^{(us)n1}_{(i1)}\right)\bar{Z}_n}{\bar{X}^{(0)}_{(i1)}} \ \frac{\bar{Z}^r_n}{\bar{Z}_n} \ , \qquad (42.19)$$

We begin by applying equation (39.8) to expand the LHS of equation (42.3), obtaining

$$\tilde{x}^{(0)}_{(i1)} = \sum_r \sum_{n\varepsilon N} \left(B^{(1)r*}_{LN}\right)_{in} S^r_{3i} z^r_n + \sum_r \sum_{m\varepsilon M} \left(B^{(1)*}_{LM}\right)_{im} S^r_{3i} z^r_m$$

$$+ \sum_r \sum_{n\varepsilon N} \left(B^{(2)r*}_{LN}\right)_{in} S^r_{3i} y^r_n + \sum_r \sum_{m\varepsilon M} \left(B^{(2)r*}_{LM}\right)_{im} S^r_{3i} y^r_m$$

$$+ \sum_{h=3,5} \sum_r \sum_{u\varepsilon U} \sum_{s=1}^{2} \left(B^{(h)r*}_{L(Us)}\right)_{iu} S^r_{3i} x^{(h)r}_{(us)}$$

$$+ \sum_r \sum_{u\varepsilon U} \left(B^{(4)r*}_{L(U1)}\right)_{iu} S^r_{3i} x^{(4)}_{(u1)} \quad , \quad i\varepsilon L, \quad r=1,\ldots,R \, , \qquad (42.11)$$

where $\tilde{x}^{(0)}_{(i1)}$ denotes the LHS of (42.3), i.e.,

$$\tilde{x}^{(0)}_{(i1)} \equiv \sum_r x^{(0)r}_{(i1)} S^r_{3i} \; .$$

(We will be using $\tilde{z}_n$ to denote the LHS of (42.1), $\tilde{y}_n$ to denote the LHS of (42.2), etc..) Equation (42.11) can be simplified by using various relationships derivable from equations (40.1) - (40.7). These imply that

$$\left(B^{(1)r*}_{LN}\right)_{in} S^r_{3i} = \left(B^{(1)*}_{LN}\right)_{in} S^r_{1n} \quad , \quad i\varepsilon L, \quad n\varepsilon N \; , \qquad (42.12)$$

$$\left(B^{(1)r*}_{LM}\right)_{im} S^r_{3i} = \left(B^{(1)*}_{LM}\right)_{im} S^r_{4m} \quad , \quad i\varepsilon L, \quad m\varepsilon M \; , \qquad (42.13)$$

$$\left(B^{(2)r*}_{LN}\right)_{in} S^r_{3i} = \left(B^{(2)*}_{LN}\right)_{in} S^r_{2n} \quad , \quad i\varepsilon L, \quad n\varepsilon N \; , \qquad (42.14)$$

typical element, a^r_{uk} , of the r^{th} such matrix shows the total input of commodity u required per unit output of industry k in region r .[1] C is a (UR × UR) matrix of sourcing coefficients arranged in R columns of R , (U × U) diagonal matrices. The u^{th} diagonal element of the r^{th} such matrix in the s^{th} column, $c_u(rs)$, is the share of the usage of commodity u in region s which is sourced from region r . Similarly, we can separate out the sourcing assumptions inherent in the vector Y by writing

$$Y = C\tilde{Y} \; , \qquad (38.5)$$

where $\tilde{Y}$ is an (UR × 1) vector of final demand originating in each region. Note that in (38.4) and (38.5) we have introduced the simplifying assumption that all users of any commodity u in region r draw their supplies from producing regions in the same proportions. The construction of the matrix C requires estimates of inter-regional flows but the need for a sector of use dimension is avoided.

Differences between alternative versions of the MRIO model often centre on the method employed for construction of the C matrix. In most cases, available primary data on inter-regional flows have been insufficient and various secondary methods have been substituted. The gravity model proposed by Leontief and Strout (1963) is a well known example.[2] It eliminates the need for data on

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1. Where region-specific technology data are not available, economy-wide input-output coefficients are usually assumed to apply at the regional level.

2. Liew (1980) is an example, in the Australian context, of a model using the gravity approach to generate inter-regional commodity flows.

inter-regional commodity flows by assuming that they are functions of inter-regional transport costs (frequently proxied by distance) and of output and absorption levels in regions of origin and destination. In the LMPST approach, data requirements are cut even further by imposing a simple dichotomy between regionally traded (national) and regionally non-traded (local) commodities. If regions can be defined so that only a small percentage of consumers are located near regional borders, local commodities (i.e., those with minimal inter-regional trade) are easy to identify. This condition is met for the case of the Australian states. In each of the six states, the principal population centre is located many miles from the state lines. Under the local/national dichotomy, intra-regional sourcing is assumed for all usage of local commodities. The local-commodity rows of the matrix C′ therefore contain only ones or zeros. In fact we have, for any local commodity ℓ ,

$$c_\ell(rs) = \begin{cases} 1 & \text{for } r = s \\ 0 & \text{for } r \neq s \end{cases} . \qquad (38.6)$$

The regional location of the production of national commodities is assumed by LMPST to be independent of the location of demand for national commodities. Each region's share in the economy-wide output of each national commodity is assumed to be exogenous. The LMPST model is equivalent, nevertheless, to a MRIO model in which, for any national commodity h , it is assumed that

$$c_h(rs) = c_h(r) \quad \text{for all } r \text{ and } s , \qquad (38.7)$$

i.e., the share of the use of good h in region s which is sourced from region r is the same for all s . It is clear that (38.7) implies that $c_h(r)$ must be region r's share in the

in the economy-wide output of industry n and S^r_{3i} is the share of region r in the economy-wide output of domestic commodity i .

Equations (42.1), (42.2) and (42.7) follow directly from equations (39.2), (39.11) and (39.26). From (39.24) it follows that

$$\sum_r x^{(5)r}_{(us)} S^r_{8(us)} = x^{(5)}_{(us)} + \sum_r q^{(5)r}_{(us)} S^r_{8(us)} , \quad u\epsilon U, \quad s=1,2 .$$

Hence, equation (42.8) is valid provided that we set the $q^{(5)}_{(us)}$ so that

$$\sum_r q^{(5)r}_{(us)} S^r_{8(us)} = 0 , \qquad u\epsilon U, \quad s=1,2 . \qquad (*)^1$$

Clearly the default setting $\left\{q^{(5)r}_{(us)} = 0 \text{ for all } u, s \text{ and } r\right\}$ is acceptable.

To establish the remaining aggregation conditions we use a simultaneous, rather than a sequential, approach. We use equations from ORES to develop expressions for the LHS's of (42.3), (42.4), (42.5), (42.6), (42.9) and (42.10). Then we establish these conditions by considering our expressions for their LHS's as a set of simultaneous equations.

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1. In this appendix the expressions marked (*) are assumptions (in addition to those already made explicit in sections 39 and 40) required for the validity of the aggregation conditions.

identical to the ORANI results? That is, do the following hold :

$$\sum_r z_n^r S_{1n}^r = z_n , \qquad n \in N , \tag{42.1}$$

$$\sum_r y_n^r S_{2n}^r = y_n , \qquad n \in N , \tag{42.2}$$

$$\sum_r x_{(i1)}^{(0)r} S_{3i}^r = x_{(i1)}^{(0)} , \qquad i \in L , \tag{42.3}$$

$$\sum_r z_m^r S_{4m}^r = z_m , \qquad m \in M , \tag{42.4}$$

$$\sum_r y_m^r S_{5m}^r = y_m , \qquad m \in M , \tag{42.5}$$

$$\sum_r x_{(us)}^{(3)r} S_{6(us)}^r = x_{(us)}^{(3)} , \qquad u \in U; \quad s = 1,2 , \tag{42.6}$$

$$\sum_r x_{(u1)}^{(4)r} S_{7u}^r = x_{(u1)}^{(4)} , \qquad u \in U , \tag{42.7}$$

$$\sum_r x_{(us)}^{(5)r} S_{8(us)}^r = x_{(us)}^{(5)} , \qquad u \in U; \quad s = 1,2 , \tag{42.8}$$

$$\sum_r v^r S_9^r = v , \tag{42.9}$$

$$\sum_r x_{(g+1,1)k}^{(1)r} S_{10,k}^r = x_{(g+1,1,)k}^{(1)} , \qquad k \in K , \tag{42.10}$$

where the variables on the LHS's of equations (42.1) - (42.10) are the results from ORES and the variables on the RHS's are economy-wide ORANI results used as an input to ORES. The S's are the appropriate weights. For example S_{1n}^r is the share of region r

economy-wide production of good h .[1] Thus under the LMPST simplifications, the total data requirements for the C matrix can be met merely by (i) assigning commodities to the local/national categories and (ii) projecting regional shares in economy-wide production of national commodities.[2]

Apart from having minimal data requirements, the LMPST approach has a second attractive feature. If we make simplifications (38.4) - (38.7) and we assume for each industry that there are no differences across regions in either the industry's technology or its shares in regional commodity outputs, then the solution of the resulting MRIO model can be decomposed into a sequence of three steps. This can be illustrated by reference to a 2-region model. Under the LMPST simplifications, the two region version of (38.1) - (38.2) can be written as

$$\begin{pmatrix} X_{1L} \\ X_{1H} \\ X_{2L} \\ X_{2H} \end{pmatrix} = \begin{bmatrix} I & & & \\ & c(1) & & c(1) \\ & & I & \\ & c(2) & & c(2) \end{bmatrix} \begin{bmatrix} a_{L\cdot} & \\ a_{H\cdot} & \\ & a_{L\cdot} \\ & a_{H\cdot} \end{bmatrix} \begin{bmatrix} d_L & d_H & & \\ & & d_L & d_H \end{bmatrix} \begin{pmatrix} X_{1L} \\ X_{1H} \\ X_{2L} \\ X_{2H} \end{pmatrix}$$

$$+ \begin{bmatrix} I & & & \\ & c(1) & & c(1) \\ & & I & \\ & c(2) & & c(2) \end{bmatrix} \begin{pmatrix} \tilde{Y}_{1L} \\ \tilde{Y}_{1H} \\ \tilde{Y}_{2L} \\ \tilde{Y}_{2H} \end{pmatrix} , \tag{38.8}$$

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1. Of course, the resulting special case of MRIO determines the regional location of the output of national commodities only in a trivial sense.

2. In LMPST the $c_h(r)$'s were projected to remain at their base period values.

where X_{rL}, X_{rH}, $r=1,2$ are the output vectors for local and national commodities in region r ; the $c(r)$, $r=1,2$, are diagonal matrices showing the sourcing coefficients $c_h(r)$ for national commodities; $a_{L\cdot}$ and $a_{H\cdot}$ are the local commodity and national commodity rows of the input-output coefficients matrix (assumed the same for all regions); d_L and d_H show industry shares in regional outputs of local and national commodities (d_L and d_H are also assumed constant across regions); and $\tilde{Y}_{rL}$ and $\tilde{Y}_{rH}$ are the final demands for local and national commodities originating in region r . By multiplying out the RHS of (38.8) we find that

$$X_{rL} = a_{L\cdot}\, d_L\, X_{rL} + a_{L\cdot}\, d_H\, X_{rH} + \tilde{Y}_{rL}\ , \qquad r=1,2 \qquad (38.9)$$

and

$$X_{rH} = c(r) \sum_{s=1}^{2} \left(a_{H\cdot}\, d_L\, X_{sL} + a_{H\cdot}\, d_H\, X_{sH}\right) + c(r) \sum_{s=1}^{2} \tilde{Y}_{sH}\ , \qquad r=1,2\ . \qquad (38.10)$$

From (38.9) and (38.10) it is apparent that

$$\begin{pmatrix} \sum_{r=1}^{2} X_{rL} \\ \sum_{r=1}^{2} X_{rH} \end{pmatrix} = \begin{pmatrix} a_{L\cdot} \\ a_{H\cdot} \end{pmatrix} \begin{pmatrix} d_L & d_H \end{pmatrix} \begin{pmatrix} \sum_{r=1}^{2} X_{rL} \\ \sum_{r=1}^{2} X_{rH} \end{pmatrix} + \begin{pmatrix} \sum_{r=1}^{2} \tilde{Y}_{rL} \\ \sum_{r=1}^{2} \tilde{Y}_{rH} \end{pmatrix}\ , \qquad (38.11)$$

that is,

$$X^{ew} = adX^{ew} + \tilde{Y}^{ew} \qquad (38.12)$$

where (38.12) represents (38.11) in simpler notation, the superscripts ew denoting economy-wide. Next we note that (38.10) implies that

42. Appendix on the Compatibility of Regional and Economy-wide Results

Consider the system of equations (39.2), (39.8), (39.11), (39.12), (39.22), (39.24), (39.26) and (39.28) for $r=1,\ldots,R$, plus equations (39.18) and (39.21), that is the eight sets of equations listed at the beginning of subsection 39.3 plus the sets of equations describing regional wage bills and regional employment by industry. This system we will call ORES, ORANI regional equation system. Assume that we have from an ORANI solution results for z_K (industry outputs), y_K (industry investments), $x^{(h)}_{(US)}$ (household, h=3, and "other", h=5, final demand) $x^{(4)}_{(U1)}$ (export demands), $p^{(1)}_{(g+1,1)K}$ (industry wage rates) and $x^{(1)}_{(g+1,1)K}$ (industry employment). Assume, in addition, that we have assigned values to the regional "other" demand variables $q^{(5)r}_{(us)}$ for all (us) and r . Then, from subsection 39.3, we know how to solve ORES to obtain values for

z^r_N, z^r_M (regional outputs for national and local industries),
y^r_N, y^r_M (regional investment for national and local industries),
$x^{(h)r}_{US}$ (regional household, h=3, and "other" final demands, h=5),
$x^{(4)r}_{(U1)}$ (regional exports), $x^{(0)r}_{(L1)}$ (regional outputs of local commodities), $x^{(1)r}_{(g+1,1,)K}$ (regional industry employment) and v^R (regional wage bills). The question to be investigated in this appendix is whether these regional results coming out of ORES are necessarily consistent with the ORANI results used as an input to ORES. Are the appropriately weighted averages of the regional results

might be located in Queensland, say) and 95 per cent of SA-steel is located in South Australia, then the poor performance of the NSW-steel industry compared with that of the SA-steel industry will induce a relatively poor performance in local industries in New South Wales compared with those in South Australia. As well as allowing us some insights to the regional allocation of steel-industry activity, the inclusion of the two steel producers in our economy-wide model would make possible the analysis of regionally-based steel policies. For example, we would be able to simulate the effects of subsidizing the NSW industry without subsidizing the SA industry.

With the currently operational version of ORANI, we have taken a first step towards the introduction of regional industries into our economy-wide model. At present, the regional industries are all in the agricultural sector. With sufficient research resources and appropriate data, perhaps from regional input-output studies, regional industries could be defined for non-agricultural activities. This would not involve any theoretical problems. The introduction of further regional detail into ORANI computations could be made without changes in either the theoretical structure of our economy-wide model or in our regional disaggregation method.

$$X_{rH} = c(r) \left\{a_{H}.d\ X^{ew} + \tilde{Y}^{ew}_{H}\right\}$$

and therefore, in view of (38.12), we have

$$X_{rH} = c(r)\ X^{ew}_{H}\ , \quad r=1,2\ . \tag{38.13}$$

Equation (38.13) confirms the interpretation, following (38.7), of the $c_h(r)$'s as regional production shares. Finally, we can rewrite (38.9) as

$$X_{rL} = a_{L}.\ d_{L}\ X_{rL} + \tilde{Y}^{*}_{rL}\ , \quad r=1,2\ , \tag{38.14}$$

where

$$\tilde{Y}^{*}_{rL} = a_{L}.\ d_{H}\ c(r)\ X^{ew}_{H} + \tilde{Y}_{rL}\ . \tag{38.15}$$

Equations (38.12), (38.13) and (38.14) show the three-step decomposition. In the first step we use the conventional economy-wide input-output model (38.12) to compute X^{ew} as

$$X^{ew} = (I - ad)^{-1}\ \tilde{Y}^{ew}\ . \tag{38.16}$$

In the second step we allocate the economy-wide outputs of the national commodities to the regions in exogenously determined proportions, i.e. we compute X_{rH} , $r=1,2$, according to (38.13). In the final step we first compute the $\tilde{Y}^{*}_{rL}$'s by using (38.15). Then we complete our solution to the MRIO model (38.8) by obtaining the regional outputs of local commodities by single-region input-output computations

$$X_{rL} = (I - a_{L}.\ d_{L})^{-1}\ \tilde{Y}^{*}_{rL}\ , \quad r=1,2\ . \tag{38.17}$$

This three-step procedure is exactly the computational method employed by LMPST.

The computational advantages of decomposing the MRIO solution are obvious. In applications of the MRIO model, it is not unusual to work with as many as 50 regions and 80 industries and commodities. This gives an (I - BD) matrix in (38.3) with dimensions of approximately 4000 × 4000. Where the LMPST simplifications are used, the inversion of such a vast matrix is unnecessary. Instead we can invert the 80 × 80 matrix (I - ad) and the smaller matrix $(I - a_{L.} d_{L.})$.

Although of interest, computational advantages are not, for our purposes, the most important aspect of the LMPST decomposition. What is more critical is the idea that, under plausible simplifications, it is possible to develop a multiregional model as post-solution extension of an existing economy-wide model. For LMPST, the economy-wide model is the input-output model (38.12). For us, the economy-wide model is ORANI.

industries classified as national (i.e., industries producing national goods). Second, our model is unsuitable for analysing the effects of policy changes originating at the state level (as opposed to the effects on states of policy changes originating at the national level). One approach to overcoming these limitations is to introduce regional dimensions into national industries in the main ORANI model.[1] If the necessary data were available, any industry in the model could be split into separate regional components, each with its own technology, sales pattern, substitution parameters etc. as appropriate. The responses of regional components of the industry to national policy changes would then be endogenously determined and the state components could also be shocked individually to reflect state policy changes. Assume, for example, that our economy-wide ORANI model included two steel producing industries : SA-steel and NSW-steel with SA-steel using a more capital-intensive technology than NSW-steel. Now suppose that we use our economy-wide model to simulate the impact of a general increase in real wages. Our projections will indicate that output, employment, etc., in the NSW-steel industry will be reduced to a greater extent than in the SA-steel industry. When we append our regional disaggregation computation to our economy-wide results, we will capture the indirect regional implications of the different performance of the two steel industries. If 90 per cent of the production of NSW-steel is located in the state of New South Wales, (the rest

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1. The idea of defining regional industries, in a rectangular input-output model can be found in Hoffman and Kent (1976). See also Dixon and Parmenter (1979).

model to generate economy-wide results. Then the regional computation disaggregates these results to the state level. The disaggregation is consistent with the initial economy-wide computation in the sense that a reaggregation of the state-level results reproduces the initial economy-wide results.

Our regional computations capture two aspects of the differential impact across the states of economy-wide disturbances. First, they reflect variations in industrial composition. For example, in Chapter 7, we argue that general increases in tariffs are likely to reduce economic activity in Queensland and Western Australia because of the comparatively heavy concentration of those states on exporting industries. On the other hand, Victoria, the home of much of Australia's import-competing industry, is projected to benefit from tariff increases. The second ingredient in our regional computations is the local multiplier effects. Stimulation of the automobile industry, for example, will have a greater effect on employment and income in South Australia than in Queensland.[1] This in turn will stimulate "local" industries (service industries and producers of perishables) in South Australia relative to those in Queensland.

Two limitations of our existing regional model should be pointed out. First, it is necessary to fix by an (arbitrary) exogenous assumption the regional location of output changes for

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1. Much of Australia's automobile production is located in South Australia and comparatively little is located in Queensland.

39. The LMPST Method Modified for use with ORANI

In the last section we saw how the LMPST computations were decomposed into three parts. Regional ORANI computations can be conducted in the same three-part way. The first part, which has already been described in the previous chapters, consists of using the ORANI model to project the economy-wide effects of the exogenous shock under consideration. In the second part, the economy-wide activity levels in industries producing national commodities are allocated to the regions in exogenously given proportions. In the third part we set up and solve a system of commodity-balance equations for each region to obtain projections of regional outputs of local goods. Given our allocation rule for projecting regional activity in industries producing national goods, it will be obvious that our regional projections for these industries are consistent with the ORANI economy-wide projections. For the outputs of local goods, however, the establishment of consistency between our regional and economy-wide projections requires a little more effort. The relevant proposition is proved in section 42.

In this section, we give the equations for the second and third parts of an ORANI regional computation. In presenting this material, we attempt, as far as possible, to keep the notation consistent with that used in the description of the main ORANI model. An additional unbracketed superscript, r, is used to denote region. We will not always define symbols for regional concepts where the definition for the corresponding economy-wide concept is given in Tables 23.2 and 27.1. For example, we use the symbol $x^{(0)r}_{(i1)}$ to denote the percentage change in the output of domestic commodity i in region r . Such a symbol need not be defined in the text since a definition is given in Table 23.2 of the symbol $x^{(0)}_{(i1)}$. Other notational conventions adopted here include the use of

- U to identify the set of all commodities;
- K to identify the set of all industries;
- N to identify the set of industries producing national commodities (i.e., "national industries");
- M to identify the set of industries producing local commodities (i.e., "local industries");
- H to identify the set of national commodities;
- L to identify the set of local commodities; and
- R to identify the number of regions.

Note that ORANI's industry and commodity classifications are such that no industries produce both national and local commodities, i.e., $N \cap M$ is empty.

39.1 The regional allocation of activity levels in national industries

We allocate the activity levels of national industries by

$$Z^r_n = Z_n G^r_n , \quad \text{for all } n \epsilon N \text{ and } r=1,\ldots,R , \qquad (39.1)$$

where G^r_n is the base-period proportion of the aggregate output of industry n which is produced in region r . Since G^r_n is a

41. Summary and Concluding Remarks

For each application of the ORANI model it is possible to append an ORANI regional computation. The theory and data base for these regional computations has been described in sections 39 and 40. Our regionalizing method, based on the earlier work of LMPST, has the following three properties :

(i) it involves a minimum use of regional data,

(ii) it is sequential to the economy-wide ORANI computations, and

(iii) it is consistent with the economy-wide ORANI results.

The input of regional data is minimized by two assumptions. First, the use of inter-regional commodity flow data is obviated by assuming that commodities can be categorized as either local or national. For local commodities, it is assumed that all sales must be in the region of their production. For national commodities, it is assumed that the regional pattern of production is independent of the regional pattern of demand. Most services and perishable commodities are classified as local. Examples of national commodities are automobiles, capital equipment, clothing and other non-perishable and easily transported commodities. Second, we assumed that economy-wide input-output coefficients for inputs of "local" commodities were valid at the regional level, thus avoiding the need for information on region-specific technology. The sequential property of the regional computations means that there is no feedback from the regional level to the economy-wide level. First we run the ORANI

W_K is a vector of the shares of industries in the national wage bill. These shares can be computed directly from the national input-output data base. The W_K^r are vectors of industrial shares in regional wage bills. Given the ideal data base, figure 40.1, they could be formed from the relevant rows in the value added matrices, $\left(V_M^r , V_N^r\right)$. Our approach was to apply the uniform technology assumption. That is, we estimated the base-period wage bill in industry k in region r , $\overline{U}_k^r$, as

$$\overline{U}_k^r = \frac{\overline{U}_k \overline{Z}_k^r}{\overline{Z}_k} , \qquad r=1,\ldots,R, \quad k\epsilon K , \qquad (40.9)$$

where $\overline{U}_k$ is the base-period, national wage bill for industry k . Then we computed $\left(W_K^W\right)_k$ as

$$\left(W_K^r\right)_k = \overline{U}_k^r \Big/ \sum_{k\epsilon K} \overline{U}_k^r , \qquad r=1,\ldots,R, \quad k\epsilon K . \qquad (40.10)$$

constant, equation (39.1) implies[1]

$$z_n^r = z_n \quad \text{for all} \quad n\epsilon N \quad \text{and} \quad r=1,\ldots,R . \qquad (39.2)$$

If there is a one per cent increase in the economy's aggregate output in a national industry (pastoral-zone agriculture, say), then the output of this industry is assumed to increase by one per cent in each region.

It is worth emphasizing that in regional ORANI computations, it is regional shares in the outputs of national industries which are assumed exogenous, not regional shares in the outputs of national commodities. This leaves us free to assume (if we wish)[2] that the commodity composition of output in national industries is constant across regions. In ORANI, the assumption of constant regional shares in the outputs of national commodities would be inappropriate. For example, consider a situation in which the ORANI economy-wide projections indicate an increase in beef production by industry 4 (northern beef) and a decrease in beef production by industry 3 (high-rainfall-zone agriculture). Industry 4 is located principally in Queensland while industry 3 is located in the southern states. Thus, an attempt to exogenize state shares in beef production is likely to be incompatible with the ORANI economy-wide results.

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1. For compatibility with ORANI, we must express the equations for the regional extension in percentage change form. As usual we use the convention that the lower case symbol represents the percentage change in the variable denoted by the corresponding upper case symbol.

2. An explicit assumption on the commodity composition of national industry output at the regional level is required only if we wish to project regional commodity outputs for national commodities.

One implication of exogenizing regional shares in national-industry activity, rather than regional shares in the outputs of national commodities is that the sourcing matrix, C , cannot be considered a constant. This causes no difficulties. In fact this matrix plays no explicit role in our regional theory.

39.2 The regional balance equations for local commodities

We next impose an intra-regional sourcing constraint on users of local commodities within each region. We require that the aggregate output of any local commodity in a region be equal to the aggregate demand for the commodity in the region. That is,

$$\begin{aligned} X^{(0)r}_{(i1)} = & \sum_{n\epsilon N} A^{(1)}_{(i1)n} Z^r_n + \sum_{m\epsilon M} A^{(1)}_{(i1)m} Z^r_m + \sum_{n\epsilon N} A^{(2)}_{(i1)n} Y^r_n \\ & + \sum_{m\epsilon M} A^{(2)}_{(i1)m} Y^r_m + X^{(3)r}_{(i1)} + X^{(5)r}_{(i1)} \\ & + \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{n\epsilon N} \sum_{h=1}^{2} A^{(us)nh}_{(i1)} X^{(h)r}_{(us)n} \\ & + \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{m\epsilon M} \sum_{h=1}^{2} A^{(us)mh}_{(i1)} X^{(h)r}_{(us)m} \\ & + \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{h=3,5} A^{(us)h}_{(i1)} X^{(h)r}_{(us)} \\ & + \sum_{u\epsilon U} A^{(u1)4}_{(i1)} X^{(4)r}_{(u1)} , \qquad i\epsilon L, \quad r=1,\ldots,R , \end{aligned} \tag{39.3}$$

As a check on the calculations described by equations (40.1) - (40.7) we computed the sums

$$\begin{aligned} \mathrm{Sum}(e,r) = & \sum_{\kappa=M,N} \sum_{f\epsilon K} \sum_{h=1}^{2} \left[B^{(h)r^*}_{L\kappa}\right]_{ef} \\ & + \sum_{s=1,2} \sum_{f\epsilon U} \sum_{h=3,5} \left[B^{(h)r^*}_{L(Us)}\right]_{ef} \\ & + \sum_{f\epsilon U} \left[B^{(4)r}_{L(U1)}\right]_{ef} \\ & r=1,\ldots,R, \quad e\epsilon L . \end{aligned} \tag{40.8}$$

For local commodities $(e\epsilon L)$, each of these sums should be 1. We forced this equality for all $r=1,\ldots,R$, and $e\epsilon L$, by the obvious proportionate scaling, thus producing our final B^* estimates. However, before we applied the scaling procedure, we used (40.8) as a check on the validity of our allocation of commodities to the local and national categories. We also found (40.8) to be a useful check on our regional data, $\overline{X}^{(0)r}_{(e1)}$, $\overline{Z}^r_f$, etc..

The remaining data required for equations (39.29) and (39.30) are the vectors W^r_K and W_K : the vector of expenditure elasticities, $\epsilon_{(US)}$, is taken from the ORANI data base[1] and the scalar, γ, is treated at this stage as a user specified parameter.

1. See section 29(e).

where δ^{s}_{ef} is as defined in (40.5). The A's appearing in these formulae are economy-wide base-period coefficients derivable from the ORANI input-output data files. $A^{(h)}_{(e1)f}$ is the use of good (e1) per unit output (h=1) or investment (h=2) in industry f. $A^{(us)fh}_{(e1)}$ is the use of good (e1) as a margin service per unit flow of good (us) to industry f for use as an intermediate input (h=1) or as an input to capital creation (h=2). $A^{(3)}_{(fs)}$ is household consumption of good (fs) per unit of aggregate consumption. $A^{(fs)3}_{(e1)}$ is the use of good (e1) as a margin service per unit flow of good (fs) to the household sector and $A^{(f1)4}_{(e1)}$ is the use of good (e1) as a margin service per unit export of good (f1).

$\bar{Y}_f$ is the base period economy-wide investment in industry f. $\bar{X}^{(0)r}_{(e1)}$, $\bar{Z}^{r}_{f}$, $\bar{X}^{(3)r}$ and $\bar{X}^{(4)r}_{(f1)}$ are estimates for region r for the base period of, in turn, the regional output of domestic commodity e, the regional output of industry f, aggregate regional consumption and regional exports of the f^{th} commodity. The derivation of these estimates is described in Lawson and Parmenter (1979) and Lawson and Vincent (1979). In equation (40.7), $\bar{X}^{(5)r}_{(e1)}$ is an estimate of regional "other" final demand for domestic commodity e in the base period and the $\bar{X}^{(fs)5r}_{(e1)}$'s are estimates of regional usage of commodity (e1) as a margin on "other" final demand. Again the derivation of these estimates is described in Lawson and Parmenter (1979) and Lawson and Vincent (1979).

where $A^{(h)}_{(i1)k}$ is the direct input of domestically produced commodity i required per unit output (h = 1) or capital formation (h = 2) in industry k, $A^{(us)kh}_{(i1)}$ is the input of domestic commodity i required as a margins service per unit direct flow of commodity u from source s to industry k for purpose h, and $A^{(us)h}_{(i1)}$ is the input of domestic commodity i required as a margins service per unit direct flow of commodity u from source s to final demand category h.

Two points regarding (39.3) can be noted immediately. First, the coefficients A do not carry regional subscripts. This reflects the assumption that each industry's technology is independent of its regional location.[1] Second, (39.3) implies that the use as a margin service of local good i produced in region r is entirely in facilitating commodity export flows and commodity flows to users within region r. Such an assumption would not be reasonable if i were a transport service. On the other hand, the assumption is reasonable for wholesale and retail trade. Thus, in applications of our regional model, the latter services are classified as local, whereas transport services are classified as national.[2]

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1. This does not mean that we assume that there are no regional differences in production technologies for particular commodities. For example, ORANI has several beef producing industries. Beef in the northern states is produced mainly by the capital-intensive technique of industry 4 (northern beef). In the south, beef production is concentrated in industry 3 (high-rainfall-zone agriculture), a comparatively labor-intensive industry.

2. This classification for transport services is just the less inappropriate of the available alternatives. It is not entirely satisfactory to assume that the regional allocation of the production of transport services is independent of the regional allocation of the demand for transport services.

Various other aspects of (39.3) are more easily discussed with reference to its percentage change form which can be written as

$$x^{(0)r}_{(i1)} = \sum_{n\epsilon N} \left(a^{(1)}_{(i1)n} + z^r_n\right) B^{(1)r}_{(i1)n} + \sum_{m\epsilon M} \left(a^{(1)}_{(i1)m} + z^r_m\right) B^{(1)r}_{(i1)m}$$

$$+ \sum_{n\epsilon N} \left(a^{(2)}_{(i1)n} + y^r_n\right) B^{(2)r}_{(i1)n} + \sum_{m\epsilon M} \left(a^{(2)}_{(i1)m} + y^r_m\right) B^{(2)r}_{(i1)m}$$

$$+ x^{(3)r}_{(i1)} B^{(3)r}_{(i1)} + x^{(5)r}_{(i1)} B^{(5)r}_{(i1)}$$

$$+ \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{n\epsilon N} \sum_{h=1}^{2} \left(a^{(us)nh}_{(i1)} + x^{(h)r}_{(us)n}\right) B^{(us)nhr}_{(i1)}$$

$$+ \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{m\epsilon M} \sum_{h=1}^{2} \left(a^{(us)mh}_{(i1)} + x^{(h)r}_{(us)m}\right) B^{(us)mhr}_{(i1)}$$

$$+ \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{h=3,5} \left(a^{(us)h}_{(i1)} + x^{(h)r}_{(us)}\right) B^{(us)hr}_{(i1)}$$

$$+ \sum_{u\epsilon U} \left(a^{(u1)4}_{(i1)} + x^{(4)r}_{(u1)}\right) B^{(u1)4r}_{(i1)} \quad , \quad i\epsilon L, \quad r=1,\ldots,R \, , \qquad (39.4)$$

where the B's are sales shares.

Considerable simplification of (39.4) results if we assume that there is no technical change. Under this assumption, all of the a's are zeros. In general, ORANI allows changes in the direct input coefficients, $A^{(h)}_{(i1)k}$, via substitution between foreign and domestic sources of good i . However, where good i is classified

$$\left(B^{(2)r*}_{L\kappa}\right)_{ef} = \frac{\left(A^{(2)}_{(e1)f} + \sum_{u\epsilon U} \sum_{s=1}^{2} A^{(us)f2}_{(e1)}\right) \overline{Y}^r_f}{\overline{X}^{(0)}_{(e1)}} \, , \qquad (40.2)$$

κ=M, N, r=1,...,R, eϵL, fϵK ,

where

$$\overline{Y}^r_f = \frac{Y_f \, \overline{Z}^r_f}{\overline{Z}_f} \; ; \qquad (40.3)$$

$$\left(B^{(3)r*}_{L(Us)}\right)_{ef} = \frac{\left(\delta^s_{ef} A^{(3)}_{(e1)} + A^{(fs)3}_{(e1)} A^{(3)}_{(fs)}\right) \overline{X}^{(3)r}}{\overline{X}^{(0)r}_{(e1)}} \, , \qquad (40.4)$$

s=1,2, r=1,...,R, eϵL, fϵU ,

where

$$\delta^s_{ef} = \begin{cases} 1 & \text{if } e=f \text{ and } s=1 \, , \\ 0 & \text{otherwise} \; ; \end{cases} \qquad (40.5)$$

$$\left(B^{(4)r*}_{L(U1)}\right)_{ef} = \frac{A^{(f1)4}_{(e1)} \; \overline{X}^{(4)r}_{(f1)}}{\overline{X}^{(0)r}_{(e1)}} \, , \qquad (40.6)$$

r=1,...,R, eϵL, fϵU ;

and

$$\left(B^{(5)r*}_{L(Us)}\right)_{ef} = \frac{\delta^s_{ef} \; \overline{X}^{(5)r}_{(e1)} + \overline{X}^{(fs)5r}_{(e1)}}{\overline{X}^{(0)r}_{(e1)}} \, , \qquad (40.7)$$

s=1,2, r=1,...,R, eϵL, fϵU , assigned to the national and local categories on the basis of whether significant inter-regional transfer of them can occur. This assignment can be accomplished, however, without any detailed knowledge of the origins and destinations of inter-regional flows. For technological and/or institutional reasons some commodities can be assigned a priori to the local category. As explained below, manipulation of our data on regional production and absorption of commodities yields relevant empirical evidence for some other cases. The general criterion is that we wish to treat as local those commodities for which an increase in demand within any region will be met mainly by an increase in production within that region.

40.2 The construction of the parameter matrices for implementation of equations (39.29) and (39.30)

The B^* matrices appearing in equations (39.29) and (39.30) all refer to the sales patterns of local goods in region r. Given the ideal data base, the B^*'s could be constructed from the information contained in matrices of the form X_{LM}^{rr}, X_{LN}^{rr}, FD_{L}^{rr}, (see figure 40.1). In the absence of consistent regional input-output tables, we computed preliminary estimates of the ef^{th} elements of $B_{L\kappa}^{(1)r*}$ and $B_{L\kappa}^{(2)r*}$ for $\kappa = M$ and N, of $B_{L(Us)}^{(3)r*}$ and $B_{L(Us)}^{(5)r*}$ for s=1,2, and of $B_{L(U1)}^{(4)r*}$, according to the formulae

$$\left(B_{L\kappa}^{(1)r*}\right)_{ef} = \frac{\left(A_{(e1)f}^{(1)} + \sum_{u\epsilon U} \sum_{s=1}^{2} A_{(e1)}^{(us)f1}\right) \bar{Z}_f^r}{\bar{X}_{(e1)}^{(0)r}} , \quad (40.1)$$

$$\kappa = M,N, \quad r=1,\ldots,R, \quad e\epsilon L, \quad f\epsilon K ;$$

as local, no substitution takes place since imports are zero. In the case of the margins-related, coefficients, $A_{(i1)}^{(us)kh}$ and $A_{(i1)}^{(us)h}$, ORANI recognizes technical change as the only source of variation (see section 17).

Further simplification is possible if we assume that

$$B_{(i1)}^{(us)khr} = B_{(i1)}^{(u\cdot)khr} S_{(us)k}^{(h)r} , \quad i\epsilon L, \quad u\epsilon U, \quad s=1,2 ,$$

$$k\epsilon K, \quad h=1,2, \quad r=1,\ldots,R . \quad (39.5)$$

With (39.5) we are saying that, in the base period,[1] the relative quantities of good (i1) used in facilitating the domestic (s=1) and import (s=2) components of each commodity flow to each industry in each region reflect the relative values of these components. If two-thirds of the costs of the steel input to the automotive industry in region r are accounted for by domestic steel and one-third by imported steel, then under (39.5) we assume that two-thirds of the wholesale margins involved in delivering steel to the automotive industry in region r are associated with the delivery of domestic steel and one-third is associated with the delivery of imported steel. If we now assume that (12.23) and (13.4) are valid at the regional level, (i.e., with superscript r's appended), then, in the absence

1. It is only in a Johansen-style computation that we can take advantage of particular relationships (such as (39.5)) between base period cost and sales shares. In an Euler-style computation we must recognize that these relationships need not remain valid as cost and sales shares move away from their base period values.

of technical change, we have

$$\sum_{s=1}^{2} x^{(1)r}_{(us)k} B^{(us)k1r}_{(i1)} = z^r_k B^{(u\cdot)k1r}_{(i1)} , \quad i\epsilon L, \quad u\epsilon U, \quad k\epsilon K, \quad r=1,\ldots,R , \quad (39.6)$$

and

$$\sum_{s=1}^{2} x^{(2)r}_{(us)k} B^{(us)k2r}_{(i1)} = y^r_k B^{(u\cdot)k2r}_{(i1)} , \quad i\epsilon L, \quad u\epsilon U, \quad k\epsilon K, \quad r=1,\ldots,R . \quad (39.7)$$

On setting the a's at zero and using (39.6) and (39.7) we find that (39.4) simplifies to

$$x^{(0)r}_{(i1)} = \sum_{n\epsilon N} \left[B^{(1)r}_{(i1)n} + \sum_{u\epsilon U} B^{(u\cdot)n1r}_{(i1)} \right] z^r_n$$

$$+ \sum_{m\epsilon M} \left[B^{(1)r}_{(i1)m} + \sum_{u\epsilon U} B^{(u\cdot)m1r}_{(i1)} \right] z^r_m$$

$$+ \sum_{n\epsilon N} \left[B^{(2)r}_{(i1)n} + \sum_{u\epsilon U} B^{(u\cdot)n2r}_{(i1)} \right] y^r_n$$

$$+ \sum_{m\epsilon M} \left[B^{(2)r}_{(i1)m} + \sum_{u\epsilon U} B^{(u\cdot)m2r}_{(i1)} \right] y^r_m$$

$$+ B^{(3)r}_{(i1)} x^{(3)r}_{(i1)} + B^{(5)r}_{(i1)} x^{(5)r}_{(i1)}$$

$$+ \sum_{u\epsilon U} \sum_{s=1}^{2} \sum_{h=3,5} B^{(us)hr}_{(i1)} x^{(h)r}_{(us)} + \sum_{u\epsilon U} B^{(u1)4r}_{(i1)} x^{(4)r}_{(u1)} ,$$

$$i\epsilon L, \quad r=1,\ldots,R . \qquad (39.8)$$

Figure 40.1 : Ideal data base for a 2 region input-output model[1]

		Intermediate Flows				Final Demands		Total Usage
		Region 1		Region 2		Region 1	Region 2	
		kεM	kεN	kεM	kεN			
REGION 1	iεL	X^{11}_{LM}	X^{11}_{LN}	X^{12}_{LM}	X^{12}_{LN}	FD^{11}_{L}	FD^{12}_{L}	X^1_L
	iεH	X^{11}_{HM}	X^{11}_{HN}	X^{12}_{HM}	X^{12}_{HN}	FD^{11}_{H}	FD^{12}_{H}	X^1_H
REGION 2	iεL	X^{21}_{LM}	X^{21}_{LN}	X^{22}_{LM}	X^{22}_{LN}	FD^{21}_{L}	FD^{22}_{L}	X^2_L
	iεH	X^{21}_{HM}	X^{21}_{HN}	X^{22}_{HM}	X^{22}_{HN}	FD^{21}_{H}	FD^{22}_{H}	X^2_H
Value Added		V^1_M	V^1_N	V^2_M	V^2_N			
Total Outputs		Z^1_M	Z^1_N	Z^2_M	Z^2_N	FD^1	FD^2	

1. The assumptions underlying our modified LMPST method eliminate the need for data on the cross-hatched commodity flows.

40. Implementation of the Modified LMPST Method

40.1 General comments on data requirements

In section 38 we pointed out that MRIO models can absorb information on flows of each commodity from each region to each user (industrial and final) in each region. Figure 40.1 illustrates an ideal data base for a model with just two regions, showing all inter-industry and inter-regional flows. Because such a data base is not normally available, builders of multiregional models usually adopt various strong assumptions, especially with respect to inter-regional commodity flows. The assumptions underlying our modified LMPST method reduce the requirements for regional data to a very modest level. For those commodities which we classify as local, we assume that all sales are to local users. Using the notation of figure 40.1, we are saying that X_{LN}^{rs} , X_{LM}^{rs} and FD_{L}^{rs} are zero for all $r \neq s$. For those commodities which are classified as national, we require no information on regional sales patterns, i.e., our model does not use the matrices X_{HM}^{rs} , X_{HN}^{rs} , FD_{H}^{rs} , $r,s=1,2$. We assume that, irrespective of the sales pattern of its output, the national industry n in region r maintains its share in the economy-wide output of industry n . Thus, of the commodity flows shown in figure 40.1, only the intra-regional sales of local commodities are required data, that is the matrices X_{LM}^{rr} , X_{LN}^{rr} and FD_{L}^{rr} , which are not cross-hatched in figure 40.1. Notice that data on inter-regional commodity flows are largely unnecessary. It is true that commodities must be

We will assume that the simplified version, (39.8), of the regional balance equations for local goods is adequate. No data is available which would contradict (39.5) and, to date, in ORANI applications in which the regional dimension has been of interest, zero technical change has been assumed. We continue the description of our modified LMPST method by providing in parts (a) - (e) of this subsection (39.2) equations for variables on the RHS of (39.8) ; z_m^r, y_n^r, y_m^r, $x_{(i1)}^{(3)r}$, $x_{(i1)}^{(5)r}$, $x_{(us)}^{(h)r}$ and $x_{(u1)}^{(4)r}$. Notice that z_n^r has already been catered for by equation (39.2). By expressing each of these variables as functions of (i) variables which are exogenous at the regional level (usually variables endogenous to a national ORANI computation) and (ii) regional outputs of local commodities, we will be able, eventually, to solve (39.8) for percentages changes in regional outputs of local commodities.

(a) Regional investment

We assume that investment in any industry is allocated to the regions in the same proportions as current output. This assumption can be written as

$$Y_n^r = Y_n G_n^r , \qquad n\varepsilon N, \quad r=1,\ldots,R , \qquad (39.9)$$

and

$$Y_m^r = Y_m Z_m^r / Z_m , \qquad m\varepsilon M, \quad r=1,\ldots,R . \qquad (39.10)$$

The G_n^r , $n\varepsilon N$, are treated as constants (see subsection 39.1) so that the percentage change forms of equations (39.9) and (39.10) are

$$y_n^r = y_n , \qquad n\varepsilon N, \quad r=1,\ldots,R , \qquad (39.11)$$

and

$$y_m^r = y_m + z_m^r - z_m , \qquad m\varepsilon M, \quad r=1,\ldots,R . \qquad (39.12)$$

(b) Household consumption at the regional level

We assume that in each region there is a link between regional consumption and regional labour income.[1] In particular, we assume that

$$X^{(3)r}_{(us)} = f^r_{(us)}\left(X^{(3)}_{(us)}\ ,\ V^r/V\right)\ ,\quad u\epsilon U,\quad s=1,2,\quad r=1,\ldots,R\ , \qquad (39.13)$$

where V^r is the total wage bill in region r, and V is the economy-wide aggregate wage bill. The percentage change form of equation (39.13) is

$$x^{(3)r}_{(us)} = \alpha^r_{(us)}\, x^{(3)}_{(us)} + \gamma^r_{(us)}\,(v^r - v)\ ,\quad u\epsilon U,\quad s=1,2,\quad r=1,\ldots,R\ . \qquad (39.14)$$

where $\alpha^r_{(us)}$ is the elasticity in region r of consumption of good u from source s with respect to the aggregate consumption of that good, and $\gamma^r_{(us)}$ is the elasticity in region r of the consumption of good u from source s with respect to the share of region r in the economy's aggregate wage bill. In our computations we assume that

$$\alpha^r_{(us)} = 1 \qquad (39.15)$$

and

$$\gamma^r_{(us)} = \epsilon_{(us)}\gamma\ ,\quad 0 \le \gamma \le 1\ ,\quad u\epsilon U,\quad s=1,2,\quad r=1,\ldots,R\ , \qquad (39.16)$$

---

1. An alternative treatment might relate regional consumption to total factor payments in the region rather than to labor income alone. However, the employment of non-labor factors in the region does not necessarily generate income payments in that region. In any case, movements in labor income and in total income are likely to be similar.

households and commodity demands by "other" users. All of the variables on the RHS of equation (39.29) are either outputs of an economy-wide ORANI computation or are exogenously determined. Equation (39.2) provides the percentage changes in the regional outputs of <u>national</u> industries. Thus (39.2), (39.28) and (39.29) together are sufficient to describe, via ORANI, the effects of national policy changes on all industry outputs at the regional level. Other variables which are of interest in describing the regional impacts of national policy can be derived by simple calculations. Equations (39.11) and (39.12) give us regional investment by industry. We could also compute, for example, regional employment and income changes.

subvectors. Finally, note that in the matrix version of equation (39.28), S^r_{ML} is a square matrix. Its i^{th} row contains zeros in all positions except for the column corresponding to the commodity produced by the i^{th} industry. In that column is a one.

Equations (39.2), (39.11), (39.12), (39.22), (39.24) and (39.26) are substituted into (39.8) and equation (39.28) is used to eliminate z^r_M. The system is solved for $x^{(0)r}_{(L1)}$, yielding

$$\begin{aligned} x^{(0)r}_{(L1)} = \psi^r \Big\{ & B^{(1)r*}_{LN} z_N + \left[- B^{(2)r*}_{LM} - \gamma B^{(3)r*}_{L(US)} \varepsilon_{(US)} W^{r'}_M \right] z_M \\ & + B^{(2)r*}_{LN} y_N + B^{(2)r*}_{LM} y_M + B^{(3)r*}_{L(US)} x^{(3)}_{(US)} \\ & + B^{(3)r*}_{L(US)} \varepsilon_{(US)} \left[W^{r'}_K - W'_K \right] \left[p^{(1)}_{(g+1,1)K} + x^{(1)}_{(g+1,1)K} \right] \\ & + B^{(4)r*}_{L(U1)} x^{(4)}_{(U1)} + B^{(5)r*}_{L(US)} x^{(5)}_{(US)} + B^{(5)r*}_{L(US)} q^{(5)r}_{(US)} \Big\} , \end{aligned} \quad (39.29)$$

where

$$\psi^r = \left[I - \left(B^{(1)r*}_{LM} + B^{(2)r*}_{LM} + \gamma B^{(3)r*}_{L(US)} \varepsilon_{(US)} W^{r'}_M \right) S^r_{ML} \right]^{-1} . \quad (39.30)$$

The set of equations (39.29), for $r=1,\ldots,R$, are the solution equations for the regional computation. They express the percentage changes in regional outputs of local goods as functions of percentage changes in various economy-wide variables : industry output levels, industry investment levels, industry employment levels industry wage rates, commodity export demands, commodity demands by

where γ is a user-specified parameter, the value of which reflects the dependence of aggregate regional consumption on regional income, and $\varepsilon_{(us)}$ is the economy-wide household expenditure elasticity of demand for good u from source s. If, in addition to setting the α's to unity, we also set γ at unity, then changes in regional consumption levels fully reflect changes in the regional allocation of labor income.[1] This might be appropriate for long-run simulations. At the other extreme, setting $\gamma = 0$ insulates regional consumption from changes in regional income. This assumption might be more suitable for the short run where the effects of fluctuations in wage income on consumption are moderated by changes in social security payments and in savings.

The wage-bill variables which appear in equations (39.13) and (39.14) can be expressed in terms of wage rates and industry-specific employment levels. First, the regional wage bills, V^r, are

$$V^r = \sum_{k\varepsilon K} P^{(1)}_{(g+1,1)k} X^{(1)r}_{(g+1,1)k} , \quad r=1,\ldots,R . \quad (39.17)$$

Notice that we do not append a regional superscript to the wage-rate variable, $P^{(1)}_{(g+1,1)k}$. This reflects an assumption that wage rates do not vary regionally. In percentage-change form, equation (39.17) becomes

$$v^r = \sum_{k\varepsilon K} \left(p^{(1)}_{(g+1,1)k} + x^{(1)r}_{(g+1,1)k} \right) W^r_k , \quad r=1,\ldots,R , \quad (39.18)$$

1. The treatment of consumption in the original LMPST article corresponds closely to this case.

where W_k^r is the share of industry k in the aggregate wage bill of region r. Second, the economy-wide wage bill is

$$V = \sum_{k \epsilon K} P^{(1)}_{(g+1,1)k} \, X^{(1)}_{(g+1,1)k} , \tag{39.19}$$

and, in percentage changes,

$$v = \sum_{k \epsilon K} \left(p^{(1)}_{(g+1,1)k} + x^{(1)}_{(g+1,1)k} \right) W_k , \tag{39.20}$$

where the weights W_k are industries' shares in the national, rather than the regional, wage bill.

Equation (39.18) includes employment by region as well as by industry. To determine regional employment we assume that the percentage change in employment per unit of output in industry k in each region r is the same as the percentage change in employment per unit of output in industry k for the nation. That is,

$$x^{(1)r}_{(g+1,1)k} - z^r_k = x^{(1)}_{(g+1,1,)k} - z_k , \quad k \epsilon K, \quad r=1,\dots,R . \tag{39.21}$$

This assumption is clearly more satisfactory for long-run analysis than for short-run. In the short run, we could expect exogenous shocks, such as tariff changes, to have different impacts on capacity utilization and labor-output ratios in different regions. For example, industries supplying local goods in regions which benefited from an exogenous shock, could, in the short run, be expected to move towards higher capacity utilization (and higher labor-output ratios). Labor-output ratios in similar industries in regions in which activity is depressed by the exogenous shock would move in the opposite direction.

In the matrix representation of equation (39.8) the matrices carrying the asterisk (*) as a superscript contain the shares of both direct and margins sales of the local commodities. This is straightforward for the first four such matrices in the equation. The fifth matrix, $B^{(3)r*}_{L(US)}$, can be written as

$$B^{(3)r*}_{L(US)} = \left(B^{(3)r*}_{L(U1)} , \; B^{(3)r*}_{L(U2)} \right)$$

with the ef^{th} elements of the two submatrices being given by

$$\left(B^{(3)r*}_{L(U1)} \right)_{ef} = \delta_{ef} \, B^{(3)r}_{(e1)} + B^{(f1)3r}_{(e1)} ,$$

where

$$\delta_{ef} = \begin{cases} 1 & \text{for } e = f \\ 0 & \text{for } e \neq f \end{cases} , \quad \text{and}$$

$$\left(B^{(3)r*}_{L(U2)} \right)_{ef} = B^{(f2)3r}_{(e1)} .$$

Similar definitions apply to $B^{(5)r*}_{L(US)}$. In the case of $B^{(4)r*}_{L(U1)}$, the typical element is

$$\left(B^{(4)r*}_{L(U1)} \right)_{ef} = B^{(f1)4r}_{(e1)} .$$

(There are no direct export sales of local commodities.)

In the matrix representation of equation (39.22), $\epsilon_{(US)}$ is the vector of household expenditure elasticities for domestic and imported commodities. Because in ORANI we assume that expenditure elasticities for each commodity are independent of source, $\epsilon_{(US)}$ consists of two identical

39.3 Solving for the regional outputs of local commodities

The eight sets of equations (39.2), (39.8), (39.11), (39.12), (39.22), (39.24), (39.26) and (39.28) are the computational core for our regional model. They are repeated below in matrix notation for region r using the same equation identifiers as before. Upper case subscripts U, K, N etc. are used to indicate that the components of the relevant vectors and matrices include all commodities, all industries, all national industries, etc..

$$z_N^r = z_N \; ; \qquad (39.2)$$

$$x_{(L1)}^{(0)r} = B_{LN}^{(1)r*} z_N^r + B_{LM}^{(1)r*} z_M^r + B_{LN}^{(2)r*} y_N^r + B_{LM}^{(2)r*} y_M^r + B_{L(US)}^{(3)r*} x_{(US)}^{(3)r} + B_{L(US)}^{(5)r*} x_{(US)}^{(5)r} + B_{L(U1)}^{(4)r*} x_{(U1)}^{(4)r} \; ; \qquad (39.8)$$

$$y_N^r = y_N \; ; \qquad (39.11)$$

$$y_M^r = y_M + z_M^r - z_M \; ; \qquad (39.12)$$

$$x_{(US)}^{(3)r} = x_{(US)}^{(3)} + \gamma\, \varepsilon_{(US)} \left[\left(W_K^{r'} - W_K'\right) \left(p_{(g+1,1)K}^{(1)} + x_{(g+1,1)K}^{(1)}\right) + W_M^{r'} \left(z_M^r - z_M\right) \right] \; ; \qquad (39.22)^1$$

$$x_{(US)}^{(5)r} = x_{(US)}^{(5)} + q_{(US)}^{(5)r} \; ; \qquad (39.24)$$

$$x_{(U1)}^{(4)r} = x_{(U1)}^{(4)} \; ; \qquad (39.26)$$

$$z_M^r = S_{ML}^r \, x_{(L1)}^{(0)r} \; . \qquad (39.28)$$

---

1. The primes in this equation denote row vectors.

In the long run, however, capital is inter-regionally mobile. If, as we have already assumed, wage rates move uniformly across regions, it is reasonable to assume that in the long run, labor-input ratios move uniformly across regions.

To summarize, (39.21) appears to be satisfactory for use in a model concerned with the long-run regional implications of an exogenous shock. Even for the short run, it could be justified for national goods (i.e., $k \epsilon N$) . On the other hand, for short-run analyses, the use of (39.21) may lead to some underestimation of the employment created in regions where the percentage expansion of industries producing local goods is greater than for the nation as a whole. This is because it fails to allow for declining productivity associated with the more intensive use of fixed capital stocks. An offset to this bias, however, is the tendency for our regional model to overestimate short-run regional output responses. Notice that we are not allowing for short-run regional variations in product prices for local goods. Hence the use of (39.14) will lead to overestimates of household demand for local goods in region r if the expansion in the production of these goods in region r is greater than the national average. (39.14) ignores short-run increases, relative to the national average, in the production costs and output prices of local commodities in regions experiencing greater than average increases in activity. While keeping these biases in mind, for simplicity, we adopt (39.21) even for local industries in short-run analyses.

On substituting from (39.20) into (39.14), from (39.21) into (39.18) and then into (39.14), and on using (39.2), (39.15) and (39.16), we obtain

$$x^{(3)r}_{(us)} = x^{(3)}_{(us)} + \varepsilon_{(us)}\gamma \left[\sum_{k\in K} \left(W^r_k - W_k\right)\left(p^{(1)}_{(g+1,1)k} + x^{(1)}_{(g+1,1)k}\right) + \sum_{m\in M} W^r_m \left(z^r_m - z_m\right)\right] , \quad u\in U, \quad s=1,2, \quad r=1,\ldots,R . \qquad (39.22)$$

(c) Other final demand at the regional level

Equation (39.23) describes the regional allocation of "other" final demand.

$$X^{(5)r}_{(us)} = X^{(5)}_{(us)} Q^{(5)r}_{(us)} , \quad u\in U, \quad s=1,2, \quad r=1,\ldots,R , \qquad (39.23)$$

where $Q^{(5)r}_{(us)}$ is the share of the total "other" final demand for commodity u from source s which is accounted for by region r. The percentage-change form of equation (39.23) is

$$x^{(5)r}_{(us)} = x^{(5)}_{(us)} + q^{(5)r}_{(us)} , \quad u\in U, \quad s=1,2, \quad r=1,\ldots,R . \qquad (39.24)$$

Notice that we do not treat the regional shares in "other" final demand, the $Q^{(5)r}_{(us)}$, as necessarily constant. This demand category comprises mainly government current expenditure and we allow for exogenous changes in its regional allocation. The default setting is, however, $q^{(5)r}_{(us)} = 0$.[1]

1. In section 42 we note some restrictions on the choice of the $q^{(5)r}_{(us)}$ which are necessary to preserve consistency with ORANI economy-wide results.

(d) International exports from regions

We simply assume fixed regional sourcing for international exports. That we we write

$$X^{(4)r}_{(u1)} = X^{(4)}_{(u1)} Q^{(4)r}_{(u1)} , \quad u\in U, \quad r=1,\ldots,R , \qquad (39.25)$$

where $Q^{(4)r}_{(u1)}$ is the share of region r in the aggregate supply of domestic commodity u for export. The $Q^{(4)r}_{(u1)}$ are treated as constants so that the percentage change form of equation (39.25) is

$$x^{(4)r}_{(u1)} = x^{(4)}_{(u1)} , \quad u\in U, \quad r=1,\ldots,R . \qquad (39.26)$$

(e) The commodity composition of the outputs of local industries

The problem of expressing percentage changes in the regional output of local industries[1] as functions of the percentage changes in the regional outputs of local commodities is simplified by the fact that in the implemented version of ORANI all the local industries are single-product industries and each of the local commodities is produced by only one local industry. Thus we can write

$$Z^r_m = X^{(0)r}_{(m'1)} , \quad r=1,\ldots,R, \quad m\in M \text{ and } m'\in L \qquad (39.27)$$

where m' is the local commodity produced by local industry m.

The percentage-change form of (39.27) is

$$z^r_m = x^{(0)r}_{(m'1)} . \qquad (39.28)$$

1. Recall that local industries are those producing local goods. Our industry and commodity classifications are such that no industry produces both local and national goods.