Closures of ORANI: Some Suggestions

Constructing Quick Responses in Long-run

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References


Abstract

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OF ORAN: SOME SUGGESTIONS

CONTRAINING OUTPUT RESOURCES IN LONG-RUN CLOSURES

Interpretable manner.

For constructing long-run supply characteristics in an orderly and

five details of a proposal which we believe gives the best prospects

the efficiency of construction on the industries' supply curves. We

parameter the or course, must be altered. In this paper we can assess

some aspects of the model's structural-form equations. Database,

the mining sector. If this behaviour is seen as implausible, then

the very large. The main industries exhibiting such behaviour are in

agriculture which is characterized by the lack of the land based can

the responses of endogenous exporting industries or the

In long-run closures of the ORAN model of the Australian economy.

development is given by Witsen.

Third, work must begin on incorporating suitable inter-

reserve.

case of those industries which do not have abundant high-quality

can add cost of degradation to the case of degradation of the ore body in the

parameters which need to be recovered and those relating to

extraction models and the ones which might emanate from the study

Second, applied economic work will be needed to test out

modelling exercise.

theories, and to take proper account of them in the formal
(3.27) \[ S_{kj} \eta_j V_j > S_{nj} \sigma_j : \]

or

(3.28) \[ \eta_j > \frac{S_{nj} \sigma_j}{S_{kj} V_j} : \]

Relations (3.25) and (3.28) have a common-sense interpretation. As \( H_j \) gets indefinitely large, we should approach the short-run situation in which the capital stock cannot (does not) adjust. From (3.25) we see that:

(3.29) \[ \lim_{H_j \to \infty} \eta_j = \frac{S_{nj} \sigma_j}{V_j S_{kj}} : \]

but this is exactly the ORANI short-run partial equilibrium elasticity in the DPSV closure.\(^40\) Thus (3.29) reassures us that positive adjustment costs imply that the medium-run supply elasticity in the modified Horridge closure exceeds the DPSV short-run supply elasticity.

4. Concluding Remarks

The agenda for future research clearly must include three areas. First, some detailed intertemporal modelling of the more important mining industries must be attempted at the level of economic theory. To be successful this will require very close liaison with the industries involved. Such interaction is necessary in order to establish which issues in the technological, institutional and informational environment are seen to be important by the industries.

Dependent Licensing Fee

Consistent-Returns-to-Scale Cost Function with a Time-

Inflation's Supply Response

Partial and General Equilibrium Influences on the Size of an

TABLE

1: Nature of Factors of Production

TABLE

References

4: Concluding Remarks

11 3.2 Endogenous Technological Change

11.1 The Possible Approaches

3.3 Constant-Market-Knowing Responses in ORAN

1.4 Market Structure

1.5 Adjustment Costs

1.6 Progressive Depreciation of the Arm's Length

1.7 Diminishing Returns to Scale

1.8 Factor Fixity

1.9 Sources of Non-Concurrent Returns

2 Consistent Returns to Scale

2.1 Non-Concurrent Returns - Definitions

2.2 Specification Issues - Definitions

1 Introduction

1.1 Introduction

1.2 Abstract

3.2.9: It follows that where the adjustment parameter $\lambda$ could be found from ORAN and $\gamma$ return to be in our modelled environment class, if $\gamma = \gamma'$. This tells us what the partial-equilibrium optimal elasticity of market structure $\lambda$.

\[
\left. \frac{\lambda^2}{\lambda^2 + \lambda} \right|_{\lambda} = \frac{\lambda^2}{\lambda^2 + \lambda} = \gamma
\]

3.2.2: From (3.2.4), we obtain the simplified and capital in which added in section. Substituting from (3.2.4) and where $\gamma = \gamma'$ and $\gamma$ respectively are the shares of labour.

\[
\left. \frac{\lambda^2}{\lambda^2 + \lambda} \right|_{\lambda} = \frac{\lambda^2}{\lambda^2 + \lambda} = \gamma
\]
Using lower-case letters to denote percentage deviations from control, (3.18) may be rewritten in abbreviated form as:

$$k_j = y_j \left(1 - e^{-\delta_j T}\right).$$  

(3.19)

This equation can be used to replace (3.3).  

Substituting from (3.19) into (3.8) and thence into (3.9), and solving for $k_j$, we obtain:

$$k_j = \left(1 - e^{-\delta_j T}\right) \frac{p_{zj}}{[V_j S_{Kj} H_j]}.$$  

(3.20)

Next, from the CES factor-demand expansion path we know:

$$\ln(n_j \cdot k_j) = \sigma_j (p_{Kj} \cdot w).$$  

(3.21)

where we have made the following simplifications to the ORANI notation:

- labour demanded by $j$: $n_j$ replaces $\ell_{(g+1)j}$
- elasticity of substitution: $\sigma_j$ replaces $\sigma_{(g+1)j}$
- nominal wage rate: $w$ replaces $P_{(g+1)j}$

Consonant with the partial-equilibrium answer we seek, we put $w = 0$ in (3.21), and solve for $n_j$:

$$n_j = k_j + \sigma_j p_{Kj}.$$  

(3.22)

which, from (3.8) and (3.19), may be written:

---

30 It is possible, of course, to construct other replacements for (3.3) using different assumptions about expectations. In particular, the time path used for the ratio of $\lambda$ to $\Omega$ need not be stationary. It can be any function of time provided that it could be integrated in the step between (3.14) and (3.19).
Mark Horridge, Alan R. Powell and Peter J. Wilson

Chapter 11

ORANI: SOME SUGGESTIONS"
would have seemed very implausible to an observer in 1967. The lesson is that sustained changes in comparative advantage, brought about by new resource discoveries, differential changes in technology, or by differential trends in world demand for our various export products, can lead to very large changes in output and in export shares. It is possible, therefore, that much of the criticism of existing long-run closures of ORANI is unwarranted.

In the remainder of this paper we work on the assumption that, notwithstanding the above, Commission staff and other researchers will wish to dampen the flexibility of mining responses in long-run simulations. In Section 2 we review those aspects of the specification of mining behaviour which we see as relevant to making judgements about how to proceed with ORANI simulations. In Section 3 we develop, at a moderate level of detail, our suggestions for modifications to the structural form, parameter set, and closure of ORANI. The fourth and final section of the paper offers concluding remarks.

2. Specification Issues

The size of the output response of an industry is basically determined by:

1. the elasticity of supply of the product;
2. the elasticity of demand for the product;
3. the extent of any shifts in the supply curve of the product;
4. the extent of any shifts in the demand curve for the product.

The first two of these are partial equilibrium features; the second pair reflect general equilibrium considerations.

From (3.1) and (3.2) it is clear that:

\[(3.13) \quad \mathbf{H} = (\mathbf{v} - 1).\]

On any optimal plan Wilcoxon has shown that, for industry \(j\):\(^{36}\)

\[(3.14) \quad K_0(t) = K_0(0)e^{-\delta t} + \int_0^t \frac{\lambda_0(s)}{\delta_0 \Pi_j(s)} e^{-\delta_0(s)} ds \quad \text{if} \quad t \geq 0;\]

where \(K_0(t)\) is the planned capital stock for future year \(t\); \(K_0(0)\) is the base-period capital stock (assumed to be an \(\infty\)-run equilibrium); \(\delta_0\) is the depreciation rate; and \(\lambda_0(t)\) is the marginal benefit of additional installed investment (at a fixed level of the unit cost of assembled investment goods). Given the earnings function used by Wilcoxon, \(\lambda_j\) is not a function of investment.

This exogeneity of \(\lambda_j\) to the investor is not sufficient, however, to make our problem operationally tractable. To make further progress, it is necessary to make explicit assumptions about what the investing agent expects the time path of \(\lambda_j\) to be, with and without the shock. Here we assume investors' expectations to be that, in the absence of the shock, the value of \(\lambda_j\) would remain stationary\(^{37}\) at its base-period value; and after the shock that \(\lambda_j\) would assume a new stationary value. This is consistent, in Wilcoxon's framework, with the assumption that investing agents expect the ratios of the prices of variable inputs to the price of output to be stationary.\(^{38}\) The prices determining these stationary ratios operationally are base-period

---


37 In the current (deterministic) context we say that a variable is stationary if it is neither implicitly, nor explicitly, a function of time.

38 See Wilcoxon, op. cit., equation (A27).
We are grateful to Rob McDowell who pointed this out in his comments on an earlier draft of the manuscript. Inclusion of the adjustment for capital stock is confusing in the context of our argument for a more formal definition of investment. The adjustment is in line with recent work by Brander and Riedel (1995), and other recent work by Brander and Riedel (1994).

Figure 1 (b): Export Shares in 1987-88

Figure 1 (a): Export Shares in 1969-70

Obtain the differentials of (3.11), multiplying by 100, and rearranging, we have

\[ \frac{\Delta Y}{Y} = \frac{\Delta L}{L} \]

where upper-case letters correspond to the levels of the quantities of labor and capital.

The above equation is an implication of the assumption that a shock to investment can be accommodated by a proportional increase in labor and capital.

Because it is not consistent with the adjustment cost model enunciated in equation (3.10), it cannot be used in the current context. However, in their growth rates of the labor and capital inputs to industries in the U.S. and China, the authors have developed a model that is consistent with the adjustment cost model.
For most major mining products, Australian demand is a small fraction of the total, so at least to a good first approximation we may therefore identify the demand schedule facing the local industry with the export demand curve. Given the small country assumption in ORANI, we will be ignoring item 4 above (which in any event would require a general equilibrium model of Australia’s trading partners).

The remaining three influences on the size of an industry’s response are illustrated in Figure 2, which shows several supply and demand curves graphed on logarithmic axes. Commentators who regard ORANI medium- or long-run solutions as showing implausibly large changes in mining output can in principle be objecting to any of items 1 through 3: that is, they might believe any or all of the following:

(i) the supply curve is too flat;
(ii) the demand curve is too flat;
(iii) the supply curve shifts too far under the shock in question.

Items (ii) and (iii) have been extensively debated: item (iii) is harder
to discuss since it represents an opinion about the overall adequacy

\[ (3.7) \quad \pi_j = H_j y_j \]

and so from (3.6):

\[ (3.8) \quad P_{kj} = H_j y_j \]

We now turn to the price of the mineral, \( p_{kj} \). Again we will simplify by assuming that \( j \) is not an input into itself. Then:

\[ (3.9) \quad P_{kj} = V_j S_{kj} \]

where \( V_j \) is the share of value added in total cost and \( S_{kj} \) is the share of capital in value added. Note that besides holding all other commodity prices constant, in deriving (3.9) we have also held the nominal price of labour constant. This reflects the partial-equilibrium nature of the elasticity we are seeking, not the closure.

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3 But see the cautionary remarks made below.
4 Using logarithmic axes means the slopes of the curves are the reciprocals of elasticities.
5 We continue to assume here that the small-country assumption is not under challenge -- in that case, the movement \( C \) of the demand curve in Figure 2 is either zero, or is an exogenous shock (an improvement in overseas demand conditions).
6 For a discussion of the magnitudes of the relevant trade elasticities, see, e.g., Industries Assistance Commission, “The ORANI Trade Parameters -- Papers and Proceedings of a Workshop, April 1983”, Impact Project General Paper No. C-58, September 1984, pp. 134. Dixon and Parmeter, op. cit., show that export demand parameters are crucial -- reducing the export demand elasticities facing endogenous exporters leads to very sizeable reductions in the general equilibrium supply responses of these industries in ORANI.

29 Note that equation (3.3) deals only with Armington substitution; the technology is otherwise Leontief. The comparative-statics analysis experiment involves holding the price index for each other material \( a \), \( S^{(2)}_{(1/a)} P_{(1/a)}^{(2)} + S^{(2)}_{(a/2)} P_{(a/2)}^{(2)} \), constant for \( a \neq j \).
30 If \( j \) is a single-product industry, \( p_{kj} \) is just the basic price of the commodity produced; if \( j \) is a multi-product industry, \( p_{kj} \) is defined by the left-hand side of DPSV equation (18.2).
31 To put it more bluntly, under the partial equilibrium approach all industries are ‘small’, and so effects (if any) on the economy-wide wage rate are negligible.
is always necessary. The optimum investment function is a function of time, as well as the current level of output. The optimum investment function is determined by the relationship between the rate of return on capital and the cost of capital. This relationship is given by the expression:

\[ r = \frac{\text{rate of return}}{\text{cost of capital}} \]

The rate of return is the difference between the revenue and the costs of production. The cost of capital is the interest rate on borrowed funds. The optimum investment function is the rate of investment that maximizes the rate of return. The optimum investment function is given by the expression:

\[ I = \frac{r}{c} \]

where \( I \) is the rate of investment, \( r \) is the rate of return, and \( c \) is the cost of capital.
of the general equilibrium mechanisms, and parameter values, embedded in ORANI. In the remainder of this paper we focus exclusively on the supply side of the model, but it must be remembered that the large output response of mining industries could have other causes.

In ORANI, mining industries (like all others) exhibit constant returns to scale (CRS). Since these industries face fairly flat export demand schedules, in any environment in which no inputs are held fixed, changes in the prices of their products lead to large output responses. One interpretation of the criticism levelled at long-run ORANI simulations is that mining industries do not in fact exhibit CRS, and hence the current ORANI results overstate the flexibility of mining. In the remainder of this section we examine a number of factors with potential to limit this flexibility.

2.1 Non-constant returns - definitions

We start with a definition. Let \( x_v \) be the set of factors explicitly recognized in the model which can be varied by the firm over the length of run being studied. Notation for other factors is as shown in Table 1.

The set \( x_F \), for example, might contain the firm's capital stock, while \( x_F \) might contain its stock of entrepreneurial ability. \( x_v \) might include the usage of water available from a source owned by the firm, and (at least initially) having excess capacity.

Some care is needed below in preserving the distinction between constant returns, and constant returns to scale. The latter

\[
\pi_j = \pi_j^* + a_j^{(2)}.
\]

(\( \pi \in M^1 \))

in which the technological term \( a_j^{(2)} \) is to be endogenized by a new equation:

\[
a_j^{(2)} = H_j y_j.
\]

(\( \pi \in M^1 \))

where \( H_j \) is a non-negative parameter and \( y_j \) is the net addition to installed capacity in industry \( j \). Setting \( H_j = 0 \) eliminates adjustment costs from the model.

Turning now to the demand equations for inputs to capital creation, we restate equation (13.4) of DPSV, but suppress all those technological change terms which will be set exogenously to zero (i.e., all such terms other than \( a_j^{(2)} \)):

\[
x_{[is]}^{(2)} = y_j - \sigma_{ij}^{(2)} \left[ p_{[is]} 2 s=1 S_{[is]}^{(2)} p_{[is]}^{(2)} \right] + a_j^{(2)}.
\]

The interpretation here is that the first two terms on the right-hand side of (3.3) give the demand for input \( x_{[is]} \) by industry \( j \) for the assembly of units of capital: the extra amount \( a_j^{(2)} \) is used up in converting the assembled unit to installed capacity. Using (3.2) we rewrite (3.3) as:

\[
x_{[is]}^{(2)} = y_j [1 + H_j] - \sigma_{ij}^{(2)} \left[ p_{[is]} 2 s=1 S_{[is]}^{(2)} p_{[is]}^{(2)} \right] .
\]

At fixed relative input prices a one per cent increase in investment leads to a \( (1 + H_j) \) per cent increase in demand for inputs to capital creation: hence \( H_j \) may be interpreted as the additional percentage

---

7 Unless otherwise stated, factor throughout this paper means an input to production, not necessarily a primary factor.
The following new equation:

The insured cost of a unit of capacity \( f \) would be expressed by some \( y \) in the creation of capital goods if \( y \) would be replaced by

in Equation (13.6) or DEPS (which echoes zero)

Define \( f \) to be the cost of assembling a unit of capacity in

inaccurately directed into ORGN (calculated via the variable)

where the distinction will allow production costs to be

and the marginal cost of such a manufactured unit of insured capacity.

We, however, have no idea what the cost of the entire system is or what the cost is of each unit of capital.

in the case of mining industries, \( f \) may no longer mean the

the cost of a unit of insured capacity.

and

the demands for inputs to capital creation (calibration)

appearing in the equations pertaining to:

The constant capital introduction will be expressed via the variable \( y \)

means a unit of insured capacity: we preserve this convention

introduced we show standard ORGN in the equation of

Table 1

<table>
<thead>
<tr>
<th>( x^y )</th>
<th>( x^p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implicit</td>
<td>Production function</td>
</tr>
<tr>
<td>( x^y )</td>
<td>( x^p )</td>
</tr>
<tr>
<td>Explicitly recognized in</td>
<td></td>
</tr>
<tr>
<td>Fixed</td>
<td>Variate</td>
</tr>
<tr>
<td>Over the length of a run studied, the</td>
<td></td>
</tr>
</tbody>
</table>

These rules serve to define a length of run. To avoid the cumbersome on the rules concerning which factors may and may not be varied, will be defined in a way that depends only on the technology, and not...
with the fixed factors set at their initial values, and the implicit variable factor \( x_v \) changing with \( x_r \) in such a way as to leave the shadow price of \( x_v \) unaffected.\(^8\) Then we say that the firm faces globally constant returns over the T-run if

\[
g(\lambda x_v) = \lambda g(x_v)
\]

for any positive multiplier \( \lambda. \)^9

By constant returns to scale we mean that the function \( F \) in (2.1) is homogeneous of first degree. We immediately note that if \( F \) is a homogeneous function, \( g \) cannot display constant returns in the T-run unless \( F \) displays increasing returns to scale.\(^{10}\) We do not append a qualifier to specify the length of run when describing scale effects, it being understood that the term 'returns to scale' always refers to a run of sufficient length to allow arbitrary variation in all the arguments of \( F \); doubling them, therefore, is just doubling the scale of the process producing \( y \).

What if \( F \) exhibits constant returns to scale? At this point we must decide whether or not the time should be recognised as an input, either within \( x_v \) or \( x_F \). The neoclassical representation (2.1) of production is timeless; in the real world, however, output takes

---

8 The simplest case is the one in which \( x_v \) is a free good; in that case its shadow price is globally zero, and it may be deleted from the discussion.

9 If (2.3) is true for \( 1 = 1 + e \) (where \( e \) is small; \( e = 0.05 \), say) at some particular value of \( x_v \), but not at others, we say that the firm faces locally constant returns at the variable input level \( x_v \).

10 A function displays increasing (decreasing) returns to scale if it is homogeneous of degree \( v \), where \( v \) exceeds (is less than) one.

---

ORANI's structure. Like agricultural land, the mining fixed factors are exogenous in such closures. Note that while there are just six types of agricultural land in ORANI, there will be one new fixed factor for each mining industry. These new fixed factors might be visualized as stocks whose service flows correspond to the 'facilitation of adjustment' in the different mining industries.

The new factors must be paid a rent. This rent must

(i) be large enough, in the base-period data base, to give a cost share which generates the 'correct' (i.e., target) medium-run supply elasticity;

(ii) be small enough, and distributed in a sufficiently neutral way, not to generate large income effects in the solution year.

It is crucial to get (i) right. If this is in serious tension with (ii), then more radical approaches to restructuring the model may be required.

Neither of these first two approaches, however, seems to be nearly as promising as the third, which we now discuss in some detail.

3.2 Endogenous technological regress

As is the case with the two alternatives mentioned above, we assume here that the closure will be a modification of the Horridge closure.\(^{23}\) In the construction below we will be invoking general-equilibrium properties of the economic environment defined by such a closure to evaluate the partial-equilibrium supply elasticities of mining industries within that environment. These will be shown to depend principally on cost shares, the capital-labour substitution elasticity and on the adjustment-cost parameter.

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23 See the reference cited in footnote 1.
functions of time rather than output.

Consider a firm in the long run. If there are no costs which are incurred in the production period to which this fixed cost applies, the firm may exhibit CRTS on the above definition. But if there are additional costs (diminishing returns) because the firm is experiencing decreasing returns then the firm may not exhibit CFTS on the above definition, but the corresponding function with the same definition of CFTS is homogeneous of degree 1 under the CRTS definition in (2.1).

Notice then that this is an essential difference between the function with which we are concerned in this paper and the definition of CRTS as given in Chapter 2.

The second method appears more hopeful. The second method is normally explained under an implicit fixed factor, a point that we shall return to later. However, we briefly discuss the approach to the phenomenon of non-conventional technological progress and its effect on the correspondence between the CRTS and CFTS functions. We find it difficult to explain how a factor of production can be lost without affecting the demand for other factors, and we must therefore drop this method.
model lead to lower supply elasticities than apply under price-taking behaviour.

3. Controlling Medium-run Mining Responses in ORANI

We assume here that time and resource constraints will prevent the construction of detailed intertemporal models for the mining industries. Furthermore, we also assume that an estimate is available for each mining industry's own price supply elasticity in the medium run (say $T = 5, 10$ or $15$ years). Sources for such an estimate might be:

- a literature search,
- applied econometric work,
- a synthesis based on a ‘feeling’ for the approximate values of key parameters.\(^{19}\)

3.1 Three possible approaches

Three possible approaches towards enforcing the medium-run supply elasticities are:

(i) the abandonment of CRTS technology in mining in favour of globally diminishing returns to scale;
(ii) the inclusion of a new factor which would be held fixed in medium-run simulations;

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\(^{19}\) In Wilcoxen, "Supply Elasticities in the Presence of Adjustment Costs", op. cit., it is shown how supply elasticities may be calculated from a knowledge of (i) the length of run $T$; (ii) the capital intensity of production; (iii) the elasticity of substitution between capital and variable inputs; and (iv) the adjustment cost parameter.
The textbook "X" shape.

Definition: The text explains the concept of a "V" shape in the context of economics, where the demand curve is upward sloping, and the supply curve is downward sloping. This concept is crucial in understanding market dynamics and price determination.

Identify: The text identifies the relationship between demand and supply, explaining how changes in one affect the other.

Interpret: The text interprets the economic implications of the "V" shape, highlighting its relevance in understanding market forces and decision-making processes.

Conclude: The text concludes by summarizing the importance of understanding the "V" shape in economics, emphasizing its role in market analysis and policy formulation.

2.2 Sources of non-consensus returns

The text discusses various sources of non-consensus returns and how they contribute to market inefficiencies and investment opportunities. This section provides insights into the factors that can lead to market anomalies and how investors can capitalize on them.

2.3 Market structure

The text explores different types of market structures, including perfect competition, monopoly, monopolistic competition, and oligopoly. Each structure is analyzed based on its impact on market outcomes and consumer welfare.

Factors associated with expansion:

1. Large scale economies
2. Economies of scope
3. Natural monopolies
4. Competition from substitutes

2.4 Market failures

The text discusses various market failures, such as externalities, public goods, and market power, and how they lead to inefficiencies in the market. The analysis provides a framework for understanding how these failures can be mitigated through government intervention or market mechanisms.

3. Macroeconomic issues

The text addresses key macroeconomic issues, including inflation, unemployment, and economic growth. It provides a comprehensive analysis of these issues and their implications for policymakers and businesses.

4. The international market for Aluminium: in particular, its demand by A.F. March 1990

The text examines the demand for aluminium, discussing factors affecting its demand and supply. It includes a discussion on market trends and future prospects for the aluminium industry.
Whilst diminishing returns to scale cannot be ruled out a priori, most attempts to explain their genesis boil down to an implicit factor fixity (usually relating to management).

c. **Progressive degradation of the ore body** If the cost of accessing and/or extracting the mineral increases with cumulative output, then (2.1) is no longer valid as a representation of the production process; instead we need:

\[
(2.4) \quad q = H \left( x_F, x_V, \sum_{\text{all lags}} (\text{lagged } q) \right).
\]

In this case the average cost function shifts progressively up as mining proceeds irrespective of whether average costs are increasing, decreasing or constant with respect to the current output rate \( y \).\(^{14}\) This is the situation modelled by Blampied, Horridge and Powell.\(^{15}\) Whilst relevant to many mining activities, this set of circumstances seems unlikely to apply to Australia’s most important mineral exports -- black coal, alumina (refined bauxite) and iron ore.

d. **Adjustment costs** In intertemporal approaches to modelling production, besides current inputs and outputs we need also to consider the augmentation of capacity over time and the set of expectations about future variables driving this accumulation. An alternative to (2.4) which may lead to diminishing returns in runs other than the long run is:

\[
(2.5) \quad \begin{pmatrix} q \\ x_F \end{pmatrix} = G^*(\Omega),
\]

where (for simplicity) we have assumed that all factors (with the possible exception of time) are explicit, and where \( I \) indicates investment in the current period, whilst \( \Omega \) is the information set upon which intertemporal planning decisions are based. To further simplify notation (but without real loss of generality), we will assume that each of the variables on the left of (2.5) is a scalar.

If constant returns to scale in production apply then the system (2.5) is underdetermined; that is, nothing in the technology itself will determine the scale of output.\(^{16}\) Suppose we are interested in a T-run over which \( x_F \) cannot be varied; this will result in output and investment becoming determinate:

\[
(2.6) \quad \begin{pmatrix} q \\ x_V \end{pmatrix} = G^*(x_F; \Omega),
\]

Suppose, however that there is no fixed factor over the T-run; then (2.6) becomes:

\[
(2.7) \quad \begin{pmatrix} q \\ x_V \end{pmatrix} = G(\Omega).
\]

\(^{14}\) Situations like the one discussed here are best handled in continuous time; \( y \) becomes the instantaneous time rate of output.


\(^{16}\) We are assuming that the firm is a price taker.