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AUSTRALIAN ESTIMATES OF WORKING'S MODEL UNDER ADDITIVE PREFERENCES:

Estimates of a Consumer Demand System for Use by CGE Modelers and Other Applied Economists

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Preliminary Working Paper No. OP-61 Melbourne April 1987

ISSN 0813 - 7986

ISBN 0 642 10144 2

This paper is issued simultaneously by the University of Florida as McKethan-Matherly Discussion Paper MM24 of the McKethan-Matherly Eminent Scholar Chair within the Graduate School of Business Administration

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Abstract: In 1943 Holbrook Working postulated a declining linear relationship between the share of food in a household's budget and the logarithm of its (real) income. In a monograph to be published this spring, Theil and Clements (forthcoming (1987)) show how Working's empirical generalization can be integrated into a complete system of demand equations suitable both for time-series and for cross-sectional work. Empirical evidence from both types of data lends overwhelming support to the relationship fitted by Working, with food's budget share consistently estimated to decline by 0.13 to 0.15 percentage points for each one per cent increase in real per capita income. Against this backdrop the current paper uses annual Australian time series data, 1953-54 through 1985-86, to fit Working's Model under additive preferences to a six commodity classification of consumption. It is found that to obtain consistency with Working's Model, the quantity of 'Rent' (i.e., the services of the housing stock, largely owner-occupied in Australia) must be treated exogenously, and the (shadow) price treated as the endogenous variable. The results for food are broadly consistent with those found by Theil and co-workers.

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*Powell was on a special studies program from the University of Melbourne, Australia, when this paper was written. Research supported in part by the McKethan-Matherly Eminent Scholar Chair, University of Florida.

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1. Introduction

The ORANI model (Dixon, Parmenter, Sutton and Vincent (1982) -- hereinafter DPSV) has been used extensively as an aid to policy analysis in Australia. (For a review of applications, see Powell and Lawson (1986).) Most applications have been based on Johansen (i.e., log-linearized differential) solutions of the model under various policy shocks. Full non-linear solutions of ORANI have been computed much less frequently. The latter are advisable when the simulated economy is subjected to large shocks of a type likely to cause major compositional changes in it. When full non-linear solutions are computed, the functional form of the underlying consumer demand system assumes an importance which it does not have in Johansen solutions -- for the latter, all that

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*With the usual caveat, the authors are very grateful to Henri Theil for critical discussion and comments. Powell acknowledges support of the University of Melbourne and the McKethan-Matherly Eminent Scholar Chair at the University of Florida, where this paper was written during Powell's tenure of a McKethan-Matherly Senior Research Fellowship.

matters is the local (viz., initial) values of the various demand elasticities.

The consumer demand coefficients in the standard ORANI parameter file were obtained by Tulpulé and Powell (1978) using the twice-extended linear expenditure system (within which the linear expenditure system -- the LES -- is nested). The ORANI model itself is not tied to this specification (although the theory as presented in DPSV does assume preference independence). Clements and Smith (1983) have demonstrated the feasibility of allowing ORANI to encompass more general utility specifications in situations in which the available data will permit the estimation of specific substitution parameters.

The aim of the present paper is to lay out a consumer demand specification for the ORANI model which is suitable for use globally. By 'globally' here is meant:

(a) both for Johansen and for full non-linear solutions of the model;

(b) both for contemporaneous comparative static (i.e., policy-analytic) and for forecasting solutions of the model;

and

(c) both for short-run and for long-run simulations.

As well, the specification sought is one sufficiently general to allow the orderly accretion of information about specific substitution effects into the parameter file if and when reliable new data make this possible. Finally, since the larger matrix of parameters into which such new information is to be imbedded can, with convenience, only be changed once every 5 or so years, the specification sought is one which

(d) is valid over substantial variations in income;

and

(e) is amenable to the modeling of demographic change.

Working's (1943) Model provides a parsimonious yet empirically successful parameterization of Engel responses, and has been incorporated successfully into modern approaches to the analytics of demand systems by Deaton and Muellbauer (1980) and by Theil and Clements (forthcoming (1987)). Theil has demonstrated the robustness of Working's relationship between (average) budget shares of broadly defined commodities and real per capita income. Marrying an additive preference postulate to Working's model, and hence making due allowance within this expanded framework for variation in prices, he estimated a complete system of consumer demand equations from the following data set:

(A) a cross section of 30 countries from the international comparisons project (Kravis et al. (1982)).

With co-workers (Finke et al. (1984); Flood et al. (1984)) he estimated the same model also from

(B) a time-series of annual Japanese data (1951-1972).

These two data sets produced estimates of the flexibility of the marginal utility of total expenditure in fairly close agreement (-0.526 and -0.642, respectively). The method was successful in capturing the variation in marginal budget shares over the very wide dispersion of real income per head within these two samples; in particular, the marginal share of food in total household expenditure was estimated from data set A to decline by 0.154 for each 1 per cent increase in real income per head, and by 0.153 from data set B. This represents a substantial advance in intuitive appeal and empirical performance over models (such as those in the LES family) in which marginal budget shares are parametrically constant.

The remainder of this paper is structured as follows. In Section 2, Theil's development of Working's model is briefly recapitulated. Sections 3 and 4, respectively, contain a description of the Australian data base and an outline of the proposed methods of estimation. In Section 5 we report initial estimates of our demand system for 6 broadly defined commodity groups. Section 6 contains a discussion of difficulties associated with estimating a demand equation for the services of the housing stock ('Rent'), which in Australia is largely owner-occupied. In Section 7 the Rent equation is respecified to reflect more nearly the realities of the Australian data. Then in Section 8 revised estimates are presented. In the ninth, concluding, section, we offer our final remarks and a perspective for future research.

2. Working's Model under Additive Preferences

This section recapitulates the development of the topic by Theil in Theil and Clements (forthcoming 1987). Working's (1943) model is chosen to represent the response of the consumption of broadly defined commodities to changing real income levels mainly because of its outstanding empirical success. Apart from the evidence mentioned above under (A) and (B) in Section 1, this success is documented:

(a) for a 1970 cross section of 15 countries by Theil and Suhm (1981);

(b) for a 25-year Dutch and a 21-year Belgian time series, by Finke and Flood (1984);

(c) for estimates based separately on three cross sections comprising 16, 34 and 58 countries in 1970, 1975 and 1980, respectively, by Fiebig, Seale and Theil (forthcoming 1987);

(d) for a pooled subset of the above data based on 30 countries which are common to all three cross sections, by the same authors (ibid.).

Moreover, the results mentioned above for the budget share of food were confirmed by Musgrove (1985) who fitted Working's Model to a 1976-77 survey of some 4,000 households in the Dominican Republic (he found -0.14, rather than -0.15, however, for the coefficient in question). Applying the AIDS model to American time-series data, Blanciforti and Green (1983) report a slightly lower fall (0.13) in the budget share of food per one per cent increase in real expenditure levels.

We adopt the notation set out in Table 2.1. For the moment we deal with the behavior of one <u>representative</u> agent on whom we will impose the demand-theoretic constraints associated with the behavior of an <u>individual</u> consumer. (For some arguments in favor of the validity of such aggregation, see Barnett (1979).) Apart

Table 2.1

Notation Used for Working's Model under Additive Preferences

Prices:	
p_i	Price of broadly-defined commodity i $(i = 1, \ldots, n)$ (observation subscript suppressed).
p_{it}	Price of broadly-defined commodity i at observation t.
$p^{(t)}$	The set of prices $(p_{it}, \ldots, p_{nt})$ prevailing at t.
p_t	Column vector format of $p^{(t)}$.
$p^{(\tau)}$	A reference set of prices; $p^{(\tau)} = (p_{i\tau}, \ldots, p_{n\tau})$.
$\text{dlog } P_t$	Divisia price index at t: $\text{dlog } P_t = \sum_{j=1}^{n} w_{jt} \text{ dlog } p_{jt}$.
$\text{dlog } P'_t$	Frisch price index a t: $\text{dlog } P'_t = \sum_{j=1}^{n} (w_{jt} + \beta_j) \text{dlog } p_{jt}$.
P_t	A consumer price index.
Total Expenditure:	
M_t	Total per capita consumer expenditure at observation t.
M_τ	A particular reference value of M_t.
Q_t, Q_τ	Real per capita total expenditure indexes corresponding, respectively, to M_t and M_τ: $Q_t = M_t/P_t$; $Q_\tau = M_\tau/P_\tau$.
Quantities:	
Q_t, Q_τ	See above.
q_{it}	Per capita quantity of commodity i consumed at t.
$\text{dlog } Q_t$	Divisia volume index: $\text{dlog } Q_t = \sum_{j=1}^{n} w_{jt} \text{dlog } q_{jt}$.

...continued

Table 2.1 (continued)

Shares:

w_{it}	Share of commodity i in consumer's budget at t: $w_{it} = p_{it}q_{it}/M_t$.
$\bar{w}_{it}$	An average of the budget shares at t and (t-1): $\bar{w}_{it} = \tfrac{1}{2}(w_{i,t-1} + w_{it})$
$\hat{w}_{i\tau}$	Budget share of commodity i at real expenditure level Q_τ and at relative prices prevailing in the reference set $p^{(\tau)}$.

Operators, Functions:

log ( )	Natural logarithm.
d	Ordinary differential.
Δ	Ordinary backward difference operator.
D	Discrete approximation to dlog ( ); e.g., $Dp_{it} = \Delta \log p_{it} = \log (p_{it}/p_{i,t-1})$. Note that DQ_t is defined by: $DQ_t = \sum_{j=1}^{n} \bar{w}_{jt} \Delta \log q_{jt}$.
bar over symbol	See $\bar{w}_{it}$ above under 'shares'.

Parameters:

α_i	Intercept in Working's Model: See equation (2.1).
β_i	Change in budget share of i (in percentage points) per one per cent change in real expenditure per head: see equation (2.1).

---

from random errors which will be appended later, the Engel response of our representative agent is assumed to be:

$$(2.1) \qquad \hat{w}_{i\tau} = \alpha_i + \beta_i \log Q_\tau \qquad (i=1,\ldots, n \text{ commodities}),$$

which is Working's model. Eqn (2.1) is supposed to hold at fixed relative prices; here we suppose that (2.1) holds at the relative prices prevailing in the reference set $P^{(\tau)} = (p_{1\tau}, p_{2\tau}, \ldots, p_{n\tau})$. Let M_τ be the nominal value of total expenditure which yields a real value of Q_τ when prices are as in $P^{(\tau)}$. If w_{it} is the value of the budget share for commodity i at some arbitrary set of prices $P^{(t)} = (p_{1t}, p_{2t}, \ldots, p_{nt})$ and nominal income M_t in the neighborhoods of $P^{(\tau)}$ and M_τ respectively, then the deviation $(w_{it} - \hat{w}_{i\tau})$ by construction is a differential equal to the sum of the income and substitution effects involved in the movements from $(P^{(\tau)}, M_\tau)$ to $(P^{(t)}, M_t)$. The differential in the i^{th} budget share can be written:

$$(2.2) \qquad dw_i = (w_{it} - \hat{w}_{i\tau}) = d(p_i q_i/M)$$

$$= w_i \left[d \log p_i + d \log q_i - d \log M \right]$$

$$= w_i \left[(d \log p_i - \sum_{j=1}^{n} w_j d \log p_j) \right.$$

$$\left. - (d \log M - \sum_{j=1}^{n} w_j \, d \log p_j) + d \log q_i \right].$$

We note that the Divisia aggregate quantity (or real total expenditure) index is:

$$(2.3)\qquad d \log Q = d \log M - \sum_{j=1}^{n} w_j d \log p_j .$$

If, in moving from $(P^{(\tau)}, M_\tau)$ to $(P^{(t)}, M_t)$, we sterilize any real income change by giving an increment dM to total expenditure, where

$$(2.4)\qquad dM/M = \sum_{j=1}^{n} w_j d \log p_j ,$$

then (2.2) becomes:

$$(2.5)\qquad dw_i = (w_{it} - \hat{w}_{i\tau})$$

$$= w_i \;(d \log p_i - \sum_{j=1}^{n} w_j d \log p_j) + w_i d \log q_i .$$

In eqn (2.5), the first right-hand term is the change in the i^{th} budget share which we would observe as the result of the change in relative prices in moving from $P^{(\tau)}$ to $P^{(t)}$ if there were no quantity adjustments to these changes. The second right-hand term is due solely to the adjustment of quantities demanded as a result of relative price changes at the constant real total expenditure level Q. By construction, therefore, the $(d \log q_i)$ are substitution effects. Under additive preferences these may be written (see, e.g., Theil (1967), pp. 197-198):

(2.6) $$w_i d \log q_i = \phi \frac{\partial(p_i q_i)}{\partial M}\left[d \log p_i - \sum_{j=1}^{n} \frac{\partial(p_j q_j)}{\partial M} d \log p_j\right].$$

An alternative way of writing (2.1) is:

(2.7) $$p_{i\tau} q_{i\tau} = M_\tau \alpha_i + \beta_i M_\tau \log Q_\tau \qquad (i=1,\ldots, n).$$

Hence the marginal budget shares are:

(2.8) $$\partial(p_i q_i)/\partial M = \alpha_i + \beta_i \log Q + M\beta_i \; \partial \log Q/\partial M.$$

(where the τ subscript is implicit). But, by definition

(2.9) $$Q = M/P.$$

With prices fixed (as they must be in evaluating the marginal budget shares), $\partial \log Q/\partial M = 1/M$; and so

(2.10) $$\partial(p_i q_i)/\partial M = w_i + \beta_i.$$

Substituting (2.10) into (2.6), the substitution component of the differential in commodity i's budget share may be written

(2.11) $$w_i d \log q_i = \phi(w_i + \beta_i) [d \log p_i - \sum_{j=1}^{n} (w_i + \beta_i) d \log p_j].$$

Substituting from (2.11) into (2.5), we obtain

$$(2.12)\quad dw_i = (w_{it} - \hat{w}_{i\tau}) = w_i(d \log p_i - \sum_{j=1}^{n} w_j \, d \log p_j) + \phi(w_i + \beta_i)\left[d \log p_i - \sum_{j=1}^{n} (w_j + \beta_j) \, d \log p_j\right]$$

Finally substituting for $\hat{w}_{i\tau}$ from (2.12) into (2.1), we obtain:

$$(2.13)\quad w_{it} = \alpha_i + \beta_i \log Q_t + w_i(d \log p_i - d \log P) + \phi(w_i + \beta_i)(d \log p_i - d \log P'),$$

where we have abbreviated the notation for the Frisch and Divisia price indexes as in Table 2.1, and have used the fact that $Q_t = Q_\tau$ (by construction). Equation (2.13) tells us the following:

(i) if, at observation t, relative prices are as in the reference set $P^{(\tau)}$, the budget share of commodity i is simply linear in real per capita expenditure Q -- no explicit price terms appear (Working's model).

(ii) if, on the other hand, relative prices at t differ from those prevailing in $P^{(\tau)}$ by a 'small' differential vector, dp, then to the component (i) we must add the corrections C_1(dp) and C_2(dp), where;

$$(2.14a)\quad C_1(dp) = w_i(d \log p_i - d \log P)$$

and

$$(2.14b)\quad C_2(dp) = \phi(w_{it} + \beta_i)\left[d \log p_i - d \log P'\right].$$

In (2.14), C_1(dp) (which is just the third right-hand term of (2.13)) represents the direct effect of the price changes dp on budget shares <u>before</u> any allowance is made for adjustments by consumers to quantities demanded; $C_2(p)$ represents quantity adjustments by consumers at given Q; i.e., substitution effects.

How can we make (2.13) operational? The most obvious method is to apply this equation directly to discrete data. In that case, (2.13) is replaced by:

$$(2.15)\quad w_{it} = \alpha_i + \beta_i \log Q_t + w_{it}\left[\log (p_{it}/p_{i\tau}) - \sum_{j=1}^{n} w_{jt} \log (p_{jt}/p_{j\tau})\right]$$
$$+ \phi(w_{it} + \beta_i)\left[\log (p_{it}/p_{i\tau}) - \sum_{j=1}^{n} (w_{jt} + \beta_j) \log (p_{jt}/p_{j\tau})\right]$$
$$+ \epsilon_{it} \qquad (i=1,\ldots,n).$$

The following points about this operational version should be noted:

1. The budget shares on the right of (2.15) ideally would be evaluated at suitable average values such as w^*_{it}, (where $w^*_{it} = \frac{1}{2} [w_{it} + w_{i\tau}]$).
2. Differentials in logs of prices have been replaced by differences.
3. The money flexibility ϕ has been treated as a constant, since the empirical evidence in favor of its variation, given (2.1), is so weak (see Theil, Section 2.13 in Ch. 2 of Theil and Clements (forthcoming 1987)).
4. Stochastic errors ϵ_{it} have now been appended.
5. To guarantee that budget shares sum to unity, it is sufficient that:

(2.16a) $$\sum_{i=1}^{n} \alpha_i = 1;$$

(2.16b) $$\sum_{i=1}^{n} \beta_i = 0;$$

and that

(2.16c) $$\sum_{i=1}^{n} \epsilon_{it} = 0 \text{ for all } t.$$

We require these three restrictions to be satisfied in all that follows.

6. In consequence of (2.16c), at any t, the variance-covariance matrix among the ϵ_{it} is of rank n-1.

In fact, (2.15) is the form of Working's model fitted by Theil (in Ch. 2 of Theil and Clements (forthcoming 1987)) to international cross-section data. The reference set of prices chosen, $P^{(\tau)}$, was the set of geometric means (across countries) of commodity prices. Note, however, that since the $w_{i\tau}$ are unobservable, the means w^*_{it} were not available for use in (2.15); the w_{it} were used instead.

For time series work, we can commence with (2.13), whose discrete analog is:

(2.17) $$\Delta w_{it} = \beta_i DQ_t + \bar{w}_{it}(Dp_{it} - \sum_{j=1}^{n} w_{jt} Dp_{jt})$$ [continued]

$$+ \phi(\bar{w}_{it} + \beta_i)\left[Dp_{it} - \sum_{j=1}^{n}(\bar{w}_{jt} + \beta_j)Dp_{jt}\right] ,$$

$$(i=1, \ldots, n)$$

where the w_{it}'s on the right have been treated as constants and set equal to their average values at (t − 1) and t. From the development following (2.2) we have seen that:

$$(2.18) \quad dw_i = w_i\left[(d \log p_i - \sum_{j=1}^{n} w_j \, d \log p_j) - d \log Q + d \log q_i\right].$$

Therefore we may write:

$$(2.19) \quad w_{it} - \bar{w}_{it}\left[Dp_{it} - \sum_{j=1}^{n} \bar{w}_{jt} \, Dp_{jt}\right] \approx \bar{w}_{it}(Dq_{it} - DQ_t).$$

Using (2.19) in (2.17), we obtain:

$$(2.20) \quad y_{it} = \beta_i DQ_t + \phi(\bar{w}_{it} + \beta_i)\left[Dp_{it} - \sum_{j=1}^{n}(\bar{w}_{jt} + \beta_j)Dp_{jt}\right] + e_{it},$$

in which

$$(2.21) \quad y_{it} = \bar{w}_{it}(Dq_{it} - DQ_t),$$

and e_{it} is a random disturbance. Eqn (2.20) is the form of Working's model used by Theil and colleagues to analyze time series data (e.g., Finke, Flood and Theil (1984)).

3. The Australian Data

In this paper we report estimates based on national time-series data; these estimates are later compared with synoptic evidence from the 1984 official Household Expenditure Survey (Australian Bureau of Statistics (1986)). It was desirable that the time-series data be as long as possible: first, in order to provide a wide measure of variation in real per capita expenditure; second, in order to provide a workable sample size for maximum likelihood estimation. Disaggregated constant-price consumption data first become available -- at the six-commodity level -- in 1953-54; disaggregation to the sixteen-commodity level is possible from 1969-70 onwards. The process of knitting these data together will only be described briefly here; full details are available in Appendix 1.

The principles behind our data handling were these: (A) all index numbers should respect the (price) x (quantity) = (value) identity. (B) Price indexes, to be computed as implicit deflators from data on expenditures in current prices, and in constant prices, should be computed only from strictly matched series; i.e., from data for these variables published in the same issue of the same publication. (This is required because substantial revisions of these data are made over time. Mismatched series would produce spurious apparent price variations.) (C) Where the relevant price information can be inferred from more than one matched pair of series, the most recently published matched data are used. (D) Series requiring linking are spliced using an OLS regression through the origin which utilizes all the available overlapping observations. (E) The current-price expenditure data in the final data base is taken from the most recently published statistics. (F) Where accuracy in the value of the level of a variable, and accuracy in its percentage change over time, become competing goals, the latter objective is given precedence (after all, our model is in log changes).

The assembly of our time series data base proceeded in the following stages:

(1) The identification of matched series on expenditure, disaggregated as fully as possible, in current and constant prices. Three such pairs of series, each based on constant prices of a different year, were required to span our sample of 33 annual observations, 1953-54 through 1985-86.

(2) The conversion of all of the constant-price expenditure data to the basis of the same year (1979-80) via a simple index-linking procedure. Notice that the data converted in this way are still matched against their original nominal expenditure series.

(3) The imputation of a time series of commodity-specific price indexes (base 1979-80 = 100) from the matched series (where the level of commodity disaggregation varies throughout the sample in line with the maximum number of constant-price expenditure series available).

(4) The aggregation of these price index numbers to the

level of the six commodities available throughout the 33-year sample; namely:

1. Food
2. Tobacco, cigarettes, alcoholic drinks
3. Clothing, footwear
4. Household durables
5. Rent
6. All other expenditures

In line with principle (A) above, component price indexes were aggregated using their shares in the value of the budget in 1979-80.

(5) The aggregation of the most recently published current-price expenditure data to the six commodity categories listed above, and the computation of budget shares $(w_{1t}, \ldots, w_{6t})$.

(6) The division of the six current-price expenditure series from step (5) by the corresponding price index series from step (4), followed by the further deflation of these commodity-specific quantity indexes by an estimate of population, to obtain per capita quantity indexes $(q_{1t}, \ldots, q_{6t})$.

Further details of the manipulations performed, citation of primary sources, and tabulations of the data can be found in Appendix 1.

4. Estimation Methods

The estimation method employed was conventional maximum likelihood. Following Barten (1969), we note that the maximum likelihood estimation of a complete system of demand equations is easily handled by the deletion of an arbitrarily chosen equation (we chose i = 6, Other), and that the MLE's are invariant to this choice. The likelihood function may be written (see, e.g., Theil, p. 54 in Theil and Clements (forthcoming (1987)) as:

$$L = \text{constant} + 16 \log |\Sigma^{-1}| - \frac{1}{2} \sum_{t=1}^{32} (y_t - \hat{y}_t) \Sigma^{-1} (y_t - \hat{y}_t), \tag{4.1}$$

where

$$y_t = (y_{1t}, \ldots, y_{5t})' \qquad \text{[see (2.21)]} \tag{4.2}$$

and

$$\hat{y}_t = (\hat{y}_{1t}, \ldots, \hat{y}_{5t})'$$

$$= DQ_t\beta + \phi[\mathrm{diag}(X)(\bar{w}_t + \beta) - (\bar{w}_t + \beta)(\bar{w}_t + \beta)'X], \tag{4.3}$$

in which

$$\beta = (\beta_1, \ldots, \beta_5)', \tag{4.4}$$

(4.5) diag (vector) = diagonal matrix made from that vector,

$$X = (Dp_{1t} - Dp_{6t}, Dp_{2t} - Dp_{6t}, \ldots, Dp_{5t} - Dp_{6t})', \tag{4.6}$$

and

$$\bar{w}_t = (\bar{w}_{1t}, \ldots, \bar{w}_{5t})'. \tag{4.7}$$

Notice that (4.3) is just a matrix notation for the non-stochastic part of the right-hand side of (2.20).

The asymptotic standard errors of the estimates of the model's six free parameters $\beta_1, \ldots, \beta_5$ and ϕ (recall that

$\beta_6 \equiv -\sum_{j=1}^{5} \beta_j$) are obtained from the information matrix in the conventional manner (see, e.g., Theil in Theil and Clements (forthcoming 1987), p. 54).

5. Initial Results

The results of our estimation of (2.20) are shown in Table 5.1. Also shown there are implicit estimates of elasticities with respect to total expenditure, as well as estimated first serial correlation coefficients. The complete set of residuals is shown in Table 5.2.

The first noteworthy feature of these results is the estimated β_i for Food, and its apparently low asymptotic standard error. This point estimate of -0.12 is somewhat less in absolute value than those commonly found (around -0.14 to -0.15) from the wide variety of data sources mentioned above in Sections 1 (see points (A) and (B)) and 2 (points (a), (b), (c), (d), and (e)). The Australian data, however, include restaurant meals, while the other data sources apparently do not. (Certainly the International Comparisons data do not.) Since the total expenditure elasticity for restaurant meals presumably exceeds unity, the Australian β_{FOOD} should be less than estimates based on just food for eating at home.

Other aspects of Table 5.1 cause concern. The errors are not remotely well-behaved, as a glance either at the last column of

Table 5.1

Maximum Likelihood Estimates of Working's Model under Additive Preferences from Australian Data 1953-54 through 1985-86

Commodity i	MLE of β_i	Ratio of MLE to Asymptotic Std Error	Implied Demand Elasticity with Respect to Total Expenditure in[(a)]			Estimated First Serial Correlation of Residuals
			1953-54 η_{i1}	1969-70 η_{i17}	1985-86 η_{i33}	
1. Food	-0.1211	9.7	0.548	0.367	0.239	0.234
2. Tobacco, cigarettes, alcohol	-0.0327	-4.3	0.690	0.642	0.518	0.447
3. Clothing, footwear	-0.0265	-2.4	0.802	0.708	0.572	0.532
4. Household durables	0.0696	4.2	1.872	1.902	2.030	0.622
5. Rent	-0.0101	-0.3	0.860	0.919	0.954	0.797
6. Other	0.1208	1.8	1.355	1.284	1.285	0.335

MLE of ϕ, the reciprocal of the elasticity with respect to total expenditure of the marginal utility of total expenditure: -0.4926. Ratio of MLE of ϕ to asymptotic standard error: 7.5.

(a) Computed as [1 + (MLE of β_i)/(actual w_{it})].

Table 5.2

Residuals from Fitted Equations Corresponding to Table 5.1

Year*	Food	Tobacco, cigarettes, alcohol	Clothing, footwear	Household durables	Rent	Other
1953-55	-0.0019	0.0006	0.0020	-0.0037	0.0002	0.0029
1954-56	0.0000	0.0001	-0.0020	-0.0011	0.0020	0.0012
1955-57	0.0009	0.0004	-0.0038	-0.0005	0.0024	0.0006
1956-58	-0.0011	-0.0004	-0.0037	0.0035	0.0017	0.0001
1957-59	-0.0009	-0.0020	-0.0021	0.0016	0.0024	0.0011
1958-60	-0.0043	-0.0022	0.0000	-0.0025	-0.0040	0.0130
1959-61	-0.0046	0.0000	-0.0001	-0.0043	0.0040	0.0051
1960-62	0.0036	-0.0009	-0.0038	-0.0043	0.0041	0.0013
1961-63	0.0011	-0.0020	-0.0020	-0.0031	0.0011	0.0050
1962-64	-0.0024	-0.0002	0.0024	-0.0024	0.0003	0.0023
1963-65	-0.0005	0.0003	-0.0001	-0.0004	0.0015	-0.0008
1964-66	0.0019	0.0004	-0.0009	-0.0041	0.0028	0.0000
1965-67	0.0023	0.0000	-0.0010	-0.0043	0.0023	0.0005
1966-68	-0.0015	0.0015	-0.0002	-0.0008	0.0028	-0.0018
1967-69	-0.0018	-0.0003	-0.0007	0.0018	0.0011	-0.0002
1968-70	-0.0013	0.0004	-0.0011	-0.0024	0.0041	0.0002
1969-71	0.0003	0.0001	-0.0003	-0.0018	0.0035	-0.0018
1970-72	0.0025	-0.0013	-0.0012	-0.0004	0.0024	-0.0019
1971-73	0.0004	0.0005	0.0022	0.0002	0.0011	-0.0044
1972-74	-0.0025	0.0009	0.0006	0.0067	-0.0004	-0.0052
1973-75	0.0028	-0.0014	-0.0038	0.0021	0.0013	-0.0011
1974-76	0.0032	-0.0019	-0.0049	0.0011	0.0030	-0.0005
1975-77	0.0027	0.0005	-0.0034	-0.0033	0.0063	-0.0029
1976-78	0.0024	0.0001	-0.0007	-0.0070	0.0079	-0.0027
1977-79	0.0010	-0.0029	-0.0015	-0.0066	0.0047	0.0053
1978-80	0.0000	-0.0007	-0.0015	-0.0031	0.0043	0.0010
1979-81	0.0029	0.0004	0.0008	0.0006	0.0032	-0.0078
1980-82	0.0015	-0.0005	-0.0005	-0.0029	0.0048	-0.0024
1981-83	-0.0008	-0.0036	-0.0005	-0.0047	0.0089	0.0007
1982-84	-0.0018	-0.0026	-0.0010	-0.0013	0.0076	-0.0009
1983-85	0.0000	-0.0018	-0.0009	-0.0032	0.0074	-0.0015
1984-86	0.0057	-0.0005	-0.0001	-0.0024	0.0084	-0.0111

---

*For example, 1973-75 refers to the first differences in equation (2.20) for the fiscal years 1973-74 and 1974-75.

Table 5.1, or at the residuals displayed in Table 5.2, will confirm. In Theil's extension of Working's model, in the absence of strong countervailing relative price effects the total expenditure elasticities of luxuries (η_{it} > 1) approach unity as real per capita total expenditure grows without limit ($Q_t \rightarrow \infty$), while those of necessities (η_{it} < 1) decline as the living standard improves. Table 5.1 gives two cases (Household durables and Rent) where, apparently either

(i) the model was not consistent with the data;

or,

(ii) the relative price effects were very strong, causing the budget shares of an apparent luxury (Household durables) and of an apparent necessity (Rent) both to increase over the sample period.

Relative price effects indeed are strong in our sample. The ratios of values of commodity-specific price indexes in 1985-86 to their initial values in 1953-54 (taken from Appendix Table A1.6) are:

Food	Clothing	Tobacco, etc.	Household durables	Rent	Other
5.679	7.626	5.212	3.939	15.870	8.017

Thus the two anomalous cases relate, respectively, to the relative price which has risen most (Rent) and to the price which has fallen most (Household durables). Note, however, that in neither case is the price movement countervailing: the share of Rent increased too fast in relation to changes in income before any account is taken of the massive increase in the relative price of Rent over the sample; while the share of Household durables declined despite an expenditure elasticity in excess of one and a big fall in price relative to other goods. We shall pursue further of the puzzling case of Rent in Sections 6 and 7.

At a more general level, considerable disagreement is evident between the Table 5.1 estimates of total expenditure elasticities and earlier estimates based on the linear expenditure system and its extensions. These conflicts are displayed in Table 5.3. They are serious for Rent, Clothing, and Household durables, in that order.

As a test of the specification (2.20) this equation was modified to include autonomous trends in budget shares; thus instead of (2.20), we fitted:

$$(5.1) \qquad y_{it} = \theta_i + \beta_i DQ_t + \phi(\bar{w}_{it} + \beta_i)\left[Dp_{it} - \sum_{j=1}^{n}(\bar{w}_{jt} + \beta_j)Dp_{jt}\right] + e_{it},$$

where y_{it} is as defined in (2.20) and θ_i is the autonomous change in i's budget share (100 θ_i is the percentage point change per annum). The results are shown in Table 5.4.

Key features of these results are as follows:

(i) An asymptotic likelihood ratio test (Theil (1971), pp. 396-7) of the joint restrictions

$$\theta_1 = \theta_2 = \ldots = \theta_6 = 0$$

yields a χ^2_5 value of 62.6, which is highly significant, indicating that the trend parameters are playing an important role.

Table 5.3

Estimates of Expenditure Elasticities Compared with Earlier Estimates for Australia

Commodity	Source of Estimate/Approx. Year to Which Estimate Applies				
				Present Study	
	A/1961	B/1961-62	C/1969-70	1961-62	1969-70
Food	0.45		0.34	0.47	0.37
Tobacco, cigarettes, alcohol	0.48	0.43	0.57	0.67	0.64
Clothing, footwear	0.52	0.45	0.29	0.75	0.71
Household durables	1.22	1.06	1.58	1.91	1.90
Rent	1.89	1.73	1.89	0.90	0.95
Other(a)	1.35	1.43	1.18	1.31	1.28

Sources: A. Powell (1973).
B. Lluch, Powell, and Williams (1977). p. 312.
C. Tulpulé and Powell (1978).

(a) Reaggregations from the original results are necessary to obtain the first three items in this row. These are done using budget share data derived from Appendix 1.

Table 5.4

Maximum Likelihood Estimates of Working's Model Under Additive Preferences from Australian Data with Autonomous Trends, 1953-54 through 1985-86

Commodity i	MLE of		Ratio of MLE to Asymptotic Std Error for		Implied Demand Elasticity with respect to Total Expenditure in (a)		First Serial Correlation of Residuals
	θ_i	β_i	θ_i	β_i	1969-70	1985-86	
1. Food	-0.0001	-0.0961	-0.3	-5.6	0.498	0.396	0.249
2. Tobacco, cigarettes, alcohol	-0.0006	-0.0282	-2.3	-3.2	0.691	0.595	0.296
3. Clothing, footwear	-0.0025	0.0342	-6.0	2.5	1.377	1.553	0.004
4. Household durables	-0.0020	0.0864	-2.5	3.5	2.115	2.278	0.542
5. Rent	0.0037	-0.0556	8.0	-3.2	0.552	0.747	0.706
6. Other	0.0015	0.0592	1.4	1.6	1.139	1.140	0.230

MLE of ϕ, the reciprocal of the elasticity with respect to total expenditure of the marginal utility of money = -0.5126.
Ratio of MLE of ϕ to asymptotic standard error = 9.2.

(a) Computed as [1 + (MLE of β_i)/(actual w_{it})].

(ii) The estimated values of the β_i are sensitive to this specification; Clothing (reckoned in Table 5.1 as a necessity) now appears as a luxury.

(iii) The serial properties of the residuals improve only slightly, except for Clothing, where the improvement is spectacular.

(iv) The Frisch parameter ϕ is relatively stable under the changed specification.

(v) The results for Rent are even more difficult to reconcile with earlier work.

6. The Strange Case of Rent

The category 'Rent' in our data base consists of housing rental payments and imputed rents by home owners. The latter are quantitatively much more important than the former: in the 1984 Household Expenditure Survey (Australian Bureau of Statistics (1986a) more than 71 per cent of respondents either owned their dwellings outright, or were purchasing them. The budget share of Rent has increased consistently throughout the sample (Figure 6.1): on average, each 1 per cent increase in real expenditure per head led to an increase of 0.17 percentage points in the budget share of Rent. Within the framework of Working's Model, these stylized facts would imply that the total expenditure elasticity of Rent at the sample midpoint is (.12 + .17)/(.12); namely 2.4. This is qualitatively similar to results obtained with the Klein-Rubin utility function from earlier Australian data (e.g., Tulpulé and Powell (1978) report a value of 1.9.

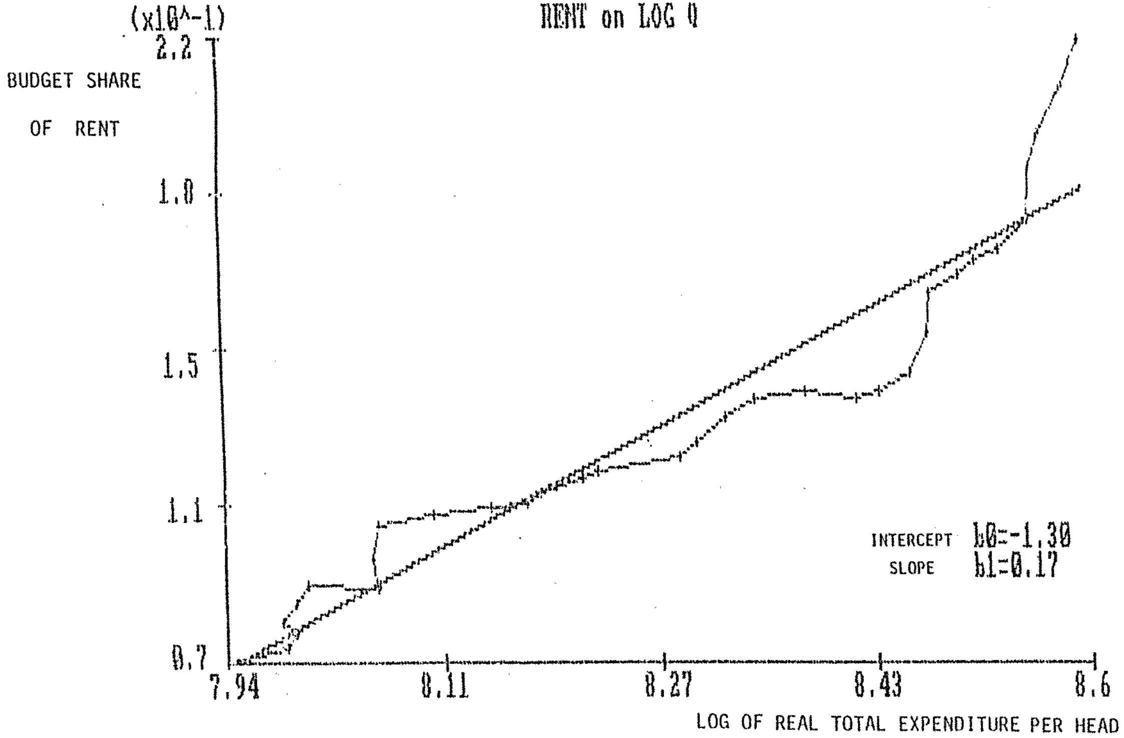


Figure 6.1: Plot of time-series data 1953-54 through 1985-86 on the budget share of Rent against per capita total real expenditure, and ordinary least squares regression

Of course, in the results reported above in Table 5.1, the response of Rent's budget share is measured by fitting Theil's extended version of Working's Model in the first-difference form (2.20). The sign of the point estimate of -0.01 for β_{RENT} is surprising. That this sign is not simply the result of the differencing operation can be verified by fitting a regression through the origin of the changes in Rent's share, w_{RENT}, on the changes log Q in the logarithm of total real expenditure per head: the resultant estimate of β_{RENT} is +0.11 (nominal t value = 3.02), yielding a total expenditure elasticity of 1.9 at the sample mid-point. It seems, then, that the negative sign for β_{RENT} has been produced by the price effects in eqn (2.21). Yet as we have seen above, these strong price effects are an embarrassment so far as consistency with Working's Model goes.

Can information from other sources be of help? Unfortunately, the 1984 Household Expenditure Survey (Australian Bureau of Statistics (1986a)) cannot shed definitive light on the question, since the survey data contain records only of market expenditures, but no information on the value of the rental services consumed on own account. However, it is possible to make an imputation of the value of rental services of houses which are owned by occupiers (see Appendix 2). The synoptic cross section data give average values for deciles of the income (NOT the total expenditure) distribution; thus it was possible to fit regressions which, apart from an additive error term, were of the form:

(6.1) $$w_{5d} = \alpha_5 + \beta_5 \log Q_d . \qquad (d = 1, \ldots, 10; \text{ deciles})$$

Results are given below for two bases of imputation of owner-occupied rent:

(i) with mortgage payments on fully owned housing imputed simply on the ratio of housing "owned outright" to "being bought" in each decile;

(ii) in a manner similar to (i) but, with imputed mortgage payments scaled by a multiplier which reconciles the Australia-wide budget share for Rent so obtained to its value in the time-series database in 1984.

The form (6.1) was modified to the extent of including an additive dummy for the lowest income decile group, which was an extreme outlier (although (6.1) was fitted first). The apparent reason for the anomalous behavior of the lowest income decile is the very high proportion of dwellings occupied by one person only (the average number of persons per household for the decile is only 1.3; the next lowest value for any decile is 2). The results are shown in Table 6.1.

The results in Table 6.1 lend support to the negative sign of the β_{RENT} value estimated from the time-series; they are not of help, however, either in

(a) reconciling the behavior of the time-series data on Rent to Working's Model;

or in

(b) reconciling the earlier high estimated total expenditure elasticities of Rent with the values estimated in the present study.

Evidently, a more radical reappraisal of the method is required.

Table 6.1

Estimates of Working's Model for Rent from Synopsis of 1984 Household Expenditure Survey*

Status of Imputed Rent Data:	1. Regressions of form: $w_{5d} = \alpha_5 + \beta_5 \log Q_d$
(i) scaled for consistency with the time-series	$w_{5d} = 0.8709 - 0.1318 \log Q_d$ $\lvert t \rvert$ values: 4.75 3.54 $R^2 = 0.226$
(ii) not so scaled	$w_{5d} = 0.6992 - 0.1062 \log Q_d$ $\lvert t \rvert$ values: 10.01 7.42 $R^2 = 0.345$
	2. Regressions of the form:[(a)] $w_{5d} = \alpha_5 + \gamma_5 X_d + \beta_5 \log Q_5$
(i) scaled for consistency with the time-series	$w_{5d} = 0.9008 + 0.1584 X_d - 0.1411 \log Q_d$ $\lvert t \rvert$ values: 153.07 201.05 118.17 $R^2 = 0.975$
(ii) not so scaled	$w_{5d} = 0.6387 + 0.1010 X_d - 0.0959 \log Q_d$ $\lvert t \rvert$ values: 193.34 211.71 141.74 $R^2 = 0.969$

* Based on the official 1984 Household Expenditure Survey, Australia. Primary source of data is Australian Bureau of Statistics (1986). For a description of data handling methods see Appendix 2. Regressions are across values of variables for the ten deciles of the distribution of cash income per household.

(a) X_d = 1 if d = 1 (the decile with the lowest income); = 0 otherwise.

7. Treating Rent as 'Rent'

7.1 Background Theory

The demand system (2.20) was estimated above on the conventional assumption of exogenous market prices. Exogeneity, there, was invoked in two contexts: first, an underlying microbehavioral postulate: households take prices as given, and maximize utility subject to a budget constraint; second, as an econometric assumption allowing the identification of our data as tracing out a set of demand functions. The latter assumption, which is a statement about market (rather than household) data, does not necessarily follow from the former; its main justification is the pragmatic one that usually it yields sensible estimates on the basis of aggregative time-series data.

The above approach is not the only one which has been used successfully to identify demand equations. For instance, there is a voluminous literature in applied agricultural economics in which the quantity of commodity i available on day t in a given market is predetermined (reflecting, among other things, prices prevailing on days t-1, t-2, ..., etc.). Daily prices then adjust to clear the market. Day to day movements in quantity thus trace out a demand curve. This is the polar extreme from the assumption of exogenous current prices. The two procedures, however, are very much alike: in one case, the supply schedule is depicted as horizontal; in the other as vertical; in either case, movements of the supply schedule trace out demand curves.

In the case of year to year movements in the Australian market for the services of the housing stock, the situation is better approximated by one in which the stock of housing is predetermined rather than by one in which the price of rental services is predetermined. This holds at both levels of the argument: first, the majority (more than 70 per cent) of households own the domicile they inhabit, and therefore, in the short run, face a predetermined quantity; second, the aggregate housing stock adjusts only slowly to relative commodity prices. We therefore feel that our data are better handled by departing from the simple "endogenous q, exogenous p" paradigm in favor of one in which the prices of commodities other than Rent are treated exogenously (as before), but in which the quantity rental services (rather than its price) is treated exogenously. That is, at the micro level, we postulate that Problem A, now described, underlies our data:

<u>Problem A</u> Choose $(q_1, \ldots, q_6)$ to maximize

$$U(q) \equiv \sum_{j=1}^{6} U_j(q_j) \tag{7.1}$$

subject to

$$\sum_{j=1}^{5} q_j p_j = M^\circ, \tag{7.2}$$

and

$$q_6 = \bar{q}_6, \tag{7.3}$$

where M° is a predetermined total ('cash expenditure'); where p_1, ..., p_5 are predetermined prices; where $\bar{q}_6$ is the predetermined quantity of housing services consumed; and where U is strictly quasi-concave.

Notice that in (7.1) we have continued to maintain, as in Section 2, that the utility function is additive. Also notice that, in order to simplify notation in what follows, we have reordered the commodities, for the rest of the paper, so that "Rent" comes last (i=6).

Problem B Choose $(q_1, \ldots, q_6)$ to maximize

$$U(q) = \sum_{j=1}^{6} U_j(q_j) \tag{7.4}$$

subject to

$$\sum_{j=1}^{6} q_j p_j = M, \tag{7.5}$$

where M (total expenditure) and $p_1, \ldots, p_6$ are predetermined; and where U is strictly quasi-concave.

Problem B is important because it provides the context in which Houthakker's (1960) result on additive preferences was derived: this is the result which allowed the simplification (2.6) above. Does this result apply equally to Problem A? Fortunately, other than the requirement that q_6 be treated as an exogenous variable, Problems A and B have identical first order conditions; this in turn implies that the result (2.6) is available to us under formulation B, since this restriction was obtained simply by manipulating the first-order conditions for Problem A (for details of these manipulations see e.g., Phlips (1974) pp. 60-63 or Powell (1974) pp.24-29).

To see the equivalence (in the sense defined above) of Problems A and B, write their respective Lagrangeans as:

(7.6) $$L_A(q, \lambda°, p_6; M°, p_1, \ldots, p_5, \bar{q}_6)$$

$$= U(q) + \lambda°(M° - \sum_{j=1}^{5} p_j q_j) + p_6 \lambda° (\bar{q}_6 - q_6),$$

and

(7.7) $$L_B(q, \lambda ; M, p_1, \ldots, p_6) = U(q) + \lambda (M - \sum_{j=1}^{6} p_j q_j).$$

L_B is straightforward; L_A requires some explanation. $\lambda°$ is the Lagrange multiplier on the 5-commodity budget constraint on cash expenditure, while $(p_6 \lambda°)$ is the Lagrange multiplier on the stock of housing constraint (7.3). Equivalently, p_6 is the Lagrange multiplier on

(7.8) $$\lambda°\bar{q}_6 = \lambda° q_6 ;$$

since (because of the non-satiation axiom, itself covered above by strict quasi-concavity) $\lambda° > 0$, (7.8) is equivalent to (7.3). Handling the multiplier on (7.3) this way has advantage that p_6 is the shadow price (in the most literal neo-classical sense, the "rent") of the given housing stock <u>expressed in dollars</u> (rather than in utils). The marginal utility of cash expenditure, $\lambda°$, being dimensioned as utils per dollar, when multiplied by p_6 produces a multiplier whose dimensions are utils per unit of housing stock.

Let U_i' denote the partial derivative of U with respect to q_i. The first-order conditions for Problem A are:

(7.9) $$\partial L_A/\partial q_i = U_i' - \lambda° p_i = 0; \qquad (i=1, \ldots, 6)$$

(7.10) $$\partial L_A/\partial \lambda° = (M° + p_6\bar{q}_6) - \sum_{j=1}^{6} p_j q_j = 0;$$

and

(7.11) $\partial L_A/\partial p_6 = \lambda^\circ \bar{q}_6 - \lambda^\circ q_6 = 0.$

The first of these, (7.9), looks very much like the first of the first-order conditions for Problem B, which are:

(7.12) $\partial L_B/\partial q_i = U_i' - \lambda p_i = 0$; (i=1, ..., 6)

and

(7.13) $\partial L_B/\partial \lambda = M - \sum_{j=1}^{6} p_j q_j = 0.$

Indeed, the only difference between (7.9) and (7.12) is that λ in the latter replaces λ° in the former; however, each is interpreted as the marginal utility of a dollar optimally spent. If we define M as the sum of cash expenditures on commodities 1, ..., 5 plus the imputed expenditure on Rent, reckoned at the shadow price of housing p_6, then (7.10) and (7.13) are equivalent. (7.11) merely states that q_6 must assume its given value $\bar{q}_6$ -- equivalent, in terms of manipulations of the system, to erasing the bar and treating q_6 as an exogenous variable. This result, of course, also applies in the case when all of the prices are endogenous, and all of the quantities exogenous -- see e.g., Theil (1976), p. 122.)

If 100 per cent of the Australian housing stock were owner-occupied, the above arguments would not be of much benefit, since p_6 would be an unobservable variable. However, the 30 per cent of the market in which cash rental transactions take place renders p_6 observable provided we assume that these prices are equal to the shadow prices of owner-occupied stock (a market equilibrium condition). This is the assumption underlying the imputation used to construct the official data with which we are working.

Lest there be any confusion over the context in which exogeneity of q_6 is invoked by us, let us state explicitly:

(a) our 'representative' agent is an owner-occupier who takes q_6 as fixed;

and

(b) the market data reflect a total housing stock which is exogenous.

Also notice that even if we replace (a) with the conventional theory (applicable to the 30 per cent of households which rent accommodation), (b) would still justify our treating q_6 as exogenous in our econometric work with market data. Finally, to recapitulate, there is no conflict in the form of the behavioral relations induced by (a) and by the more conventional alternative.

A final result needed below is the following: given the additivity of U, the conditional demand function for commodities 1 through 5, given the quantity of Rent consumed, may be found as the solution to:

<u>Problem C</u> Choose $(q_1, \ldots, q_5)$ to maximize

$$U^* = \sum_{j=1}^{5} U_j(q_j) \tag{7.14}$$

subject to

$$\sum_{j=1}^{5} p_j q_j = M°, \tag{7.15}$$

where M° is the total expenditure which would be found as the optimal amount to spend on items 1 through 5 by solving Problem B. It follows that if M° (total cash expenditure, in the case of our database) is exogenous, the demand system may be consistently estimated in two steps:

(i) fit a subsystem of 5 equations in which quantities are endogenous, and prices p_1, ..., p_5 and cash expenditure M^o are exogenous;

(ii) fit a single equation in which p_6 is endogenous and M^o, p_1, ..., p_5 and q_6 are exogenous.

To be sure, it might be possible to improve efficiency of estimation by handling the two steps simultaneously, but this did not prove to be feasible.

The development in the next section of the rental price equation is able to go a long way just on the basis of additive preferences; finally, however, this equation's parameterization depends on the form of the 5-commodity expenditure subsystem. We shall fit Theil's extension of Working's Model to the subsystem (not to the system as a whole) in our empirical work.

7.2 Deriving the Rental Price Equation

We start with Houthakker's (1960) result on additive preferences, which may be written as:

(7.16) $$\sigma_{ij} = -\phi \eta_i \eta_j, \qquad (i \neq j;\ i,\ j = 1, \ldots, 6)$$

in which σ_{ij} is the Allen-Uzawa partial substitution elasticity between i and j; ϕ, as before, is:

(7.17) $$\phi = 1/(\partial \log \lambda / \partial \log M);$$

and η_i is the elasticity of demand for i with respect to total spending:

(7.18) $$\eta_i = \partial \log q_i / \partial \log M \Big|_{p_1, \ldots, p_6 \text{ constant}} .$$

(Note that equations (7.16) and (2.6) are equivalent.) Next we note that the ordinary uncompensated cross-price elasticity of demand for i with respect to the price of j is:

(7.19) $\eta_{ij} = w_j(\sigma_{ij} - \eta_i)$ $(i \neq j)$.

In (7.19), $w_j\sigma_{ij}$ may be interpreted as the compensated cross elasticity of demand for i with respect to the price of j (see e.g., Powell (1974), p. 13); summing such terms over j corresponds to the thought experiment in which all prices rise by one per cent with a simultaneous one per cent relaxation of the budget constraint; thus

(7.20) $$\sum_{j=1}^{6} w_j\sigma_{ij} = 0, \qquad (i = 1, \ldots, 6)$$

which implies

(7.21) $$\sum_{j=1}^{6} \eta_{ij} = -\eta_i . \qquad (i = 1, \ldots, 6)$$

Hence

$$\eta_{66} = -(\eta_6 + \sum_{j=1}^{5} \eta_{6j}).$$

[from (7.19)]

$$= -[\eta_6 + \sum_{j=1}^{5} w_j(\sigma_{6j} - \eta_6)].$$

(7.22) $$= -[w_6\eta_6 + \sum_{j=1}^{5} w_j\sigma_{6j}].$$

Under additive preferences we may use (7.16), obtaining:

(7.23) $$\eta_{66} = -[\eta_6 w_6 - \phi\eta_6 \sum_{j=1}^{5} w_j\eta_j].$$

However, the weighted average of all elasticities with respect to total expenditure must be unity; therefore, (7.24) $$\sum_{j=1}^{5} w_j\eta_j = 1 - w_6\eta_6 ;$$

substituting this expression into (7.23), we obtain:

(7.25) $$\eta_{66} = \eta_6[\phi - w_6(\phi\eta_6 + 1)].$$

For finite differentials we can write, in the context of Problem B,

(7.26) $$\text{dlog } q_6 = \sum_{j=1}^{6} \eta_{6j} \text{ dlog } p_j + \eta_6 \text{dlog } M.$$

Therefore,

(7.27) $$\text{dlog } p_6 = (\text{dlog } q_6 - \eta_6 \text{ dlog } M - \sum_{j=1}^{5} \eta_{6j} \text{ dlog } p_j)/\eta_{66}.$$

(7.27) is the equation needed to complete our demand system. We need not attempt unconstrained estimation of the linear combination of prices on the right, as we will know a good deal about the values of the η_{6j} coefficients from step 1 -- i.e., from our estimation of the five-commodity subsystem. In particular, we will have estimated demand elasticities with respect to subsystem total expenditure M°; i.e., estimates of the coefficients

(7.28) $$\eta_i^\circ = \partial\log q_i/\partial\log M^\circ \Big|_{p_1, \ldots, p_5 \text{ const.}} \qquad (i=1, \ldots, 5)$$

will be available. We can find the elasticities with respect to total expenditure M using the chain rule:

(7.29) $$\eta_i = (\partial\log q_i / \partial\log M^\circ)(\partial\log M^\circ/\partial\log M), \qquad (i=1, \ldots, 5)$$

where the partials are taken with <u>all</u> prices held constant (and we are, therefore, in the world of Problem B).

In fact

(7.30) $$M^\circ = M - p_6q_6 ;$$

since

$$(7.31) \qquad \frac{\partial \log M^\circ}{\partial \log M} = \frac{M}{M^\circ} \left(1 - p_6 \frac{\partial q_6}{\partial M}\right)$$

$$= (1 - w_6)^{-1}(1 - w_6\eta_6),$$

thus .

$$(7.32a) \qquad \eta_i = \eta_i^\circ(1 - w_6\ \eta_6)/(1 - w_6)$$

$$(7.32b) \qquad = \eta_i^\circ \eta^* \text{ , (say)} \qquad (i=1, \ldots, 5)$$

where η^* is the elasticity of demand for the subgroup $(q_1, \ldots, q_5)$ as a whole with respect to total expenditure. Using (7.32) and (7.16) in (7.19), for i = 6 we obtain:

$$(7.33) \qquad \eta_{6j} = - w_j \eta_6[\eta_j^\circ \eta^* \phi + 1]. \qquad (j \neq 6)$$

Now we substitute from (7.33) into (7.27), obtaining:

$$(7.34) \qquad \text{dlog } p_6 = \text{dlog } q_6/\{\eta_6[\phi - w_6(\phi\eta_6 + 1)]\}$$

$$- \text{dlog } M/[\phi - w_6(\phi\eta_6 + 1)]$$

$$+ \sum_{j=1}^{5} w_j \text{dlog } p_j/[\phi - w_6(\phi\eta_6 + 1)]$$

$$+ \phi\eta^* \sum_{j=1}^{5} w_j \eta_j^\circ \text{ dlog } p_j/[\phi - w_6(\phi\eta_6 + 1)].$$

Recalling that

$$(7.35) \qquad w_j = w_j^\circ(1 - w_6),$$

where w_j° is the share of commodity j (j=1, ...,5) in the budget for the five-commodity subsystem, and recognizing that $(w_j^\circ \eta_j^\circ)$ is just the marginal budget share of commodity j within the subsystem, we can see that the last two terms on the right of (7.34), respectively, involve the Divisia and the Frisch price indexes for the five-commodity aggregate. Writing

$$(7.36) \qquad \text{dlog } (P^\circ) = \sum_{j=1}^{5} w_j^\circ \text{ dlog } p_j \qquad \text{[Divisia price index]}$$

and

(7.37) $$\text{dlog } (P^{o})' = \sum_{j=1}^{5} (w_j^{o} \eta_j^{o}) \text{dlog } p_j \,, \qquad \text{[Frisch price index]}$$

(7.34) after clearing fractions becomes:

(7.38) $$[\phi - w_6(\phi\eta_6 + 1)] \text{ dlog } p_6$$

$$= (\text{dlog } q_6)/\eta_6 - \text{dlog } M$$

$$+ (1 - w_6) \text{ dlog } (P^{o})$$

$$+ \phi (1 - w_6\eta_6) \text{ dlog } (P^{o})'.$$

Notice that (7.38) possesses the following homogeneity property: if q_6 is held constant and each of p_1, ..., p_5 and M increases by one per cent, then so also do the rental price and M°, thus leaving relative prices and all quantities undisturbed. Notice also that once the estimation of the subsystem is complete, (7.38) contains only observable variables plus ϕ and η_6.

To this point, our results have not utilized any information on the form of the Engel curves. However, given that we have adopted Working's Model for the subsystem, the operational analog of dlog (P°)' is:

(7.39) $$D(P^{o})' = \sum_{j=1}^{5} (\bar{w}_j^{o} + \hat{\beta}_j^{o}) \text{ log } p_j \,,$$

in which $\hat{\beta}_j^{o}$ is estimated from Working's Model applied to the subsystem. For the sake of explicitness, we note that the latter's operational form is:

$$(7.40)\qquad y^{\circ}_{it} = \beta_i\ DQ^{\circ}{}_t + \phi^{\circ}(\bar{w}^{\circ}_{it} + \hat{\beta}^{\circ}_i)[Dp_{it} - \sum_{j=1}^{5} (\bar{w}^{\circ}_{jt} + \hat{\beta}^{\circ}_j)Dp_{jt}] + e^{\circ}{}_{it}\ ,$$

in which

$$(7.41)\qquad y^{\circ}_{it} = \bar{w}^{\circ}_{it}\ (Dq_{it} - DQ^{\circ}_t).$$

In these equations DQ°_t is the Divisia volume index for the five-commodity aggregate, while ϕ° is the reciprocal of the elasticity with respect to expenditure totalled over the five commodities of the marginal utility of money.

How does ϕ relate to ϕ°? Since, at fixed prices, a one per cent increase in M° translates into an $1/\eta^*$ per cent increase in M, we have that

$$(7.42)\qquad \phi = [\partial \log \lambda / \partial \log M]^{-1}$$
$$= [(\partial \log \lambda / \partial \log M^{\circ})(\partial \log M^{\circ} / \partial \log M)]^{-1}$$
$$= \phi^{\circ}/\eta^* = (1 - w_6)\phi^{\circ}/(1 - w_6\eta_6).$$

Notice that with ϕ° assumed globally constant (from step 1), ϕ must be treated as a variable. Once ϕ° is estimated in step 1, substitution from (7.42) into (7.38) yields an equation whose only unknown is η_6. It is our task to estimate it. At this stage we must, therefore, specify how the shares w_6 and $(1- w_6)$ vary with income at constant prices. Again we adopt Working's Model (which consequently now is nested two deep). Then

$$(7.43)\qquad w_6\eta_6 = w_6 + \beta_6\ .$$

To obtain the final form of our estimating equation from (7.34) we must do the following:

(i) Make substitutions from (7.42) and (7.43) to obtain an expression in which the only parameter not already known from the subsystem estimation is β_6.

(ii) Restore the data-point subscript t to all variables.

(iii) Replace logarithmic differentials with differences of logarithms, and replace commodity six's share, w_{6t}, by its average values $\bar{w}_{6t}$.

(iv) Express DM_t in terms of exogenous variables and Dp_{6t}.

(v) Append an error structure.

The result of steps (i) - (iii) is:

$$(7.44) \qquad [\phi^\circ(1 - \bar{w}_{6t}) - \bar{w}_{6t}]Dp_{6t} \approx [\bar{w}_{6t}/(\bar{w}_{6t} + \beta_6)]Dq_{6t} - DM_t + (1 - \bar{w}_{6t})D(P^\circ_t) + \phi^\circ\ (1- \bar{w}_{6t})D(p^\circ_t)' \ .$$

But

$$(7.45) \qquad DM_t \equiv \Delta\log\ (M^\circ_t + p_{6t}q_{6t}) \approx (1 - \bar{w}_{6t})\Delta\log\ M^\circ_t + \bar{w}_{6t}(\Delta\log\ q_{6t} + \Delta\log\ p_{6t}) \ ;$$

that is,

$$(7.46) \qquad DM_t \approx (1-\bar{w}_{6t})DM^\circ_t + \bar{w}_{6t}Dq_{6t} + \bar{w}_{6t}Dp_{6t} \ .$$

Ignoring second-order approximation errors, we see that (7.44) and (7.46) jointly imply:

$$(7.47) \qquad \phi^\circ Dp_{6t} = (\bar{w}_{6t}[(\bar{w}_{6t} + \beta_6)^{-1} - 1]/(1 - \bar{w}_{6t}))Dq_{6t} - DM^\circ_t + DP^\circ_t) + \phi^\circ D(P^\circ_t)' \ .$$

An ideal estimation procedure would take account of the fact that ϕ°, $D(P^\circ_t)$ and $D(P^\circ_t)'$ have sampling errors generated by the subsystem disturbances (e°_{it}) in (7.40); here, however, we will simply append an additive zero mean error v_t to (7.47), and compute a (non-linear) least-squares estimator of β_6 which minimizes the criterion

$$(7.48) \qquad \sum_{t=1}^{32} \hat{v}_t^2 = \sum_{t=1}^{32} [\phi^\circ Dp_{6t} - f(\bar{w}_{6t}, \beta_6)\, Dq_{6t} + D(Q_t^\circ) - \phi^\circ D(P_t^\circ)']^2,$$

where $f(\bar{w}_{6t}, \beta_6)$ is the expression within curly parentheses on the right of (7.47). If we assume that v_t is normally distributed with classical serial properties, then, conditional on the values of the subsystem parameters, we can compute an asymptotic standard error for our estimate of β_6 from:

$$(7.49) \qquad s_{\beta 6} = [\hat{\sigma}^2 / \partial^2 (\sum_{t=1}^{32} \hat{v}_t^2)/(\partial \beta_6)^2]^{\frac{1}{2}} ,$$

where

$$(7.50) \qquad \hat{\sigma}^2 = \sum_{t=1}^{32} \hat{v}_t^2/32.$$

Some of the properties of the estimator defined by minimizing (7.48) can be inferred a priori. First, when the marginal budget share of Rent passes through zero (the dividing line between normal and inferior goods), there is a positive infinite discontinuity in the criterion function. From (7.47) it can be seen that f experiences such singular points whenever β_6 is set equal to the negative of any of the sample values of $\bar{w}_{6t}$. However, since in the world of additive preferences inferiority is ruled out, when optimizing β_6 we need only scan the half line whose lower bound is minus the minimum sample value of $\bar{w}_{6t}$ (namely, 0.0742). This constraint then places immediate prior bounds on the total expenditure elasticity of Rent at any date. Consider 1985, for example, where the relevant $\bar{w}_{6t}$ (also the sample maximum) is 0.2136. Then the above proposition establishes that $\eta_{6,1985}$ lies in the interval (1 - 0.0742/0.2136, ∞) = (0.6526, ∞). The corresponding bound for 1954, which is both the start of the sample and the point at which the minimum value of $\bar{w}_{6t}$ occurs, is just the positive half line (0, ∞). Notice that the estimates of β_6 obtained in Section 6 from the synoptic cross-section data would, in light of the early time-series values of $\bar{w}_{6t}$, imply that Rent were an inferior good in the early part of the time-series. But our maintained hypothesis -- additive preferences -- rules such an eventuality out.

8. Revised Estimates

8.1 Five-Commodity Subsystem

Subsystem estimates of Working's Model, with and without trends are given in Table 8.1. Although serial correlation remains a problem, these estimates are satisfactory in that broadly they conform to Working's Model, although the elasticity of demand for Household durables (a luxury) still increases slightly over the sample period. Collectively, the trends are still significant at the five per cent level according to the asymptotic likelihood ratio test (X^2_4 = 18.77). Notice, however, that relative to the earlier results, these trends provide less conclusive evidence of misspecification, Clothing being the only case in which the expenditure elasticity is still very sensitive to their inclusion or exclusion. Because unexplained trends are not viable in CGE modeling , from this point on we work with the trend-free estimates. It is to be hoped that further econometric work might

Table 8.1

Estimates of Working's Model under Additive Preferences

Fitted to A Five-Commodity Subsystem,

with and without Trends

Commodity i	Without Trends:					With Trends:						
	MLE of β_i°	Precision Ratio for β_i° (a)	First Serial Correlation of Residuals	See (b) η_{i17}°	See (b) η_{i33}°	MLE of θ_i°(c)	MLE of β_i°	Precision Ratios for (see note (a)) θ_i°	Precision Ratios for (see note (a)) β_i°	First Serial Correlation of Residuals	See (b) η_{i17}°	See (b) η_{i33}°
1. Food	-.1370	9.1	.2595	.37	.33	.0009	-.1489	1.5	7.7	.2134	.32	.27
2. Cigarettes, tobacco, alcohol	-.0417	4.9	.3772	.60	.52	-.0005	-.0334	1.7	3.5	.2505	.68	.62
3. Clothing, footwear	-.0364	2.9	.4645	.65	.54	-.0017	.0043	4.3	0.3	.1540	1.0	1.0
4. Household durables	.0573	3.0	.5224	1.6	1.7	-.0021	.1066	2.6	4.2	.4723	2.2	2.2
5. Other	.1578	4.9	.4702	1.3	1.3	.0034	.0714	3.4	2.1	.2208	1.1	1.1

MLE of ϕ°; the elasticity with respect to subsystem total expenditure of the marginal utility of money: -0.5870 (d) -0.5560 (e)

Precision Ratio (a) for ϕ°: 6.898 (d), 8.189 (e).

(a) Ratio of absolute value of MLE to estimated value of asymptotic standard error.

(b) η_{i17}° is the implied value of the elasticity of demand for i with respect to "cash expenditure" in 1969-70; η_{i33}° is the corresponding value in 1985-86.

(c) 100 θ_i° is the autonomous annual percentage point change in the share of i within the subsystem budget M_t°.

(d) Without trends.

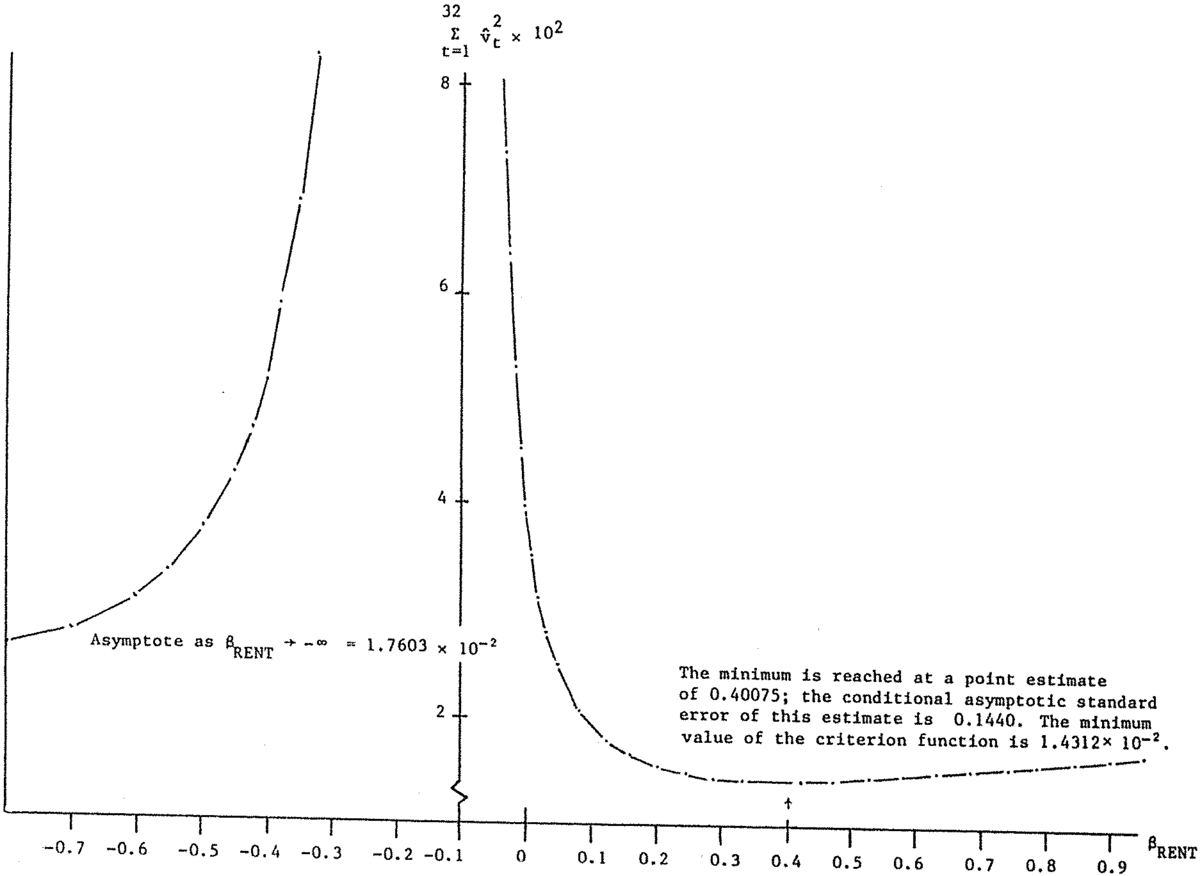
(e) With trends. [<u>Note</u>: "Cash expenditure" and "subsystem total expenditure" are synonyms for the subsystem budget M_t°.]

unravel the mechanisms responsible for the remaining trends (especially in the case of Clothing), so that they can be incorporated explicitly into the modeling exercise.

8.2 Rental Price Equation

The price-of-Rent equation (7.47) has only one parameter, β_6. The sum-of-squares function, conditional on the subsystem estimation without trends, is shown in Figure 8.1. The estimated value of β_6 is 0.40075 with an estimated conditional asymptotic standard error of 0.143997. There is, however, evidence of misspecification in the residuals (Table 8.2), which have a first-order serial correlation of 0.363. The sign pattern of the residuals indicates that equation (7.47) underpredicts price changes in the early years of the sample (1953-54 through 1961-62), overpredicts in the middle (1962-63 through 1975-76) and underpredicts in the last part of the sample (1976-77 through 1985-86). The increasingly deregulated nature of housing interest rates in the 'eighties might explain the last episode, but is of no help in explaining the 'fifties. Nevertheless, the residuals from (7.43) are much better behaved than those for Rent displayed in Table 5.2, and now are qualitatively comparable to the residuals from the other fitted equations.

The point estimate of β_6 implies very high income elasticities for housing by Australians. If we take Working's Model in the outer nest of our 2-tiered system literally, then the total expenditure elasticity for Rent in 1953-54 had the extraordinarily high value of 6.5; by the end of the sample it had declined to the still very high value of 2.8. And of course, these values are



Asymptote as $\beta_{RENT} \to -\infty = 1.7603 \times 10^{-2}$

The minimum is reached at a point estimate of 0.40075; the conditional asymptotic standard error of this estimate is 0.1440. The minimum value of the criterion function is 1.4312×10^{-2}.

Figure 8.1: Criterion function for fitting the Rental price equation (7.47)

Table 8.2

Residuals from Fitted Rental Price Equation

Year[*]	Residual	Year[*]	Residual
1953-55	-.002093		
1954-56	-.015587	1969-71	-.004816
1955-57	-.020557	1970-72	.005847
1956-58	-.013419	1971-73	.019602
1957-59	-.019167	1972-74	.036622
1958-60	.032733	1973-75	.021520
1959-61	-.052299	1974-76	.003343
1960-62	-.031232	1975-77	-.021372
1961-63	.014063	1976-78	-.028320
1962-64	.019891	1977-79	-.000089
1963-65	.022738	1978-80	.007855
1964-66	-.001863	1979-81	.012264
1965-67	.014950	1980-82	-.002409
1966-68	.017671	1981-83	-.031007
1967-69	.024917	1983-84	-.019540
1968-70	.018363	1983-85	-.015533
		1984-86	-.019667

First-order Serial Correlation Coefficient: 0.363

Mean Residual: .000831

Mean of Absolute Value of Residual as Proportion of Left-hand Variable (per cent): 44.8

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[*] For example, 1964-66 refers to the data point generated by the differences in the relevant variables between 1964-65 and 1965-66.

widely divergent from the Australian synoptic cross-sectional evidence of Table 6.1, which suggests a total expenditure elasticity for Rent in 1984 in the range 0.30 to 0.53.

We should not take the point estimate of $ß_6$ literally, however. The two standard-error band spans the interval (0.1128, 0.6887). The weight of external evidence favors a low, rather than a high value; for example, using data from the International Comparisons Project, Theil in Theil and Clements (forthcoming (1987), p. 52) finds a value of 0.032 for the category 'Gross Rent and Fuel'; and as we have seen above, our preliminary examination of the 1984 Household Expenditure Survey points to a negative value of ß for Rent.

We take the lower bound of our two standard error band as our preferred estimate of $ß_{RENT}$. This, in conjunction with the subsystem estimates (estimated without trends) of Table 8.1, leads to the results displayed in Table 8.3.

The results in Table 8.3 exhibit the following (relatively) satisfactory characteristics:

(a) The behavior over time of the total expenditure elasticity for Rent is consistent with Working's Model.

(b) Although Frisch's money flexibility ϕ necessarily varies over time in our treatment, this variation is minimal (2.7 per cent in 33 years).

Next we turn to the integration of the results at the two levels of estimation into a single set of systems estimates. The total expenditure elasticities for the five commodities of the subsystem are found simply by multiplying their elasticities with

Table 8.3

Estimated 2-Level Working's Model: Main Characteristics of Outer Nest

Year t	Budget Share of Rent w_{6t}	Total Expenditure Elasticity for Rent η_{6t}	Total Expenditure Elasticity for Rest(a) η^*_{6t}	Total Expenditure Elasticity of M.U. of Money ϕ_t	Substitution Elasticity between Rent & Rest(b)	Own Price Elasticity(c)	
						Rent $\eta_{66,t}$	Rest(a) $\eta^*_{..,t}$
1953-54	0.0724	2.558	0.8784	-0.516	1.16	-1.26	-0.90
1969-70	0.1241	1.909	0.8712	-0.511	0.85	-0.98	-0.87
1985-86	0.2194	1.514	0.8555	-0.502	0.65	-0.84	-0.81

Descriptive measure of goodness of fit (d): 0.411

(a) "Rest" is the 5-commodity subsystem aggregate whose volume is measured by Q^o_t; its components are the commodities listed in column 1 of Table 8.1.

(b) Minus the product of the previous three columns; see eqn (7.16).

(c) See eqn (7.25).

(d) The R^2 from a simple regression of the left-hand variable in eqn (7.47) on its estimated values.

respect to M° by the elasticity of M° with respect to M:

$$(8.1) \qquad \eta_{it} = \eta^{\circ}_{it}\,\eta^{*}_{t}$$
$$= \eta^{\circ}_{it}(1-w_{6t}\eta_{6t})/1-w_{6t}) \ . \qquad (i=1, \ldots, 5)$$

Own and cross price elasticities for the system may be worked out using equations (7.19), (8.1), (7.21), and (7.42). We obtain:

$$(8.2a) \qquad \eta_{ijt} = \partial \log q_{it}/\partial \log p_{jt}|M = \text{const}$$
$$= -\eta^{*}_{t}\eta^{\circ}_{it}w_{jt}[\phi^{\circ}(\eta^{*}_{t})^{2}\eta^{\circ}_{jt} + 1] \ ; \qquad (i,j \neq 6, \ i \neq j)$$

$$(8.2b) \qquad \eta_{i6t} = -\eta^{*}_{t}\eta^{\circ}_{it}w_{6t}[\phi^{\circ}\eta^{*}_{t}\eta_{6t} + 1] \ ; \qquad (i \neq 6)$$

$$(8.3a) \qquad \eta_{iit} = \eta^{*}_{t}\eta^{\circ}_{it}(\phi^{\circ}(\eta^{*}_{t})^{2}[1 - w_{6t} - w_{it}\eta^{\circ}_{it})] - w_{it}$$
$$+ w_{6t}\phi^{\circ}\eta^{*}_{t}\eta_{6t}) \ ; \qquad (i \neq 6)$$

$$(8.3b) \qquad \eta_{66t} = \eta_{6t}[\phi^{\circ}\eta^{*}_{t} - w_{6t}(\phi^{\circ}\eta^{*}_{t} + 1)]$$

and

$$(8.4) \qquad \eta_{6jt} = -\eta_{6t}w_{jt}[\phi^{\circ}(\eta^{*}_{t})^{2}\eta^{\circ}_{jt} + 1] \ . \qquad (j \neq 6)$$

These formulae contain only ϕ°, elasticities with respect to cash expenditure M°_{t} or total expenditure M_{t}, and observed shares in the total budget. The elasticities are obtained directly from the latter and the estimated parameters from Working's Model at the appropriate level:

$$(8.5) \qquad \eta^{\circ}_{it} = 1 + \beta^{\circ}_{i}/\bar{w}^{\circ}_{it} \ ; \qquad (i=1, \ldots, 5)$$

$$(8.6) \qquad \eta_{6t} = 1 + \beta_{6}/\bar{w}_{6t}.$$

Putting i = 1 in the above formulae and t = 33, we can, for example, compute the estimated own and cross-price elasticities for Food in 1985-86. We obtain:

Estimated Elasticities of Demand for Food in 1985-86 with respect to Price of:

Food	Tobacco, Cigs, Alcohol	Clothing	H'Hold Durables	Other	Rent	Sum Previous Columns	Total Expenditure M
-.1789	-.0148	-.0133	-.0054	-.0530	-.0147	-.2801	.2801

Note that the own and cross price elasticities sum, as they should, to minus the total expenditure elasticity.

How do our estimates for Food compare with the commonly found result mentioned in Section 1? Recall that when Working's Model was fitted at a single level of commodity disaggregation, the $ß_i$ parameter for Food was estimated variously at values from -0.13 (Blanciforti and Green (1983), from U.S. time series 1948 through 1978) through -0.15 (Theil and associates -- see Sections 1 and 2 for references -- from Japanese time series 1951 through 1972, and from a cross section of 30 countries from the International Comparisons Project). Because we have used a nested version of Working's Model, there is no directly comparable $ß_i$ for Food. However, whereas in the standard model described in Section 2, the marginal budget share of every commodity takes the form $(w_{it} + ß_i)$, in our nested model the marginal share of component i of major commodity s in the total budget M_t becomes:

$$(8.7) \qquad \partial(p_{it}^{(s)} q_{it}^{(s)})/\partial M_t = w_{(s,i)t} + [ß_i^{(s)}(ß_s + w_{st}) + ß_s w_{it}^{(s)}] ,$$

in which $p_{it}^{(s)}$ and $q_{it}^{(s)}$, respectively, are the price and quantity of component i within s; $w_{(s,i)t}$ is the share <u>in the total budget</u> of the i^{th} component of s; $\beta_i^{(s)}$ is the β parameter for i as estimated from the subsystem in which the quantity index log $Q^{(s)}$ for the group s as a whole appears on the right as the "income" variable; β_s is the corresponding parameter for the major group s as a whole as estimated from the outer-nest Working's Model, in which log Q appears as the "income" variable; w_{st} is the budget share of major group s; and finally, $w_{it}^{(s)}$ is the share of component commodity i within the subsystem expenditure total for s: in our example, M_t^o. In our application s = 2 indicates the miscellaneous commodity comprising everything except Rent; thus $\beta_2 = -\beta_1 \equiv -\beta_{RENT}$, and the $\beta_i^{(2)}$ (i = 1, ..., 5) are just the β_i^o in the notation of Section 7, while the subsystem shares written above as $w_{it}^{(s)}$ were, in the notation of the latter Section, written w_{it}^o.

The expression in square parentheses on the right of (8.7), although a variable, is the entity most directly comparable with the corresponding parameter β_i in an unnested system. Call this expression β_{it}^*. Then the values of β_{it}^* implied for Food (i = 1) by our 2-level estimates are as follows:

1953-54: $\beta_{1t}^* = -0.144$; 1969-70: $\beta_{1t}^* = -0.129$; 1985-86: $\beta_{1t}^* = -0.114$

Given the existence of restaurant and take-away (viz., "carry-out") prepared food in the Australian data, and the increasing share of this component in total Food, these Australian estimates are thus consistent with the weight of international evidence. The results also point to the greater flexibility of the Engel responses possible under a nested specification of Working's Model.

8.3 Further Disaggregation -- An Illustration

Finally, there is the possibility of using the results obtained from the 5-commodity subsystem and the Rental price equation to estimate further subsystems. Data are available at the 16-commodity level of disaggregation for the most recent 17 years of the sample (Australian Bureau of Statistics (1986b)). For a selection of some of these commodities, and of some aggregates thereof which are still finer than the broad 6-commodity level so far employed, it would be reasonable to continue to maintain the additive preference postulate. To make matters concrete, consider the disaggregation of Tobacco, cigarettes and alcohol into Tobacco products and Alcoholic beverages. Under additive preferences, we can write:

$$(8.8) \qquad \sigma_{ij}^{(2)} = -\phi^{(2)} \eta_i^{(2)} \eta_j^{(2)} \quad . \qquad (i \neq j)$$

Here the notation is as follows. $\sigma_{ij}^{(2)}$ is the elasticity of substitution between component commodities i and j within major commodity 2 (Tobacco, cigarettes, and alcoholic beverages). $\phi^{(2)}$ is the reciprocal of the elasticity with respect to total spending on commodity 2 of the marginal utility of money. The $\eta_i^{(2)}$ are the elasticities with respect to total spending on major commodity 2 of the demand for subcommodity i (i = 1, in our example, is Tobacco products, while i = 2 indicates alcoholic beverages). Using arguments similar to those used in Section 7 we see that if ϕ° is a parameter, then $\phi^{(2)}$ (and more generally, $\phi^{(s)}$ (s = 1, ..., 6)) is a variable whose value at t is

$$(8.9) \qquad \begin{aligned} \phi_t^{(2)} &= [\partial \log \lambda/\partial \log (P_t^{(2)}Q_t^{(2)})]^{-1} \\ &= [\partial \log \lambda/\partial \log Q_t^{(2)}]^{-1} \\ &= [(\partial \log \lambda/\partial \log M_t^\circ) \div (\partial \log Q_t^{(2)}/\partial \log M_t^\circ)]^{-1} \\ &= \phi^\circ n^\circ_{2t} \ . \end{aligned}$$

We can use Working's Model at a third level of nesting for the disaggregation of a major commodity into its components. In the case of Tobacco, cigarettes, and alcohol, we would write (analogously with (2.20)):

$$(8.10) \qquad y_{it}^{(2)} = \beta_i^{(2)} DQ_t^{(2)} + (\phi^\circ n^\circ_{2t})(\bar{w}_{it}^{(2)} + \beta_i^{(2)}) \times$$

$$[Dp_{it}^{(2)} - \sum_{j=1}^{2}(\bar{w}_{jt}^{(2)} + \beta_j^{(2)})\ Dp_{jt}^{(2)}] + e_{it}^{(2)} \ ,$$

in which

$$(8.11) \qquad y_{it}^{(2)} = \bar{w}_{it}^{(2)}(Dq_{it}^{(2)} - DQ_t^{(2)}) \ .$$

The notation is as follows: the superscript within parentheses (in this case 2) identifies the major commodity being disaggregated; the i and j subscripts identify the components thereof (in this case there are 2 components, but more generally, n_s (say), where s = 1, ..., 6). $\beta_i^{(2)}$ is defined such that $(\bar{w}_{it}^{(2)} + \beta_i^{(2)})$ is the *marginal* share of i in the value of major commodity 2, where $\bar{w}_{it}^{(2)}$ is the corresponding average budget share (in the microeconomic sense). $Dp_{it}^{(2)}$ and $Dq_{it}^{(2)}$, respectively, are the logarithmic first differences of the price and quantity of component i of major commodity 2, while $DQ_t^{(2)}$ is the (operational version of) the Divisia index for the quantity of major commodity 2.

The price elasticities for the components of major commodities can be derived using methods analogous to those underlying equations (8.2) to (8.6). In general, we can use (7.19); namely,

$$\eta_{ijt} = \bar{w}_{jt}(\sigma_{ijt} - \eta_{it}) \qquad (i \neq j),$$

where σ_{ijt} and η_{it} have to be computed using the appropriate choice of parameters from the different levels of estimation. In general:

$$(8.12) \qquad \sigma_{ijt} = -\phi_t \eta_{it} \eta_{jt} \text{ ,}$$

where ϕ_t is defined by (7.42); i.e.,

$$(8.13) \qquad \phi_t = (1 - \bar{w}_{6t})\phi^{\circ}/(1 - \bar{w}_{6t}\eta_{6t});$$

and, for Rent:

$$(8.14) \qquad \eta_{6t} = (1 + \beta_6/\bar{w}_{6t}) \text{ ;}$$

while for other major commodities i, which are not further disaggregated, (see (7.32b)),

$$(8.15) \qquad \eta_{it} = (1 - \bar{w}_{6t}\eta_{6t})(1 + \beta^{\circ}_i/\bar{w}^{\circ}_{it})/(1 - \bar{w}_{6t}) = \eta^{*}_t \eta^{\circ}_{it} \text{ ;}$$

and finally, for commodities i which themselves are components of major commodities s at the 5-commodity level,

$$(8.16) \qquad \eta_{it} = \eta^{(s)}_{it} \eta^{\circ}_{st} \eta^{*}_t \text{ ,} \qquad (i \in s)$$

where η°_{st} is $(1 + \beta^{\circ}_s/\bar{w}^{\circ}_{st})$ and $\eta^{(s)}_{it}$ is

$$(8.17) \qquad \eta^{(s)}_{it} = \partial \log q_{it}/ \partial \log Q^{(s)}_t$$

$$= (1 + \beta^{(s)}_i/\bar{w}^{(s)}_{it}) \text{ .} \qquad (i \in s)$$

Own price elasticities are computed using (7.21) (with the upper limit of summation changed to reflect the total number of commodities finally distinguished in the 3-tiered system).

For reasons of space we do not pursue the matter further here; we simply report the result of the estimation of our chosen example; namely, the disaggregation of major commodity s = 2

(Tobacco, cigarettes, and alcohol) into its components, i = 1: Tobacco products; and i = 2: Alcoholic beverages. We obtain $\beta_1^{(2)} = -0.2127$ (and hence, $\beta_2^{(2)} = +0.2127$). The standard error is of the order of 4.5; therefore, these point estimates are not reliable. However, we illustrate the method by using these values in (8.17), and thence calculating the cross elasticity of Food with respect to Tobacco products and Alcoholic beverages separately for 1985-86. The relevant shares are $w_{1t}^{(2)} = 0.2829$ (the share of Tobacco products within Tobacco, cigarettes and alcohol in 1985-86), and its complement $w_{2t}^{(2)}$ (the share of Alcoholic beverages within the same major group). From Table 8.1 we see that η_{2t}° in 1985-86 is 0.52 (to four places, 0.5207) and from Table 8.2 that η_t^* is 0.8555; using (8.16) we obtain the following values of the total expenditure elasticities:

Estimated Elasticities in 1985-86 with Respect to Total Expenditure for:

Tobacco Products	Alcoholic beverages	Tobacco, cigarettes and alcohol
$\eta_{(2,1)t}$	$\eta_{(2,2)t}$	η_{2t}
0.1105	0.5776	0.4455

Above, $\eta_{(s,i)t}$ is the total expenditure elasticity for component i of major commodity s at t. Notice that weighting these elasticities by their shares (0.2829 and 0.7171, respectively) within major commodity 2, and adding them up, we obtain a value of 0.4455 for

η_{2t}, which is identical with the value estimated from the 5-commodity subsystem via eqn (8.15). That is, in general,

$$(8.18) \qquad \sum_{i \in s} w_{it}^{(s)} \eta_{(s,i)t} = \eta_{st} . \qquad (s=1, \ldots, 6 \text{ major commodities})$$

Having obtained the total expenditure elasticities for the components of major commodity 2, we conclude our illustration by computing the cross elasticity of the demand for Food with respect to the prices of these components. For the first of these cross elasticities we obtain:

$$(8.19) \qquad \partial \log q_{1t} / \partial \log p_{1t}^{(2)} = -[w_{1t}^{(2)} \; w_{2t}] \; \eta_{1t} \; (\phi_t \eta_{(2,1)t} + 1)$$
$$= -0.0051;$$

and for the second:

$$(8.20) \qquad \partial \log q_{1t} / \partial \log p_{2t}^{(2)} = -[w_{2t}^{(2)} \; w_{2t}] \; \eta_{1t} \; (\phi_t \eta_{(2,2)t} + 1)$$
$$= -0.0097.$$

Notice that these elasticities add to the cross elasticity of Food with respect to the price of major commodity 2 in the panel displayed after equation (8.7); namely, -0.0148. More generally if we define:

$$(8.21) \qquad \eta_{(r)(s,i)t} = \partial \log q_{rt} / \partial \log p_{it}^{(s)} ,$$

where r is a major or a minor commodity and i is a component of s, then we require:

$$(8.22) \qquad \sum_{i \in s} \eta_{(r)(s,i)t} = \eta_{rst} ,$$

which simply says that increasing the prices of all components of major commodity s by one per cent has the same effect on the demand for any commodity r as increasing the overall price (index) of s by one per cent. This property, and (8.18), are preserved under applications of formulae (8.12) through (8.17).

9. Summary, Concluding Remarks and Perspective for Future Work

Computable general equilibrium models are parameter hungry. The ORANI model (Dixon et al., (1982)), for example, routinely distinguishes a hundred or more commodities; the number of own- and cross-price elasticities which are required by the model's parameter file, therefore, is of the order 10^4. Since the pioneering work of Johansen (1960), it has been the standard practice of CGE modelers to approach this daunting requirement by first working at a high level of aggregation at which the assumption of additive preferences is used. Typically, this has been done within the framework of the Klein-Rubin utility function and the associated Linear Expenditure System and its extensions. The main drawback of this approach is the constancy of marginal budget shares; at least in the case of Food (and, therefore, of the complementary miscellaneous category) there is now overwhelming evidence to the contrary (especially, Theil, Ch. 2 in Theil and Clements (forthcoming 1987)).

In this paper we have estimated a nested version of Theil's additive preference extension of Working's Model, in which marginal budget shares depend on real total expenditure per head. We did this at the level of six broad commodities on the basis of a relatively long time series (33 years), and suggested how the available supply of data might support the extension of our results to cover a total of about a dozen commodities.

A critical phase of our work was our treatment of the commodity "Rent," which, in Australia, denotes the imputed value of a predominantly owner-occupied stock of housing. We found that treating the quantity of this commodity as exogenous and its (shadow) price as endogenous produced estimates which were consistent with Working's Model, whereas the conventional treatment (in which all quantities are endogenous) did not. Nevertheless, a quick cross check against a synopsis of the most recent household expenditure survey (Australian Bureau of Statistics (1986a)) left a question mark on the size of the elasticity of demand for Rent with respect to total expenditure. Hopefully this can be unraveled with more extensive econometric work on unit records, although the cross-section data at best will be patchy on details suitable for imputation of the value of the services of the owner-occupied housing stock.

Further work should proceed on two fronts: first, disaggregation of our single representative consumer into several agents representative of different household types. (Given the dearth of Australian panel data, this will have to be based mainly on the 1984 Household Expenditure Survey.) Second, disaggregation by commodity. The preliminary step here is along the lines mentioned towards the end of Section 8 above: it simply involves an extension of Working's Model to a third level of nesting, and the use of the available comparatively short (16-year) time series. Further steps then require price and quantity series on a more finely disaggregated basis, for which additive preferences would not provide a useful framework. Possible approaches are to be found in Clements (Ch. 4 of Theil and Clements (forthcoming (1987)) and Clements and Smith (1983)). Finally, it is to be hoped that a synthesis of the cross-sectional and time-series work will uncover the mechanisms responsible for the unexplained autonomous trends evident in the latter (especially the persistent, frequently encountered negative trend in the demand for Clothing).

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Appendix 1

The Time Series Database

The general principles underlying the collation and editing of the database are described in Section 3 of the text. Here we give full details and tabulate the data. For future reference, we identify the following primary sources (all of which are publications of the Australian Bureau of Statistics, Canberra) from which we extracted time series data on consumer expenditures:

I. Australian National Accounts: National Income and Expenditure 1967-68 (published February 1969)

II. Australian National Accounts: National Income and Expenditure 1969-70 (published April 1971)

III. Australian National Accounts: National Income and Expenditure 1975-76 (published March 1977; reference No. 7.1)

IV. Time Series Data on Magnetic Tape and Microfiche September Quarter 1986 (Australian Bureau of Statistics Catalogue No. 1311.0)

We now proceed step by step, as described in Section 3 of the text.

The matched pairs of series, required for calculating commodity-specific deflators, were obtained from these sources according to the scheme laid out in Appendix Table A1.1. So far we have covered step (1) of Section 3 of the text. Next we describe how the constant-price data for the four sub-intervals identified in column 1 of Appendix Table A1.1 were spliced to yield series in constant prices of a single year (1979-80). To link the series we commenced by fitting regressions of the form

$$\text{(A1.1)} \qquad y_{it} = \beta_i x_{it} + u_{it}, \qquad (i = 1,\ldots,6)$$

where y_{it} is the value of expenditure on i in prices of 1966-67, and x_{it} is its value in prices of 1959-60, for the nine years (1959-60 through 1967-68) for which we had overlapping data. The y data for this exercise came from sources II and III (the former for 1959-60 through 1963-64; the latter for 1964-65 through 1967-68); the x data came from

Appendix Table A1.1

Sources of Matched Pairs of Time Series in Current and in Constant Prices

Sub-Period	Source	Year of Base of Constant Price Series	Level of Disaggregation Available
1953-54 through 1958-59	I, p.61	1959-60	6 commodities; (1) Food; (2) Tobacco, cigarettes, alcholic drinks; (3) Clothing, footwear; (4) Household durables; (5) Rent; (6) All other expenditure.
1959-60 through 1963-64	II, p. 61	1966-67	7 commodities: same as above, except that the last is split into (6) 'Travel and communication' and (7) 'All other expenditure.'
1964-65 through 1968-69	III, p. 52-53	1966-67	Same as last item.
1969-70 through 1985-85	IV	1979-80	16 commodities: As listed in Appendix Table A1.2

source I. Notice that for the y-data it was necessary to aggregate 'Travel and communications' with 'All other expenditure'. (This aggregation was by simple addition of constant-dollar values). The resultant estimates of the conversion factors were:

$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$
1.1467	1.2322	1.0911	1.5365	1.0830	1.2160

Next, conversion factors for the transition from prices of 1966-67 to prices of 1979-80 were calculated from regressions of the form

(A1.2) $$v_{it} = \gamma_i \, y_i + e_{it}, \qquad (i = 1,\ldots, 7)$$

where v_{it} is the value of expenditure on i in prices of 1979-80, and y_{it} is its value in prices of 1966-67, for the seven years (1969-70 through 1975-76) for which we had data. The v-data required aggregation from 16 to 7 commodities. (The key to the latter aggregation is shown in Appendix Table A1.2). The resultant conversion factors were:

$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{\gamma}_5$	$\hat{\gamma}_6$	$\hat{\gamma}_7$
2.7451	2.8717	2.9559	2.3262	3.5694	2.8267	3.4762

Conversion of 1959-60 constant-price expenditure data into prices of 1979-80 was achieved simply by compounding the conversion factors β_i and γ_i. Because of the transition from a maximum disaggregation level of 7 to one fewer commodities at the interface of 1959-60 and 1958-59, we need an additional γ to implement the link for the miscellaneous category of the first sub-period. Since expenditure data are available for 1959-60 through 1967-68 for this category in prices of both 1966-67 and 1979-80, we have no trouble in computing an OLS value of $\hat{\gamma}_{6/7} = 3.2255$. The cumulative factors for conversion from prices of 1959-60 to prices of 1979-80 then are $\alpha_i = \hat{\beta}_i \hat{\gamma}_i$ (i = 1,..., 5) and $\hat{\alpha}_6 = \hat{\beta}_6 \hat{\gamma}_{6/7}$. The primary data identified in Appendix Table A1.1, after the constant-price expenditures for the first

Appendix Table A1.2

Commodity Disaggregation Available in Current and Constant Prices, 1969-70 and Later*

No.	Description	No.	Description
1	Food ^{1}	9.	Other household durables ^{4}
2.	Alcoholic drinks ^{2}	10.	Fares ^{6}
3.	Cigarettes and tobacco ^{2}	11.	Purchase of motor vehicles ^{6}
4.	Clothing, footwear and drapery ^{3}	12.	Operation of motor vehicles ^{6}
5.	Health ^{7}	13.	Postal and telephone service ^{6}
6.	Dwelling rent ^{5}	14.	Entertainment and recreation ^{7}
7.	Gas, electricity, fuel ^{7}	15.	Financial services ^{7}
8.	Household appliances ^{4}	16.	Other goods and services ^{7}

Source: Source IV identified above.

* Aggregation to the seven commodities shown for the second sub-period of Appendix Table A.1 is obtained by simple summation of the constant-price dollar values of the items having the same superscript; e.g., commodities 8 and 9 in data Source IV aggregate to commodity (4) in data sources II and III.

sub-period are scaled by the α_i and the data for the subsequent two sub-periods are scaled by the γ_i, are shown in Appendix Tables A1.3 and A1.4. This completes step (2) of text Section 3.

Step 3 involves the imputation of price indexes by dividing the expenditure data in current prices in Table A1.3 through by the matched value of the corresponding item of expenditure in constant prices from Table A1.4. For example, since t = 5 corresponds to 1957-58, $p_{25} = 826/2986.5 = 0.2766$ is the price index for Tobacco, cigarettes and alcohol in 1957-58. Likewise, the price index for Travel and communication in 1964-65 is 1780/5314.2 = 0.3350, while the price index for Financial services in 1985-86 is 4240/2576 = 1.646.

To implement step 4, we must aggregate these price indexes from 7 commodities to 6 for the sub-period 1959-60 through 1968-69; and from 16 commodities to 6 for the sub-period 1969-70 through 1985-86. Since (the inevitable imperfections of the linking procedure aside) our quantity (i.e., constant-price expenditure) data are in prices of 1979-80, in order to preserve the price x quantity = value identity among our indexes, the appropriate weights for aggregating our price indexes are 1979-80 budget shares. These are shown in Appendix Table A1.5. For convenience, the resultant price indexes at the 6-commodity level (the p_{it} of our econometric model) are shown in Appendix Table A1.6.

In step 5 we identified source IV as the most recently published series for data on expenditures at current prices over the sub-period 1959-60 through 1985-86. For 1953-54 through 1958-59, source I was identified as the most recent available. These data are presented at the 6-commodity level of aggregation in Appendix Table A1.7.

Appendix Tables A1.6 and A1.7 respectively contain our econometric database on prices and on values. In step 6 we obtain quantity indexes by taking the quotient of Appendix Tables A1.7 and A1.6; for example, the value of the quantity index for household durables in 1959-60 is 778/0.4378 = 1777.1 ($ of 1979-80 purchasing power). Notice that although (apart from rounding error) this particular example agrees exactly with the

Appendix Table A1.3

Part A

Current Price Expenditure Data 1953-54 through 1968-69

($ million)

Fiscal Year	Food	Tobacco, cigarettes, alcohol	Clothing, footwear	Household durables	Rent	Travel and communica-tion	All other expenditure
1953-54	1556	613	778	464	421	1979	
55	1633	655	842	501	480	2208	
56	1759	709	871	531	537	2379	
57	1871	795	885	558	596	2548	
58	1904	826	907	620	654	2699	
59	1990	845	930	668	724	2854	
1959-60	2111	905	1020	778	807	1193	1984
1960-61	2251	946	1062	772	926	1243	2124
62	2290	970	1063	758	1027	1256	2255
63	2390	1000	1103	808	1136	1453	2420
64	2510	1056	1199	873	1248	1599	2626
65	2666	1145	1271	962	1359	1780	2961
66	2838	1267	1316	973	1483	1847	3198
67	3026	1351	1389	1021	1647	2020	3501
68	3199	1472	1479	1123	1830	2300	4272
1968-69	3342	1575	1553	1202	2042	2539	4255

Source: See Appendix Table A1.1 for details of sources.

Appendix Table A1.3 (continued)

Part B

Current-Price Expenditure Data 1969-70 through 1985-86

($ million)

Fiscal Year	Commodity Number and Brief Descriptor*															
	(1) Food	(2) Alcohol	(3) Cigar	(4) Cloth	(5) Health	(6) Rent	(7) Gas	(8) House1	(9) House2	(10) Fares	(11) Veh1	(12) Veh2	(13) Communi	(14) Rec	(15) Financ	(16) Other
1969-70	3570	513	1191	1690	1070	2314	454	559	886	596	975	1106	173	663	425	2467
71	3819	559	1321	1830	1240	2680	486	620	971	654	1045	1272	203	738	465	2715
72	4144	608	1429	1986	1449	3071	527	708	1091	699	1150	1418	251	849	511	3028
73	4569	668	1575	2255	1638	3502	558	832	1251	782	1247	1532	288	963	619	3429
74	5393	745	1836	2670	1878	4080	620	1075	1642	907	1420	1764	340	1172	762	4086
1974-75	6213	881	2175	3156	2392	5017	769	1492	1959	1034	1706	2213	433	1495	872	5156
1975-76	7104	1069	2639	3547	2970	6215	917	1890	2332	1267	1827	2777	603	1769	1057	6254
77	8203	1170	2942	3956	3326	7550	1075	2097	2573	1415	2146	3031	697	2005	1253	7107
78	9339	1185	3236	4394	3595	8924	1208	2020	2750	1569	2211	3332	772	2253	1385	7940
79	10585	1345	3653	4756	4169	10362	1397	2055	2946	1766	2504	3874	869	2518	1570	8886
1979-80	12083	1465	4033	5114	4588	11889	1549	2234	3225	2043	2787	4874	971	2847	1792	9853
1980-81	13821	1581	4517	5735	5095	13666	1769	2624	3697	2303	2916	5461	1113	3165	2124	11168
82	15478	1740	5059	6318	6069	16208	2127	2969	4096	2613	3297	6024	1277	3604	2439	12671
83	17160	1886	5540	6871	6971	19578	2574	3263	4248	2914	3746	7076	1461	3963	2871	13994
84	18549	2200	5860	7349	7507	22641	2856	3576	4588	3202	4119	7452	1671	4440	3282	15450
85	19854	2389	6268	7861	7760	26220	3073	3745	4967	3698	4751	7902	1868	4893	3663	17252
1985-86	22399	2705	6857	8712	8536	30891	3354	3969	5548	4044	4870	8233	2116	5460	4240	18894

* For fuller descriptions of commodities, see Appendix Table A1.2.

Appendix Table A1.4

Part A

Constant-Price Expenditure Data 1953-54 through 1968-69

($ million at prices of 1979-80)

Fiscal Year	Food	Tobacco, cigarettes, alcohol	Clothing, footwear	Household durables	Rent	Travel and communica-tion		All other expenditure
1953-54	5581.3	2675.1	2718.8	1282.3	3515.5		9519.2	
55	5792.0	2837.9	2931.7	1380.6	3597.8		10382.1	
56	5936.6	2905.1	3002.7	1428.4	3751.3		10739.0	
57	6090.7	2901.6	2999.4	1453.6	3899.4		10762.5	
58	6288.8	2986.5	2986.5	1589.7	4036.5		11135.1	
59	6424.0	3021.9	3025.2	1723.2	4195.6		11558.7	
1959-60	6783.1	3156.0	3281.0	1777.2	4204.8	3691.7		8600.1
1960-61	6728.2	3239.3	3357.9	1749.3	4329.7	3779.3		8885.2
62	7065.9	3291.0	3322.4	1714.4	4533.1	3864.1		9170.2
63	7400.8	3380.0	3434.8	1849.3	4758.0	4432.3		9709.0
64	7592.9	3540.8	3709.7	2040.1	4975.7	4898.7		10261.7
65	7702.8	3687.3	3875.2	2265.7	5279.1	5314.2		11057.8
66	7952.6	3733.2	3969.8	2282.0	5543.3	5399.0		11551.4
67	8306.7	3879.7	4105.7	2375.1	5878.8	5709.9		12170.2
68	8496.1	4080.7	4274.2	2596.0	6260.7	6331.8		12872.4
1968-69	8773.3	4287.4	4407.2	2737.9	6660.5	6755.8		13706.7

Source: Sources I, II and III as identified above. For methods, see text of this appendix.

Appendix Table A1.4 (continued)
Part B

Constant-Price Expenditure Data 1969-70 through 1985-86

($ million at prices of 1979-80)

Fiscal Year	Commodity Number and Brief Descriptor*															
	(1) Food	(2) Alcohol	(3) Cigar	(4) Cloth	(5) Health	(6) Rent	(7) Gas	(8) House1	(9) House2	(10) Fares	(11) Veh1	(12) Veh2	(13) Communi	(14) Rec	(15) Financ	(16) Other
1969-70	9009	1402	3109	4635	2944	7219	1081	807	2273	1467	2228	3434	437	1822	1064	7450
71	9326	1427	3239	4821	3198	7610	1168	874	2398	1481	2301	3646	482	1897	1109	7540
72	9768	1434	3330	4944	3449	7999	1221	978	2504	1423	2418	3801	518	2045	1138	7770
73	10053	1439	3557	5291	3685	8398	1255	1150	2704	1532	2526	3911	573	2154	1290	8211
74	10102	1452	3835	5518	3774	8822	1338	1433	3187	1693	2706	3933	674	2304	1445	8623
1974-75	10616	1491	3879	5403	3985	9253	1396	1796	3161	1662	2797	3981	721	2478	1369	8776
1975-76	11098	1497	3786	5222	4233	9662	1432	2171	3334	1716	2485	4306	690	2585	1370	9364
77	11431	1519	3956	5037	4217	10146	1517	2324	3302	1775	2617	4358	736	2633	1474	9441
78	11715	1502	4128	5023	4178	10694	1540	2166	3220	1826	2459	4436	807	2693	1550	9485
79	11942	1493	3938	5067	4561	11259	1613	2120	3254	1972	2653	4578	883	2778	1690	9867
1979-80	12083	1465	4033	5114	4588	11889	1549	2234	3225	2043	2787	4874	971	2847	1792	9853
1980-81	12521	1508	4195	5340	4583	12540	1571	2545	3340	1957	2781	4930	1104	2877	1905	10063
82	12948	1554	4291	5515	4937	13233	1627	2752	3438	1998	2853	5038	1186	2941	1974	10412
83	13110	1439	4183	5642	5145	13942	1597	2968	3205	2088	2991	5174	1229	2870	2108	10330
84	13150	1385	4079	5666	5217	14644	1644	3234	3203	2085	3067	5009	1301	2962	2232	10834
85	13395	1365	4050	5725	5043	15418	1668	3481	3306	2321	3325	4947	1393	3095	2360	11447
1985-86	14146	1384	4101	5840	5200	16248	1717	3559	3383	2376	3048	4873	1506	3199	2576	11436

Source: Reference IV identified above.

* For fuller descriptions of commodities, see Appendix Table A1.2.

Appendix Table AI.5

Weights Used for Aggregating Price Indexes

A. 1979-80 Budget Shares, 16 Commodity Level(a)

i =	1	2	3	4	5	6	7	8
	0.1694	0.0205	0.0565	0.0717	0.0643	0.1666	0.0217	0.0313
	9	10	11	12	13	14	15	16
	0.0452	0.0286	0.0391	0.0683	0.0136	0.0399	0.0251	0.1381

B. 1979-80 Budget Shares, 7 Commodities Level(b)

i =	1	2	3	4	5	6	7
	0.1694	0.0771	0.0717	0.0765	0.1666	0.1496	0.2891

(a) For key to commodity classification, see Appendix Table AI.2.

(b) For key to commodity classification, see Appendix Table AI.1. The numbers in Part B can be found by aggregating those in Part A according to the scheme set out in Appendix Table AI.2.

Appendix Table A1.6

Price Indexes Used for Econometric Analysis

(p_{it})

Fiscal Year	Commodity 1 Food	2 Tobacco, cigarettes, alcohol	3 Clothing, footwear	4 Household durables	5 Rent	6 All other expenditure
1953-54	0.2788	0.2291	0.2862	0.3618	0.1198	0.2079
55	0.2819	0.2308	0.2872	0.3629	0.1334	0.2127
56	0.2963	0.2441	0.2901	0.3717	0.1431	0.2215
57	0.3072	0.2740	0.2951	0.3839	0.1528	0.2367
58	0.3028	0.2766	0.3037	0.3900	0.1620	0.2424
59	0.3098	0.2796	0.3074	0.3877	0.1726	0.2469
1959-60	0.3112	0.2868	0.3109	0.4378	0.1919	0.2622
1960-61	0.3346	0.2920	0.3163	0.4413	0.2139	0.2697
62	0.3241	0.2947	0.3199	0.4421	0.2266	0.2729
63	0.3229	0.2959	0.3211	0.4369	0.2388	0.2760
64	0.3306	0.2982	0.3232	0.4279	0.2508	0.2799
65	0.3461	0.3105	0.3280	0.4246	0.2574	0.2907
66	0.3569	0.3394	0.3315	0.4264	0.2675	0.2991
67	0.3643	0.3482	0.3383	0.4299	0.2802	0.3102
68	0.3765	0.3607	0.3460	0.4326	0.2923	0.3426
69	0.3809	0.3674	0.3524	0.4390	0.3066	0.3327
1969-70	0.3963	0.3785	0.3646	0.5137	0.3205	0.3621
1970-71	0.4095	0.4035	0.3796	0.5295	0.3522	0.3868
72	0.4242	0.4278	0.4017	0.5537	0.3839	0.4166
73	0.4545	0.4485	0.4262	0.5694	0.4170	0.4406
74	0.5339	0.4879	0.4839	0.6114	0.4625	0.4886
75	0.5852	0.5687	0.5841	0.7061	0.5422	0.5916
76	0.6401	0.7016	0.6792	0.7695	0.6432	0.6923
77	0.7176	0.7508	0.7854	0.8296	0.7441	0.7682
78	0.7972	0.7853	0.8748	0.8862	0.8345	0.8384
79	0.8864	0.9205	0.9386	0.9315	0.9203	0.9007
1979-80	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1980-81	1.1038	1.0692	1.0740	1.0758	1.0898	1.1057
82	1.1954	1.1632	1.1456	1.1453	1.2248	1.2179
83	1.3089	1.3207	1.2178	1.2329	1.4042	1.3607
84	1.4106	1.4771	1.2970	1.2987	1.5461	1.4575
85	1.4822	1.6016	1.3731	1.3278	1.7006	1.5452
1985-86	1.5834	1.7473	1.4918	1.4252	1.9012	1.6667

Source: Quotient of Appendix Tables A1.3 and A1.4, with result aggregated (where necessary) using weights given in Appendix Table A1.5.

Appendix A1.7

Current-Price Expenditure Data Used for Econometric Analysis

Fiscal Year	Food	Tobacco, cigarettes, alcohol	Clothing, footwear	Household durables	Rent	Other
1953-54	1556	613	778	421	464	1979
55	1633	655	842	480	501	2208
56	1759	709	871	537	531	2379
57	1871	795	885	596	558	2548
58	1904	826	907	654	620	2699
59	1990	845	930	724	668	2854
1959-60	2088	909	1019	778	807	3435
1960-61	2226	949	1062	772	926	3650
62	2265	973	1063	758	1027	3799
63	2364	1004	1103	810	1136	4186
64	2482	1060	1199	878	1248	4600
1964-65	2666	1145	1271	962	1359	5005
1965-66	2838	1266	1316	973	1483	5319
67	3026	1351	1389	1021	1647	5826
68	3199	1472	1479	1121	1830	6480
69	3342	1575	1580	1313	2042	7052
1969-70	3570	1704	1690	1445	2314	7929
1970-71	3819	1880	1830	1591	2680	8818
72	4144	2037	1986	1799	3071	9882
73	4569	2243	2255	2083	3502	11056
74	5393	2581	2670	2717	4080	12949
1974-75	6213	3056	3156	3451	5017	16070
1975-76	7104	3708	3547	4222	6215	19441
77	8203	4112	3956	4670	7550	22055
78	9339	4421	4394	4770	8924	24265
79	10585	4998	4756	5001	10362	27553
1979-80	12083	5498	5114	5459	11889	31304
1980-81	13821	6098	5735	6321	13666	35114
82	15478	6799	6318	7065	16208	40121
83	17160	7426	6871	7511	19578	45570
84	18549	8060	7349	8164	22641	49979
85	19854	8657	7861	8712	26220	54860
1985-86	22399	9562	8712	9517	30891	59747

Source: 1953-54 through 1958-59, Source I.
1959-60 through 1985-86, Source IV.

matching element in Table A1.4 (1777.2), there is no reason in principle why the two numbers should be exactly equal. This is because revisions in the nominal value of an expenditure item (which are reflected in differences among its recorded values in sources I through IV) can imply revisions to the corresponding real quantity.

To complete step 6, we must obtain a suitable population series. Our population data were taken from four publications of the Australian Bureau of Statistics:

A1 Demography 1961, Bulletin No. 79 (Canberra, 1963);

A2 Demography 1967 and 1968, Bulletin No. 85 (Canberra, 1970);

A3 Demography 1971, Bulletin No. 87 (Canberra, 1974) (Reference No. 4.9);

IV Time Series Data on Magnetic Tape and Microfiche September Quarter 1986 (Australian Bureau of Statistics Catalogue No. 1311.0)

All of these sources provide quarterly estimates of population. A1, A2 and A3 form a continuum, and we use them for the March quarter of 1954 through the June quarter of 1971. For the period September quarter 1971 through the June quarter of 1986 we use Source IV. Relative to trend, there is a jump of about 0.3 million (on a base of about 13 million) at the transition between the A3 and IV sources. This represents a new series which (among other things) takes into account census underenumeration (discovered relatively recently to be a problem). Since for our purposes the level of the population is unimportant, but the annual percentage changes in it are crucial, we have converted the IV series to a basis compatible with the A series by multiplying each value of the former by 0.9783. This value was obtained by regressing the values of the A series on the corresponding values of the IV series for four overlapping quarters (Sept. 1971 through June 1972). The quarterly series were then averaged for each fiscal year; the resultant values are given in Appendix Table A1.8.

Appendix Table A1.8

Population Data Used for Econometric Analysis*

Fiscal Year	Population (millions)	Fiscal Year	Population (millions)
1953-54	8.967		
1954-55	9.117		
1955-56	9.342	1970-71	12.686
57	9.560	72	12.935
58	9.768	73	13.140
59	9.975	74	13.344
1959-60	10.191	1974-75	13.540
1960-61	10.430	1975-76	13.681
62	10.669	77	13.825
63	10.872	78	13.990
64	11.086	79	14.142
1964-65	11.306	1979-80	14.307
1965-66	11.527	1980-81	14.516
67	11.727	82	14.756
68	11.936	83	14.973
69	12.175	84	15.152
1969-70	12.433	85	15.339
		1985-86	15.544

*Sources: See text.

Note: The above values are known to underestimate the population level, at least since 1971. See text.

To complete our data handling exercise, the nominal expenditure values in Appendix Table A1.7 were divided by the population values in Appendix Table A1.8. Further division by the elements of Appendix Table A1.6 gives the per capita quantities $\{q_{it}\}$ required for modeling.

Appendix 2

Synoptic Cross-Section Data on Consumption of Rental Services

The 1984 Australian Household Expenditure Survey has recently become available on magnetic tape; our scope here, however, is confined to the synoptic tabulations published in Australian Bureau of Statistics (ABS) (1986a). The following recorded items are relevant to housing: (i) Current Housing Costs (selected dwelling); (ii) Mortgage payments - Principal (selected dwelling) and (iii) Other Capital Housing Costs. The first of these is an aggregate of nine one-line items, and the last an aggregate of ten such items.

The most useful of the synoptic tables, from our point of view, is Table 2 of ABS (1986a), which gives household averages of expenditure on several hundred items separately for each decile of the household distribution of cash income. The latter concept, unlike the time-series national accounts data described in Appendix 1, does not contain any imputation for the services of the owner-occupied housing stock. The parts of the ABS tabulation relevant to the present discussion are reproduced in Appendix Table A2.1.

Current Housing Costs (101-109) include the interest portion of mortgage repayments for those households in the process of purchasing a dwelling; however 39.42 percent of all households occupied premises which were owned outright by the occupiers. A crude imputation of the mortgage interest notionally accruing to this occupier-owned stock can be computed, for each decile, by taking the ratio of the percentage of houses owned outright to the corresponding percentage of houses being bought, and multiplying this quotient by item 102, Mortgage payments--interest payments.

Two other adjustments were made in the hope of bringing the conceptual basis of these data closer to that of the time series. First, the flow represented by depreciation of the owner-occupied housing stock conceptually is part of consumption. No data on this item are available other than items 106-107 (the total of which seems too low); however, the capital item 756 Internal renovations was added to total expenditure to make some additional allowance for this depreciation. Second, as well as imputed mortgage payments

Appendix Table A2.1

Selected Items from the 1984 Australian Household Expenditure Survey

			Weekly Household Gross Income Distribution ($) (b)										
	Household Characteristics		*Lowest 10%*	*Second decile*	*Third decile*	*Fourth decile*	*Fifth decile*	*Sixth decile*	*Seventh decile*	*Eighth decile*	*Ninth decile*	*Highest 10%*	*All households*
	Average weekly household income ($) (b)		*85.16*	*147.03*	*198.80*	*278.15*	*349.73*	*427.99*	*517.70*	*619.01*	*757.19*	*1156.80*	*453.60*
	Average age of household head (years)		*62.40*	*57.08*	*53.71*	*45.39*	*42.33*	*41.75*	*40.62*	*41.25*	*41.97*	*44.01*	*47.05*
	Average number of persons per household:												
	Total	No.	1.30	2.04	2.61	2.74	3.02	3.10	3.18	3.21	3.33	3.82	2.84
	Proportion of households with nature of housing occupancy being:												
	Owned outright	%	58.90	55.08	54.58	38.23	30.79	30.37	27.63	31.25	32.32	35.02	39.42
	Being bought	%	7.48	10.96	15.02	26.59	36.56	39.34	44.47	44.79	48.86	46.76	32.08
	Renting-government	%	9.83	11.36	8.46	5.94	7.01	4.70	2.91	3.33	2.67	.90	5.71
	Renting-private	%	16.61	19.12	17.56	25.28	22.76	22.87	21.92	18.99	14.00	15.72	19.49
	Occupied rent free	%	7.18	3.47	4.38	3.97	2.88	2.72	3.07	1.65	2.15	1.60	3.31
	Total	%	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Proportion of households with family composition of the household being:												
	Single person household	%	84.48	23.29	11.45	22.37	18.43	13.15	7.71	6.24	2.51	1.61	19.08
	Number of households in the sample		*886*	*886*	*890*	*921*	*953*	*980*	*1,021*	*996*	*1,010*	*1,028*	*9,571*
	Estimated total number in the population:												
	Households	('000)	500.5	505.0	506.6	506.5	500.2	507.0	501.7	506.0	502.9	502.8	5,039.2
	Persons	('000)	651.7	1,031.6	1,321.8	1,388.6	1,511.3	1,570.0	1,597.7	1,625.0	1,672.9	1,920.3	14,290.8
			Average weekly household expenditure ($)										
	Current Housing Costs (selected dwelling)												
101	*Rent Payments*		*8.83*	*12.53*	*14.34*	*17.22*	*18.04*	*17.23*	*16.56*	*15.06*	*12.70*	*14.64*	*14.72*
102	*Mortgage Payments - interest component*		*2.44*	*2.59*	*4.15*	*10.02*	*16.03*	*17.99*	*21.20*	*22.34*	*26.56*	*26.43*	*14.97*
103	Water and sewerage rates		1.93	2.09	2.43	2.62	3.04	3.09	3.39	3.66	4.18	4.92	3.14
104	General rates		2.60	2.84	3.00	3.59	3.82	3.94	4.34	4.71	5.07	5.97	3.99
103-104	*Rate Payments*		*4.54*	*4.93*	*5.43*	*6.20*	*6.87*	*7.03*	*7.72*	*8.37*	*9.24*	*10.89*	*7.12*
105	*House and Contents Insurance*		*1.80*	*2.03*	*2.28*	*2.14*	*2.40*	*2.76*	*2.97*	*3.20*	*3.62*	*4.55*	*2.77*
106	Repairs and maintenance - payments to contractors (materials and labour)		* 3.80	* 2.66	* 1.06	* 2.79	* 1.48	* 3.01	* 1.36	* 2.23	* 2.72	* 6.19	2.73
107	Repairs and maintenance - materials only		2.00	2.55	1.97	1.95	2.46	3.70	3.30	3.11	5.16	4.70	3.09
106-107	*Repairs and Maintenance Payments*		*5.80*	*5.21*	*3.03*	*4.74*	*3.94*	*6.71*	*4.66*	*5.34*	*7.88*	*10.89*	*5.82*
108	Interest payments on loans for alterations and additions		.16	.13	.12	.29	.82	.69	.95	1.48	1.23	2.69	.56
109	Body corporate payments		*	*	*	*	*	*	*	*	*	*	.20
108-109	*Other Current Housing Costs*		*.79*	*.34*	*.17*	*.67*	*.82*	*.69*	*1.20*	*1.62*	*1.56*	*2.74*	*1.06*
101-109	**Total Current Housing Costs (selected dwelling)**		**24.19**	**27.64**	**29.40**	**41.00**	**48.10**	**52.41**	**54.31**	**55.94**	**61.56**	**70.14**	**46.46**

... continued

Appendix Table A2.1 (continued)

752	Mortgage Payments - Principal (selected dwelling)	2.01	1.52	2.19	4.44	6.44	7.02	8.43	9.17	10.76	12.90	6.49
Other Capital Housing Costs												
753	Principal component of mortgage payment for other property	.00	.06	.09	.27	.39	.43	1.38	1.27	1.38	1.84	.71
754	Purchase of selected dwelling or other property (excl. mortgage payments)	*	*	*	*	*	*	*	*	*	*	*
755	Additions and extensions	* 1.89	* .63	* .97	* 2.24	* 4.00	* 4.63	* 3.27	* 6.08	* 6.33	* 16.67	4.67
756	Internal renovations	1.61	1.68	2.56	2.53	4.51	3.33	4.06	4.91	5.75	6.90	3.78
757	Insulation	.09	.10	.12	.15	.24	.36	.27	.23	.28	.27	.21
758	Inground swimming pool	* .13	* .18	* .15	* .21	* .18	* .82	* .76	* 1.82	* 1.68	* 2.03	.80
759	Outside building	.50	.97	.99	1.13	1.98	1.70	1.86	2.46	4.03	4.01	1.96
760	Landscape contractor	* .02	* .03	* .21	* .02	* .13	* .15	* .25	* .37	* .50	* .99	.27
761	Outside improvements, n.e.c.	.72	.87	1.15	1.16	1.49	2.00	2.24	3.61	3.00	3.99	2.02
762	Capital housing costs, n.e.c.	*	*	*	*	*	*	*	*	*	*	* 1.41
753-762	Total Other Capital Housing Costs	(13.29)	1.27	7.75	7.53	15.83	17.48	18.40	24.36	27.18	47.15	18.01
Superannuation and Life Insurance												
771	Superannuation and annuities	.26	.09	.65	2.78	6.02	7.94	10.82	12.88	16.81	22.25	8.05
101-735	TOTAL COMMODITY OR SERVICE EXPENDITURE	140.79	187.70	239.54	284.92	326.35	369.12	412.78	443.61	527.12	686.96	361.84

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* An asterisk preceding an entry in the table indicates that the estimated sampling standard error is between 30 and 50 percent of the estimated value. Where this percentage exceeds 50, the estimated value is suppressed and is replaced by an asterisk.

Source: Australian Bureau of Statistics (1986a).

and the last-mentioned depreciation item 711, Superannuation and annunities contributions (i.e., the payment of subscriptions to retirement income plans) were added to the subtotal for items 101-735 ("Total Commodity or Service Expenditure") to obtain our grand total estimate of consumption expenditure. These adjustments, together with the imputation of mortgage interest, are shown in Appendix Table A2.2.

It will be noted that the data in rows e and g of Appendix Table A2.2 yield a budget share for Rent (i.e., 'Total Housing') of 65.33/394.80, or 16.5 percent; the value in the time series data base is 20 percent. It is a matter of simple algebra to solve for a multiplier λ for row b such that if 'Total Housing' is defined as

$$(\text{row a}) + \lambda\,(\text{row b}) + \text{row c},$$

and if (row b) is replaced by λ (row b) in computing Total Expenditure, the resulting budget share is 20 percent. λ turns out to have the value 2.11. Whilst such a magnitude might be amenable to rationalization in terms of the historical and institutional features of the mortgage market in 1984, it does seem rather high.

Appendix Table A2.2

Imputation of Mortgage Interest and Other Data Adjustments

($ per household per week)

Item	Decile of the Income Distribution										All Households
	1	2	3	4	5	6	7	8	9	10	
a. 101-109 Total Current Housing Costs	24.19	27.64	29.40	41.00	48.10	52.41	54.31	55.94	61.56	70.14	46.46
b. Imputed Housing Interest	19.21	13.02	15.08	14.41	13.50	13.89	13.17	15.59	17.57	19.79	18.40
c. 756 Internal Renovations	1.61	1.68	2.56	2.53	4.51	3.33	4.06	4.91	5.75	6.90	3.78
d. Total Housing (a + b + c)	45.01	42.34	47.04	57.34	66.11	69.63	71.54	76.44	84.88	96.83	65.33
e. 101-735 Total Commodity or Expenditure Service	140.79	187.70	239.54	284.92	326.35	369.12	412.78	443.61	527.12	686.96	362.66
f. Superannuation & Annuities	0.26	0.09	0.65	2.78	6.02	7.94	10.82	12.88	16.81	22.25	8.05
g. Total Expenditure*	162.62	203.25	259.14	306.82	353.49	397.93	445.53	481.38	572.91	744.05	394.80
h. Alternative Imputation of d	66.39	56.83	63.83	73.98	81.14	85.09	86.20	93.79	104.44	118.86	83.48
i. Alternative Imputation of f	184.00	217.74	275.93	322.86	368.52	513.39	460.19	498.73	592.47	766.08	416.89

* Sum of b, c, c and f.

Note: The data for the first pair of regressions in Table 6.1 are obtained from rows d and g above; the data for the second pair of regressions in that Table come from lines h and i.