The Commonwealth Government.

The opinions of the participating agencies, not of
The views expressed in this paper do not necessarily reflect


University of Melbourne
IMPACT Research Centre

Alan A. Powell

and

Nisha Agarwal

By

SOME EARLY ESTIMATES

MADs Under Additive Preferences:

Project IMPACT

IMPACT Project

153 Emu Street, Carlton, Victoria 3053, Australia
Impact Research Centre, The University of Melbourne.
In terms of the average elasticity of substitution, the parameter is estimated (along the lines suggested by Patinkin) to yield a CCE model. The system is successful in explaining over the period 1953-63 through 1965-66. The F to a live consumer model approach of Australian consumption yields a range of variation in real income. The system is successful with parameters in the use of parameters of regular over a very long period. (Houthakker (1960)) yield a demand system with preference (Houthakker (1960) and from additive rotation analysis (e.g., Thiel (1967)) and from additive rotation analysis (e.g., Thiel (1967)).)

MAIDS in different utilitarian forms with synthesized ideas from the Mincer-Warner model, and Mincer's MAIDS demand system (1967).

Variables, cooperative and competitive. MAIDS demand system (1967). General equilibrium (GCE) modeling often needs demand for labor at large values of real income. Computable general equilibrium (CGE) models are more applicable, however. Since budget shares are driven outside the model, substantial variations in real income, these budget curves cannot be well approximated. (1943) yields budget curves which are a

ABSTRACT
that individual shares responsibility. The in the unit: interior, that is, that
W!'s probability and access, to utility; they do not. However, guarantee
Cermonis (1987). These approaches ensure that the budget shares
Demands, (1967) and the model: program (1969) and in the parameters
(1969) and by Theil and expenditure. This insight is based on the
consumption budget is driven in the integration of real world
consumption, that the share W's of a commodity (e.g., ...). In the
workings law (1943) postulates as an empirical
Introduction
impact, research center, University of Melbourne
M. S. M. Powell and Alan W. Powell
Some early estimates
MADs under additive preferences
Cambridge: Massachusetts: Ballinger
T. E. (1967) "Economical and Information Theory
Economic Journal 64 (255): 551-557 (September)
Stanley Richard (1959) "Linear Expenditure Systems and Demand
Economic (801): 102-124 (January-February)
D. R. (1972) "Additive Utility Functions with Double Log
L. P. L. (1977) "Utility Functions with Double Log...
$0 \leq W_i \leq 1$ for all $i$ at any positive value of real total expenditure. Indeed, this is one reason why AIDS is only almost ideal.

Recent work by Cooper and McLaren (1987, 1988) offers some generalizations of AIDS which do not suffer from the above defect. These are examples of the fractional demand systems first explored by Lewbel (1987). The Modified AIDS, or MAIDS, system of Cooper and McLaren

"allows the imposition of all regularity conditions [i.e., those listed below] over an extensive region of expenditure-price space..." (Cooper and McLaren (1988), p. 3).

The regularity conditions in question require that the indirect utility function be:

(i) homogeneous of degree zero in nominal total consumption expenditure and in the vector of commodity prices;

(ii) non-decreasing in nominal total consumption expenditure (at fixed prices);

(iii) non-increasing in every price argument (at a fixed value of consumption);


Frisch, Ragnar (1959) "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in A Model with Many Sectors". Econometria 27: 177-96.


we offer conducting remaining a percentage of quality research. In Section 5 of the econometric estimates obtained by full-information methods are reported. In Section 4 the econometric estimates obtained by full-information methods are reported. In Section 3 we give some further background material on the data. In Section 2 we describe the version of WAIDS which we use. In the remainder of this paper is structured as follows.

Section 1 allows us to avoid estimating one of the variables of interest in the model. This estimated in levels difference form (rather than the levels). The model is estimated to just one of the factors parameters. Also, the model is estimated to the number of substitution parameters to be estimated without the number of additional parameters down. We invoke additively preferences, which is clear from Roy's identity that satisfaction of (4) and (5) levels.

REFERENCES

2. A Differential Form of MAIDS Under Additive Preferences

2.1 The MAIDS System

MAIDS (Cooper and McLaren (1988), p. 8) is generated from an underlying cost or total expenditure function \( M(u,P) \) which satisfies:

\[
(2.1) \quad \ln M(u,P) = \ln \Pi_1 + u \Pi_2 / M(u,P)
\]

In which \( M \) is the household's total spending on commodities; \( u \) is the utility level attained; \( P \) is the \( n \)-vector of commodity prices; and \( \Pi_1 \) and \( \Pi_2 \) are arbitrary concave functions, both homogeneous of first degree in \( P \). The indirect utility function dual to (2.1) is:

\[
(2.2) \quad U(M,P) = [\ln(M/\Pi_1)] [M/\Pi_2]
\]

As Cooper and McLaren point out, property (i) listed above is guaranteed by the first-degree homogeneity of \( \Pi_1 \) and \( \Pi_2 \), while (ii) and (iii) will be satisfied provided:

\[
(2.3) \quad M \geq \Pi_1
\]

(In our work below, we will insist that (2.3) be satisfied.) Moreover, if:

\[
(2.4) \quad \Pi_1, \Pi_2 \text{ are concave in } P
\]

\[
\sum_{l=1}^{n} \frac{W_l(1 - W_l)}{(1 - P_lW_l)}
\]

in most examples the sum is likely to be close to 1.

Ex post one could compute

\[
(A4) \quad \sigma = \phi / \sum_{l=1}^{n} \frac{W_l(1 - W_l)}{(1 - P_lW_l)}
\]

at some set of co-ordinates, and report this estimate as “the average substitution elasticity” at these coordinates. The weights implicit in this definition would be:

\[
(A5) \quad N_{ij} = \frac{W_iW_j}{(1 - E_iW_j)} / \sum_{k=1}^{n} \frac{W_k(1 - W_k)}{(1 - E_kW_k)}
\]

so that

\[
(A6) \quad \sigma = \sum_{i=1}^{n} \sum_{j \neq i} N_{ij} \sigma_{ij}
\]
Thus far, we have not established the upper and lower bounds for

\[ t = \frac{M^{1/2} - 1}{2} \sum \frac{1}{u} \quad (2.7) \]

Notice that the first degree homogeneity of \( \frac{1}{u} \) and \( \frac{1}{u^2} \) implies:

\[ \left( \frac{1}{u} \right)^2 \text{ in } \frac{1}{u} \text{ and } \frac{1}{u^2} \text{ are the following expressions:} \]

where the \( \epsilon \) and \( \eta \) are the following expressions:

\[ \frac{1}{(1/\epsilon)(M - 1)} \sum \frac{1}{u} \eta \]

\[ \frac{1}{(1/\epsilon)(M - 1)} \sum \frac{1}{u^2} \eta \]

Consider the following two (arbitrary) examples (with \( n = 3 \)):

\[ \frac{1}{(1/\epsilon)(M - 1)} \sum \frac{1}{u} \quad (2.8) \]

\[ \frac{1}{(1/\epsilon)(M - 1)} \sum \frac{1}{u^2} \]

Thus, the following two equations are obtained from the homogeneity of \( \frac{1}{u} \) and \( \frac{1}{u^2} \) (e.g., the property above also holds when over the region defined by \( \epsilon \)).
total expenditure by the representative household. We make the following four assumptions:

\[(2.8a)\] Apart from scaling, \( \Pi_1 \) is the implicit price deflator for consumption. Its use to deflate nominal per capita consumption expenditure produces an index \( Q \) of real consumption expenditure per head; thus

\[ Q = M / (\Pi_1) \, \text{, where} \, \Pi_1 = \Pi_1^0 e^\alpha \]

in which \( \alpha \) is a scaling parameter and \( M, \Pi_1^0 \), and \( Q \), respectively, are nominal per capita consumption, the implicit price deflator and (a resealed version of) per capita consumption in constant prices. (The raw data on real per capita consumption below will be written \( Q' \).)

\[(2.8b)\] \( \Pi_2 = \gamma \prod_{i=1}^{n} p_i^{e_{i1}} \) \, [i.e., \( \Pi_2 \) is Cobb-Douglas]:

\[(2.8c)\] the direct utility function is directly additive; and

\[(2.8d)\] the Frisch parameter \( \phi \) is an absolute constant.

The last of these assumptions is almost tantamount to assuming that the average elasticity of substitution (over different pairs of

\[ = \frac{-\phi E_i}{(1 - W_j)} \sum_{j \neq i} W_j E_j \]

\[ (A3) \]

\[ = \frac{-\phi E_i}{(1 - W_j)} (1 - W_i E_j) \, . \]

[Notice that the weights \( W_j / (1 - W_j) \) in the first line of (A3) do sum to unity over \( i \neq j \).] We would like to show that a weighted average of the \( S^i \) is equal to \( \phi \). We have not been able to do this. However, consider the following positive linear combination of the \( S^i \):

\[ (A4) \]

\[ \sum_{i=1}^{n} \frac{W_i (1 - W_j)}{(1 - E_i W_j)} S^i \]

\[ = \sum_{i=1}^{n} \sum_{j \neq i} \frac{W_i W_j}{(1 - E_i W_j)} \sigma_{ij} \]

\[ = \sum_{i=1}^{n} \frac{W_i}{(1 - E_i W_j)} \sum_{j \neq i} W_j \sigma_{ij} \]

[from (A1) and (A2)]

\[ = -\phi \sum_{i=1}^{n} E_i W_i = -\phi \, . \]
Income effects. (2.11) becomes:

\[ \frac{d}{d\theta} \frac{\partial \pi}{\partial \theta} \bigg|_{\theta = \bar{\theta}} \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} = 1 \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} \]

Now

\[ \frac{d}{d\theta} \bigg|_{\theta = \bar{\theta}} \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} = 1 \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} \]

The share-weighted average of substitution elasticities over all parts

\[ \sum_{u} \frac{1}{u} = 1 \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} \]

where "l" is the total expenditure elasticity of l, and that

\[ \sum_{f} \frac{1}{f} \frac{d}{df} \phi = 0 \]

follows. First, we note that

Theorem (1977)." We note that the change in the u budget share is:

and following the differential approach in demand analysis (c.e.

case letters. Next, using the equation of the quantity of a commodity,
differential of the variables desired by the corresponding upper
where in (2.9) and from now on lower case letters indicate log.

\[ \frac{d}{d\theta} \bigg|_{\theta = \bar{\theta}} \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} = 1 \bigg( \frac{f}{f} \bigg) \sum_{u} \frac{1}{u} \]

Differential is:

We note that the definition of \( \frac{d}{d\theta} \) implies that its log

process 2. Commodities \( \phi \) does not vary in response to changes in income and

Appendix. The SATO Insight.
\[(2.12) \quad q_t = \sum_{j=1}^{n} \eta_{ij}^{\text{subst}} \cdot W_j E_j p_j + E_t m \quad .\]

The term on the right of (2.12) carrying the superscript "subst" is the utility-compensated derivative of \(\ln Q_i\) with respect to \(\ln P_j\); while \(E_t\) is the \(t\)th total expenditure elasticity \(\partial \ln Q_t / \partial \ln M\). Using (2.9) and substituting from (2.12) into (2.10), we obtain:

\[(2.13) \quad w_t = (p_t - \pi_t) + \sum_{j=1}^{n} \eta_{ij}^{\text{subst}} \cdot W_j E_j p_j \quad .\]

\[\quad + (\pi_t + q)E_t - q \quad .\]

Under directly additive preferences, the substitution term may be written (Theil (1967), pp. 197-198):

\[(2.14) \quad \sum_{j=1}^{n} \eta_{ij}^{\text{subst}} p_j = \phi E_i (p_t - \sum_{j=1}^{n} W_j E_j p_j) \quad .\]

where \(\phi\) is the reciprocal of the elasticity of the marginal utility of total expenditure with respect to total expenditure (the Frisch 'parameter'). Under MAIDS, we find from (2.5) that the total expenditure elasticities \(E_t\) are:

\[(2.15) \quad E_t = (W_t R + \varepsilon_0) / [(1 + R) W_t] \quad .\]
The three right hand terms of (2.18) may be interpreted as follows:

\[ I = 1 \]

\[ b \left( \frac{I}{I + 1} \right) \frac{M}{k^2 + k^2 M} + \left\{ \int_0^1 \varphi \varphi \left( \frac{I}{I + 1} \right) \frac{M}{k^2 + k^2 M} \right\} - \]

\[ \left\{ \int_0^1 \varphi \varphi \left( \frac{I}{I + 1} \right) \frac{M}{k^2 + k^2 M} \right\} = b \]  

(2.18)

This form of the relationship (15-18) between \( \phi \) and \( \theta \) should be investigated. The strength of the estimated empirical function should be allowed to vary with real income and \( \phi \), i.e., \( \phi \) and \( \theta \). (1977)  

Pownell and Williams' comparative work by Judge, Powell, and Williams'  

comparisons with international  

desirable to reexamine the French (1959)  

In this world where budget curves are both more  

flexible than in Workshop's model and better  

interrelated with demand theory, it would be  

in the world wide available from 1969-70 on.  

Some further dislunctuation of commodities,  

service as predetermined.  

endogenous variable and the low of rental  

Powell (1985), who keep the rental price an  

show how these disequilibrated by demand, chins and  

Resolving Rent to the overall system, probably  

As far as the current thread of time-series work goes.
(presumably small) between movements in the implicit price deflator $\Pi_j$ and in the Divisia index $P_{Div}$ defined by:

$$
(2.19) \quad d \ln P_{Div} = \sum_{j=1}^{n} W_j d \ln P_j = \sum_{j=1}^{n} W_j p_j.
$$

Notice that the system (2.18) is not subject to an exact linear constraint of the type that usually leads, in demand systems, to the dropping of one equation before estimation. For suppose that we add an explicit error term $u_t$ to the right of (2.18), then multiply each side by the budget share $W_t$ and sum over all commodities. The resultant equation, after restoring time subscripts, is:

$$
(2.20) \quad q_{Div}^t = \pi_{It} \cdot P_{Div}^t + q_t + \sum_{i=1}^{n} W_{ii} u_{it}
$$

in which $q_{Div}^t$ and $P_{Div}^t$ are the Divisia quantity and price indexes, respectively. It is usual at this point in the analysis of a demand system to find that all variables in the analogue of (2.20) are predetermined except for the sum $\sum_{i=1}^{n} W_{ii} u_{it}$: it is then concluded that this sum must similarly be predetermined (and usually with a value of zero); and so, that the rank of the covariance matrix of the $u$'s can be at most $(n-1)$. Equation (2.20) differs from the standard case, however, because the Divisia quantity index $q_{Div}^t$ commodities) from time-series data. This required the maintenance of strong assumptions; given the quality of the data, however, it is doubtful whether a more demanding informational load could reasonably be put on them.

Many areas are ripe for further explorations with MAIDS. With richer time-series data sources (especially panel data) it may prove possible to go beyond the Cobb-Douglas specification for $\Pi_2$ and to adopt an explicit functional form for $\Pi_1$. However, with cross-sectional data, the lack of price variation and/or availability of price data is likely to mean that the framework adopted in this paper (or some variant of it) is optimal, at least in the case of broad commodity groupings. In particular, the variation of consumption patterns across households is likely to enable the $\varepsilon_{2i}$ elasticities to be estimated, since these magnitudes hold the key to the responsiveness of commodity demand to changes in income (see (4.4)). To be sure, an extraneous estimate of the Frisch parameter will be needed, but as we have seen above, such estimates are easily obtained from time-series estimation.

In the cross-section work planned for the 1984 Australian Household Expenditure Survey there is scope to reformulate the $\varepsilon_{2i} s$ as functions of demographic variables. Approaches to this kind of problem are given in the pioneering work of Barten (1964), in Jorgenson, Lau and Stoker (1982) and Jorgenson, Slesnick and Stoker (1983). A particularly promising approach is the recent work of Chung (1987).
I} is not such that \( d \in \mathcal{D} \) has the Division form.

(2.10)

assumption: variable \( \mathcal{D} \).

Provided we make the following additional

Note: however, that any system of equations does not contain the

\[ \eta \in \mathcal{D} \]

(2.11)

systematic part of the Division quantity index.

Comparing (2.20) with (2.22), we find the following identity for the

\[ \sum_{i=1}^{n} \eta_{i} w_{i} + \sum_{i=1}^{n} \eta_{i} b_{i} = \sum_{i=1}^{n} \eta_{i} a_{i} \]

(2.22)

Finally, we note that the Frechet parameter once again

For the systematic increased by \( 2 \% \) for each

between these two points of time, recall the exponential per head

between \( r \) and \( Y \) are not great at the sample midpoint.

This differential behaviour of the two systems is quite

\[ \text{as difference increases, so does correlation.} \]

converted to (2.8), as a result, \( Y \) declines by

as local real expenditure grows without limit, so does \( Y \), while \( W \)

\[ (L \times (-1/2) + L / \eta_{2} + L) = \]

(4.1)
### Table 4.2

<table>
<thead>
<tr>
<th>Year</th>
<th>Tobacco, cigarettes, alcohol</th>
<th>Clothing, footwear</th>
<th>Household durables</th>
<th>Other (non-rent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>2.94</td>
<td>3.16</td>
<td>1.70</td>
<td>1.65</td>
</tr>
<tr>
<td>1970</td>
<td>1.27</td>
<td>1.37</td>
<td>1.54</td>
<td>1.29</td>
</tr>
<tr>
<td>1980</td>
<td>0.92</td>
<td>0.86</td>
<td>0.79</td>
<td>0.90</td>
</tr>
</tbody>
</table>

#### Notes:

(a) 'Cash' expenditure refers to consumption expenditure on home excluding housing. (A very large component of 'Rent' in the national accounts data is imputed.)

(b) Values shown in square parentheses are comparable values from Chung and Powell (1987), Table 8.1, for 1969-70 and 1968-69, respectively.

---

**2.3 Discrete-time Analogue**

The discrete-time analogues of the variables appearing in (2.18) selected for empirical work were as follows:

- \( q_t = \ln(\frac{Q_t}{Q_{t-1}}) \)
- \( W_t = \frac{1}{2} \left( W_{t-1} + W_t \right) \)
- \( \pi_t = \frac{1}{1 + \frac{1}{2} \ln \left( \frac{Q_t}{Q_{t-1}} \right) } \)
- \( \theta = \frac{\ln \left( \frac{Q_t}{Q_{t-1}} \right) }{\ln \left( \frac{Q_t}{Q_{t-1}} \right) - \alpha} \)

**2.24d**

- \( q_t = \ln \left( \frac{Q_t}{Q_{t-1}} \right) \)
- \( \theta = \frac{\ln \left( \frac{Q_t}{Q_{t-1}} \right) }{\ln \left( \frac{Q_t}{Q_{t-1}} \right) - \alpha} \)
- \( \pi_t = \frac{1}{1 + \frac{1}{2} \ln \left( \frac{Q_t}{Q_{t-1}} \right) } \)
- \( W_t = \frac{1}{2} \left( W_{t-1} + W_t \right) \)
In MAIDS the total expenditure elasticities take the form:

\[ f^w_x = \begin{cases} 1 + \frac{\beta_q}{\gamma_q} & \text{if } q > \gamma_q \vspace{1em} \\
1 & \text{otherwise} \end{cases} \]

We compare these elasticities with those obtained by Chinn and Powell (1987). Under the assumption of constant returns to scale, we compare our estimated elasticities at the middle and point of the sample with the estimated elasticities at the middle of MAIDS. In Table 4.3 we report only the expenditure elasticities in the bottom (compare with the estimated industrial expenditure elasticities).
these variables published in the same issue of the same publication. (This is required because substantial revisions of these data are made over time. Mismatched series would produce spurious apparent price variations.) (C) Where the relevant price information can be inferred from more than one matched pair of series, the most recently published matched data are used. (D) Series requiring linking are spliced using an OLS regression through the origin which utilizes all the available overlapping observations. (E) The current-price expenditure data in the final data base is taken from the most recently published statistics. (F) Where accuracy in the value of the level of a variable, and accuracy in its percentage change over time, become competing goals, the latter objective is given precedence (after all, our model is in log changes).

Consistent with these principles it proved possible to obtain a relatively long (33-year) sample on per capita expenditures in current and in constant prices for the following six-commodity split-up of consumption expenditures:

1. Food
2. Tobacco, cigarettes, alcoholic drinks
3. Clothing, footwear
4. Household durables
5. Rent
6. All other expenditures

A fitted system represented by the last row of Table 4.1 will not cause any regularity problems in CGE simulations involving real total per capita expenditures (excluding rent) exceeding $1,197.88 1461 per annum. Other versions of MAIDS (for example, generalized MAIDS (GMAIDS) -- Cooper and McLaren (1988)) may exhibit regularity over a wider range of variation in Q.

Next, we should note that the serial properties of the residuals from our fitted equations are far from ideal. Using the Durbin-Watson statistic (DW) as a descriptive device, our equations yielded:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Food</td>
<td>1.24</td>
</tr>
<tr>
<td>2. Tobac., cigs, alcohol</td>
<td>1.09</td>
</tr>
<tr>
<td>3. Clothing and footwear</td>
<td>1.09</td>
</tr>
<tr>
<td>4. Household durables</td>
<td>0.98</td>
</tr>
<tr>
<td>5. Other (excluding Rent)</td>
<td>1.08</td>
</tr>
</tbody>
</table>

These results (not surprisingly) are nevertheless much less pathological than those obtained for a version of generalized MAIDS (GMAIDS) fitted in the levels to much the same data by Cooper and McLaren (1987, Table 2), where the DWs had the following values:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Food</td>
<td>0.58</td>
</tr>
<tr>
<td>2. Tobac., cigs, alcohol</td>
<td>0.15</td>
</tr>
<tr>
<td>3. Clothing and footwear</td>
<td>0.36</td>
</tr>
</tbody>
</table>
with the remaining elasticity to be recovered from property (2.17). The scaling parameter α is estimated from the elasticities $e_1, e_2, e_3, e_4, e_5, e_6$. The price parameter $\phi$ is estimated from the demand system (2.18).

Section 3 (but with rent removed). The parameters to be estimated are absolute values of the coefficients in which they are linked above in $R$ and $L$, but are indicated by the order in which they are held above in $T$ (version 4.1b on a VAX computer). The command queue 1 through 5 was estimated by full-information-maximum-likelihood (FIML) using the five commodity expenditure system (2.18). We found the estimation of the parameters to be stable.

4. Economic Estimates

China and Powell (1989) estimated the coefficients of primary sources and tabulations of the data, see Annex for further details of the manipulation performed. Also, the computation of consumer prices and expenditure was performed in that way that a large part of the expenditure includes the price of the commodity included in the estimation, whereas the price of the commodity included in the estimation is not included in the price of the commodity excluded. Because the data are very sparse, the estimation is based on a limited number of observations, whereas in the case of the price, the sample is larger and the data are more reliable.
Some difficulty was experienced in locating suitable initial values. To find a good starting point (i.e., one from which convergence could be achieved), we proceeded in four stages:

1. Treating the values of \( \phi \) and \( \varepsilon_{21}, \ldots, \varepsilon_{24} \) as 'known', we estimated \( \alpha \), using \( \alpha = 7 \) as initial value.

2. Using the estimate of \( \alpha \) so obtained and the same 'known' values of \( \varepsilon_{21}, \ldots, \varepsilon_{24} \) as starting values, we reestimated \( \alpha \) and simultaneously estimated \( \phi \).

3. Taking the estimated values of \( \phi \) and \( \alpha \) from the last round as starting values, we estimated simultaneously all of the parameters listed above. This led to a negative estimate of \( \varepsilon_{21} \).

4. The value of \( \varepsilon_{21} \) was set to zero and the remaining parameters reestimated. (A likelihood ratio test revealed that \( \varepsilon_{21} \) did not differ significantly from zero, the values of the log likelihood function with and without the constraint being 492.031 and 492.141, respectively).

Where did the 'known' values of \( \alpha \) and the \( \varepsilon \)'s come from? We took \( \alpha = 7 \). This value of \( \alpha \) was chosen in the light of the requirement (2.3). Using (2.8a), we see that (2.3) implies that:

\[
(4.1) \quad Q^o e^{\alpha} > 1 ,
\]

where \( Q^o \) is unscaled real per capita expenditure (i.e., \( Q^o = M/N^o_1 \)); (4.1) in turn implies that:

\[
(4.2) \quad Q^o_{min} > e^{\alpha} ,
\]

where \( Q^o_{min} \) is the smallest value of total real per capita consumption expenditure (excluding Rent) for which the model is expected to 'work' (in the sense of staying in a regular region). Then we could take \( Q^o_{min} \) equal to the smallest sample value of \( Q^o \) (namely, the value in 1953-54, which was 2.428 in 1979-80 dollars). From (4.2) this yields \( \alpha = 7.795 \) as a maximum value; in practice we used the smaller value \( \alpha = 7.00 \) (corresponding to a \( Q^o_{min} \) of \$1979-80 1096.6).

The 'known' values of \( \varepsilon_{21}, \ldots, \varepsilon_{24} \) were calculated from Chung and Powell's (1987) estimates of Working's Law from the same data. These values were:

\[
\varepsilon_{21} = 0.0214 \quad \varepsilon_{22} = 0.0455 \quad \varepsilon_{23} = 0.0528 \quad \varepsilon_{24} = 0.1686 ;
\]

they were chosen so as to make the total expenditure elasticities in our system at the sample mid-point (1969-70) the same as those reported by Chung and Powell in column 4 of their Table 8.1.

The results in Table 4.1 demonstrate that there may be practical limitations on the applicability of MAIDS; however, the