Solving intertemporal CGE model in parallel using Singly Bordered Block Diagonal ordering technique*

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Contents

INTRODUCTION

The Rationale

- Inter-temporal CGE models are big and difficult to solve.
- Solution to CGE model usually involves solution of a big first order derivative matrix of a non-linear system.
- The most popular CGE software packages in the market now are GEMPACK and GAMS, but they rely on serial matrix solvers, which have limited power in solving very big models.
- The paper proposes a direct reordering method for the first order differential matrices arising from the CGE models' solution. The ordering method facilitates parallel solution of the matrices and, therefore, reduces computing time for the solution of inter-temporal CGE models.

SOLUTION METHODS FOR CGE MODELS

The GEMPACK's linearize method

- The GEMPACK's linearize method involves linear approximation and Richardson extrapolation.

$$x_1 = x_0 + f_x^{-1} f_y dy \tag{1}$$

- GEMPACK use the direct serial solver MA48 or MA28 from The HSL Mathematical Software Library (see HSL, 2013).
- More accurate result can be obtained by "chopping" down the shock.
- Advantage of the method is faster solution speed and the user can control the speed and accuracy by changing the number of sub-steps.
- The downside is no convergence guarantee, the user should check for convergence of the solution.

The GAMS's iterative methods

- GAMS is a flexible system and how it solve the model depends on the solver involved.
- PATH (Ferris and Munson, n.d.; Dirkse and Ferris, 1995) and MILES Rutherford (n.d.) or optimiser MINOS (Bruce et al., n.d.) are the usual choices for CGE modelers.
- PATH is Newton-based solver. PATH uses LUSOL (Saunders et al., 2013) as a linear system solver to solve the system involving Jacobian matrix. LUSOL is a serial direct linear system solver.
- MINOS is an optimisation solver, CGE model can be solved by setting-up non-linear constraints and a dummy objective function.
- MINOS also uses LUSOL for Jacobian matrix solution.

Parallel solution

- To our knowledge, current CGE software packages use serial solver to solve big linear system.
- The idea of parallel solution in CGE modelling has been introduced in GEMPACK version 10, but the parallel resources are used only to solve different steps at the same time, not for joint solution of a linear system.
- The parallel software libraries are available but they can not employ special feature of CGE models to solve efficiently.
- We will introduce Singly Bordered Block Diagonal ordering method to solve inter-temporal CGE models efficiently.

SBBD MATRIX AND DIRECT METHOD FOR SOLVING LINEAR SYSTEM

SBBD matrix

- The Singly Bordered Block Diagonal matrix:

$$\begin{pmatrix} A_1 & & & & C_1 \\ & A_2 & & & C_2 \\ & & \dots & & \dots \\ & & & A_K & C_K \end{pmatrix} \tag{2}$$

- The linear equation system will have the form:

$$\begin{pmatrix} A_1 & & & & C_1 \\ & A_2 & & & C_2 \\ & & \dots & & \dots \\ & & & A_K & C_K \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ . \\ x_K \\ x_L \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ . \\ b_K \end{pmatrix} \tag{3}$$

How it can be solved fast in parallel

- Factorisation individual sub-matrices (see Duff and Scott, 2004, for more details):

$$\begin{pmatrix} A_i & C_i \end{pmatrix} = P_i \begin{pmatrix} L_i & \\ \widetilde{L}_i & I \end{pmatrix} \begin{pmatrix} U_i & \widetilde{U}_i \\ & S_i \end{pmatrix} Q_i \tag{4}$$

- Forward elimination:

$$P_i \begin{pmatrix} L_i & \\ \widetilde{L}_i & I \end{pmatrix} \begin{pmatrix} y_i \\ \widetilde{y}_i \end{pmatrix} = \begin{pmatrix} \hat{b}_i \\ \widetilde{b}_i \end{pmatrix} \tag{5}$$

- And solve interface problem: $\widetilde{y}_i$ will be summing-up to form the right hand side $\widetilde{y}_L$ and S_i to S.

$$Sx_L = \widetilde{y}_L \tag{6}$$

- Backward substitution:

$$U_i Q_i x_i = y_i - \widetilde{U}_i Q_i x_L \tag{7}$$

INTER-TEMPORAL CGE MODEL AND SBBD FORM

Inter-temporal CGE model

$$
\begin{aligned}
x_t &= f(x_t, z_t, \lambda_t, y_l) && (8)\\
z_t &= k(x_t, z_t, \lambda_t, y_l) && (9)\\
\dot{\lambda} &= h_t(x_t, z_t, \lambda_t, y_l) && (10)\\
y_l &= g(z_t, \lambda_t, y_l) && (11)
\end{aligned}
$$

First order differential matrix and SBBD form

$$\begin{pmatrix} f^1_{x_t} & f^1_{z_t} & f^1_{\lambda_t} & f^1_{y_l} \\ k^1_{x_t} & k^1_{z_t} & k^1_{\lambda_t} & h^1_{y_l} \\ h^1_{x_t} & h^1_{z_t} & h^1_{\lambda_t} & h^1_{y_l} \\ 0 & g^1_{z_t} & g^1_{\lambda_t} & g^1_{y_l} \end{pmatrix} \tag{12}$$

NUMERICAL ANALYSIS

Vietnamese regional inter-temporal CGE model

Table 1: Model descriptions

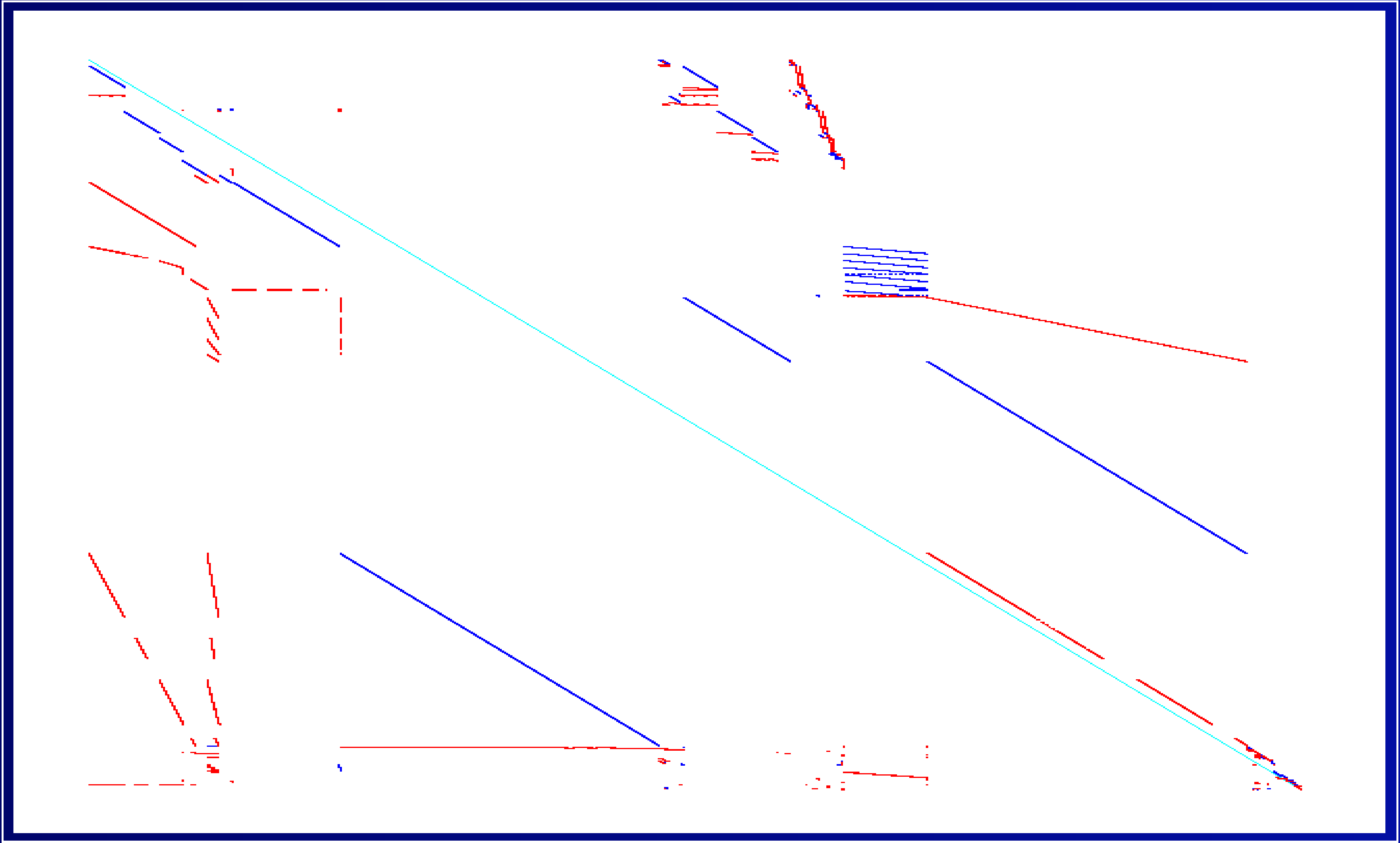
ID	Mode's Size	Number of endogenous variables	Number of exogenous variables	Number of non-zeros
1	3 sectors, 3 commodities, 8 regions and 10 time periods	246833	55467	779466
2	3 sectors, 3 commodities, 8 regions and 50 time periods	1144433	257067	3614846
3	8 sectors, 8 commodities, 8 regions and 50 time periods	5720368	1298272	16806197
4	28 sectors,28 commodities, 8 regions and 20 time periods	26422686	7944170	70680341

Source: Author's calculation.

Note: The Vietnamese CGE model is from (Ha and Kompas, 2009).

Direct ordering vs no ordering

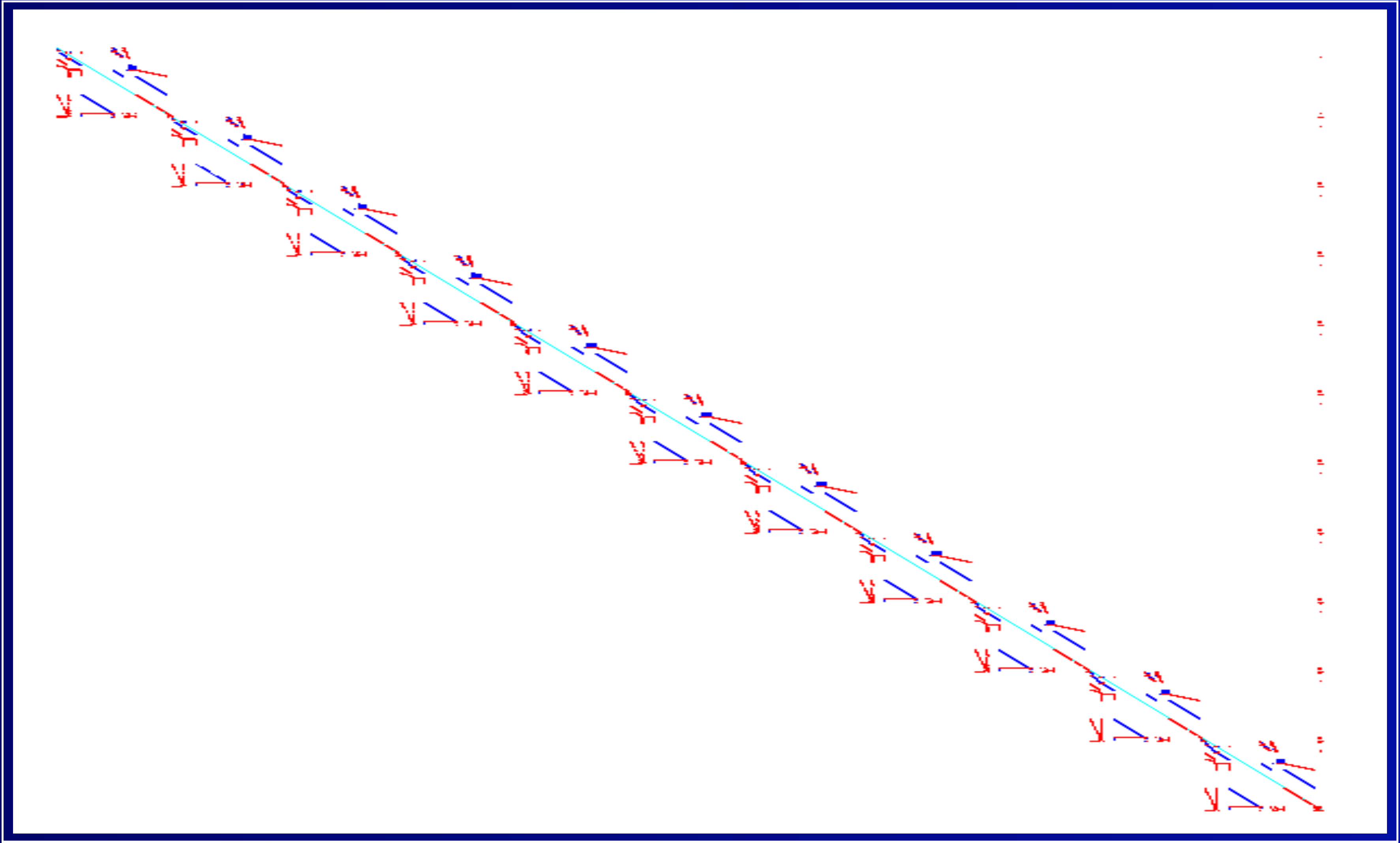
Figure 1: The matrix without ordering.



Source: Author's calculation.
Note: Matrix drawing function from PETSC (Balay et al., 1997, 2012b,a).

Direct ordering vs no ordering

Figure 2: The reordered matrix.



Source: Author's calculation.
Note: Matrix drawing function from PETSC (Balay et al., 1997, 2012b,a).

Direct ordering vs automatic ordering (MC66)

Table 2: Matrix ordering

		Netcut		Row Difference	
ID	No. of blocks	HSL_MC66	Direct reordering	HSL_MC66	Direct reordering
1	11	595	719	0.01%	0.00%
2	51	3011	3359	1.01%	0.00%
3	51	26563	7399	62.94%	0.00%
4	21	failed	9579	failed	0.00%

Source: Author's calculation.

Note: HSL_MC66 is an ordering function from HSL (2013).

Serial computing performance

Table 3: Calculation time in sec.

ID	HSL_MC66 + HSL_MP48	Direct reordering + HSL_MP48	MA48
1	4.517662	2.111207	1.533605
2	30.507594	10.647396	21.338317
3	245.775494	116.342514	541.252467
4	failed	2090.518025	13664.878297

Source: Author's calculation.

Note: HSL_MC66, HSL_MP48, and MA48 are from HSL (2013).

Parallel computing performance

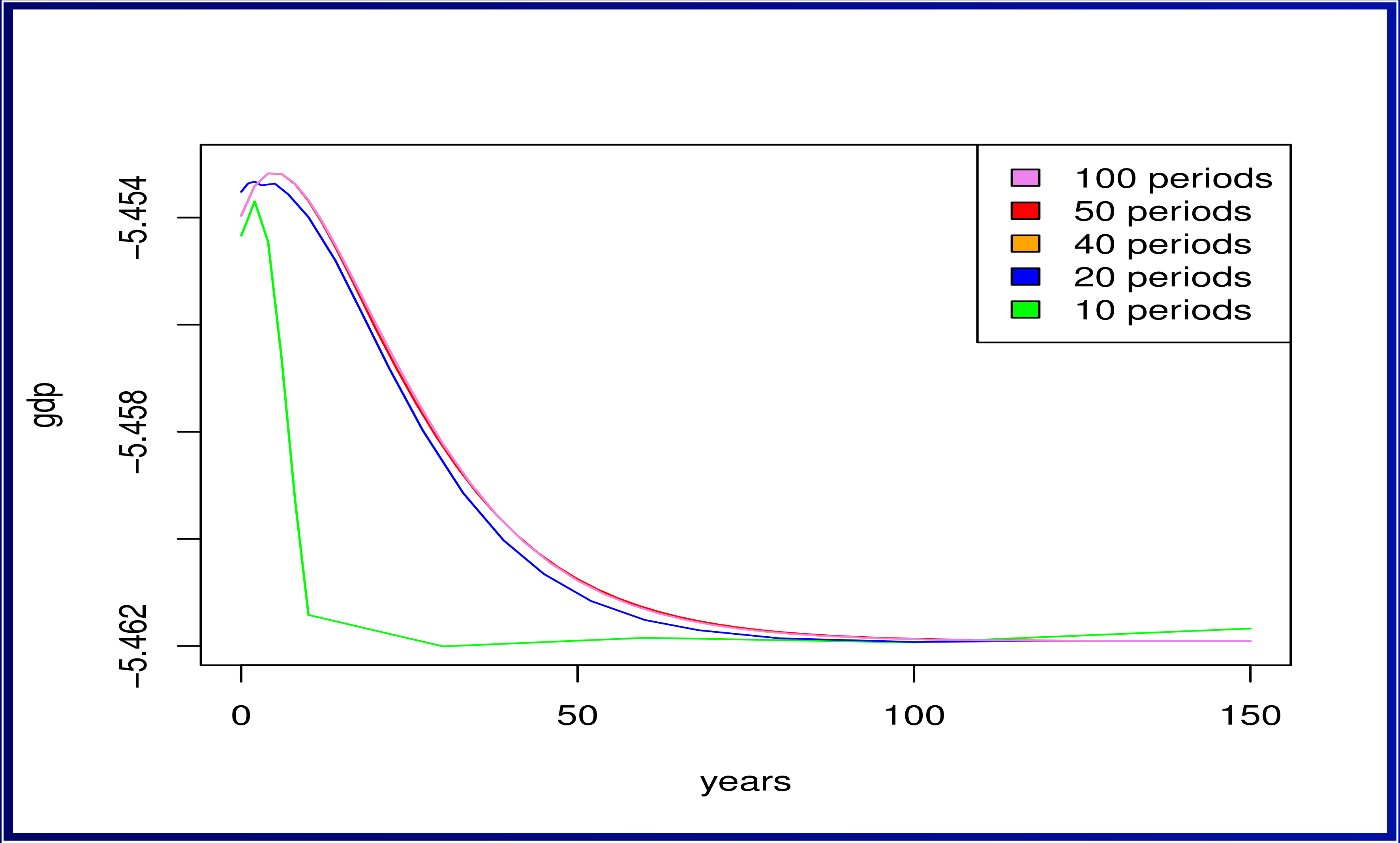
Table 4: Parallel computing performance.

	3 processes on machine 1	4 machines and 4 processes	4 machines and 6 processes	4 machines and 10 processes
1	1.353008	0.740006	0.666306	0.791391
2	6.719306	4.142068	3.615683	3.855334
3	67.698225	37.883721	31.359824	36.274224
4	1226.153309	635.964948	450.738670	528.355081

Source: Author's calculation.

The Accuracy of Finite Difference Method in Inter-temporal CGE Model

Figure 3: The finite difference grid and calculation accuracy (% change in GDP).



Source: Author's calculation.

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CONCLUSION

Concluding remarks

- The direct reordering matrix into SBBD form method proves to be very efficient in solving inter-temporal CGE model in parallel.
- The efficiency of the method depends on the interface matrix. Researchers can try to substitute out variable y_l to reduce the interface problem.
- Nevertheless, the size of interface matrix will increase moderately as the finite grid size increases, hence our method will be efficient in solving inter-temporal CGE models.

REFERENCES

Cited References

Satish Balay, William D. Gropp, Lois Curfman McInnes, and Barry F. Smith. Efficient management of parallelism in object oriented numerical software libraries. In E. Arge, A. M. Bruaset, and H. P. Langtangen, editors, *Modern Software Tools in Scientific Computing*, pages 163–202. Birkhäuser Press, 1997

Satish Balay, Jed Brown, Kris Buschelman, Victor Eijkhout, William D. Gropp, Dinesh Kaushik, Matthew G. Knepley, Lois Curfman McInnes, Barry F. Smith, and Hong Zhang. PETSc users manual. Technical Report ANL-95/11 - Revision 3.3, Argonne National Laboratory, 2012a

Satish Balay, Jed Brown, Kris Buschelman, William D. Gropp, Dinesh Kaushik, Matthew G. Knepley, Lois Curfman McInnes, Barry F. Smith, and Hong Zhang. PETSc Web page, 2012b. http://www.mcs.anl.gov/petsc

A. Murtagh Bruce, A. Saunders Michael, Murray Walter, and E. Gill Philip. Minos. MINOS Solver Manual, http://www.gams.com/solvers/solvers.htm, Accessed date: 5 April 2013, n.d

Steven P. Dirkse and Michael C. Ferris. The path solver: A non-monotone stabilization scheme for mixed complementarity problems. *Optimization Methods and Software*, 5:123–156, 1995

Michael C. Ferris and Todd S. Munson. Path 4.7. PATH Solver Manual, http://www.gams.com/dd/docs/solvers/path.pdf, Accessed date: 3 October 2013, n.d

Iain S. Duff and Jennifer A. Scott. A parallel direct solver for large sparse highly unsymmetric linear systems. *ACM Trans. Math. Softw.*, 30(2): 95–117, June 2004. ISSN 0098-3500. doi: 10.1145/992200.992201. URL `http://doi.acm.org/10.1145/992200.992201`

Pham Van Ha and Tom Kompas. Xay dung mo hinh can bang tong the dong lien vung cho nen kinh te Viet Nam. Chuyen de nghien cuu cap Vien, Vien Khoa hoc Tai chinh, Hoc vien Tai chinh, Bo Tai chinh, 2009

HSL. A collection of fortran codes for large scale scientific computation. http://www.hsl.rl.ac.uk, 2013

Michael Saunders, Philip Gill, Walter Murray, Margaret Wright, Michael O'Sullivan, Kjell Eikland, Yin Zhang, and Nick Henderson. Lusol: Sparse lu for ax = b. Dept of Management Science and Engineering (MS&E), Stanford University, Online at: http://www.stanford.edu/group/SOL/software/lusol.html, Accessed: 6 April 2013, 2013

Thomas F. Rutherford. Miles. MILES Solver Manual, http://www.gams.com/dd/docs/solvers/miles.pdf, Accessed date: 3 October 2013, n.d