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# An iterative method for solving CGE models

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This paper presents a solution method for computable general equilibrium (CGE) models, an alternative to the conventional methods based on matrix inversion and mathematical solvers. This method uses iterative price adjustments to equate supplies and demands in markets. It is intuitively simple because it resembles a bargaining process, such as an auction, in which different prices are tried in a structured way until the quantities supplied and demanded clear. This paper explains how such an iterative search method works and how it can be applied to solving CGE models. The paper uses a stylised CGE model to illustrate the procedure, starting by setting up the equation system, and solving it using the iterative method. The paper is concluded with a summary of the key steps required for this procedure and a discussion of the benefits of using this method for CGE models.

## Solving CGE models

CGE models have been widely used as a tool for policy analysis. These models include sets of variables and equations that describe the behaviours of economic agents, and those agents' relationships with each other. The models are calibrated using a database assembled from national accounts and trade statistics. The models' equations are then treated as a simultaneous equation system (SES) and solved using mathematical and computational methods.

Algorithms for solving SES problems are well developed. In the early stage of CGE model development, however, due to the constraint of computing capacity, many of these methods were not readily available to model users. In the 1960s, a Norwegian economist, Leif Johansen, pioneered a method that could solve large scale CGE models as a linear equation system (Johansen, 1960). His method was adopted by the Centre of Policy Studies to develop its GEMPACK, a specialised software suite for building and solving CGE models (Cods

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and Pearson, 1988). This software suite, together with its linearised solution method, became a popular solution tool for CGE model users around world.

In recent years, as personal computers became faster and more powerful, non-linear solution methods have attracted more CGE model users. The algorithms used to solve non-linear simultaneous equation systems, are readily available from many popular general mathematical software suites, such as GAMS and Matlab. These software suites have become popular among CGE modellers in recent years.<sup>1</sup>

Solving CGE models mathematically as a *linear* SES has advantages.<sup>2</sup> It is easy to know if such a model has a unique feasible solution so long as the number of endogenous variables equal the number of equations. This is because, such a general equilibrium model can be represented by a matrix of  $m$  equations and  $n$  variables. Of all the variables,  $m$  is the number of endogenous variables and  $m - n$  is that of exogenous variables. Exogenous variables are set and fixed by the model operator, while endogenous variables are solved to determine the equilibrium. When all exogenous variables are given, the solution of the model is just the equilibrium values of all the endogenous variables, which can be found by inverting the  $m \times m$  matrix.<sup>3</sup> Solving CGE models in this way implicitly treats all endogenous variables indiscriminately: the equilibrium value of every endogenous variable, no matter what they are, is determined in a similar fashion by the equilibrium values of all the other variables simultaneously. So long as the numbers of endogenous variables and equations are equal, the model should have a unique solution, no matter whether it is the right solution that conforms with economic theory. This is because the SES approach treats a CGE model as a generic mathematical problem. The meaning of its internal structure makes no difference to this solution method, equations are not linked in any way to particular variables.<sup>4</sup>

Solving CGE models as large sets of simultaneous equations without regard to the specific equations driving each variable contributes to the image of CGE models as a “black box”. As Horridge and Pearson (2011) indicate, it was the intension of the authors of these modelling packages to insulate the model users from solution methods to allow them to concentrate their efforts on model theory. One way to open this “black box” and demystify the model solution process is to move away from the math method and search for a natural solution that comes from the internal structure of CGE models themselves.

Every CGE model consists of markets for goods and for factors of production, and the clearing these markets by a set of equilibrium prices leads to the solution. In each of these markets, the quantities supplied and demanded adjust to the price of the product. The model theory uses supply and demand functions to describe the responses of market participants to

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<sup>1</sup> Horridge and Pearson (2011) provides a useful and informative comparison between two different solution methods, linear and non-linear, represented by two popular software suites GEMPACK and GAMS.

<sup>2</sup> Some CGE models are linearised to facilitate their solution.

<sup>3</sup> In practice, the solutions of large equation systems could be approximated by some iterative methods, such as the Jacobi or Gauss-Seidel methods. However, it will become clear in the following sections that these are not the iterative method discussed in this paper for solving CGE models.

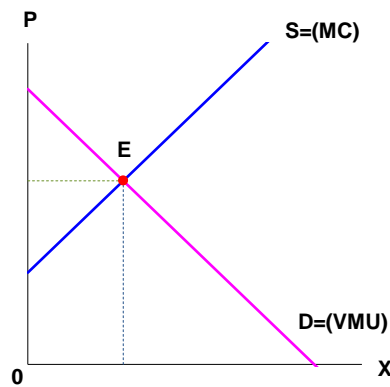
<sup>4</sup> That said, some modellers write equations so as to define certain variables as a matter of convenience.

the changes in the market price. Although it is outside the control of the buyer and seller to set the price in a competitive market, the market price of a product is influenced by their separate supply and demand functions. The final transaction could be the result of a bargaining process, in which a mutually agreed price level is reached to satisfy both parties. This bargaining process can be seen as a solution method to solve for an equilibrium price, which ensures the equality of supply and demand in the market. This solution method involves a mechanism that iteratively adjusts a price according to the excess demand in a market toward its equilibrium.

### Iteratively searching for an equilibrium price

For a market price to be solved iteratively, there must be a separation of the buyer and the seller in the market. If they are price takers, independently deciding their buying and selling in response to a given market price, an iterative method can be introduced to adjust the price to mitigate the gap between supply and demand. A single-good market is illustrated as an example (figure 1). In this market, the supply is determined by the marginal cost (MC) of its production, which is depicted as an upward-sloping line. The demand for the product is determined by the value of marginal utility of consumption (VMU) of the good, which is depicted as a downward-sloping line.

Figure 1      **Supply and demand in the market of a product with DRTS**



Notes: DRTS denotes diminishing return to scale.

Figure 1 is used to represent a single market with two actors. The upward sloping supply curve implies diminishing returns to scale (DRTS) technology in production. If a production function displays such a technology, a profit-maximising supply function can be derived for the seller of the product. Since the supply and demand decisions are made separately by the seller and the buyer, an independent market price is required to reduce the difference between supply and demand. For example, a bargaining process may be used to adjust the price according to the gap between the two curves in this market. If the initial price is higher than the equilibrium level, the iterative process should drive the price downward. The equilibrium is reached at point E, where the supply meets the demand to clear the market. The adjustment is an iterative interaction between the price and the quantities of supply and

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demand. If both parties act according to the shapes of the supply and demand curves that summarise their behaviours, the convergence of the price toward an equilibrium level should be guaranteed.

This simple example illustrates that iteration is a natural way of finding an equilibrium price because it is an imitation of the bargaining process in marketplace. However, this solution method requires iteratively searching for the equilibrium price in each market individually. If all goods and factor prices in a CGE model need to be solved iteratively, this solution method could be time-consuming. To apply this iterative method, the number of variables required to be solved should be kept to a minimum. Otherwise, this method could be too costly to be used as a solution method for most CGE models.

### **Iteration as a solution method for CGE models: what is required?**

CGE models have many variables and equations. However, the variables and equations responsible for its general equilibrium solution normally account for a small portion of their total number. We refer to this group as *CORE* variables and equations in the remainder of this paper.<sup>5</sup>

The solution of a CGE model lies in a set of general equilibrium prices required to clear their markets. There are many goods and factors in CGE models. If every good or factor has an independently determined supply and demand function, as shown in figure 1, their equilibrium price can only be determined through bargaining between its seller and buyer in the market. In other words, every price needs to be solved iteratively to obtain a full solution for the CGE model.

Fortunately, this is not the case for all goods in many CGE models. This is because, unlike the endowments of factors of production, goods in CGE models are produced by firms using intermediate inputs and factors. The technology is described by production functions. These include popular functional forms, such as Leontief and constant elasticity of substitution (CES) production functions. These functions exhibit constant returns to scale (CRTS) technology, which has two properties that are useful for simplifying the solution of CGE models.

The first property is constant marginal cost (CMC) in the production of goods. This means that regardless of the level of output, the cost to produce a unit of output in an industry is the same (that is, supply curves are neither upward nor downward sloping). The second property is the zero profit condition (ZPC). This means that the price of a product is equal to the unit cost of all the inputs used in its production, including normal returns to capital. In other words, the price of the product can be simply calculated if the cost of its inputs and the

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<sup>5</sup> All other variables and equations are used mainly for presentation purposes, such as various price indexes or quantity aggregates. As these variables and equations are irrelevant to a model's solution, they are not discussed in this paper.

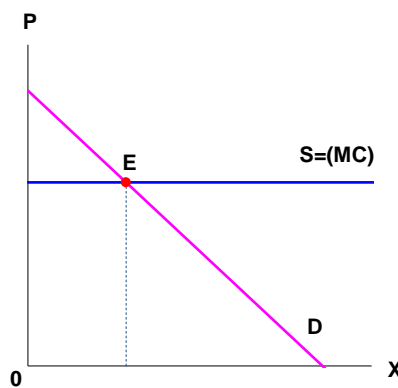
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quantity of its output are known. These two properties imply that, with a given price, the supply of a product is determined solely by its demand.

The market equilibrium for a CRTS product can be shown in a single-good market (Figure 2). The only difference between Figures 2 and 1 is the shape of the supply curve. Under CRTS technology, an increase in the supply does not raise the marginal cost because it is given by the unit cost of its production. For any given price, the output supply is determined by its demand. With given input prices, the marginal cost of a product does not change with its demand. In other words, supply can satisfy any demand at a constant price.

**Figure 2      Supply and demand in the market of a product with CRTS**

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Notes: CRTS denotes constant return to scale.

Under CRTS technology, the price of each product can be calculated as a function of its total cost, divided by the quantity of its supply. As the unit cost can be readily derived from the relevant cost components, the demand determines the quantity to be supplied and no iterations are required to determine their equilibrium.

Unlike goods, however, factors are usually endowed and not produced within the model. There is no technology involved in the supply of factors.<sup>6</sup> If factor supplies are given from outside the model, their supplies usually assumed to be constant, which can be depicted as a vertical curve in Figure 3. In this case, a factor's supply is independent from its demand, implying that a flexible price is required to adjust factor demand toward its given supply.

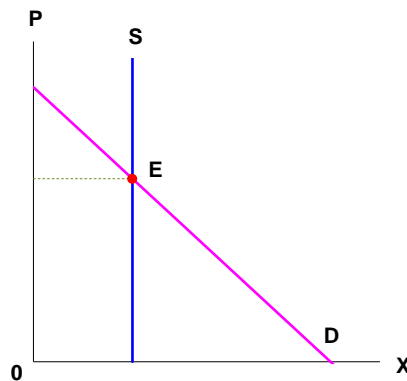
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<sup>6</sup> In a dynamic model, capital stock may be cumulated through annual investment. In a given period, however, the current capital stock is given and not affected by current investment.

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**Figure 3      Supply and demand in the market with a fixed factor**

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The final solution of a CGE model with CRTS technology can, therefore, be traced back to just a few factor prices. This implies that the number of endogenous variables, needed to be iteratively solved, can be reduced to just factor prices. Once these factor prices are solved, the equilibrium values for all goods prices and other endogenous variables can be derived from their core equations. Solve for a few factor prices through iterative adjustments could be an extremely simple but powerful method for solving CGE models.

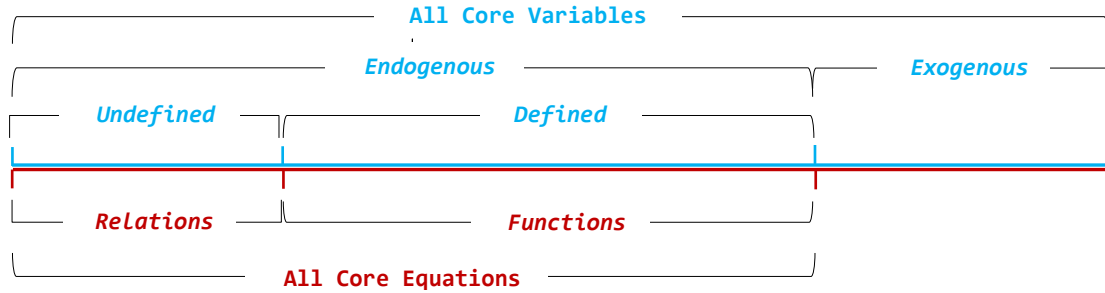
In the remainder of this paper, a stylised model and database will be used to illustrate the procedure of building a CGE model and using an iterative solution. The implementation of this solution method relies heavily on the transparency of the structure in its equation system, in which every variable has a meaning and every equation has a clearly defined role to play in the solution process. This clear structure allows model users to quickly locate the key variables that are required to be solved iteratively.

Before moving to the details of the model and database, it would be useful to have an overview in the next section of the general structure of the core equation system of a CGE model. Understanding such a system would shed a light on the internal working relations between model equations. CGE models and their solutions are not “black box”: they can be explained not only by their underlying theories, but also, more importantly, by their equation systems.

## General structure of the core equation system

A core equation system contains only the variables and equations that are essential for finding the model’s general equilibrium solution. The internal structure of the core equation system can be shown in figure 4. The horizontal axis of the figure depicts the numbers of variables and equations. The upper and the lower sides of the axis divides the variables and equations into groups.

Figure 4      **Types of core variables and equations in a CGE model**



There are two broad types of core variables in a CGE model: exogenous and endogenous. The values of the former are given from outside the model, while the values of the latter are determined within the model system. Most endogenous variables can be defined by an equation as a function of other variables and they may, therefore, be referred to as defined variables. A small group of endogenous variables cannot be defined as a function of any other variables. We refer to them as undefined variables. These undefined variables play a crucial role in solving CGE models.

There are two types of equations: one type is used to define endogenous variables as *functions* of other variables, while the other is used to specify *relations* between variables. The number of endogenous variables is equal to the number of core equations in the system, a necessary condition for a general equilibrium solution. Moreover, the number of defined variables matches the number of function equations. This is because each equation can define only one endogenous variable. Once all definable endogenous variables are exhausted in the core equation system, the remaining endogenous variables are the undefined ones. They cannot be defined as a function of other variables. Their equilibrium values are conditional on some general equilibrium conditions and can only be determined through interactions with those conditions. As each undefined variable is determined by its own general equilibrium condition, the number of undefined variables should also match the number of the equations of relations.

The core equation system can be used to reduce a CGE model to just the few equations that describe the equilibrium conditions for the undefined variables. This group of variable-equation pairs holds the key to the solution of the entire CGE model. On the left-hand side of these equations are those undefined variables because they are unknown to the system. On the right-hand side are all other variables, endogenous or exogenous, because they are all known to the system. Solving this system means finding the equilibrium values of the undefined variables. The core equation system provides all the required information to construct these few equations to solve the undefined variables.

The above structure of the core equation system demonstrates the roles that different variables play in solving a CGE model. It also shows the way in which model equations should be written. As most endogenous variables have clearly defined meanings in the model theory, they should be written explicitly as a function of their explanatory variables, as

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specified in the model theory. That is, only the variable to be defined should be placed on the left-hand side of an equation, while all other explanatory variables on the right-hand side.

To build a CGE model from scratch, one could start with those defined variables according to the model theory. Once all the endogenous variables that could be defined are included in the equation system, the remaining endogenous variables must be undefined. Each of those undefined variables should have its own equilibrium condition, which can be traced easily. For example, if a price variable cannot be defined, its corresponding equation must be a market clearing condition for that product. This is the equation that specifies the relation between supply and demand in equilibrium.

Once the undefined variables and their corresponding equations of relations are specified, the core equation system is complete. An iterative mechanism can then be introduced to solve this model. This solution method starts with a set of arbitrarily selected initial values for the undefined variables and gradually adjusts these variables according to their equations of relations that specify the equilibrium conditions. After each iteration, the results are fed back to the equation system to update the values of other endogenous variables and check the equilibrium conditions. This iterative process stops when the equilibrium conditions are satisfied. This implies that the undefined variables have converged to their equilibrium levels. The equilibrium values of the previously undefined variables can now be used to calculate the defined variables. This provides the general equilibrium solution for the entire model.

In the following four sections, a stylised CGE model will be presented as an example to illustrate a three-step procedure of building a CGE model suitable to be solved by the iterative method. The first section shows how the model's theory is formulated conceptually from the data. The second section builds the core equation system and how it can be solved using an iterative method. In the third section, the database is used to parametrise the core equation system initially and the calibration process consists of finding equilibrium prices that are consistent with the values in the initial database by using the iterative solution method, which produces quantity based shares. The fourth section shows the completed model ready for use in policy simulations.

## **Model data and interpretation**

The first step toward building a CGE model is an examination of its database. CGE models can have many forms, but they are all built on economic data, collected from national and international statistical sources. A model database is a snapshot of an economy or a group of economies, whose economic data are placed in a particular format. This format sets the boundary of the types of the model to be built. It is important for the model builder to understand the database so that a theoretical framework can be found to provide a logical interpretation of the economic activities that underpin the database. This database interpretation lays the foundation for the type of CGE model to be built.



## Model database

The CGE model used to illustrate the solution method in this paper is a single country open-economy model, with two single-product industries, two factors of production, and four final users. Despite its small size, this model has all the ingredients that a single country model should have and can, therefore, be used as a good example to represent many similar CGE models. This is a model of a one country with two goods and two factors. It is referred to as the 122 model in the following.

The model database is presented in the form of a social accounting matrix (SAM) in figure 5.<sup>7</sup> A SAM is a matrix representation of national accounts for a given country. It has a square table with equal numbers of rows and columns recording income and expenditures. The sums of the same rows and columns should be equal in equilibrium.

Figure 5 **Social Accounting Matrix for the 122-model database**

		BRD	MLK	CAP	LAB	PTX	TRF	HOU	GOV	INVBRD	INVMLK	EXT	
		1	2	3	4	5	6	7	8	9	10	11	
BRD	1	21	8					10.97	20	6.4	9.6	16.03	92
MLK	2	17	9					16.45	15	6	9	16.55	89
CAP	3	20	30										50
LAB	4	15	25										40
PTX	5	5	4										9
TRF	6	1	2										3
HOU	7			27.42	40								67.42
GOV	8					9	3	23					35
INVBRD	9							6.8				5.6	12.4
INVMLK	10							10.2				8.4	18.6
EXT	11	13	11	22.58									46.58
		92	89	50	40	9	3	67.42	35	12.4	18.6	46.58	

Notes: BRD: bread; MLK: milk; CAP: capital; LAB: labour; PTX: production tax; TRF: import tariff; HOU: household; GOV: government; INVBRD: investment in bread; INVMLK: investment in milk; EXT: external account.

Source: Hosoe (2004).

The economy has two industries, one producing bread (BRD) and one milk (MLK). Firms use intermediate inputs (BRD, MLK) and capital (CAP) and labour (LAB) to produce the two goods (Columns 1 and 2). The total outputs are used for domestic consumption (HOU, GOV), investment (INVBRD, INVMLK), and export (EXT) (Rows 1 and 2). The country also imports foreign bread and milk (Cells 11:1 and 11:2). The country uses also foreign capital in its production and therefore sends income to their foreign owner (Cell 11:3). Row and Column 11 (EXT) show the information from external accounts in the balance of payments. The other rows and columns show the information from internal accounts.

<sup>7</sup> The author thanks John Salerian for introducing this SAM database. It was used in Salerian and Murray (2019), originally sourced from Hosoe (2004).

In addition to producers, there are two other users of the products in the economy: household (HOU) and government (GOV). Household receives factor income (Row 7) to pay for its consumption (Column 7). Government expenditure (Column 8) is funded by taxes on income, output production and imports (Row 8). The two goods are also used as capital goods to invest in the two industries (INVBRD, INVMLK), which are financed by household saving (Cells 9:7 and 10:7) and foreign investment (Cells 9:11 and 10:11).

## Interpretation of the model's database

A SAM can be interpreted to show the production of goods and the distributions of income between various economic sectors and agents. Although they are usually compiled by model developers, the SAM data are originally sourced from official statistics, such as national accounts and input-output (I-O) tables. An I-O table describes the production and primary distribution of a country's national income by sector and user. It forms the core of a SAM database, on which, additional data on secondary or even tertiary distributions of national income can be incorporated. Such an I-O table is also embodied in the SAM database in Figure 5. Describing the I-O table structure is important for the building of a CGE model because the latter is based directly on the I-O structure.

The information in the first two rows and columns of the SAM in Figure 5 can be rearranged to form an I-O table, shown in Figure 6. This table shows the cost structure of the two industries and how their outputs are purchased by users. It reveals the distribution of national income between factors of production and redistributions between private users and the government.

**Figure 6 An implied input-output table for the 122-model database**  
Implied by the SAM – non-standard i-o table, because it contains mixed prices

	BRD	MLK	HOU	GOV	INVBRD	INVMLK	EXP	TRF	IMP	
BRD	21	8	10.97	20	6.4	9.6	16.03	-1	-13	<b>78</b>
MLK	17	9	16.45	15	6	9	16.55	-2	-11	<b>76</b>
CAP	20	30								
LAB	15	25								
PTX	5	4								
	<b>78</b>	<b>76</b>								

Source: Data from figure 5.

The above I-O table is not a standard I-O table, as described by *System of National Accounts* (SNA) (Commission of the European Communities, *et al.* 1993 and of European Commission, *et al.* 2008). A standard I-O table should be valued at basic prices. As a result, domestically produced goods and imported goods are presented in separate sections of rows because they are valued at different basic prices. For domestically produced goods, their basic prices include all the factor or intermediate input costs of production plus production tax. For imported goods, their basic prices include the cost-insurance-freight (*c.i.f.*) of imports plus import tariffs, converted into domestic currency by an exchange rate.

Unlike the standard table, the I-O table in Figure 6 is not valued on one price. This is because its origin, the SAM database, does not have a consistent price structure. As the basic price is the foundation for any CGE model, it is important that a basic price I-O table is constructed first.

There is no detailed I-O information on a basic price I-O table available in Figure 6. However, it does have useful information about this missing I-O table. Figure 7 shows the structure of this basic price table and how the information in Figure 6 is related to this basic price table. The missing information in this basic price table is the sales of domestic and imported goods to individual domestic users, shown as the shaded areas in Figure 7. However, the information about the row and column sums are available from the I-O table in Figure 6. The row sums provide the sales of domestic and imported goods to all domestic users,<sup>8</sup> while the column sums provide the sales of composite goods (composed of domestic and imported goods) to individual domestic users, valued at their composite prices.

**Figure 7 Commodity usage data, aggregated from a missing basic price I-O table for the 122 model's database**

Domestic sales of domestic and imported goods in basic prices						1. Aggregate sales in basic price				
	BRD	MLK	HOU	GOV	INVBRD	INVMLK	DOM	EXP	SALE	
DOMBRD							=	61.97	16.03	78
DOMMLK							=	59.45	16.55	76
	BRD	MLK	HOU	GOV	INVBRD	INVMLK	IMP+TRF			
IMPBRD							=	14		
IMPMLK							=	13		
	II	II	II	II	II	II	II			
2. Aggregation of domestic and imported goods (DOM+IMP) in composite prices										
	BRD	MLK	HOU	GOV	INVBRD	INVMLK	TOTAL			
BRD	21	8	10.97	20	6.4	9.6	=	75.97		
MLK	17	9	16.45	15	6	9	=	72.45		
3. Factor inputs and production tax										
CAP	20	30								
LAB	15	25								
PTX	5	4								
COST	78	76								

Source: Data from Figure 6. The shaded parts represent a detailed table in basic prices, that is not available and cannot be derived from the SAM, because the SAM contains only row and column totals.

Figure 7 reveals that the original I-O table in Figure 6 can be divided into three related tables: the first is a table of the total sales of domestically produced and imported goods, all valued

<sup>8</sup> The sales of domestic goods to domestic users (highlighted in yellow colour in Figure 7) is derived by subtracting imports (inclusive of tariff) from the sales of composite goods to domestic users, shown as the row sums of the second table in Figure 7.

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at basic prices. The second is a table of the sales of composite goods to all domestic users, valued at their composite prices. The third table shows the value added produced in the two industries.

The structure of Figure 7 re-establishes the balancing of the I-O table in their basic prices, which is a vital requirement for building a CGE model. The row sums of the second table are linked with the column sum of the first table. As a result, the two tables show the overall balance between supply and demand: the total sales of the two goods, valued at the basic price, (row sums of the first table) are equal to the total costs of all inputs plus production tax in the two industries (column sums of the second and third tables). The three tables in Figure 7 provide all the basic information required to build a CGE model.

Comparing with the SAM, the only information not in these I-O tables is the additional data on the distribution of primary income, such as income tax, household saving (investment) and foreign investment, shown in the yellow cells in figure 5. This information is collected from national accounts and balance of payment statistics. They complement the I-O tables and allow CGE models to address issues such as changes in income taxes or other redistributional policies.

## **Model's core equations and iterative solution**

A clear model theory follows from the above interpretation of the database, particularly the data from the I-O tables in figure 7. It can now be implemented with a model equation system.

### **Model theory and the core equation system**

The first table in figure 7 shows that the country has an aggregate demand for two composite goods (bread and milk), each composed of a domestically produced product and an imported variety. With the demands for composite goods and their prices given, the country is assumed to choose its demands for the two goods from domestic and import sources to minimise the cost of purchasing through a CES function.

The second table in Figure 7 shows that these CES composites are aggregated from the demands of all domestic users: the producers of the two industries, an aggregate household, a government, and investors in two industries. The second table also shows how these users spend their income on the two composite goods.

The industries produce their output using composite goods as intermediate inputs and the two primary factors. The industries' demands for inputs can be specified as a function of a Leontief production technology. As capital and labour are imperfect substitutes in production, the cost-minimising demands for the two factors can be derived from a CES production function with an assumed CRTS technology. This implies that the basic prices of all goods equal their unit costs of production plus production tax (assuming zero pure profits)

and the supply quantity of the goods are determined by their aggregate demands at those prices.

The household receives all factor income after deducting capital income, which accrues to foreigners, and pay income tax to the government at a fixed rate. The household's post-tax income is divided between consumption expenditure and savings by a fixed savings rate. The household demand for composite goods can be derived from a Cobb-Douglas utility function subject to its budget constraint.

The government uses tax revenue to pay for its expenditure on composite goods. Investments in the two industries are funded by household savings and foreign investment. The investment demands for composite goods can be defined to be a fixed-proportion system based on the input data.

The exported and domestically consumed goods are assumed to be the same. The exports of the two goods are determined by the foreign demands, which are formulated by the constant elasticity of demand functions of their prices.

For given factor supplies, a general equilibrium solution for the model requires solutions for only three variables: the basic prices of capital and labour, and the foreign exchange rate. These variables reach their equilibrium levels when three equilibrium conditions are satisfied: demand equals supply in both factor markets, and the balance of payments is zero (the balances of the current and financial accounts are equal).

At this point, the model equation system is over-determined. This means that one of the three endogenous price variables required for model solution must be exogenously set as a benchmark against which all other variables are measured,<sup>9</sup> and one corresponding equation can be removed from the system. If the foreign exchange rate is chosen as a numeraire for the system, the equation for the external balancing condition becomes redundant and can be removed from the core equation system.<sup>10</sup>

The above model theory can be implemented in a core equation system of 25 equation blocks and their associated 25 endogenous variable blocks<sup>11</sup> (Table 1). The table lists the names of the sets, the parameters and the exogenous variables used in the equation system. The names of the endogenous variables are given by the names of the equations.

**Table 1      The core equations of the 122 model**

Parameters and exogenous variables
1. Sets

<sup>9</sup> This is often referred to as the numeraire.

<sup>10</sup> The difference between the balances of the current and financial accounts is a useful check of the overall balance of the model solution. Sometimes referred to as a Walras slack.

<sup>11</sup> To simplify the expression, equation or variable blocks, mentioned in the following, may be referred to as equations or variables.

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- (1) COM: commodities (*brd, mlk*);  
(2) IND: industries (*brd, mlk*);  
(3) FAC: factors (*cap, lab*);
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## 2. Parameters

- (1)  $\alpha_{(c,i)}^{int}$  Leontief share of intermediate com  $c$  in ind  $i$  ( $c \in \text{COM}; i \in \text{IND}$ )  
(2)  $\alpha_{(i)}^{fac}$  Leontief share of composite factors in ind  $i$  ( $i \in \text{IND}$ )  
(3)  $\sigma_{(i)}^{fac}$  CES elasticity of substitution between factors in ind  $i$  ( $i \in \text{IND}$ )  
(4)  $\delta_{(f,i)}^{fac}$  CES share of fac  $f$  in the demand of ind  $i$  for factor  $f$  ( $f \in \text{FAC}; i \in \text{IND}$ )  
(5)  $A_{(i)}^{fac}$  CES scale parameters for composite factor in ind  $i$  ( $i \in \text{IND}$ )  
(6)  $s^{kinc}$  Share of domestic capital income  
(7)  $\alpha_{(c)}^{hou}$  Cobb-Douglas shares for household demand for composite com  $c$  ( $c \in \text{COM}$ )  
(8)  $\alpha_{(c)}^{gov}$  Leontief share of government demand for composite com  $c$  ( $c \in \text{COM}$ )  
(9)  $s^{hinv}$  Share of household investment in ind  $i$  ( $i \in \text{IND}$ )  
(10)  $\alpha_{(c,i)}^{inv}$  Leontief share for investment demand for com  $c$  in ind  $i$  ( $c \in \text{COM}; i \in \text{IND}$ )  
(11)  $\sigma_{(c)}^{dom}$  CES elasticity of substitution between domestic and imported com  $c$  ( $c \in \text{COM}$ )  
(12)  $\delta_{(s,c)}^{dom}$  CES share parameters for the demand for com  $c$  from src  $s$  ( $s \in \text{SRC}; c \in \text{COM}$ )  
(13)  $A_{(c)}^{dom}$  CES scale parameters for composite com  $c$  ( $c \in \text{COM}$ )  
(14)  $\sigma_{(c)}^{exp}$  Constant elasticity of demand for export com  $c$  ( $c \in \text{COM}$ )  
(15)  $\alpha_{(c)}^{exp}$  Scale parameter for export demand for com  $c$  ( $c \in \text{COM}$ )
- 

## 3. Exogenous variables

- (1)  $r_{(c)}^{mtax}$  Rate of tariff on import com  $c$  ( $c \in \text{COM}$ )  
(2)  $r_{(i)}^{ptax}$  Rate of tax on output of ind  $i$  ( $i \in \text{IND}$ )  
(3)  $r^{htax}$  Rate of tax on household  
(4)  $r^{hsav}$  Household saving rate  
(5)  $r^{ex}$  Foreign exchange rate (foreign price of domestic currency)  
(6)  $P_{(c)}^{cif}$  World price of import com  $c$  ( $c \in \text{COM}$ )  
(7)  $X_{(f)}^{fac}$  Supply of factor  $f$  ( $f \in \text{FAC}$ )  
(8)  $V_{(i)}^{finv}$  Foreign investment in ind  $i$  in foreign currency ( $i \in \text{IND}$ )
- 

## Core equations for endogenous variables (1-25)

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### 1. Industries (1-6)

- (1) Demand of ind  $i$  for composite com  $c$   

$$Q_{(c,i)}^{ind} = \alpha_{(c,i)}^{int} X_{(i)} \quad (c \in \text{COM}; i \in \text{IND})$$
(2) CES price for composite factor for ind  $i$   

$$P_{(i)}^{fac} = \frac{1}{A_{(i)}^{fac}} \left( \sum_f \delta_{(f,i)}^{fac} \sigma_{(i)}^{fac} P_{(f)}^{fac} \right)^{\frac{1}{1-\sigma_{(i)}^{fac}}} \quad (i \in \text{IND})$$
(3) CES demand for fac  $f$  used in ind  $i$   

$$Q_{(f,i)}^{fac} = \frac{Q_{(i)}^{ind}}{A_{(i)}^{fac} \sigma_{(i)}^{fac}} \left( \delta_{(f,i)}^{fac} \frac{P_{(i)}^{fac}}{P_{(f)}^{fac}} \right)^{\sigma_{(i)}^{fac}} \quad (f \in \text{FAC}, i \in \text{IND})$$
(4) Total demand for factor  $f$   

$$Q_{i(f)}^{fac} = \sum_i Q_{(f,i)}^{fac} \quad (f \in \text{FAC})$$
-

---

(5) Unit cost of output in ind  $i$

$$C_{(i)}^{ind} = \sum_c \alpha_{(c,i)}^{int} P_{s(c)}^{dom} + \alpha_{(i)}^{fac} P_{(i)}^{fac} \quad (i \in IND)$$

(6) Basic price of output in ind  $i$

$$P_{(i)}^{ind} = C_{(i)}^{ind} (1 + r_{(i)}^{ptax}) \quad (i \in IND)$$


---

## 2. Household (7-10)

(7) Household factor income

$$Y^{fhou} = s^{kinc} P_{("cap")}^{fac} X_{("cap")}^{fac} + P_{("lab")}^{fac} X_{("lab")}^{fac}$$

(8) Household disposable income

$$Y^{dhou} = Y^{fhou} (1 - r^{htax})$$

(9) Household spending income

$$Y^{hou} = Y^{dhou} (1 - r^{hsav})$$

(10) Household demand for composite com  $c$

$$Q_{(c)}^{hou} = \alpha_{(c)}^{hou} \frac{Y^{hou}}{P_{s(c)}^{dom}} \quad (c \in COM)$$


---

## 2. Government (11-13)

(11) Government income

$$Y^{gov} = Y^{fhou} r^{htax} + \sum_i C_{(i)}^{ind} X_{(i)}^{ind} r_{(i)}^{ptax} + \sum_c P_{(c)}^{cif} / r^{ex} Q_{("imp",c)}^{dom} r_{(c)}^{imp}$$

(12) Price index for government consumption

$$P^{gov} = \sum_c \alpha_{(c)}^{gov} P_{(c)}^{dom}$$

(13) Government demand for composite com  $c$

$$Q_{(c)}^{gov} = \alpha_{(c)}^{gov} \frac{Y^{gov}}{P^{gov}} \quad (c \in COM)$$


---

## 3. Investment (14-17)

(14) Net foreign investment in ind  $i$  (domestic currency)

$$V_{(i)}^{finv} = V_{(i)}^{*finv} / r^{ex} \quad (c \in COM)$$

(15) Investment income in ind  $i$

$$Y_{(i)}^{inv} = s_{(i)}^{hinv} \left( \sum_f P_{(f)}^{fac} X_{(f)}^{fac} (1 - r^{htax}) r^{hsav} \right) + V_{(i)}^{finv} \quad (r \in REG)$$

(16) Price index for investment in ind  $i$

$$P_{(i)}^{inv} = \sum_c \alpha_{(c,i)}^{inv} P_{s(c)}^{dom} \quad (i \in IND)$$

(17) Investment demand for composite com  $c$  in ind  $i$

$$Q_{(c,i)}^{inv} = \alpha_{(c,i)}^{inv} \frac{Y_{(i)}^{inv}}{P_{(i)}^{inv}} \quad (c \in COM; i \in IND)$$


---

## 4. CES demand for domestic and imported goods (18-21)

(18) Domestic basic price of com  $c$  from src  $s$

$$P_{("dom",c)}^{dom} = P_{(c)}^{ind} \quad (c \in COM)$$

$$P_{("imp",c)}^{dom} = P_{(c)}^{cif} / r^{ex} (1 + r_{(c)}^{imp}) \quad (c \in COM)$$

(19) CES price for composite com  $c$

$$P_{s(c)}^{dom} = \frac{1}{A_{(c)}^{dom}} \left( \sum_s \delta_{(s,c)}^{dom} \sigma_{(c)}^{dom} P_{(s,c)}^{dom} \right)^{\frac{1}{1-\sigma_{(c)}^{dom}}} \quad (c \in COM)$$

(20) Aggregate demand for composite com  $c$

$$Q_{s(c)}^{dom} = \sum_i (Q_{(c,i)}^{ind} + Q_{(c,i)}^{inv}) + Q_{(c)}^{hou} + Q_{(c)}^{gov} \quad (c \in COM)$$

(21) CES demand for com  $c$  from src  $s$

$$Q_{(s,c)}^{dom} = \frac{Q_{s(c)}^{dom}}{A_{(c)}^{dom} \sigma_{(c)}^{dom}} \left( \delta_{(s,c)}^{dom} \frac{P_{s(c)}^{dom}}{P_{(s,c)}^{dom}} \right)^{\sigma_{(c)}^{dom}} \quad (s \in SRC; c \in COM)$$


---

## 5. Exports (22-24)

(22) World price of export com  $c$

$$P_{(c)}^{fob} = P_{(c)}^{ind} \cdot \mathbf{r}^{ex} \quad (c \in \text{COM})$$

(23) Foreign demand for export com  $c$

$$Q_{(c)}^{exp} = \left( \frac{a_{(c)}^{exp}}{P_{(c)}^{fob}} \right)^{\sigma_{(c)}^{exp}} \quad (c \in \text{COM})$$

(24) Supply of com  $c$

$$X_{(c)}^{ind} = Q_{("dom",c)}^{dom} + Q_{(c)}^{exp} \quad (c \in \text{COM})$$

#### 5. Market equilibrium conditions (25)

(25) Market clearing condition for fac  $f$  to determine basic price of factor  $f$ ,  $P_{(f)}^{fac}$

$$Q_{-i(f)}^{fac} = \mathbf{X}_{(f)}^{fac} \quad (f \in \text{FAC})$$

Notes:

1. Variables in red are set as exogenous in the basic closure.
2. Variables in blue are *undefined* and need to be determined by their corresponding *market clearing conditions*.
3. Equations shaded in blue are the *market clearing conditions* required to determine the *undefined* variables.

In the above equation system, the foreign exchange rate is chosen as a numeraire and its corresponding equation of the external account balancing condition is removed from the system. This missing equation is the balance of payments, which consists of the sum of the current and financial accounts.

$$\sum_c P_{(c)}^{fob} Q_{(c)}^{exp} - \sum_c P_{(c)}^{cif} Q_{(imp,c)}^{dom} - \mathbf{r}^{ex} (1 - \mathbf{s}^{kinc}) P_{("cap")}^{fac} \mathbf{X}_{("cap")}^{fac} = \sum_i \mathbf{V}_{(i)}^{*finv}$$

The left-hand side of this equation is the current account balance and the right-hand side is the financial account balance. In equilibrium the two sides will be equal, implying an overall balance of payments. If the foreign exchange rate is set as an endogenous variable, its value can only be determined by the external balancing condition through iterations, like the basic prices of factors. This is because the exchange rate cannot be defined by any equation in the system. However, if it is chosen as an exogenous variable (as the numeraire), its iteration through a market clearing condition is no longer needed. This is because the overall balance of payments is guaranteed by all the other variables and equations and, therefore, need not be included in the core equation system.

## Model solution and the iterative method

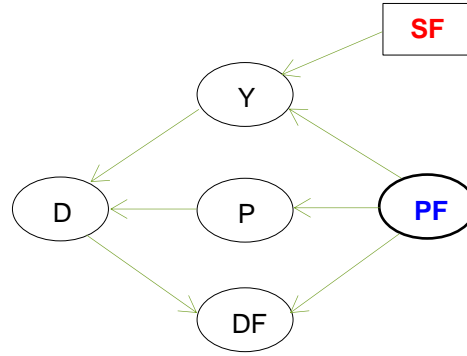
The above shows a CGE model as a network of variables, linked to each other by equations. Through these links, one can trace and find a few key variables, from which all other endogenous variables can be directly or indirectly derived. These variables are at the centre of model solution. This internal structure of CGE model solution can be illustrated using a schematic diagram in Figure 8.

In a simplified version, the 122 model can be seen as consisting of only six groups of variables: goods demand (D) and demand price (P), income (Y), factor demand (DF), supply (SF) and supply price (PF). Among all groups, SF is exogenous and the other five are endogenous. There are five groups of equations, four of which are used to define the four



endogenous variable groups: D is a function of P and Y, P is a function of PF, Y is a function of SF and PF, and DF is a function of D and PF. The undefined group of variables is PF and its associated equation group is a group of factor market equilibrium conditions:  $DF = SF$ .

Figure 8 **Schematic illustration of CGE model solution**



In this system, all endogenous variable groups (circles in the figure) can be derived from their relations with other variables, except PF. PF is therefore the foundation of this network and holds the key to the solution of the system.

The value of the undefined variable group PF can only be determined when its relationship with DF is known. DF is known for any given PF. It is, therefore, possible to start with an arbitrarily initial level of PF to measure the gap between DF and SF and then adjust PF to narrow the gap. Once PF reaches its equilibrium, all other endogenous variable groups are at their equilibrium positions too. This is the rationale for the iterative solution method.

The first step in the iterative solution method is to identify those endogenous variables that are not defined in the system and which equations are associated with these undefined variables. In this system, the first 24 equations are used to define 24 endogenous variables. This leaves just one endogenous variable block undefined or unknown to the system: the basic prices of the two factors  $P_J^{fac}$ . There is also one equation block, Equation 25, not used to define any endogenous variable. This equation block specifies the market clearing conditions for the two factors.

As the equilibrium solution of the entire model has been reduced to the equilibrium value for two variables, a simple method of iteration can be used to solve this problem. For any given factor price, the model system can automatically calculate a particular demand for the factor. This demand can then be compared with the supply of this factor to measure the distance from the market equilibrium. The factor price is determined by the excess demand in the factor market. If the demand for the factor is lower than its supply, the excess supply will drive down the factor price. Otherwise, the factor price will be pushed up by the excess demand for the factor. A continuous adjustment of the prices will ensure the factor demands eventually converge to supplies. Such a price adjustment mechanism can be set up to replace Equation 25 in Table 1.

---


$$\begin{aligned}
\text{If } X_{(f)}^{fac} &< Q_{-i(f)}^{fac}, P_{(f)}^{fac} = P_{(f)}^{fac} + \text{tiny} & (f \in \text{FAC}) \\
\text{If } X_{(f)}^{fac} &> Q_{-i(f)}^{fac}, P_{(f)}^{fac} = P_{(f)}^{fac} - \text{tiny} & (f \in \text{FAC}) \\
\text{If } X_{(f)}^{fac} &= Q_{-i(f)}^{fac}, P_{(f)}^{fac} = P_{(f)}^{fac} & (f \in \text{FAC})
\end{aligned}$$

where *tiny* is a small number, such as 0.0001, arbitrarily selected to set an adjustment step. A loop operation can be introduced for the entire core equation system to automate the price adjustment process. To begin the loop operation, initial values need be assigned to some endogenous variables to allow the calculation to proceed. These include the basic price of factors,  $P_{(f)}^{fac}$ . With a given initial factor price, the loop will use the core equation system to calculate the response of the demand for the factor. If this demand is greater than the supply, the basic price of this factor will be adjusted upward by a tiny amount. This new price is then fed back to the system to trigger a new round of iteration that updates the demands for all factors. Each price adjustment should narrow the gap between the demand and supply. This adjustment is repeated until the demands for all factors converge to their supplies. When the factor prices stop adjusting, a unique general equilibrium solution is reached for all endogenous variables in the model.

In the following section, the above core equation system will be solved to calibrate all the behavioural parameters from the model database.

## Calibration of parameters

CGE models simulate the quantitative responses in the demands of industries and other users to price changes. These responses rely on various behavioural parameters, dependent on the chosen functional forms in the model theory. A calibration process uses the model database to assign values to these parameters, so that the model's equilibrium solution can reproduce the initial database. This is the benchmark on which all the subsequent policy simulations are based. The quality of a CGE model relies on the accuracy of its calibrated parameters.

Calibration of model parameters uses the variables and equations in the model system. Take the CES production function in the model as an example. The composite factor used in industry *i* is a CES aggregate of capital and labour used in the industry.

$$Q_{-f(i)}^{fac} = A_{(i)}^{fac} \left( \sum_f \delta_{(f,i)}^{fac} Q_{(f,i)}^{fac} \right)^{\frac{\sigma_{(i)}^{fac}-1}{\sigma_{(i)}^{fac}}} \quad (i \in \text{IND})$$

The CES share parameters  $\delta_{(f,i)}^{fac}$  can be derived from this production function as

$$\delta_{(f,i)}^{fac} = \frac{P_{(f)}^{fac} Q_{(f,i)}^{fac} \frac{1}{\sigma_{(i)}^{fac}}}{\sum_{ff} P_{(ff)}^{fac} Q_{(ff,i)}^{fac} \frac{1}{\sigma_{(i)}^{fac}}} \quad (f \in \text{FAC}; i \in \text{IND})$$

The variable for the demands for factors,  $Q_{(f,i)}^{fac}$ , is not available in the database because it is a quantity, not a value.  $Q_{(f,i)}^{fac}$  can only be derived from database values if the associated prices  $P_{(f)}^{fac}$  are known.

$$\delta_{(f,i)}^{fac} = \frac{P_{(f)}^{fac} \left( \frac{v_{(f,i)}^{fac}}{P_{(f)}^{fac}} \right)^{\frac{1}{\sigma_{(i)}^{fac}}}}{\sum_{ff} P_{(ff)}^{fac} \left( \frac{v_{(ff,i)}^{fac}}{P_{(ff)}^{fac}} \right)^{\frac{1}{\sigma_{(i)}^{fac}}}} \quad (f \in \text{FAC}; i \in \text{IND})$$

In a CGE model prices are endogenous variables and cannot be obtained without a general equilibrium solution. Such a solution in turn cannot be obtained without calibrated parameters. The iterative solution method can be used to solve this dilemma. This is because the calibration of parameters does not rely on any pre-assumed prices, such as those assumed in the Harberger convention (i.e., initialising all prices to 1 in the database so that initial quantities are equal to their database values). Instead, it uses the model equation system to solve for all the general equilibrium prices first and then uses them to calibrate model parameters. These prices are derived from a general equilibrium solution and database and are internally consistent. This method of parameter calibration allows model users to introduce additional stock or quantity data (defined in physical units) into the model and have their real unit prices for the goods or factors determined endogenously by the model system.

## Data coefficients

Parameter calibration begins with the database. The data sets to be used in this model are taken from the SAM database in figure 1. They are divided into coefficient groups shown in figure 9. The detailed explanation of the coefficients is listed in Table 2.

Figure 9 Primary data coefficients for the 122-model in a SAM framework

		BRD	MLK	CAP	LAB	PTX	TRF	HOU	GOV	INVBRD	INVMLK	EXT
		1	2	3	4	5	6	7	8	9	10	11
BRD	1	$V_{(c,i)}^{int}$						$V_{(c)}^{hou}$	$V_{(c)}^{gov}$	$V_{(c,i)}^{inv}$		$V_{(c)}^{exp}$
MLK	2											
CAP	3	$V_{(f,i)}^{fac}$										
LAB	4											
PTX	5	$V_{(t)}^{ptax}$										
TRF	6	$V_{(c)}^{mtax}$										
HOU	7		$V^{kinc}$									
GOV	8							$V^{htax}$				
INVBRD	9							$V_{(t)}^{hinv}$				$V_{(i)}^{finv}$
INVMLK	10											
EXT	11	$V_{(c)}^{imp}$										

*Notations:* BRD: bread; MLK: milk; CAP: capital; LAB: labour; PTX: production tax; TRF: import tariff; HOU: household; GOV: government; INVBRD: investment in bread; INVMLK: investment in milk; EXT: external sector.

Note: See Table 2 for data set details.

There are 13 groups of primary coefficients, read into the model from the SAM database. There are also three groups of secondary data coefficients, derived from the primary data sets. They are all used to calibrate parameters in the model. There are also three uncalibrated elasticity parameters that are imposed from outside of the database.

**Table 2 The base data and parameters for the 122 model**

1. Primary data coefficients

- |   |  |
|---|--|
| (1) $V_{(c,i)}^{int}$ : Values of input com $c$ in ind $i$ at composite price;                  | $(c \in \text{COM}; i \in \text{IND})$ |
| (2) $V_{(f,i)}^{fac}$ : Values of fac $f$ used as intermediate input in ind $i$ at basic price; | $(f \in \text{FAC}; i \in \text{IND})$ |
| (3) $V_{(i)}^{ptax}$ : Values of production tax on output of ind $i$ ;                          | $(i \in \text{IND})$                   |
| (4) $V_{(c)}^{hou}$ : Household expenditure on com $c$ ;  | $(c \in \text{COM})$                   |
| (5) $V_{(c)}^{gov}$ : Government expenditure on com $c$ ;                                       | $(c \in \text{COM})$                   |
| (6) $V_{(c,i)}^{inv}$ : Investment expenditure on COM $c$ in IND $i$ ;                          | $(c \in \text{COM}; i \in \text{IND})$ |
| (7) $V_{(i)}^{finv}$ : Foreign investment in ind $i$ in domestic currency                       | $(i \in \text{IND})$                   |
| (8) $V_{(i)}^{hinv}$ : Household investment in ind $i$ ;  | $(i \in \text{IND})$                   |
| (9) $V_{(c)}^{exp}$ : Domestic value of export com $c$ ;  | $(c \in \text{COM})$                   |
| (10) $V_{(c)}^{imp}$ : Domestic value of import com $c$ ;                                       | $(c \in \text{COM})$                   |
| (11) $V_{(c)}^{mtax}$ : Tariff on import com $c$ ;  | $(c \in \text{COM})$                   |
| (12) $V^{htax}$ : Tax on household income;  |  |
| (13) $V^{kinc}$ : Domestic capital income;  |  |

2. Derived data coefficients

- |  |                      |
|--|----------------------|
| (1) Cost of production in ind $i$  |                      |
| $C_{(i)}^{ind} = \sum_c V_{(c,i)}^{int} + \sum_f V_{(f,i)}^{fac}$  | $(i \in \text{IND})$ |
| (2) Value of com $c$ from src $s$ at domestic basic price  |                      |
| $V_{("imp",c)}^{dom} = V_{(c)}^{imp} + V_{(c)}^{mtax}$   | $(c \in \text{COM})$ |
| $V_{("dom",c)}^{dom} = \sum_i (V_{(c,i)}^{int} + V_{(c,i)}^{inv}) + V_{(c)}^{hou} + V_{(c)}^{gov} - V_{("imp",c)}^{dom}$ | $(c \in \text{COM})$ |
| (3) Household income   |                      |
| $Y^{hou} = \sum_i (s^{kinc} V_{("cap",i)}^{fac} + V_{("lab",i)}^{fac})$  |                      |
| where $s^{kinc}$ is the share of domestic capital income,  |                      |
| $s^{kinc} = \frac{V^{kinc}}{\sum_i V_{("cap",i)}^{fac}}$   |                      |

3. Given parameters (elasticities)

- |  |                      |
|--|----------------------|
| (1) CES elasticity of substitution between factors in ind $i$            |                      |
| $s_{(i)}^{fac} = 0.99$   | $(i \in \text{IND})$ |
| (2) CES elasticity of substitution between domestic and imported com $c$ |                      |
| $s_{(c)}^{dom} = 2.0$  | $(c \in \text{COM})$ |
| (3) Constant elasticity of demand for export com $c$                     |                      |
| $s_{(c)}^{exp} = 5.0$  | $(c \in \text{COM})$ |

Notes:

1. All data sets are shown in green colour.
2. All parameters are shown in purple colour.

## Parameter calibration with core variables and equations

The core equation system from Table 1 can be used to calibrate the model parameters. The calibration relationships are presented in Table 3. The iterative solution method allows the system to simultaneously calculate the equilibrium prices to calibrate the parameter and use the calibrated parameters to update the equilibrium prices.

To set up this process, the model database is used first to set the values for exogenous variables and then, arbitrary values are assigned to some endogenous variables to initiate the calculation. The core equations for all 25 endogenous variables are listed along with their associated parameters. These parameters are calibrated with the relevant data coefficients and related endogenous prices. All parameter calibrations are highlighted in yellow colour.

**Table 3 Calibration of parameters with core equations**

### 1. Exogenous variables

(1.1) Rate of tariff on import com  $c$

$$r_{(c)}^{mtax} = \frac{v_{(c)}^{mtax}}{v_{(c)}^{imp}} \quad (c \in \text{COM})$$

(1.2) Rate of tax on output of ind  $i$

$$r_{(i)}^{ptax} = \frac{v_{(i)}^{ptax}}{c_{(i)}^{ind}} \quad (i \in \text{IND})$$

(1.3) Rate of tax on household income

$$r^{htax} = \frac{v_{(i)}^{htax}}{y^{hou}}$$

(1.4) Household saving rate

$$r^{hsav} = \frac{\sum_i v_{(i)}^{hinv}}{y^{hou}(1-r^{htax})}$$

(1.5) Foreign exchange rate (foreign price of domestic currency)

$$r^{ex} = 0.75$$

(1.6) World price of import com  $c$

$$P_{(c)}^{cif} = \frac{r^{ex}}{1+r_{(c,i)}^{mtax}} \quad (c \in \text{COM})$$

(1.7) Value of foreign investment in ind  $i$  in foreign currency

$$V_{(i)}^{*finv} = v_{(i)}^{finv} r^{ex} \quad (i \in \text{IND})$$

(1.8) Supply of factor  $f$

$$X_{(f)}^{fac} = \sum_i v_{(f,i)}^{fac} \quad (f \in \text{FAC})$$

### 2. Initial values for selected endogenous variables

(2.1) Supply of ind  $i$

$$X_{(i)} = \sum_c v_{(c,i)}^{int} + \sum_f v_{(f,i)}^{fac} + v_{(i)}^{ptax} \quad (i \in \text{IND})$$

(2.2) CES price of composite com  $c$

$$P_{(c)}^{dom} = 1.0 \quad (c \in \text{COM})$$

(2.3) CES price of factor in ind  $i$

$$P_{(i)}^{fac} = 1.0 \quad (i \in \text{IND})$$

(2.4) Domestic price of imported com  $c$

$$P_{(c)}^{imp} = 1.0 \quad (c \in \text{COM})$$

(2.5) Basic price of factor  $f$

$$P_{(f)}^{fac} = 1.0 \quad (f \in \text{FAC})$$

### Core equations with parameter calibration (1-25)

#### 1. Industries (1-6)

Calibration 1. *Leontief* share of intermediate com  $c$  in ind  $i$

$$\alpha_{(c,i)}^{int} = \frac{v_{(c,i)}^{int}/P_{(c,i)}}{\sum_c v_{(c,i)}^{int}/P_{(c,i)} + \sum_f v_{(f,i)}^{fac}/P_{(f)}^{fac}} \quad (c \in \text{COM}; i \in \text{IND})$$

Calibration 2. *Leontief* share of composite factor in ind  $i$

$$\alpha_{(i)}^{fac} = \frac{\sum_f v_{(f,i)}^{fac}/P_{(f)}^{fac}}{\sum_c v_{(c,i)}^{int}/P_{(c,i)} + \sum_f v_{(f,i)}^{fac}/P_{(f)}^{fac}} \quad (i \in \text{IND})$$

(1) Demand of ind  $i$  for composite com  $c$

$$Q_{(c,i)}^{ind} = \alpha_{(c,i)}^{int} X_{(i)} \quad (c \in \text{COM}; i \in \text{IND})$$

Calibration 3. *CES* share of fac  $f$  in the demand of ind  $i$  for factors

$$\delta_{(f,i)}^{fac} = \frac{P_{(f)}^{fac} \left( \frac{v_{(f,i)}^{fac}}{P_{(f)}^{fac}} \right)^{\frac{1}{\sigma_{(i)}^{fac}}}}{\sum_{ff} P_{(ff)}^{fac} \left( \frac{v_{(ff,i)}^{fac}}{P_{(ff)}^{fac}} \right)^{\frac{1}{\sigma_{(i)}^{fac}}}} \quad (f \in \text{FAC}; i \in \text{IND})$$

Calibration 4. *CES* scale parameters

$$A_{(i)}^{fac} = \left( \sum_f \delta_{(f,i)}^{fac} \sigma_{(i)}^{fac} \right)^{\frac{1}{1-\sigma_{(i)}^{fac}}} \quad (i \in \text{IND})$$

(2) *CES* price for composite factor for ind  $i$

$$P_{(i)}^{fac} = \frac{1}{A_{(i)}^{fac}} \left( \sum_f \delta_{(f,i)}^{fac} \sigma_{(i)}^{fac} P_{(f)}^{fac} \right)^{\frac{1}{1-\sigma_{(i)}^{fac}}} \quad (i \in \text{IND})$$

(3) *CES* demand for fac  $f$  used in ind  $i$

$$Q_{(f,i)}^{fac} = \frac{Q_{(i)}^{ind}}{A_{(i)}^{fac} \sigma_{(i)}^{fac}} \left( \delta_{(f,i)}^{fac} \frac{P_{(i)}^{fac}}{P_{(f)}^{fac}} \right)^{\sigma_{(i)}^{fac}} \quad (f \in \text{FAC}, i \in \text{IND})$$

(4) Total demand for factor  $f$

$$Q_{i(f)}^{fac} = \sum_i Q_{(f,i)}^{fac} \quad (f \in \text{FAC})$$

(5) Unit cost of output in ind  $i$

$$C_{(i)}^{ind} = \sum_c \alpha_{(c,i)}^{int} P_{s(c)}^{dom} + \alpha_{(i)}^{fac} P_{(i)}^{fac} \quad (i \in \text{IND})$$

(6) Basic price of output in ind  $i$

$$P_{(i)}^{ind} = C_{(i)}^{ind} (1 + r_{(i)}^{ptax}) \quad (i \in \text{IND})$$

#### 2. Household (7-10)

Calibration 5. Share of domestic capital income

$$s^{kinc} = \frac{v^{kinc}}{\sum_i v_{("cap",i)}^{fac}}$$

(7) Household factor income

$$Y^{fhou} = s^{kinc} P_{("cap")}^{fac} X_{("cap")}^{fac} + P_{("lab")}^{fac} X_{("lab")}^{fac}$$

(8) Household disposable income

$$Y^{dhou} = Y^{fhou} (1 - r^{htax})$$

(9) Household spending income

$$Y^{hou} = Y^{dhou} (1 - r^{hsav})$$

Calibration 6. *Cobb-Douglas* shares for household demand for composite com  $c$

$$\alpha_{(c)}^{hou} = \frac{v_{(c)}^{hou}}{\sum_{cc} v_{(cc)}^{hou}} \quad (c \in \text{COM})$$

(10) Household demand for composite com  $c$

$$Q_{(c)}^{hou} = \alpha_{(c)}^{hou} \frac{Y^{hou}}{p_{s(c)}^{dom}} \quad (c \in \text{COM})$$

## 2. Government (11-13)

(11) Government income

$$Y^{gov} = Y^{fhou} r^{htax} + \sum_i C_{(i)}^{ind} X_{(i)}^{ind} r_{(i)}^{ptax} + \sum_c P_{(c)}^{cif} / r^{ex} Q_{("imp",c)}^{dom} r_{(c)}^{imp}$$

Calibration 7. *Leontief* share of government demand for composite com  $c$

$$\alpha_{(c)}^{gov} = \frac{v_{(c)}^{gov} / p_{s(c)}^{dom}}{\sum_{cc} v_{(cc)}^{gov} / p_{(cc)}^{dom}} \quad (c \in \text{COM})$$

(12) Price index for government consumption

$$p^{gov} = \sum_c \alpha_{(c)}^{gov} p_{(c)}^{dom}$$

(13) Government demand for composite com  $c$

$$Q_{(c)}^{gov} = \alpha_{(c)}^{gov} \frac{Y^{gov}}{p^{gov}} \quad (c \in \text{COM})$$

## 3. Investment (14-17)

(14) Net foreign investment in ind  $i$  (domestic currency)

$$V_{(i)}^{finv} = V_{(i)}^{*finv} / r^{ex} \quad (c \in \text{COM})$$

Calibration 8. Share of household investment in ind  $i$

$$s_{(i)}^{hinv} = \frac{v_{(i)}^{hinv}}{\sum_{ii} v_{(ii)}^{hinv}} \quad (i \in \text{IND})$$

(15) Investment income in ind  $i$

$$Y_{(i)}^{inv} = s_{(i)}^{hinv} \left( \sum_f P_{(f)}^{fac} X_{(f)}^{fac} (1 - r^{htax}) r^{hsav} \right) + V_{(i)}^{finv} \quad (r \in \text{REG})$$

Calibration 9. *Leontief* share for investment demand for com  $c$  in ind  $i$

$$\alpha_{(c,i)}^{inv} = \frac{v_{(c,i)}^{inv} / p_{s(c)}^{dom}}{\sum_{cc} v_{(cc,i)}^{inv} / p_{s(cc)}^{dom}} \quad (c \in \text{COM}; i \in \text{IND})$$

(16) Price index for investment in ind  $i$

$$p_{(i)}^{inv} = \sum_c \alpha_{(c,i)}^{inv} p_{s(c)}^{dom} \quad (i \in \text{IND})$$

(17) Investment demand for composite com  $c$  in ind  $i$

$$Q_{(c,i)}^{inv} = \alpha_{(c,i)}^{inv} \frac{Y_{(i)}^{inv}}{p_{(i)}^{inv}} \quad (c \in \text{COM}; i \in \text{IND})$$

## 4. CES demand for domestic and imported goods (18-21)

(18) Domestic basic price of com  $c$  from src  $s$

$$p_{("dom",c)}^{dom} = p_{(c)}^{ind} \quad (c \in \text{COM})$$

$$p_{("imp",c)}^{dom} = P_{(c)}^{cif} / r^{ex} (1 + r_{(c)}^{imp}) \quad (c \in \text{COM})$$

Calibration 10. *CES* share parameters for the demand for com  $c$  from src  $s$

$$\delta_{(s,c)}^{dom} = \frac{p_{(s,c)}^{dom} \left( \frac{v_{(s,c)}^{dom}}{p_{(s,c)}^{dom}} \right)^{\frac{1}{\sigma_{(c)}^{dom}}}}{\sum_{ss} p_{(ss,c)}^{dom} \left( \frac{v_{(ss,c)}^{dom}}{p_{(ss,c)}^{dom}} \right)^{\frac{1}{\sigma_{(c)}^{dom}}}} \quad (s \in \text{SRC}; c \in \text{COM})$$

Calibration 11. *CES* scale parameters for composite com  $c$

$$A_{(c)}^{dom} = \left( \sum_s \delta_{(s,c)}^{dom} s_{(c)}^{dom} \right)^{\frac{1}{1 - \sigma_{(c)}^{dom}}} \quad (c \in \text{COM})$$

(19) CES price for composite com  $c$

$$P_{s(c)}^{dom} = \frac{1}{A_{(c)}^{dom}} \left( \sum_s \delta_{(s,c)}^{dom} P_{(s,c)}^{dom} \right)^{\frac{1}{1-\sigma_{(c)}^{dom}}} \quad (c \in \text{COM})$$

(20) Aggregate demand for composite com  $c$

$$Q_{s(c)}^{dom} = \sum_i (Q_{(c,i)}^{ind} + Q_{(c,i)}^{inv}) + Q_{(c)}^{hou} + Q_{(c)}^{gov} \quad (c \in \text{COM})$$

(21) CES demand for com  $c$  from src  $s$

$$Q_{(s,c)}^{dom} = \frac{Q_{s(c)}^{dom}}{A_{(c)}^{dom} \delta_{(s,c)}^{dom}} \left( \delta_{(s,c)}^{dom} \frac{P_{s(c)}^{dom}}{P_{(s,c)}^{dom}} \right)^{\sigma_{(c)}^{dom}} \quad (s \in \text{SRC}; c \in \text{COM})$$

## 5. Exports (22-24)

(22) World price of export com  $c$

$$P_{(c)}^{fob} = P_{(c)}^{ind} r^{ex} \quad (c \in \text{COM})$$

Calibration 12. Export demand parameters for com  $c$

$$\alpha_{(c)}^{exp} = P_{(c)}^{fob} \left( \frac{V_{(c)}^{exp}}{P_{(c)}^{ind}} \right)^{\frac{1}{\sigma_{(c)}^{exp}}} \quad (c \in \text{COM})$$

(23) Foreign demand for export com  $c$

$$Q_{(c)}^{exp} = \left( \frac{\alpha_{(c)}^{exp}}{P_{(c)}^{fob}} \right)^{\sigma_{(c)}^{exp}} \quad (c \in \text{COM})$$

(24) Supply of com  $c$

$$X_{(c)}^{ind} = Q_{(c)}^{dom} + Q_{(c)}^{exp} \quad (c \in \text{COM})$$

## 5. Market equilibrium conditions (25)

(25) Factor price adjustment to clear factor market

$$\begin{aligned} \text{If } X_{(f)}^{fac} < Q_{i(f)}^{fac}, P_{(f)}^{fac} &= P_{(f)}^{fac} + \text{tiny} & (f \in \text{FAC}) \\ \text{If } X_{(f)}^{fac} > Q_{i(f)}^{fac}, P_{(f)}^{fac} &= P_{(f)}^{fac} - \text{tiny} & (f \in \text{FAC}) \\ \text{If } X_{(f)}^{fac} = Q_{i(f)}^{fac}, P_{(f)}^{fac} &= P_{(f)}^{fac} = \left( \frac{1.000}{1.000} \right) & (f \in \text{FAC}) \end{aligned}$$

Notes:

1. Variables in red colour are set as exogenous in the basic closure.
2. Variables in blue colour are *undefined* and need to be determined by their correspondent *market clearing conditions*.
3. Equations shaded in yellow calibrate parameters using the base data shown in green colour. The calibrated parameters are shown in purple colour.
4. Equations shaded in blue are the *market clearing conditions* required to determine the *undefined* variables.

A loop operation together with a factor price adjustment process can still be implemented to find a general equilibrium solution for all the endogenous prices that are used in parameter calibration. However, as the supplies of factors (capital and labour) are assumed to be equal to their database values (Formula 2.5 in Table 3), the basic prices of these two factors are implicitly assumed to be equal to unity.<sup>12</sup> This assumption sets the basic prices of factors equal to one. Therefore,  $P_{(f)}^{fac}$  can be treated as an exogenous variable. This requires the exogenous variables of factor supplies,  $X_{(f)}^{fac}$  to be swapped with  $P_{(f)}^{fac}$  as an endogenous variable to be determined by their demands. This swap keeps the equation system balanced in the sense that the number of endogenous variables equals the number of equations. The factor price adjustment becomes redundant and can be removed from the calibration model.

<sup>12</sup> The Harberger convention is maintained for factors (assuming that their price equals 1) unless information about quantities in addition to values is available. See more detail below.



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The loop process remains operational because other endogenous prices still need to be updated from their initial arbitrary levels toward equilibrium with the given factor prices.<sup>13</sup>

This arbitrary assumption about factor prices and quantities in Table 3 is due to the lack of information about the quantities of factor endowments in the database, such as capital stock and labour force data. If this type of additional factor supply data is available, the unity assumption must be substituted with an alternative price, taken from that data or model solution. For example, if the value of capital stock  $V_{(i)}^{cap}$  is introduced, the supply of capital could be calculated as this value divided by a price,

$$X_{("cap")}^{fac} = \sum_i \frac{V_{(i)}^{cap}}{P_{(i)}^{inv}}$$

where  $P_{(i)}^{inv}$  is the price of investment or of capital stock. This is an endogenous variable in the model system, whose level cannot be determined without a general equilibrium solution. As capital supply  $X_{("cap")}^{fac}$  depends on the model's solution for  $P_{(i)}^{inv}$  and cannot be pre-determined the rental price of capital  $P_{("cap")}^{fac}$  also becomes an unknown variable and needs to be solved with its own market equilibrium condition.

This example shows that the setting of factor prices and quantities in the model for parameter calibration in Table 2 is an exceptional case. In a more general case where information on factor supplies is available, the factor prices become the cost per natural unit of the factor supplied, such as rental price per unit of capital stock, annual labour cost per worker, or average wage per hour. These prices are unlikely to be equal to unity. Alternatively, they can always be treated as an unknown variable and solved by their market clearing conditions.

In a model for calibration, an iterative loop operation results in a general equilibrium solution that calibrates all model parameters. This solution provides the first check for the integrity of the model's equation system. If the model is built properly as a closed system without any leak, its solution should guarantee the clearing of excess demands in all markets and ensure account balance for all agents in the model. Using the prices and quantities solved, the model user should be able to duplicate the original database, such as the SAM or the I-O table. If this is not the case, the model theory or data is flawed.

## Policy model and iterative solution

Once all parameters are calibrated, the CGE model can be used for policy simulations. The policy model is presented in Table 4. It consists of all 25 core endogenous variables and equations with their calibrated parameters and the derived exogenous variables, required for a general equilibrium solution in policy simulations. These parameters and exogenous variables absorb all the required information in the original database for the CGE model. Therefore, the initial data from the SAM or I-O data are no longer needed in the policy

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<sup>13</sup> Without this swap, the model's solution will still produce the initial equilibrium prices of factors equal to unity.

model. The calibrated values of these parameters and exogenous variables are listed in Table 4.

**Table 4 The 122 model for policy analysis: an iterative version**

1. Calibrated and given parameters

- (1) *Leontief* share of intermediate com  $c$  in ind  $i$   
 $\alpha_{(c,i)}^{int} = \begin{pmatrix} 0.27 & 0.10 \\ 0.23 & 0.12 \end{pmatrix}$  ( $c \in \text{COM}; i \in \text{IND}$ )
- (2) *Leontief* share of composite factors in ind  $i$   
 $\alpha_{(i)}^{fac} = (0.50, 0.78)$  ( $i \in \text{IND}$ )
- (3) *CES* elasticity of substitution between factors in ind  $i$   
 $\sigma_{(i)}^{fac} = 0.99$  ( $i \in \text{IND}$ )
- (4) *CES* share of fac  $f$  in the demand of ind  $i$  for factor  $f$   
 $\delta_{(f,i)}^{fac} = \begin{pmatrix} 0.57 & 0.55 \\ 0.43 & 0.45 \end{pmatrix}$  ( $f \in \text{FAC}; i \in \text{IND}$ )
- (5) *CES* scale parameters for composite factor in ind  $i$   
 $A_{(i)}^{fac} = (1.98, 1.99)$  ( $i \in \text{IND}$ )
- (6) Share of domestic capital income  
 $s^{kinc} = 0.548$
- (7) *Cobb-Douglas* shares for household demand for composite com  $c$   
 $\alpha_{(c)}^{hou} = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$  ( $c \in \text{COM}$ )
- (8) *Leontief* share of government demand for composite com  $c$   
 $\alpha_{(c)}^{gov} = \begin{pmatrix} 0.56 \\ 0.44 \end{pmatrix}$  ( $c \in \text{COM}$ )
- (9) Share of household investment in ind  $i$   
 $s_{(i)}^{hinv} = (0.4, 0.6)$  ( $i \in \text{IND}$ )
- (10) *Leontief* share for investment demand for com  $c$  in ind  $i$   
 $\alpha_{(c,i)}^{inv} = \begin{pmatrix} 0.51 & 0.51 \\ 0.49 & 0.49 \end{pmatrix}$  ( $c \in \text{COM}; i \in \text{IND}$ )
- (11) *CES* elasticity of substitution between domestic and imported com  $c$   
 $\sigma_{(c)}^{dom} = 2.0$  ( $c \in \text{COM}$ )
- (12) *CES* share parameters for the demand for com  $c$  from src  $s$   
 $\delta_{(s,c)}^{dom} = \begin{pmatrix} 0.69 & 0.69 \\ 0.31 & 0.31 \end{pmatrix}$  ( $s \in \text{SRC}; c \in \text{COM}$ )
- (13) *CES* scale parameters for composite com  $c$   
 $A_{(c)}^{dom} = \begin{pmatrix} 1.75 \\ 1.75 \end{pmatrix}$  ( $c \in \text{COM}$ )
- (14) Constant elasticity of demand for export com  $c$   
 $\sigma_{(c)}^{exp} = 5.0$  ( $c \in \text{COM}$ )
- (15) Export demand parameters for com  $c$   
 $\alpha_{(c)}^{exp} = \begin{pmatrix} 1.42 \\ 1.39 \end{pmatrix}$  ( $c \in \text{COM}$ )

2. Exogenous variables

- (1) Rate of tariff on import com  $c$   
 $r_{(c)}^{mtax} = \begin{pmatrix} 0.077 \\ 0.182 \end{pmatrix}$  ( $c \in \text{COM}$ )
- (2) Rate of tax on output of ind  $i$   
 $r_{(i)}^{ptax} = (0.069, 0.056)$  ( $i \in \text{IND}$ )
- (3) Rate of tax on household income  
 $r^{htax} = 0.341$
- (4) Household saving rate  
 $r^{hsav} = 0.383$
- (5) Foreign exchange rate (foreign price of domestic currency)  
 $r^{ex} = 0.75$

- 
- (6) World price of import com  $c$   
 $P_{(c)}^{cif} = \begin{pmatrix} 0.696 \\ 0.635 \end{pmatrix}$  ( $c \in \text{COM}$ )
- (7) Supply of factor  $f$   
 $X_{(f)}^{fac} = \begin{pmatrix} 50 \\ 40 \end{pmatrix}$  ( $f \in \text{FAC}$ )
- (8) Foreign investment in ind  $i$  in foreign currency  
 $V_{(i)}^{finv} = (4.2, 6.3)$  ( $i \in \text{IND}$ )
- 

### 3. Initial values for selected endogenous variables

- (1) Supply of output in ind  $i$   
 $X_{(i)} = (78 \ 76)$  ( $i \in \text{IND}$ )
- (2) CES price of composite com  $c$   
 $P_{(c)}^{dom} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$  ( $c \in \text{COM}$ )
- (3) CES price of factor in ind  $i$   
 $P_{(i)}^{fac} = (1.0 \ 1.0)$  ( $i \in \text{IND}$ )
- (4) Domestic price of imported com  $c$   
 $P_{(c)}^{imp} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$  ( $i \in \text{COM}$ )
- (5) Basic price of factor  $f$   
 $P_{(f)}^{fac} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$  ( $f \in \text{FAC}$ )
- 

### Core equations (1-25) and the first solution for 25 core variables

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#### 1. Industries (1-6)

- (1) Demand of ind  $i$  for composite com  $c$   
 $Q_{(c,i)}^{ind} = \alpha_{(c,i)}^{int} X_{(i)} = \begin{pmatrix} 19.283 \ 7.346 \\ 16.041 \ 8.492 \end{pmatrix}$  ( $c \in \text{COM}; i \in \text{IND}$ )
- (2) CES price for composite factor for ind  $i$   
 $P_{(i)}^{fac} = \frac{1}{A_{(i)}^{fac}} \left( \sum_f \delta_{(f,i)}^{fac} \sigma_{(i)}^{fac} P_{(f)}^{fac} \right)^{\frac{1}{1-\sigma_{(i)}^{fac}}} = (1.000 \ 1.000)$  ( $i \in \text{IND}$ )
- (3) CES demand for fac  $f$  used in ind  $i$   
 $Q_{(f,i)}^{fac} = \frac{Q_{(f,i)}^{fac}}{A_{(i)}^{fac} \sigma_{(i)}^{fac}} \left( \delta_{(f,i)}^{fac} \frac{P_{(i)}^{fac}}{P_{(f)}^{fac}} \right)^{\sigma_{(i)}^{fac}} = \begin{pmatrix} 20.0 \ 30.0 \\ 15.0 \ 25.0 \end{pmatrix}$  ( $f \in \text{FAC}, i \in \text{IND}$ )
- (4) Total demand for factor  $f$   
 $Q_{i(f)}^{fac} = \sum_i Q_{(f,i)}^{fac} = \begin{pmatrix} 50.0 \\ 40.0 \end{pmatrix}$  ( $f \in \text{FAC}$ )
- (5) Unit cost of output in ind  $i$   
 $C_{(i)}^{ind} = \sum_c \alpha_{(c,i)}^{int} P_{s(c)}^{dom} + \alpha_{(i)}^{fac} P_{(i)}^{fac} = (1.038 \ 1.016)$  ( $i \in \text{IND}$ )
- (6) Basic price of output in ind  $i$   
 $P_{(i)}^{ind} = C_{(i)}^{ind} (1 + r_{(i)}^{ptax}) = (1.109 \ 1.073)$  ( $i \in \text{IND}$ )
- 

#### 2. Household (7-10)

- (7) Household factor income  
 $Y^{hou} = s^{kinc} P_{("cap")}^{fac} X_{("cap")}^{fac} + P_{("lab")}^{fac} X_{("lab")}^{fac} = (67.419)$
- (8) Household disposable income  
 $Y^{dhou} = Y^{hou} (1 - r^{htax}) = (44.419)$
- (9) Household spending income  
 $Y^{hou} = Y^{dhou} (1 - r^{hsav}) = (27.419)$
- (10) Household demand for composite com  $c$   
 $Q_{(c)}^{hou} = \alpha_{(c)}^{hou} \frac{Y^{hou}}{P_{s(c)}^{dom}} = \begin{pmatrix} 10.071 \\ 15.523 \end{pmatrix}$  ( $c \in \text{COM}$ )
- 

#### 2. Government (11-13)

(11) Government income

$$Y^{gov} = Y^{fhou} \mathbf{r}^{htax} + \sum_i C_{(i)}^{ind} X_{(i)}^{ind} \mathbf{r}_{(i)}^{ptax} + \sum_c \mathbf{P}_{(c)}^{cif} / \mathbf{r}^{ex} Q_{("imp",c)}^{dom} \mathbf{r}_{(c)}^{imp} = (35.000)$$

(12) Price index for government consumption

$$P^{gov} = \sum_c \alpha_{(c)}^{gov} P_{(c)}^{dom} = (1.076)$$

(13) Government demand for composite com c

$$Q_{(c)}^{gov} = \alpha_{(c)}^{gov} \frac{Y^{gov}}{P^{gov}} = \left( \begin{smallmatrix} 18.365 \\ 14.154 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

### 3. Investment (14-17)

(14) Net foreign investment in ind i (domestic currency)

$$V_{(i)}^{finv} = \mathbf{V}_{(i)}^{*finv} / \mathbf{r}^{ex} = (5.60 \ 8.40) \quad (c \in \text{COM})$$

(15) Investment income in ind i

$$Y_{(i)}^{inv} = \mathbf{s}_{(i)}^{inv} \left( \sum_f \mathbf{P}_{(f)}^{fac} \mathbf{X}_{(f)}^{fac} (1 - \mathbf{r}^{htax}) \mathbf{r}^{hsav} \right) + V_{(i)}^{finv} = (12.40 \ 18.40) \quad (r \in \text{REG})$$

(16) Price index for investment in ind i

$$P_{(i)}^{inv} = \sum_c \alpha_{(c,i)}^{inv} P_{s(c)}^{dom} = (1.075 \ 1.075) \quad (i \in \text{IND})$$

(17) Investment demand for composite com c in ind i

$$Q_{(c,i)}^{inv} = \alpha_{(c,i)}^{inv} \frac{Y_{(i)}^{inv}}{P_{(i)}^{inv}} = \left( \begin{smallmatrix} 5.877 \ 8.815 \\ 5.661 \ 8.492 \end{smallmatrix} \right) \quad (c \in \text{COM}; i \in \text{IND})$$

### 4. CES demand for domestic and imported goods (18-21)

(18) Domestic basic price of com c from src s

$$P_{("dom",c)}^{dom} = P_{(c)}^{ind} = \left( \begin{smallmatrix} 1.109 \\ 1.073 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

$$P_{("imp",c)}^{dom} = \mathbf{P}_{(c)}^{cif} / \mathbf{r}^{ex} (1 + \mathbf{r}_{(c)}^{imp}) = \left( \begin{smallmatrix} 1.000 \\ 1.000 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

(19) CES price for composite com c

$$P_{s(c)}^{dom} = \frac{1}{A_{(c)}^{dom}} \left( \sum_s \delta_{(s,c)}^{dom} \sigma_{(c)}^{dom} P_{(s,c)}^{dom} \right)^{\frac{1}{1 - \sigma_{(c)}^{dom}}} = \left( \begin{smallmatrix} 1.089 \\ 1.060 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

(20) Aggregate demand for composite com c

$$Q_{s(c)}^{dom} = \sum_i (Q_{(c,i)}^{ind} + Q_{(c,i)}^{inv}) + Q_{(c)}^{hou} + Q_{(c)}^{gov} = \left( \begin{smallmatrix} 69.757 \\ 68.364 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

(21) CES demand for com c from src s

$$Q_{(s,c)}^{dom} = \frac{Q_{s(c)}^{dom}}{A_{(c)}^{dom} \frac{1}{1 - \sigma_{(c)}^{dom}}} \left( \delta_{(s,c)}^{dom} \frac{P_{s(c)}^{dom}}{P_{(s,c)}^{dom}} \right)^{\sigma_{(c)}^{dom}} = \left( \begin{smallmatrix} 55.869 \ 55.414 \\ 14.000 \ 13.000 \end{smallmatrix} \right) \quad (s \in \text{SRC}; c \in \text{COM})$$

### 5. Exports (22-24)

(22) World price of export com c

$$P_{(c)}^{fob} = P_{(c)}^{ind} \mathbf{r}^{ex} = \left( \begin{smallmatrix} 0.832 \\ 0.805 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

(23) Foreign demand for export com c

$$Q_{(c)}^{exp} = \left( \frac{\alpha_{(c)}^{exp}}{P_{(c)}^{fob}} \right)^{\sigma_{(c)}^{exp}} = \left( \begin{smallmatrix} 14.455 \\ 15.424 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

(24) Supply of com c

$$X_{(c)}^{ind} = Q_{("dom",c)}^{dom} + Q_{(c)}^{exp} = \left( \begin{smallmatrix} 70.324 \\ 70.838 \end{smallmatrix} \right) \quad (c \in \text{COM})$$

### 5. Market equilibrium conditions (25)

(25) Factor price adjustment to clear factor market

$$\text{If } \mathbf{X}_{(f)}^{fac} < Q_{i(f)}^{fac}, \mathbf{P}_{(f)}^{fac} = \mathbf{P}_{(f)}^{fac} + \text{tiny} \quad (f \in \text{FAC})$$

$$\text{If } \mathbf{X}_{(f)}^{fac} > Q_{i(f)}^{fac}, \mathbf{P}_{(f)}^{fac} = \mathbf{P}_{(f)}^{fac} - \text{tiny} \quad (f \in \text{FAC})$$

$$\text{If } \mathbf{X}_{(f)}^{fac} = Q_{i(f)}^{fac}, \mathbf{P}_{(f)}^{fac} = \mathbf{P}_{(f)}^{fac} = \left( \begin{smallmatrix} 1.000 \\ 1.000 \end{smallmatrix} \right) \quad (f \in \text{FAC})$$

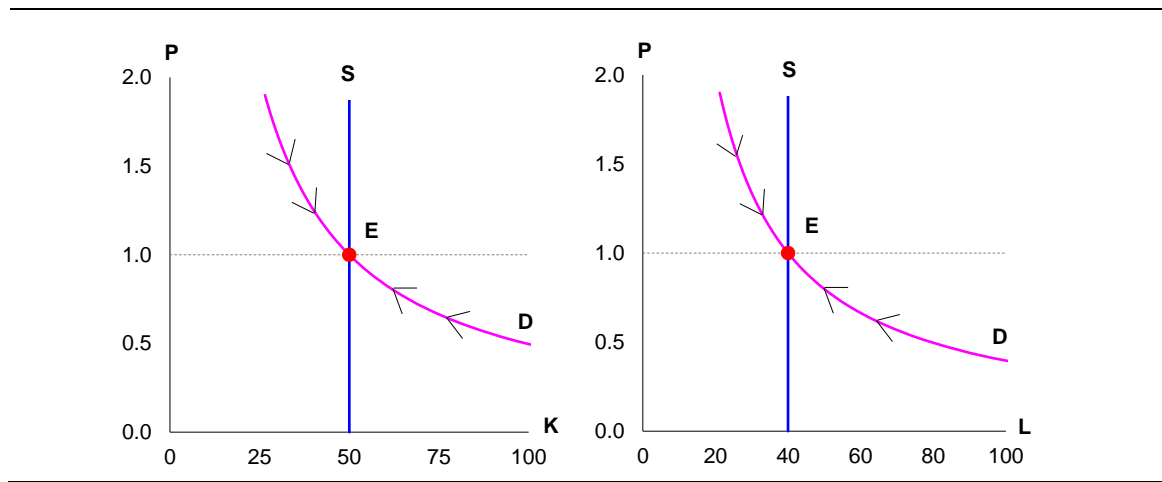
Notes:

1. Variables in red colour are set as exogenous in the basic closure.

2. Variables in blue colour are *undefined* and need to be determined by their correspondent *market clearing conditions*.
3. The parameters are shown in purple colour.
4. Equations shaded in blue are the *market clearing conditions* required to determine the *undefined* variables.

The model is solved by iteratively adjusting the two factor prices to equilibrate supply and demand in the two factor markets. The factor market equilibrium results can be depicted in figure 10. The demand curves are the aggregations of the CES demands for capital and labour used in the two industries. The model solutions for all other endogenous variables are derived from these two equilibrium factor prices.

**Figure 10 Supply-demand equilibrium in capital and labour markets**



Using only initial database values for exogenous variables, the model solution should reproduce the original database. The results of the first solution are also presented in Table 4. This solution is a benchmark against which policy simulations can be compared, to reveal the effects of the policy change on the initial pre-policy economic equilibrium. Further tests of the model can be carried out with the benchmark solution. For example, a homogeneity test should be carried out by changing the exogenous foreign exchange  $r^{ex}$ . This should raise all domestic prices and value variables by the same proportion, with no change in quantity variables.

## Summary and conclusion

This paper introduced an alternative solution method for solving CGE models iteratively without a mathematical solver. This solution method opens the door to CGE modelling in a wide range of computing programs and programming languages.

## Summary of the procedure

The previous sections outlined a procedure of writing the core equation system of a CGE model, suitable to be solved using an iterative method. The important steps in this procedure can be summarised as follows:

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1. Conceptualise theory to explain the database. Describe the behaviour and connections within the economic system required to produce the data observed.
  2. Write down the model's core equations as functions or relations. Endogenous variables should be defined, according to the theory, as a function of other variables.
  3. Once all the definable variables are linked to equations, the remaining endogenous variables are undefined, and must be solved using their respective market clearing conditions.
  4. With CRTS technology, all goods prices can be calculated from their costs, and the quantity supplied is determined by demand. As a result, the factor prices become the only undefined variables, required to be solved iteratively with their market clearing conditions. The model's general equilibrium solution for all other endogenous variables can be calculated from the solved factor prices.

The above procedure can be used to prepare a CGE model for the iterative solution method. It helps facilitate an understanding of the underlying drivers of CGE model solutions. Following the above procedure, the solution of a CGE model can be traced back to its foundation: those undefined variables and their associated equilibrium conditions. It leads model users to solve only a small number of undefined variables and lets the equation system determine the solutions for all other endogenous variables.

## **Benefits of using the iterative method**

In addition to analysing the structure of the model to reduce the problem to a small number of variables, the iterative solution method has some benefits over a large matrix inversion. Analysing the structure has the additional benefit of avoiding the 'black box' aspect of many solution processes.

### **A simple way to solve non-linear CGE models**

With conventional mathematical methods for solving CGE models in levels, a model's equations must be solved as a system of non-linear equations. This is more complex than solving a linear system, especially if the system is large. As computing technology progresses, various well-developed solvers have become available. They are designed for solving large non-linear simultaneous equation systems, suitable also for solving CGE models.

Among these solvers, Mixed Complementarity Programming (MCP) stands out as one solution for CGE model users in recent years. This is because MCP is capable of handling large numbers of corner solutions in non-linear equation systems. To use MCP a CGE model needs to be set up in a specific format. For example, model equations are not used to define variables as functions, but to specify relations between variables, written as inequality

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constraints. In MCP, each inequality needs to be paired with a complementary variable. A general equilibrium solution is reached when all complementarity variables find their equilibrium under their individually given conditions of inequalities.

The iterative method is conceptually similar to MCP. The relations between market clearing conditions and their associated undefined prices, shown in the iterative method above, closely resemble the relations in MCP models between inequality constraints and their complementary variables. In the iterative solution method described in this paper, undefined prices are adjusted through their market clearing conditions until they reach their equilibrium. This is like the complementarity variables that are paired to ensure their respective inequality constraints are satisfied at equilibrium (either by binding equality, or through slack at a corner solution). The only difference is that in MCP all endogenous variables are solved simultaneously (abstracting from the exact computer science tools ‘under the hood’ of the MCP solvers, which are not typically examined by applied CGE modellers), while in the proposed iterative method, only a handful of undefined variables need to be solved iteratively. The rest of the model system can be derived from the equilibrium variables. This is an advantage over mathematical solvers because it overcomes the so-called “curse of dimensionality” (Kompas and Ha, 2019): the solution of a CGE model is no longer limited by the large number of its equations, but only on a small number of undefined variables and their associated equations. In CGE models, most endogenous variables are well defined and can be calculated as functions of other variables. Only a small number of variables are undefined. As their equilibrium conditions are clearly specified, these undefined variables can be solved through simple iterations. The matrix inversion for the entire model equation systems could, therefore, be avoided.

At the very least, the process of reducing a CGE model (as described in this paper) imposes a discipline on applied modellers which — even in the absence of the iterative solution method — results in more transparent and easily understood model, which is smaller, thus easier to debug and more quickly solved with existing mathematical solvers.

### Easy to introduce flexible functional forms in CGE models

The iterative solution method allows for flexibility in writing model equations. This is because the computation required for iteration is a sequential calculation of all interlinked equations defining endogenous variables. There is no inverse of any matrix of simultaneous equations and no need to store large amounts of temporary data in a computer’s Random-Access Memory (RAM). Moreover, no stringent conditions are attached to what functions are acceptable in the system.

As the iteration method is based on a series of simple calculation of the endogenous variables in the equation system, a wide range of functional forms can be introduced into the equation system to accommodate complex behaviours or policy instruments. For example, conditional functions, not acceptable in matrix inversion, can be used with the iterative method to stipulate certain policy instruments with quantitative or value thresholds, such as tariff-quotas or discontinuous tax schedules. This is also useful for policy instruments that

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encourage the use of new technology in production. For example, a cost threshold may be introduced, above which a new industry begins to operate.

Model users can also set ranges for endogenous variables to handle possible corner solutions in certain markets. If they become binding during the process of iterations, the model solution could be constrained by the conditions. This implies that some market clearing conditions might not be fully satisfied. Such inequalities can easily be accommodated in iterative formulations. For example, instead of setting demand equal to supply with a strictly positive price, demand can be set as less than or equal to supply by introducing a price range with a lower bound of zero. Demand and supply will stop converging toward each other when the price hits zero during iterations.

### Easy to merge CGE models with other models

The simplicity of the iterative solution method makes it easy for a CGE model to be incorporated with other models, such as partial equilibrium (PE) or micro-simulation (MS) models. These non-CGE models have been used widely by researchers to analysis sub-industry or micro unit-record data on individual households or firms. Unlike CGE models, PE models vary widely in theory and solution method. They are all built on certain given conditions with no feedback effects on prices or incomes. To introduce these feedback effects, one must build a separate CGE model. This is because, unlike the PE model, the equations in the CGE model are typically solved simultaneously. It is this unique solution method that prevents CGE models from integrating with PE models.

The iterative solution method simplifies the solution for CGE models. It allows the equations of CGE models to be solved in sequence as a PE model. The iterative method makes it possible for a CGE model or structure to be built around a PE model and solved as one system.

### Ease of CGE modelling in any software or programming language

CGE modelling has relied on specially built modelling software suites, such as GEMPACK (for linearised percentage change models), or specialised solvers used in general-purpose mathematical software programs and languages, such as Matlab, GAMS and the Pyomo packages in the Python programming language.

The iterative solution method described in this paper is basic at its core and is more dependent on the understanding of model theory than the expertise of computer programming or software. As a result, the iteration method opens the door for CGE models to be built in all those general software programs that have basic mathematical operations. It could even be built in a spreadsheet. For example, Microsoft Excel has a calculation capacity sufficient to process and link variables defined in different cells. It is straightforward to write variables in a cell of an Excel spreadsheet as functions (formula) of other variables defined in the spreadsheet. Such a model can be solved by linking the



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undefined equilibrium prices with their associated market clearing conditions. A conditional function can be introduced to adjust these prices according to their market clearing conditions. Excel has an iteration option to recalculate all formulas in a spreadsheet. A CGE model can be solved by simply repeating these iterations until the undefined prices reaches the equilibrium required to clear their markets. Unlike other programs, Excel displays the numerical results from each iteration in real time to allow users to see how endogenous variables converge toward their equilibrium. It also has various graphical functions for users to visualise the simulation results. The capability of Excel to quickly build stylised models could be a useful tool for back-of-the-envelope (BOTE) checks of large model results. It can also be used to design new model theory or test new policy instruments in CGE models in a way more accessible to economists without a deeper programming background.

The iterative method can be implemented in any computing programming languages, including open-source languages such as Julia, Python and R. These languages all have loop functions, which can be used to set up the iterative procedure used to solve for the general equilibrium prices of a CGE model. There is no need to invoke any mathematical solver. This gives those model users with no experience with modelling solvers access to many computing programs to build and solve their CGE models.

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