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Constraining Output Responses in Long-run Closures of ORANI: Some Suggestions

by

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themselves, and to take proper account of them in the formal modelling exercise.

Second, applied econometric work will be needed to flesh out existing models and the ones which might eventuate from the study recommended above. Two groups of intertemporally relevant parameters which need to be researched are those relating to adjustment costs and to the rate of degradation of the ore body in the case of those industries which do not have abundant high quality reserves.

Third, work must begin on incorporating suitable inter-temporal paradigms into ORANI in the large. The prototype for this development is given by Wilcoxen.[41]

41 *Op. cit.*

Abstract

CONSTRAINING OUTPUT RESPONSES IN LONG-RUN CLOSURES OF ORANI: SOME SUGGESTIONS

Mark Horridge, Alan A. Powell and Peter J. Wilcoxen

In long-run closures of the ORANI model of the Australian economy, the responses of endogenous exporting industries other than agriculture (which is characterized by the fixity of the land base) can be very large. The main industries exhibiting such behaviour are in the mining sector. If this behaviour is seen as implausible, then some aspects of the model's structural-form equations, database, parameter file, or closure, must be altered. In this paper we canvass the alternatives, concentrating on the industries' supply curves. We give details of a proposal which we believe gives the best prospects for constraining long-run supply elasticities in an orderly and interpretable manner.

$$S_{Kj}\,\eta_j\,V_j \quad > \quad S_{Nj}\,\sigma_j \; : \tag{3.27}$$

or

$$\eta_j \; > \; \frac{S_{Nj}\,\sigma_j}{S_{Kj}\,V_j} \; . \tag{3.28}$$

Relations (3.25) and (3.28) have a common-sense interpretation. As H_j gets indefinitely large, we should approach the short-run situation in which the capital stock cannot (does not) adjust. From (3.25) we see that:

$$\lim_{H_j \to \infty} \eta_j \; = \; \frac{S_{Nj}\,\sigma_j}{V_j\,S_{Kj}} \; ; \tag{3.29}$$

but this is exactly the ORANI short-run partial equilibrium elasticity in the DPSV closure.[40] Thus (3.29) reassures us that positive adjustment costs imply that the medium-run supply elasticity in the modified Horridge closure exceeds the DPSV short-run supply elasticity.

4. *Concluding Remarks*

The agenda for future research clearly must include three areas. First, some detailed intertemporal modelling of the more important mining industries must be attempted at the level of economic theory. To be successful this will require very close liaison with the industries involved. Such interaction is necessary in order to establish which issues in the technological, institutional and informational environment are seen to be important by the industries

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40 See equations (A2.30) - (A2.32), p. 250 of Peter J. Higgs, *Adaptation and Survival in Australian Agriculture* (Melbourne: Oxford University Press, 1986).

$$n_j = k_j \left(1 + \sigma_j H_j / (1 - e^{-\delta_j T})\right) \quad . \qquad (3.23)$$

Given CRTS and zero materials-materials and materials-primary factor substitution, output of j obeys:

$$z_j = S_{Nj} n_j + S_{Kj} k_j \quad ; \qquad (3.24)$$

where S_{Nj} ($\equiv 1 - S_{Kj}$) and S_{Kj}, respectively, are the shares of labour and capital in value added in sector j. Substituting from (3.20) and (3.23) into (3.24), we obtain after simplification:

$$\eta_j = \frac{z_j}{p_{zj}} = \frac{[1 - e^{-\delta_j T} + S_{Nj}\, \sigma_j H_j]}{V_j S_{Kj} H_j} \quad . \qquad (3.25)$$

This tells us what the partial-equilibrium own-price elasticity of ORANI will turn out to be in our modified Horridge closure. If η_j were known, the adjustment parameter H_j could be found from (3.25):

$$H_j = \frac{1 - e^{-\delta_j T}}{S_{Kj}(\eta_j V_j + \sigma_j) - \sigma_j} \quad . \qquad (3.26)$$

For positive depreciation rates, and for $T > 0$, the numerator on the right of this equation is positive. Since H_j must exceed zero, it follows that:

$$S_{Kj}\,(\eta_j V_j + \sigma_j) \;>\; \sigma_j \quad ;$$

that is,

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Using lower-case letters to denote percentage deviations from control, (3.18) may be rewritten in abbreviated form as:

$$(3.19) \qquad k_j = y_j \; (1 - e^{-\delta_j T}) \; .$$

This equation can be used to replace (3.3).[39]

Substituting from (3.19) into (3.8) and thence into (3.9), and solving for k_j, we obtain:

$$(3.20) \qquad k_j = (1 - e^{-\delta_j T}) \; p_{zj} \; / \; (V_j \; S_{Kj} \; H_j) \; .$$

Next, from the CES factor-demand expansion path we know:

$$(3.21) \qquad (n_j - k_j) = \sigma_j(p_{Kj} - w) \; ;$$

where we have made the following simplifications to the ORANI notation:

labour demanded by j: n_j replaces $x^{(1)}_{(g+1,1)j}$

elasticity of substitution: σ_j replaces $\sigma^{(1)}_{(g+1,v)j}$

nominal wage rate: w replaces $p^{(1)}_{(g+1,1)j}$

Consonant with the partial-equilibrium answer we seek, we put $w = 0$ in (3.21), and solve for n_j:

$$(3.22) \qquad n_j = k_j + \sigma_j \; p_{Kj} \; .$$

which, from (3.8) and (3.19), may be written:

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39 It is possible, of course, to construct other replacements for (3.3) using different assumptions about expectations. In particular, the time path used for the ratio of λ to Π need not be stationary: it can be any function of time provided that it could be integrated in the step between (3.14) and (3.15).

prices (without the shock) and the solution prices for year T, respectively.

If $\lambda_j(s)$ and $\dot{\Pi}_j(s)$ in (3.14) are stationary, then for t = T this equation simplifies to:

$$(3.15) \qquad K_j(T) = K_j(0)\, e^{-\delta_j T} + \frac{1}{\delta_j}\left(\frac{\lambda_j}{\phi_j \dot{\Pi}_j}\right)^{1/(\phi_j - 1)} (1 - e^{-\delta_j T}) .$$

Using Wilcoxen's equation (A24), the variable λ is now eliminated in favour of Y (investment). The relevant equation states that, along an optimal path,

$$(3.16) \qquad Y_j(s) = \left(\frac{\lambda_j(s)}{\phi_j \dot{\Pi}_j(s)}\right)^{1/(\phi_j - 1)} \qquad (s \geq 0).$$

With the right-hand arguments stationary under our expectational assumptions, it follows that investment is also stationary (at a level which is shock-dependent). Substituting from (3.16) into (3.15), we obtain:

$$(3.17) \qquad K_j(T) = K_j(0)\, e^{-\delta_j T} + \frac{1}{\delta_j} Y_j(T)\, (1 - e^{-\delta_j T}) ;$$

where (of course) $Y_j(T)$ is the value of investment not only in the solution year T, but also for all future years t (t > 0). We now distinguish the control and the shocked solutions by appendage of the superscripts c and s. Keeping in mind that $K_j(0)$ is the same in the control and in the shocked solution, from (3.17) we see that:

$$(3.18) \qquad \frac{K_j^S(T) - K_j^C(T)}{K_j^C(T)} = \frac{Y_j^S(T) - Y_j^C(T)}{Y_j^C(T)} (1 - e^{-\delta_j T}) .$$

CONSTRAINING OUTPUT RESPONSES IN LONG-RUN CLOSURES OF ORANI: SOME SUGGESTIONS[*]

by

Mark Horridge, Alan A. Powell and Peter J. Wilcoxen

1. *Introduction*

Various commentators, both within the Industry Commission (IC) and elsewhere, have noted that in long-run closures[1] of ORANI[2] there is a tendency for mining industries to respond very flexibly to shocks affecting their price/cost situation. This volatility has been regarded as implausible by many model users, and as a liability by IC staff when presenting the model's results. This paper contains suggestions about how ORANI might be modified to lessen the offending volatility. Before advancing our suggestions, however, we offer a cautionary remark.

Two decades ago the composition of Australian exports was as illustrated in Figure 1(a). Less than twenty years later the picture had changed to that shown in Figure 1(b). The share of primary produce other than minerals in exports declined from more than half to less than one quarter. It seems likely that the latter outcome

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* Without implicating him in any remaining errors, the authors would like to thank Rob McDougall for pointing out a serious inconsistency in an early draft. Much of the territory covered by this paper was earlier surveyed by P. B. Dixon and B. R. Parmenter in "Long Run Specifications for Mining Industries in ORANI: First Progress Report", Institute of Applied Economic and Social Research, University of Melbourne, June 1987.

1 For a description of the proptotypical long-run closure of ORANI, see Mark Horridge, "Long-Run Closure of ORANI: First Implementation", Impact Project *Preliminary Working Paper* No. OP-50, February 1985, pp. 287.

2 P.B. Dixon, B.R. Parmenter J. Sutton and D.P. Vincent, *ORANI: A Multisectoral Model of the Australian Economy* (Amsterdam: North-Holland, 1982). Hereafter referred to as just DPSV.

would have seemed very implausible to an observer in 1967. The lesson is that sustained changes in comparative advantage, brought about by new resource discoveries, differential changes in technology, or by differential trends in world demand for our various export products, can lead to very large changes in output and in export shares. It is possible, therefore, that much of the criticism of existing long-run closures of ORANI is unwarranted.

In the remainder of this paper we work on the assumption that, notwithstanding the above, Commission staff and other researchers will wish to dampen the flexibility of mining responses in long-run simulations. In Section 2 we review those aspects of the specification of mining behaviour which we see as relevant to making judgements about how to proceed with ORANI simulations. In Section 3 we develop, at a moderate level of detail, our suggestions for modifications to the structural form, parameter set, and closure of ORANI. The fourth and final section of the paper offers concluding remarks.

2. *Specification Issues*

The size of the output response of an industry is basically determined by:

1. the elasticity of supply of the product;
2. the elasticity of demand for the product;
3. the extent of any shifts in the supply curve of the product;
4. the extent of any shifts in the demand curve for the product.

The first two of these are partial equilibrium features; the second pair reflect general equilibrium considerations.

From (3.1) and (3.2) it is clear that:

$$(3.13) \qquad H_j \equiv (\phi_j - 1) .$$

On any optimal plan Wilcoxen has shown that, for industry j:[36]

$$(3.14) \qquad K_j(t) = K_j(0)e^{-\delta_j t} + \int_0^{\infty} \left\{ \frac{\lambda_j(s)}{\phi_j \dot{\Pi}_j(s)} \right\}^{1/(\phi_j - 1)} e^{-\delta_j(t-s)} ds \qquad (t \geq 0);$$

where $K_j(t)$ is the planned capital stock for future year t; $K_j(0)$ is the base-period capital stock (assumed to be an ∞-run equilibrium); δ_j is the depreciation rate; and $\lambda_j(t)$ is the marginal benefit of additional installed investment (at a fixed level of the unit cost of *assembled* investment goods). Given the earnings function used by Wilcoxen, λ_j is not a function of investment.

This exogeneity of λ_j to the investor is not sufficient, however, to make our problem operationally tractable. To make further progress, it is necessary to make explicit assumptions about what the investing agent expects the time path of λ_j to be, with and without the shock. Here we assume investors' expectations to be that, in the absence of the shock, the value of λ_j would remain stationary[37] at its base-period value; and after the shock that λ_j would assume a new stationary value. This is consistent, in Wilcoxen's framework, with the assumption that investing agents expect the ratios of the prices of variable inputs to the price of output to be stationary.[38] The prices determining these stationary ratios operationally are base-period

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36 *Op. cit.*, equation (36).

37 In the current (deterministic) context we say that a variable is stationary if it is neither implicitly, nor explicitly, a function of time.

38 See Wilcoxen, *op. cit.*, equation (A27).

In the Horridge closure, following Dixon, Parmenter and Rimmer[32], newly installed capacity and the capital stock deviate from control by the same proportion in the solution year (T):[33]

$$(3.10) \qquad y_j(T) = k_j(T) \quad .$$

The above equation is an implication of the assumption that a shock now may affect the size of industries in the T run, but has no effect on their growth rates at this (relatively distant) future date. Equation (3.10) cannot be used in the current context, however, because it is not consistent with the adjustment cost model embodied in (3.1) and (3.2).[34]

In a recent paper Wilcoxen analyses investment behaviour subject to adjustment costs of the form:[35]

$$(3.11) \qquad \Pi_j Y_j \quad = \quad \Pi_j \dot{Y}_j^{\phi_j} \qquad (\phi_j \geqq 1);$$

where upper-case letters correspond to the levels of the corresponding lower-case (percentage change) variables. Taking log differentials of (3.11), multiplying by 100, and rearranging, we obtain:

$$(3.12) \qquad \pi_j - \dot{\pi}_j \quad = \quad (\phi_j - 1)\, y_j \, .$$

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32 Peter B. Dixon, B. R. Parmenter and Russell J. Rimmer, "Extending the ORANI Model of the Australian Economy: Adding Foreign Investment to a Miniature Version", Ch. 12 in H. E. Scarf and J. B. Shoven (eds), *Applied General Equilibrium Analysis* (New York: Cambridge University Press, 1984), pp. 485 - 533.

33 More formally, the investment to capital stock ratio is exogenous in the Horridge closure.

34 We are grateful to Rob McDougall, who pointed this out in his comments on an early draft.

35 Wilcoxen, *op. cit.*, equation (22).

Figure 1(a): Export Shares in 1969-70

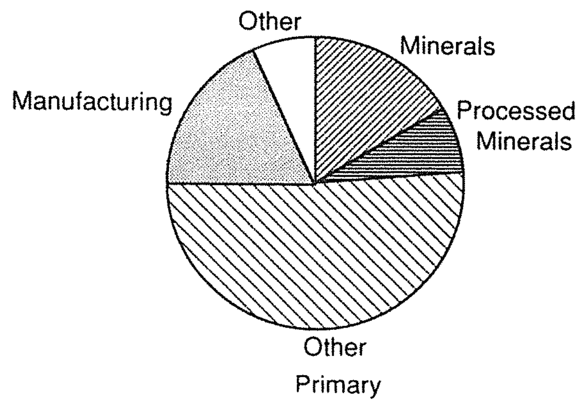
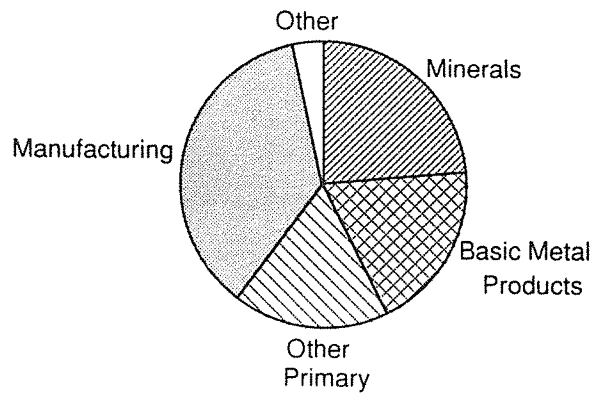


Figure 1(b): Export Shares in 1987-88



Source: Australian Bureau of Statistics, *Exports, Australia - Annual Summary Tables*, Catalogue No. 5424.0.

For most major mining products, Australian demand is a small fraction of the total, so at least to a good first approximation we may therefore identify the demand schedule facing the local industry with the export demand curve. Given the small country assumption in ORANI,[3] we will be ignoring item 4 above (which in any event would require a general equilibrium model of Australia's trading partners).

The remaining three influences on the size of an industry's response are illustrated in Figure 2, which shows several supply and demand curves graphed on logarithmic axes.[4] Commentators who regard ORANI medium- or long-run solutions as showing implausibly large changes in mining output can in principle be objecting to any of items 1 through 3:[5] that is, they might believe any or all of the following:

(i) the supply curve is too flat;
(ii) the demand curve is too flat;
(iii) the supply curve shifts too far under the shock in question.

Items (i) and (ii) have been extensively debated;[6] item (iii) is harder to discuss since it represents an opinion about the overall adequacy

3 But see the cautionary remarks made below.

4 Using logarithmic axes means the slopes of the curves are the reciprocals of elasticities.

5 We continue to assume here that the small-country assumption is not under challenge -- in that case, the movement C of the demand curve in Figure 2 is either zero, or is an exogenous shock (an improvement in overseas demand conditions).

6 For a discussion of the magnitudes of the relevant trade elasticities, see, e.g., Industries Assistance Commission, "The ORANI Trade Parameters -- Papers and Proceedings of a Workshop, April 1983", Impact Project *General Paper* No. G-58, September 1984, pp. 134. Dixon and Parmenter, *op. cit.*, show that export demand parameters are crucial -- reducing the export demand elasticites facing endogenous exporters leads to very sizeable reductions in the general equilibrium supply responses of these industries in ORANI.

the price of all other commodities constant. Given the ORANI investment technology, under which a unit of capacity in industry j is just a Leontief bundle of commodities, it follows that the cost of an assembled unit of capacity in j is fixed; i.e., that $\pi_j^* = 0$.[29] Then (3.1) becomes:

(3.7) $$\pi_j = H_j y_j \ ;$$

and so from (3.6):

(3.8) $$p_{Kj} = H_j y_j \ .$$

We now turn to the price of the mineral, p_{zj}.[30] Again we will simplify by assuming that j is not an input into itself. Then:

(3.9) $$p_{zj} = V_j S_{Kj} p_{Kj} \ ;$$

where V_j is the share of value added in total cost and S_{kj} is the share of capital in value added. Note that besides holding all other commodity prices constant, in deriving (3.9) we have also held the nominal price of labour constant. This reflects the partial-equilibrium nature of the elasticity we are seeking, not the closure.[31]

29 Note that equation (3.3) deals only with Armington substitution; the technology is otherwise Leontief. The comparative-static thought experiment involves holding the price index for each other material α, $S^{(2)}_{(i1)\alpha} p^{(2)}_{(i1)\alpha} + S^{(2)}_{(i2)\alpha} p^{(2)}_{(i2)\alpha}$, constant for $\alpha \neq j$.

30 If j is a single-product industry, p_{zj} is just the basic price of the commodity produced; if j is a multi-product industry, p_{zj} is defined by the left-hand side of DPSV equation (18.2).

31 To put it more bluntly, under the partial equilibrium approach all industries are 'small', and so effects (if any) on the economy-wide wage rate are negligible.

demand for inputs per one per cent increase in investment that is due to adjustment costs.[26]

Next we note that in Horridge-like closures the percentage change in the installed cost of a unit of capacity is equal to the percentage change in the rental price of that unit. To simplify notation in what follows, the rental price of a unit of capital in industry j, $p^{(1)}_{(g+1,2)j}$, will be simplified to p_{Kj}. Equation (19.7) of DPSV (p. 133) states that the net rate of return in industry j, $r_j(0)$, is determined as follows :

(3.5) $$r_j(0) = Q_j\,(p_{Kj} - \pi_j)\ ;$$

where Q_j is the ratio of the gross to the net-of-depreciation rate of return in industry j. With $r_j(0)$ set exogenously to zero deviation from control,[27] (3.5) implies:

(3.6) $$p_{Kj} = \pi_j\ .$$

In this expository paper, to keep matters simple, we will assume that mining industries have zero share in their own investment bundle.[28] To work out the partial-equilibrium own price elasticity of supply of j (a mineral) we take differentials in the price of j, in its output, investment, and factor demands; however we keep

26 The cost-of-adjustment penalty is treated neutrally across all inputs and sources only because data availability is likely to preclude anything more ambitious.

27 If one takes the view that each type of investment activity has its own supply price for capital, then arguably the assumption of exogenously set rates of return on some Australian mining activities needs reexamination; in the case of bauxite, for example, Australia's share of world production (about 35 per cent) is large enough to suspect that feedbacks might be important. That is, while Australia is undeniably a small part of the world capital market, it may nevertheless have a substantial share of the market for certain asset types.

28 Fairly straight forward generalization of the algebra (starting at equation (3.7)) is otherwise necessary.

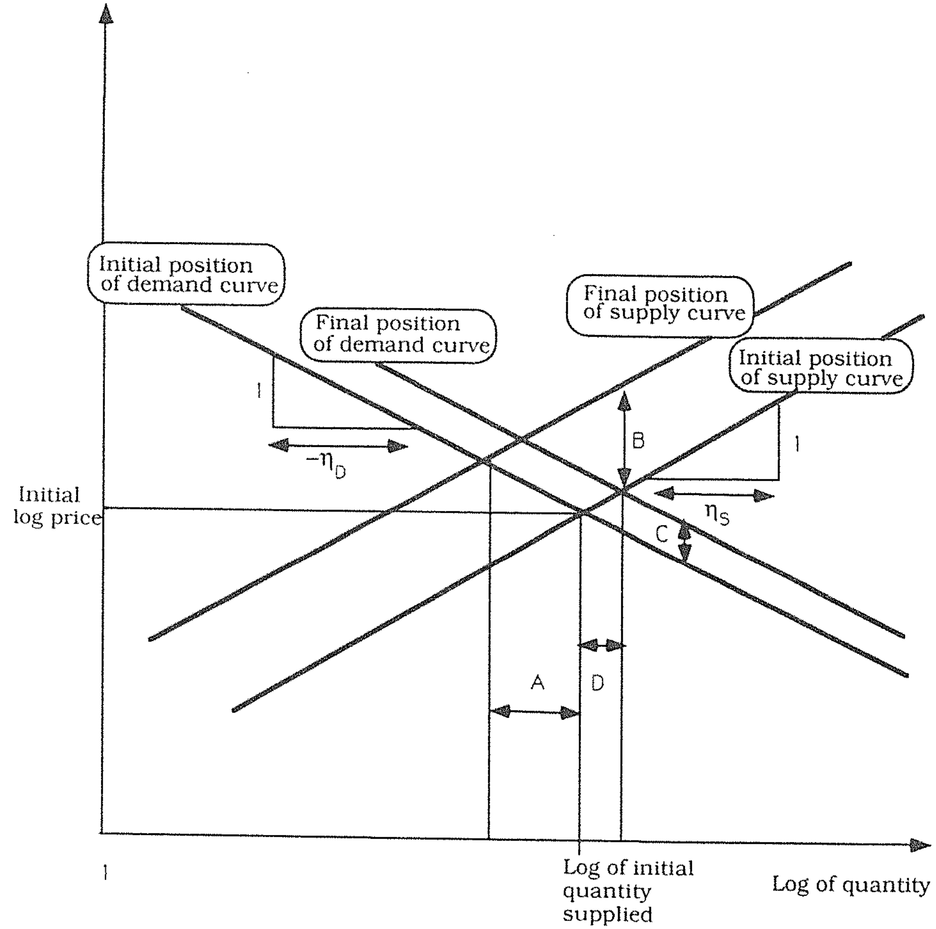


Figure 2: Partial and General Equilibrium Influences on the Size of an Industry's Supply Response

Under a shift upwards of B in the supply curve, the output response is a proportional contraction of $A = -\eta_D B$. Under a shift upwards of C in the demand curve, the output response is a proportional expansion of $D = \eta_S C$. η_S and η_D respectively are the partial equilibrium elasticities of supply and demand.

of the general equilibrium mechanisms, and parameter values, embedded in ORANI. In the remainder of this paper we focus exclusively on the supply side of the model, but it must be remembered that the large output response of mining industries could have other causes.

In ORANI, mining industries (like all others) exhibit constant returns to scale (CRTS). Since these industries face fairly flat export demand schedules, in any environment in which no inputs are held fixed, changes in the prices of their products lead to large output responses. One interpretation of the criticism levelled at long-run ORANI simulations is that mining industries do not in fact exhibit CRTS, and hence the current ORANI results overstate the flexibility of mining. In the remainder of this section we examine a number of factors with potential to limit this flexibility.

2.1 *Non-constant returns -- definitions*

We start with a definition. Let x_v be the set of factors explicitly recognized in the model which can be varied by the firm over the length of run being studied. Notation for other factors[7] is as shown in Table 1.

The set x_F, for example, might contain the firm's capital stock, while $\dot{x}_F$ might contain its stock of entrepreneurial ability. $\dot{x}_v$ might include the usage of water available from a source owned by the firm, and (at least initially) having excess capacity.

Some care is needed below in preserving the distinction between constant returns, and constant returns to scale. The latter

7 Unless otherwise stated, *factor* throughout this paper means an input to production, not necessarily a primary factor.

$$\pi_j = \dot{\pi}_j + a_j^{(2)} . \qquad (j\varepsilon M^+) \tag{3.1}$$

in which the technological term $a_j^{(2)}$ is to be endogenized by a new equation:

$$a_j^{(2)} = H_j y_j ; \qquad (j\varepsilon M^+) \tag{3.2}$$

where H_j is a non-negative parameter and y_j is the net addition to installed capacity in industry j. Setting $H_j = 0$ eliminates adjustment costs from the model.

Turning now to the demand equations for inputs to capital creation, we restate equation (13.4) of DPSV, but suppress all those technological change terms which will be set exogenously to zero (i.e., all such terms other than $a_j^{(2)}$):

$$x_{(is)j}^{(2)} = y_j - \sigma_{ij}^{(2)} \left[p_{(is)j}^{(2)} - \sum_{s=1}^{2} S_{(is)j}^{(2)} p_{(is)j}^{(2)} \right] + a_j^{(2)} . \tag{3.3}$$

The interpretation here is that the first two terms on the right-hand side of (3.3) give the demand for input $X_{(is)}$ by industry j for the assembly of units of capital; the extra amount $a_j^{(2)}$ is used up in converting the assembled unit to installed capacity. Using (3.2) we rewrite (3.3) as:

$$x_{(is)j}^{(2)} = y_j(1 + H_j) - \sigma_{ij}^{(2)} \left[p_{(is)j}^{(2)} - \sum_{s=1}^{2} S_{(is)j}^{(2)} p_{(is)j}^{(2)} \right] . \tag{3.4}$$

At fixed relative input prices a one per cent increase in investment leads to a $(1 + H_j)$ per cent increase in demand for inputs to capital creation; hence H_j may be interpreted as the *additional* percentage

Apart from some simplifications which are explained when introduced, we follow standard ORANI notation. In that notation y_j means a unit of installed capacity; we preserve this convention.[24] The technological deterioration will be expressed via the variable $a_j^{(2)}$ appearing in the equations pertaining to:

(a) the demands for inputs to capital creation (equation (13.4) of DPSV[25])

and

(b) the cost of a unit of installed capacity.

In the case of mining industries, π_j will no longer mean the (percentage change in) the cost of creating a unit of capital in industry j ($j \in M^+$, where M^+ is the set of mining industries), but will mean the cost of such a unit of installed capacity. That is, we will allow an explicit difference between the cost of *assembling* the components of a unit of capacity for j, and the *installed* cost of such a unit. Making this distinction will allow adjustment costs to be incorporated directly into ORANI equations via the variable $a_j^{(2)}$.

Define $\dot{\pi}_j$ to be the cost of assembling a unit of capacity in industry j. Then in equation (18.6) of DPSV (which enforces zero pure profits in the creation of capital goods) π_j would be replaced by $\dot{\pi}_j$. The installed cost of a unit of capacity π_j would be endogenized by the following new equation:

24 From this point on, unless noted otherwise, lower case letter are percentage deviations from control.

25 We remind the reader that DPSV is short-hand for the reference cited in footnote 2.

will be defined in a way that depends only on the technology, and not on the rules concerning which factors may and may not be varied. These rules serve to define a length of run. To avoid the cumbersome

Table 1

Notation for Factors of Production

Status	Over the length of run studied, the factor is:	
	Fixed	Variable
Explicitly recognised in production function	x_F	x_v
Implicit	$\dot{x}_F$	$\dot{x}_v$

repetition of the qualifier "being studied", we will assume that the length of run being studied has been identified as T years, and from this point on refer simply to "the T-run".

We say that the firm faces **constant returns** over the T-run if increasing all the explicit variable factors x_v by a common percentage p leads to a p per cent increase in output. More formally, let the production function be:

(2.1) $$q = F(x_F, x_v, \dot{x}_F, \dot{x}_v) \ .$$

and define an output function for the T-run by

$$q = g(x_v) \quad . \tag{2.2}$$

with the fixed factors set at their initial values, and the implicit variable factor $\dot{x}_v$ changing with x_v in such a way as to leave the shadow price of $\dot{x}_v$ unaffected.[8] Then we say that the firm faces globally constant returns over the T-run if

$$g(\lambda x_v) = \lambda g(x_v) \tag{2.3}$$

for any positive multiplier λ.[9]

By **constant returns to scale** we mean that the function F in (2.1) is homogeneous of first degree. We immediately note that if F is a homogeneous function, g cannot display constant returns in the T-run unless F displays increasing returns to scale.[10] We do not append a qualifier to specify the length of run when describing scale effects, it being understood that the term 'returns to scale' always refers to a run of sufficient length to allow arbitrary variation in **all** the arguments of F; doubling them, therefore, is just doubling the scale of the process producing y.

What if F exhibits constant returns to scale? At this point we must decide whether or not the *time* should be recognised as an input, either within x_v or $\dot{x}_F$. The neoclassical representation (2.1) of production is timeless; in the real world, however, output takes

8 The simplest case is the one in which $\dot{x}_v$ is a free good; in that case its shadow price is globally zero, and it may be deleted from the discussion.

9 If (2.3) is true for $l = 1 + e$ (where e is small; = 0.05, say) at some particular value of x_v but not at others, we say that the firm faces locally constant returns at the variable input level x_v.

10 A function displays increasing (decreasing) returns to scale if it is homogeneous of degree v, where v exceeds (is less than) one.

ORANI's structure. Like agricultural land, the mining fixed factors are exogenous in such closures. Note that while there are just six types of agricultural land in ORANI, there will be one new fixed factor for *each* mining industry. These new fixed factors might be visualized as stocks whose service flows correspond to the 'facilitation of adjustment' in the different mining industries.

The new factors must be paid a rent. This rent must

(i) be large enough, in the base-period data base, to give a cost share which generates the 'correct' (i.e., target) medium-run supply elasticity;

(ii) be small enough, and distributed in a sufficiently neutral way, not to generate large income effects in the solution year.

It is crucial to get (i) right. If this is in serious tension with (ii), then more radical approaches to restructuring the model may be required.

Neither of these first two approaches, however, seems to be nearly as promising as the third, which we now discuss in some detail.

3.2 *Endogenous technological regress*

As is the case with the two alternatives mentioned above, we assume here that the closure will be a modification of the Horridge closure[23]. In the construction below we will be invoking general-equilibrium properties of the economic environment defined by such a closure to evaluate the partial-equilibrium supply elasticities of mining industries within that environment. These will be shown to depend principally on cost shares, the capital-labour substitution elasticity and on the adjustment-cost parameter.

23 See the reference cited in footnote 1.

(iii) the introduction of an endogenous technological deterioration which is able to capture the influence of adjustment costs.[20]

We tend to favour the last approach. However, we briefly discuss the other two approaches before outlining (iii) in detail.

Approach (i) requires abandoning the marginal productivity rule for factor rewards since, by Euler's theorem, not all the product would be exhausted if factors were paid their marginal products.[21] Many familiar homogeneity properties which are routinely used as checks on correct computation of CGE models would also be lost. The database would have to be recalibrated to reflect whatever rule is used to reward factors in the non-CRTS industries. In any event, data which appear to exhibit decreasing returns to scale can normally be explained *via* an implicit fixed factor, as pointed out earlier.

The second method appears more hopeful. The basic technique is to endow the mining industries, within ORANI, with a fictitious 'fixed factor' whose base-period share in costs is chosen so as to yield the desired medium-run supply elasticity.[22] The method should otherwise be chosen so as to minimize the changes made to

20 For an *ad hoc* approach to the inclusion of adjustment costs in ORANI, see Dixon and Parmenter *op. cit.*, who argue that this approach can be very effective in limiting output responses.

21 If υ is the degree of homogeneity of the production function, then Euler's theorem states that:

$$\sum_{i=1}^{n} \frac{\partial Q}{\partial X_i} X_i = \upsilon Q \ ;$$

where Q is output and the Xs are the n inputs.

22 Dixon and Parmenter, *op. cit.*, find that recognizing fixed factors in mining industries can reduce general equilibrium output responses considerably.

time to produce, and there may be costs that vary with time (such as an annual licensing fee paid to the owner of a patent) rather than with output (such as the cost of material inputs). The most commonly used notion of (globally) constant returns to scale (CRTS) is based on the assumption that in the absence of a periodic cost, the firm would face constant returns in the long run.[11] In the case in which CRTS applies and there is an annual licensing fee f, for example, the cost curves for the firm would look as in Figure 3. Implicit in this definition of CRTS is the assumption that the period allowed for production is fixed.[12]

Notice that if time is an element of $\dot{x}_v$ then the function which is homogeneous of degree 1 under the CRTS definition is not (2.1), but the corresponding conditional function with the element of $\dot{x}_v$ corresponding to the length of the production period held fixed. If a licensing fee or other time-dependent charge applies, the technology of the firm may exhibit CRTS on the above definition, but this firm may nevertheless experience increasing returns (diminishing average costs) because the time-dependent charge acts just like a fixed cost to be amortized over whatever output level is achieved in the production period to which this fixed cost applies. Under this set of definitions *constant returns to scale* will only imply *constant returns in the long run* if there are no costs which are functions of time rather than output.

11 The long run is usually defined as a horizon of sufficient length to allow *all* factors (other than the length of the production period) to be treated as variable.

12 We shall adopt this definition of CRTS throughout.

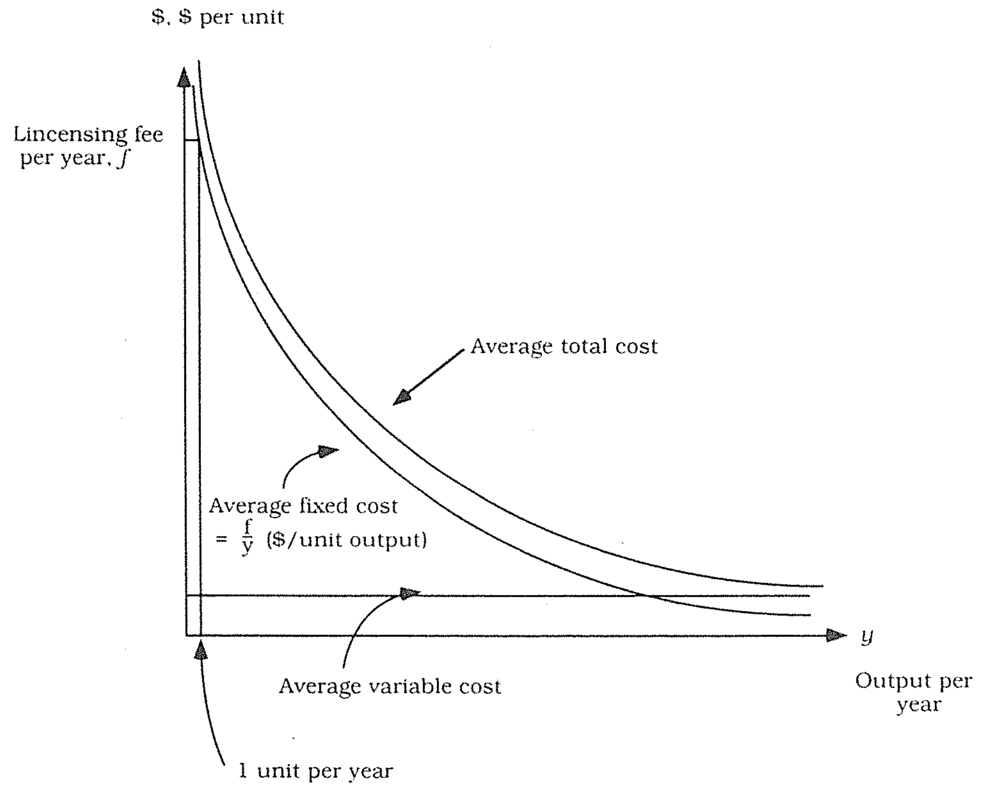


Figure 3: Constant-Returns-to-Scale Cost Function with a Time-dependent Licensing Fee

model lead to *lower* supply elasticities than apply under price-taking behaviour.

3. *Controlling Medium-run Mining Responses in ORANI*

We assume here that time and resource constraints will prevent the construction of detailed intertemporal models for the mining industries. Furthermore, we also assume that an estimate is available for each mining industry's own price supply elasticity in the medium run (say T = 5, 10 or 15 years). Sources for such an estimate might be:

- a literature search,
- applied econometric work,
- a synthesis based on a 'feeling' for the approximate values of key parameters.[19]

3.1 Three possible approaches

Three possible approaches towards enforcing the medium-run supply elasticities are:

(i) the abandonment of CRTS technology in mining in favour of globally diminishing returns to scale;

(ii) the inclusion of a new factor which would be held fixed in medium-run simulations;

19 In Wilcoxen, "Supply Elasticities in the Presence of Adjustment Costs", *op. cit.*, it is shown how supply elasticities may be calculated from a knowledge of (i) the length of run T; (ii) the capital intensity of production; (iii) the elasticity of substitution between capital and variable inputs; and (iv) the adjustment cost parameter.

which has the same indeterminacy problem as (2.5) if production is subject to CRTS. This is where adjustment costs may be helpful. As Wilcoxen has shown.[17] 5- or 10-year own-price elasticities of supply become quite modest in size even for investment cost schedules displaying only mildly increasing costs.

If we take a more aggregative approach to the problem, interpreting q as the final output delivered to the wharf from a long chain of resource allocation decisions (rather than as just the production of the mineral at the mine gate), then many factors may contribute to adjustment costs, including the following:

- delivery lags in the supply of inputs to production and investment.
- the costs of complying with government regulation.
- lags in infrastructure development.
- increasing wage costs or diminishing standards in the available workforce as expansion proceeds, particularly in remote locations.
- lags associated with exploration.

e. **Market structure** If the typical Australian firm producing a given mineral is not strictly a price-taker,[18] then responses to shifts in foreign demand curves may be governed by strategic behaviour or other alternatives to the competitive model. Whilst no fully robust generalization can be made, many departures from the competitive

17 Peter J. Wilcoxen, "Supply Elasticities in the Presence of Adjustment Costs", Impact Project *Preliminary Working Paper* No. IP-46, March 1990.

18 The international market for alumina, in particular, is dominated by a few large firms.

2.2 *Sources of non-constant returns*

a. **Factor fixity** With the above background we can identify some reasons why constant returns might not apply in mining over the length of run of interest to the IC (T = 5 or 10 years, say). The first is that while CRTS may well apply to the production functions (2.1), there will be factors which remain fixed (or nearly so) over this time horizon. Obvious candidates are:

- the supply of proven ore-bodies;
- the available managerial talent.

If we expand our definition of production to mean not just production at the mine gate, but output on the wharf at an export facility, it is clear that the activity "exporting minerals" could run into diminishing returns for reasons quite unrelated to mining. The obvious elements of the infrastructure which might lead to increasing costs are transportation, refining, or port loading facilities. Ideally, these would be modelled separately.

b. **Diminishing returns to scale** We cannot rule out, *a priori*, the possibility that the technology is one of diminishing returns to scale.[13] Then, if there were no time-dependent costs, the firm would face diminishing returns at *every* length of run. If a time-dependent cost did apply, the average total cost curve would assume the textbook "U" shape.

13 Returns to scale (like returns in the T-run) need not necessarily be globally of one type: for example, returns to scale at some initial scale of output might be locally increasing; at an intermediate scale, returns might be locally constant; at a yet larger scale still, returns to scale might be locally decreasing.

Whilst diminishing returns to scale cannot be ruled out *a priori*, most attempts to explain their genesis boil down to an implicit factor fixity (usually relating to management).

c. Progressive degradation of the ore body If the cost of accessing and/or extracting the mineral increases with cumulative output, then (2.1) is no longer valid as a representation of the production process; instead we need:

$$q = H\left(x_F, x_V, \dot{x}_F, \dot{x}_V, \sum_{\text{all lags}} (\text{lagged } q) \right) . \tag{2.4}$$

In this case the average cost function shifts progressively up as mining proceeds irrespective of whether average costs are increasing, decreasing or constant with respect to the current output rate y.[14] This is the situation modelled by Blampied, Horridge and Powell.[15] Whilst relevant to many mining activities, this set of circumstances seems unlikely to apply to Australia's most important mineral exports -- black coal, alumina (refined bauxite) and iron ore.

d. Adjustment costs In intertemporal approaches to modelling production, besides current inputs and outputs we need also to consider the augmentation of capacity over time and the set of expectations about future variables driving this accumulation. An

14 Situations like the one discussed here are best handled in continuous time; y becomes the instantaneous time rate of output.

15 Chris W. Blampied, Mark Horridge and Alan A. Powell, "The Behaviour of the Major Extractive Industries in Long-Run Closures of ORANI: A Proposal", Impact Project *Preliminary Working Paper* No. OP-55, April 1986, pp. 40.

alternative to (2.4) which may lead to diminishing returns in runs other than the long run is:

$$\begin{pmatrix} q \\ I \\ x_F \\ x_V \end{pmatrix} = G^*(\Omega) . \tag{2.5}$$

where (for simplicity) we have assumed that all factors (with the possible exception of time) are explicit, and where I indicates investment in the current period, whilst Ω is the information set upon which intertemporal planning decisions are based. To further simplify notation (but without real loss of generality), we will assume that each of the variables on the left of (2.5) is a scalar.

If constant returns to scale in production apply then the system (2.5) is underdetermined; that is, nothing in the technology itself will determine the scale of output.[16] Suppose we are interested in a T-run over which x_F cannot be varied; this will result in output and investment becoming determinate:

$$\begin{pmatrix} q \\ I \\ x_V \end{pmatrix} = G^{+}(x_F; \Omega) . \tag{2.6}$$

Suppose, however that there is no fixed factor over the T-run; then (2.6) becomes:

$$\begin{pmatrix} q \\ I \\ x_V \end{pmatrix} = G(\Omega) . \tag{2.7}$$

16 We are assuming that the firm is a price taker.